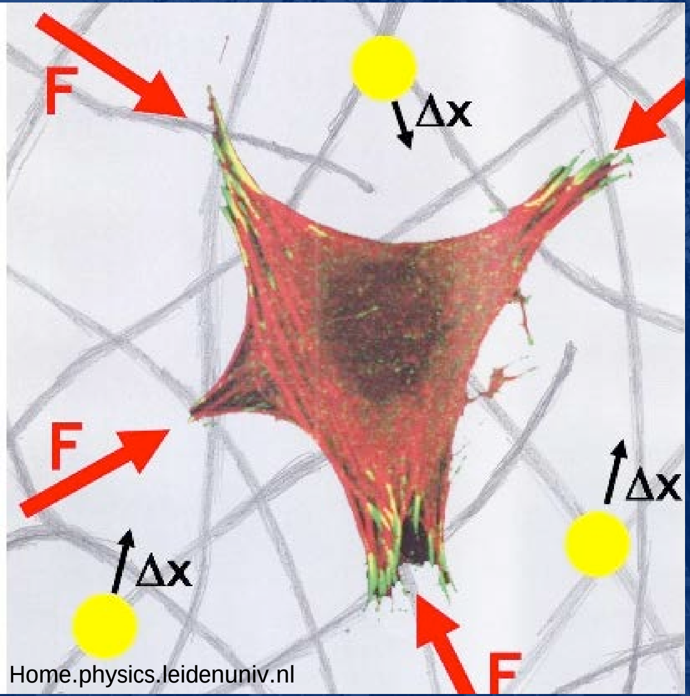
Johns Hopkins
Whiting School of Engineering

