Module 08: Cell and Tissue Mechanics

Assignment

Total Point Value = 30

Due by midnight on Day 7 of Module 8

This should be submitted to Blackboard as a pdf.

1. The Hagen-Poiseuille equation describes flow through a cylindrical tube. We discussed this in the context of blood flow but it is also applicable to respiration where air flows through cylindrical alveoli. Please use this equation to explain why breathing is so difficult for someone suffering from just mild asthma.

   The equation for Hagen-Poiseulle flow relates the volumetric flow rate to the driving force (pressure drop) and the geometry of the tube.

   $$Q = \frac{(P_0 - P_L)\pi R^4}{8\eta L}$$

   We can rewrite this equation as follows

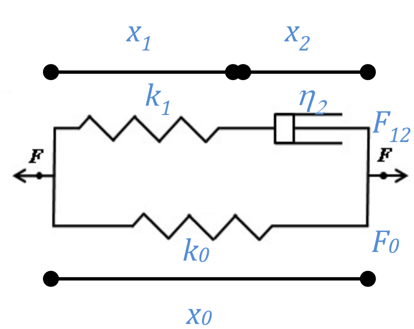
   $$\frac{\Delta P}{Q} = \frac{8\eta L}{\pi R^4}$$

   This is analogous to Ohm's law which states that resistance = voltage/current or resistance = driving force / flow.

   $$\frac{\Delta P}{Q} = \frac{8\eta L}{\pi R^4} = Resistance$$

   Someone with mild asthma will have just minor swelling in their bronchi, leading to a small decrease in the radius of those "tubes". Looking at the rearranged Hagen-Poiseuille equation we see that resistance is inversely proportionally to the radius to the 4^{th} power. A very small decrease in radius will result in a very large increase in resistance to flow. This is why even mild asthma suffers have a very difficult time breathing!

2. Derive the ordinary differential equation for the Kelvin viscoelastic solid (pictured below). Show ***all of your work*** (each step!) for full credit.



The displacement of the top and bottom parts of the system (the spring & dashpot, and spring respectively) must be equal if they are connected in parallel. Therefore you can write:

$$x_0 = x_1 + x_2$$

The force on the top and bottom however are not equal to each other but must sum to the total applied force as follows (note that the spring and the dashpot on the top feel the same force as they are connected in series):

$$F = F_0 + F_{12}$$

Equations for each element are written as follows:

Bottom spring: $F_0 = k_0 x_0$
Top spring: $F_{12} = k_1 x_1$
Top dashpot: $F_{12} = \eta_2 \dfrac{dx_2}{dt}$

Substitution of the displacement and element equations into the force balance results in the ODE:

$$F = F_0 + F_{12} \quad ==> \quad F = k_0 x_0 + \eta_2 \frac{dx_2}{dt}$$

substitute for the displacement of the dashpot: $\frac{dx_2}{dt} = \frac{dx_0}{dt} - \frac{dx_1}{dt}$

$$F = k_0 x_0 + \eta_2 \left(\frac{dx_0}{dt} - \frac{dx_1}{dt} \right)$$

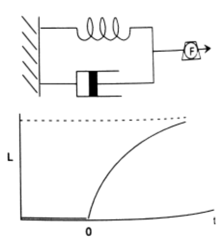
now substitute for the displacement of the spring: $\frac{dx_1}{dt} = \frac{dF_{12}}{dt} \Big/ k_1$

$$F = k_0 x_0 + \eta_2 \left(\frac{dx_0}{dt} - \frac{dF_{12}}{dt} \Big/ k_1 \right)$$

Finally put F_{12} in terms of F using: $\frac{dF_{12}}{dt} = \frac{dF}{dt} - \frac{dF_0}{dt}$ and $\frac{dF_0}{dt} = k_0 \frac{dx_0}{dt}$

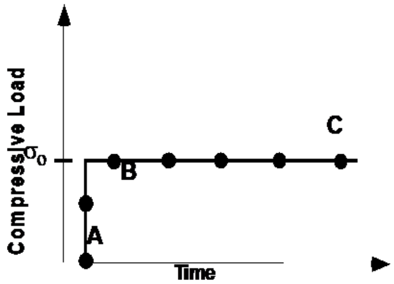
$$\boldsymbol{F = k_0 x_0 + \eta_2 \frac{dx_0}{dt} - \eta_2 \cdot \left(\frac{\frac{dF_0}{dt} - k_0 \frac{dx_0}{dt}}{k_1} \right)}$$

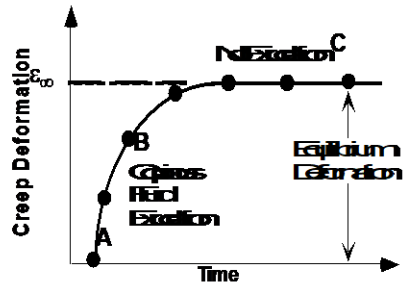
3. In lecture we discussed the biomechanics of articular cartilage and how important it is for tissue engineers to understand these mechanics in order to design successful tissue substitutes.
   a. Based on the shape of the confined compressive load test on cartilage what type of model would you use to describe it mechanically and why?



The model that best describes the behavior below is a Voight Model. This give a slow curved deformation over time when a constant force is applied.

   b. What components (dashpots and springs…) describe the behavior shown below on the right? What phase(s) of cartilage are represented by the component(s)?
   Cartilage is made of several zones which make up a biphasic material. The solid phase is represented by the spring and the fluid phase is represented by the dashpot. In terms of zones the top zone has the highest fluid content and the deep zone has the highest solid content.





c. Over the last two weeks you've read on the use of microscale topographies (Nikkhah et al. *Engineering microtopographies to control the cell-substrate interface)*. In 300 words or less please explain how microtopographies can be employed in the development of engineered tissues. What tissue properties can they influence? What cell behaviors can they control?

d.

Assignment Rubric

Question	Component	Total Point Value
1		4
2		8
3	a	4
	b	4
4		10

Total Point Value = 30