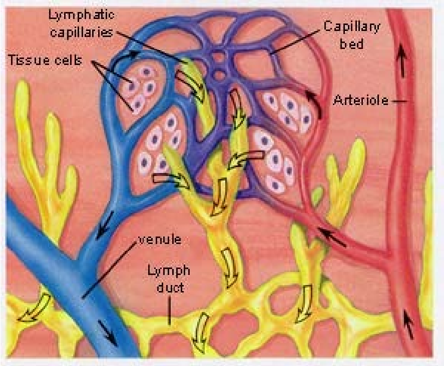
Cell and Tissue Engineering

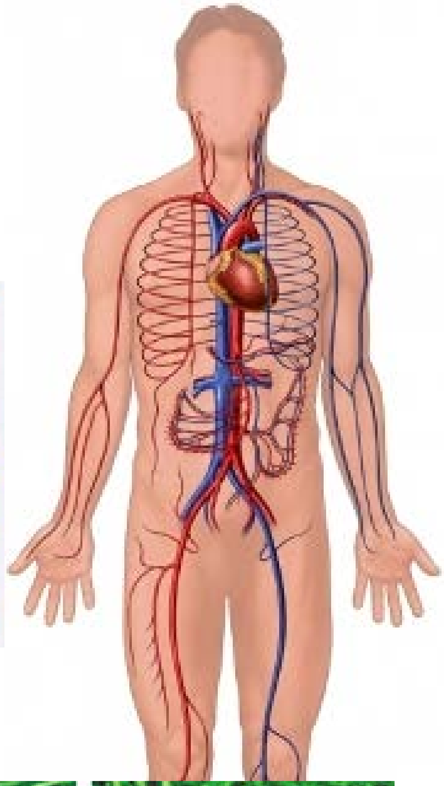
Basic Fluid Mechanics

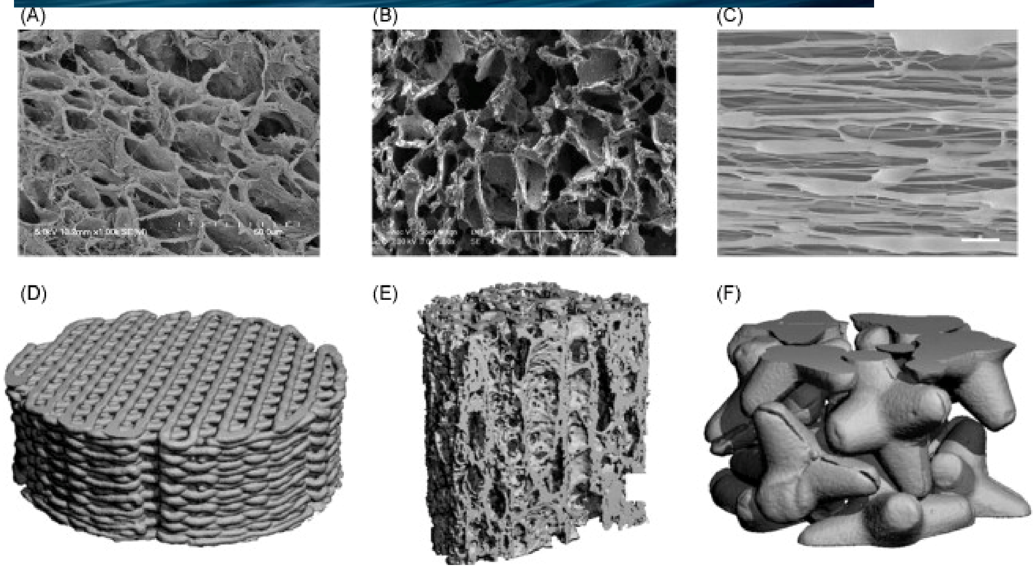


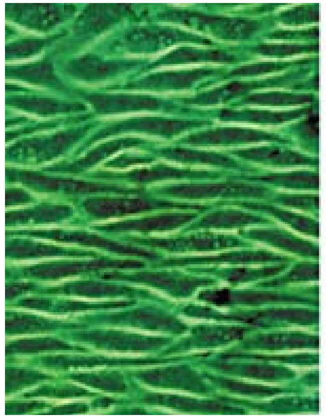
Home.physics.leidenuniv.nl

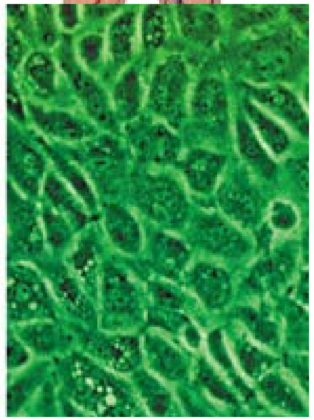
Elementary fluid mechanics







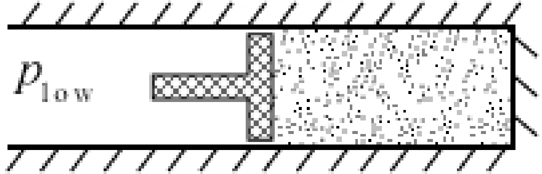


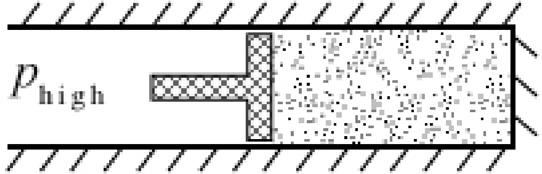


Malek et al 2009; Flickr.com; Kirbyreserach.com; Provitanutrition.com; Grayson et al 2009

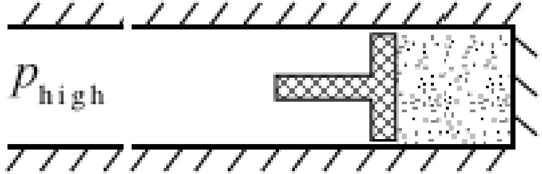
Elementary fluid mechanics (cont.)

Incompressible – constant density, ρ



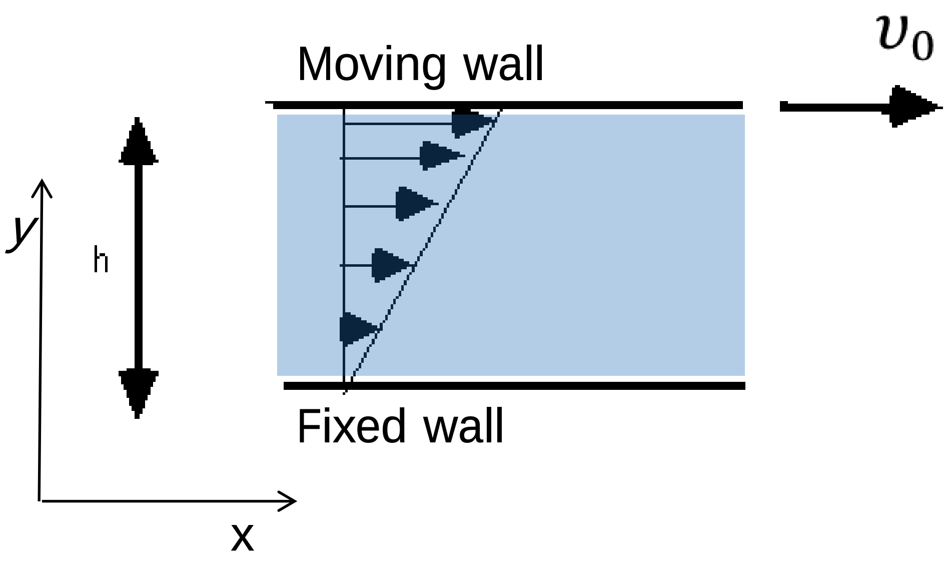


Compressible – ρ(P)



Flickr.com; Kirbyreserach.com; Provitanutrition.com

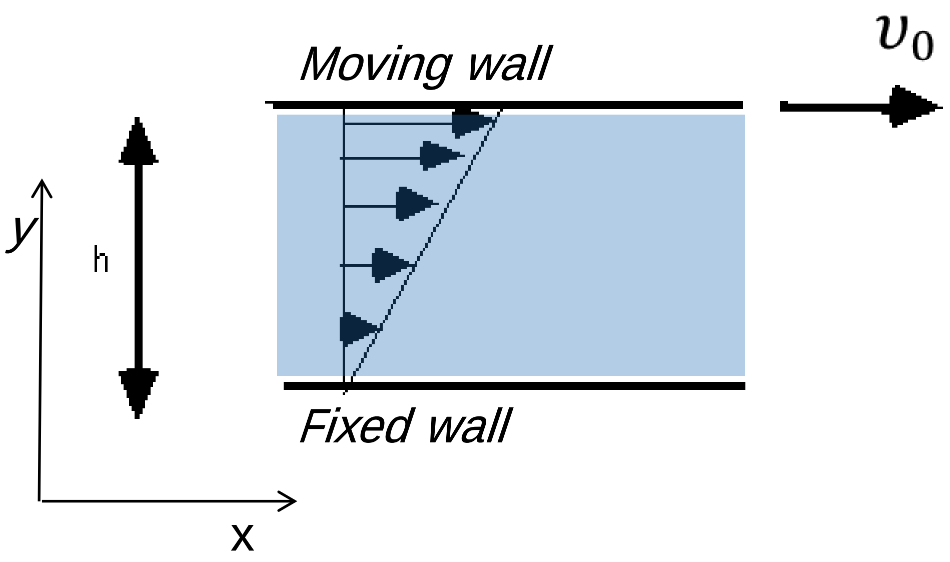
Shear stress is proportional to viscosity



Tangential shear stress

$$\tau = \frac{F}{A} \qquad \tau \propto \frac{v_0}{h}$$

Shear stress is proportional to viscosity



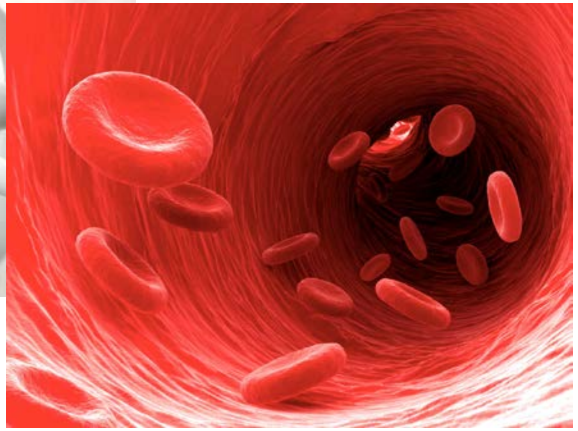
Tangential shear stress

$$\tau = \frac{F}{A} \qquad \tau \propto \frac{v_0}{h}$$

$$\tau_{xhy} = -\eta \frac{dv_x}{dy}$$

η, viscosity

The viscosity of Newtonian fluids is not a function of shear rate



Tangential shear stress

$$\tau = \frac{F}{A} \qquad \tau \propto \frac{v_0}{h}$$

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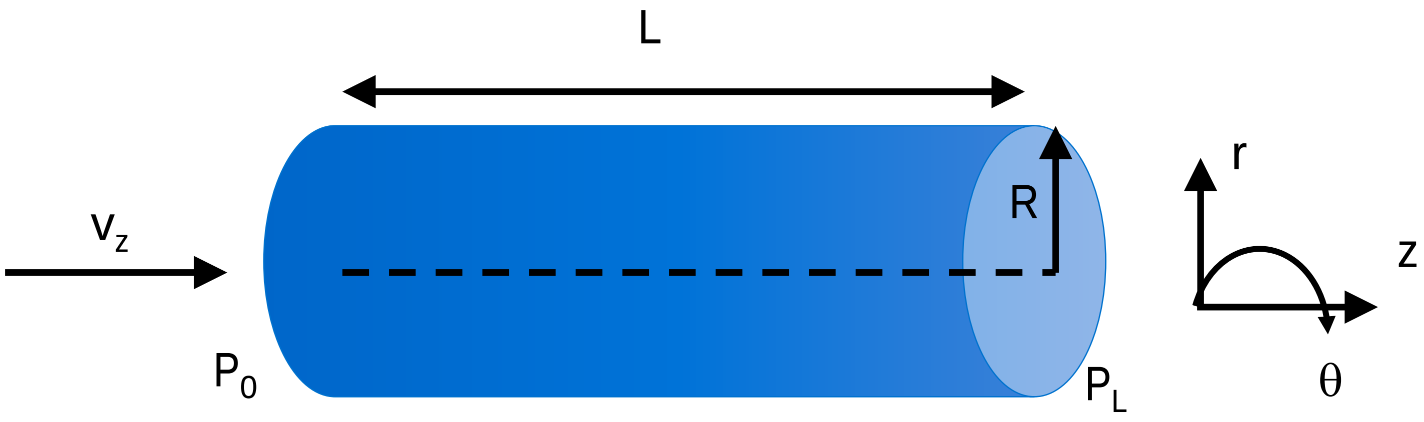
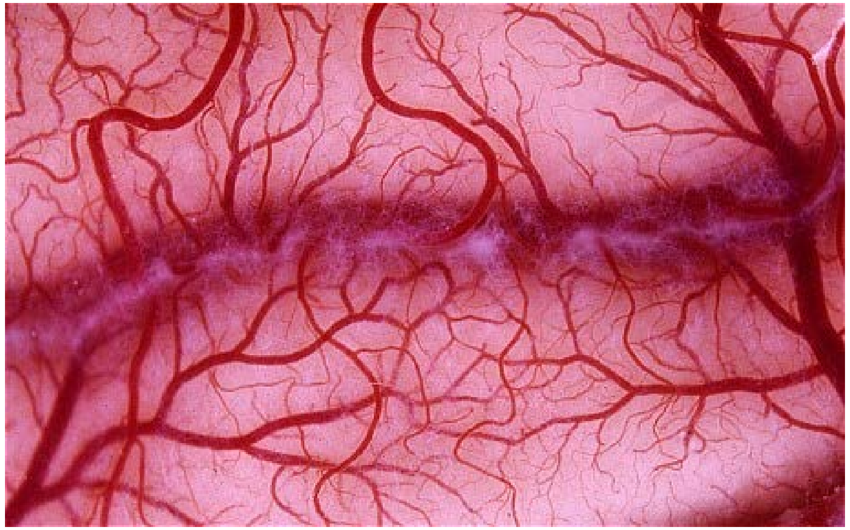
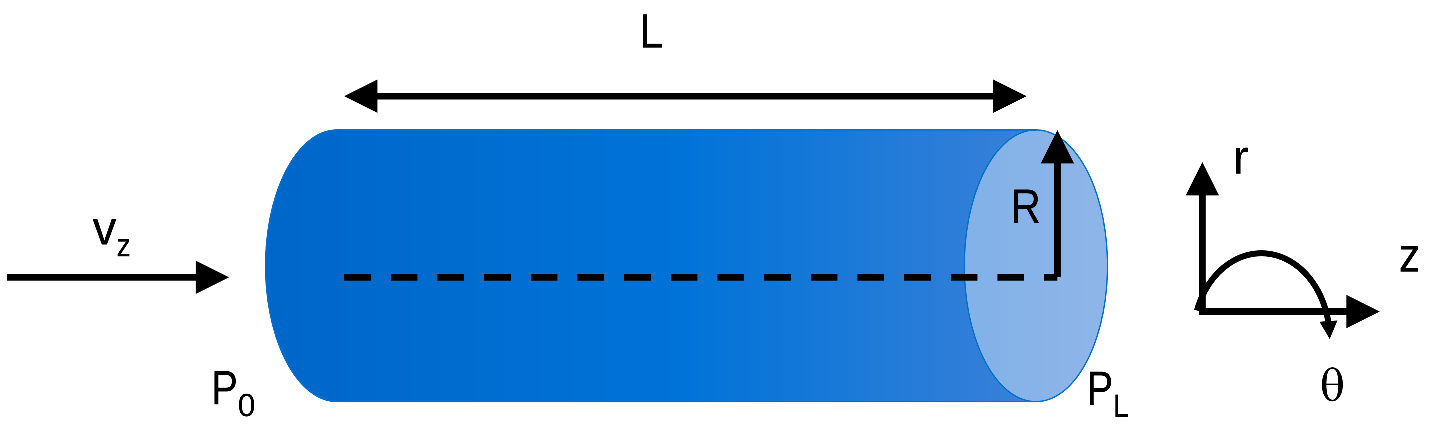
Consider flow in a cylindrical tube





Telegraph.co.uk

Consider flow in a cylindrical tube (cont.)



Navier-Stokes Equation

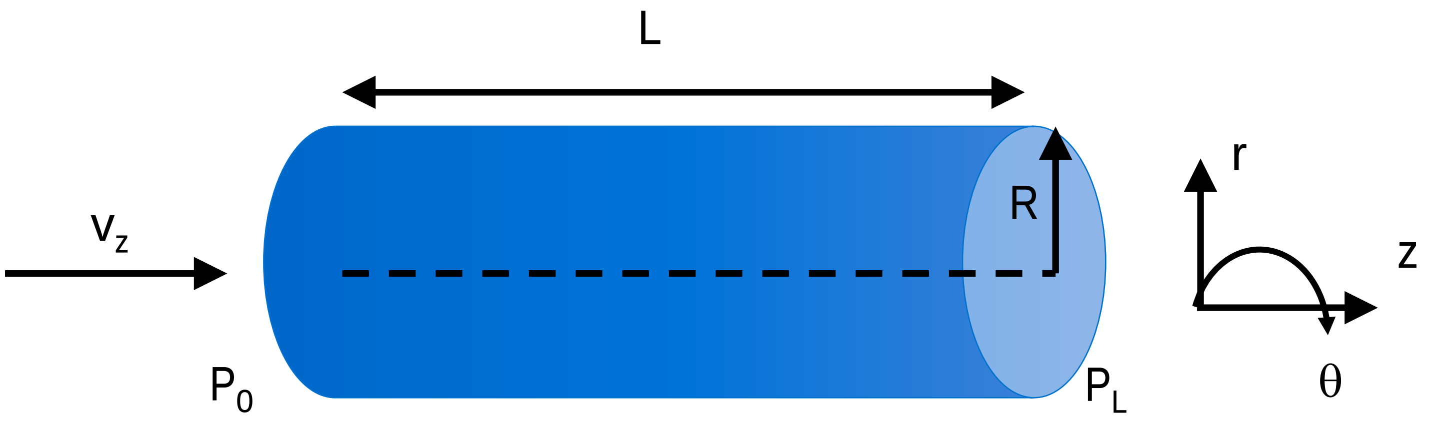
$$\rho(\frac{\delta v_z}{\delta t} + v_r \frac{\delta v_z}{\delta r} + \frac{v_\theta}{r}\frac{\delta v_z}{\delta \theta} + v_z \frac{\delta v_z}{\delta z}) = -\frac{\delta p}{\delta z} + \eta\left[\frac{1}{r}\frac{\delta}{\delta r}\left(r\frac{\delta v_z}{\delta r}\right) + \frac{1}{r^2}\frac{\delta^2 v_z}{\delta^2 r} + \frac{\delta^2 v_z}{\delta^2 z}\right] + \rho g_z$$

Transient inertia + Convective inertia = Body force + Pressure force + Viscous force

A continuum version of $F = ma$

ρ – *density*
υ – *velocity*
t – *time*
η – *viscosity*
g – *gravitational forces*

Consider flow in a cylindrical tube (cont.)



Navier-Stokes Equation

$$\rho\left(\frac{\delta v_z}{\delta t}+v_r\frac{\delta v_z}{\delta r}+\frac{v_\theta}{r}\frac{\delta v_z}{\delta\theta}+v_z\frac{\delta v_z}{\delta z}\right)=-\frac{\delta p}{\delta z}+\eta\left[\frac{1}{r}\frac{\delta}{\delta r}\left(r\frac{\delta v_z}{\delta r}\right)+\frac{1}{r^2}\frac{\delta^2 v_z}{\delta^2 r}+\frac{\delta^2 v_z}{\delta^2 z}\right]+\rho g_z$$

1. Stead flow, no transient inertia
2. <u>Flow is axial, neglect convective inertia</u>
3. Gravity is negligible

ρ – *density*
υ – *velocity*
t – *time*
η – *viscosity*
g – *gravitational forces*

Reynold’s number tells us about the type of flow

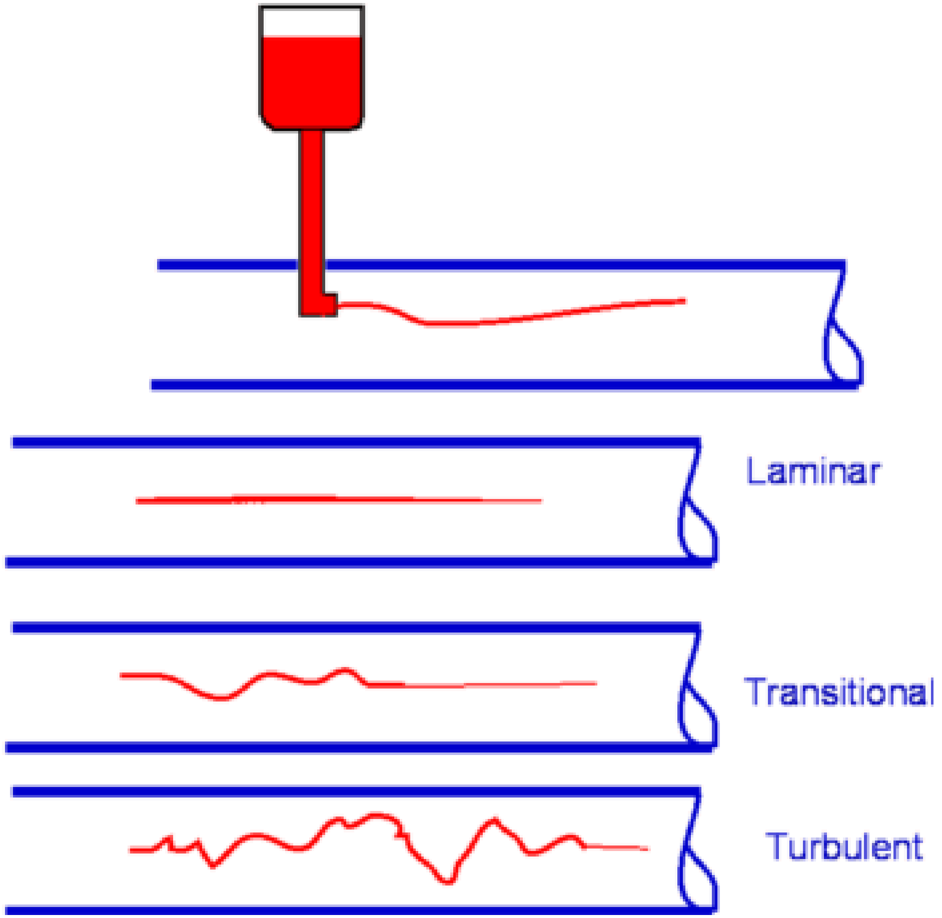
$$\mathrm{Re} = \frac{\rho v^2 / L}{\mu v / L^2} = \frac{\rho v L}{\mu}$$

$$\frac{\text{inertial force}}{\text{viscous force}}$$

Non-dimensional number

If Re << 1, we can neglect inertial forces

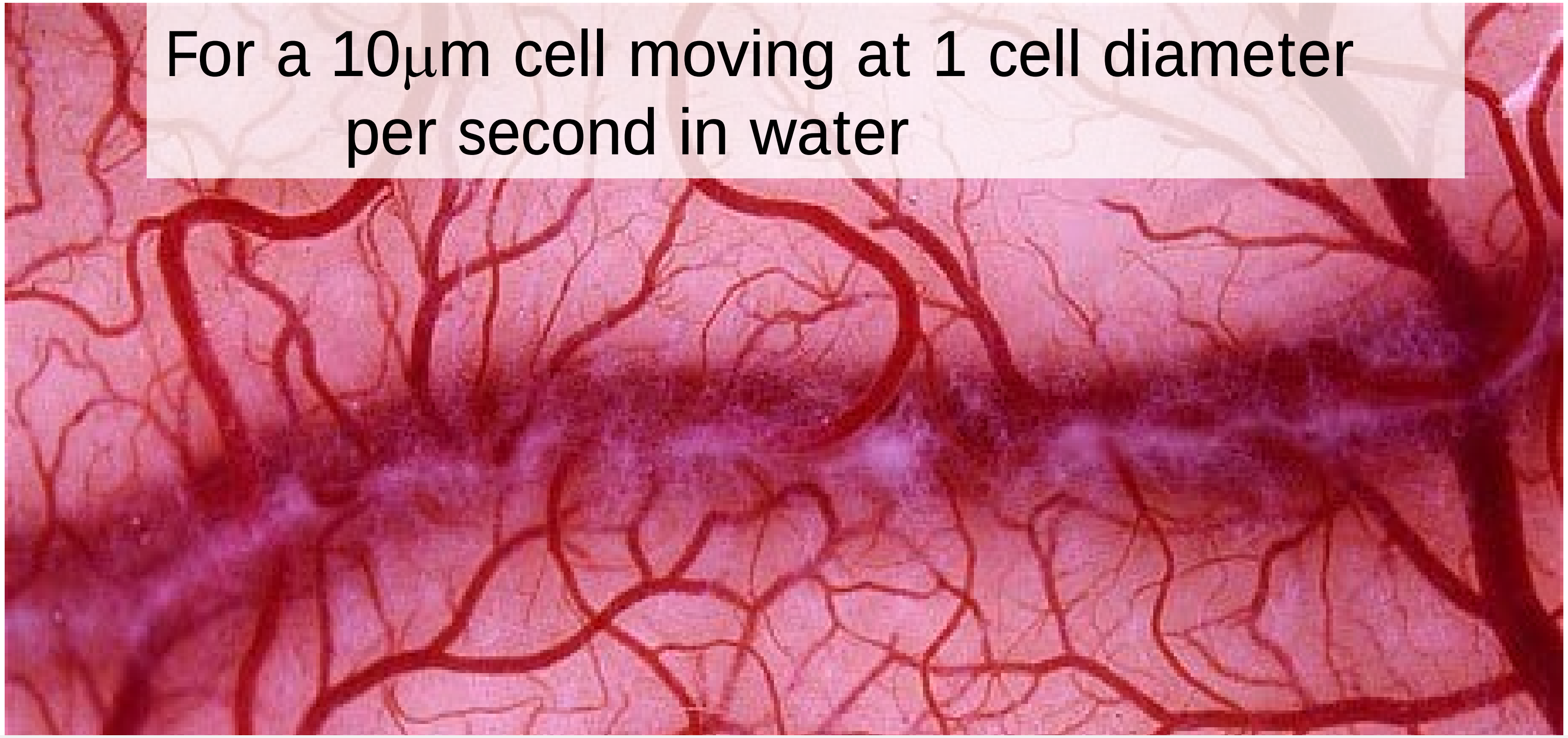
If Re >> 1, inertial forces dominate



Reynold's number tells us about the type of flow

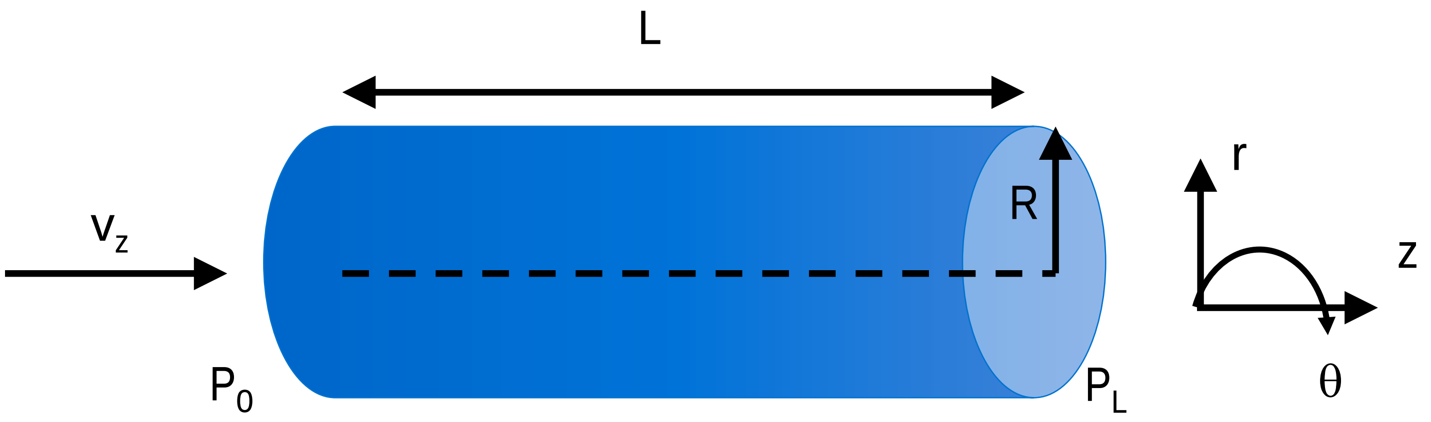
$$\mathrm{Re} = \frac{\rho v L}{\mu} = \frac{(1g/cm^3)\times(100cm/\sec)\times 30cm}{0.01dyn-\sec/cm^2} = 300,000$$

Reynold's number tells us about the type of flow



$$\mathrm{Re} = \frac{\rho v L}{\mu} = \frac{(1g/cm^3)\times(0.001cm/\sec)\times 0.001cm}{0.01dyn-\sec/cm^2} = 10^{-4}$$

Consider flow in a cylindrical tube



Navier-Stokes Equation

$$\rho\left(\frac{\delta v_z}{\delta t}+v_r\frac{\delta v_z}{\delta r}+\frac{v_\theta}{r}\frac{\delta v_z}{\delta\theta}+v_z\frac{\delta v_z}{\delta z}\right)=-\frac{\delta p}{\delta z}+\eta\left[\frac{1}{r}\frac{\delta}{\delta r}\left(r\frac{\delta v_z}{\delta r}\right)+\frac{1}{r^2}\frac{\delta^2 v_z}{\delta^2 r}+\frac{\delta^2 v_z}{\delta^2 z}\right]+\rho g_z$$

1. Stead flow, no transient inertia
2. <u>Flow is axial, neglect convective inertia</u>
3. Gravity is negligible

$$\frac{\delta p}{\delta z}=+\frac{\eta}{r}\frac{\delta}{\delta r}\left[r\frac{\delta v_z}{\delta r}\right]$$

Consider flow in a cylindrical tube (cont.)

$$\frac{\delta p}{\delta z} = +\frac{\eta}{r}\frac{\delta}{\delta r}\left[r\frac{\delta v_z}{\delta r}\right]$$

Apply two boundary conditions:

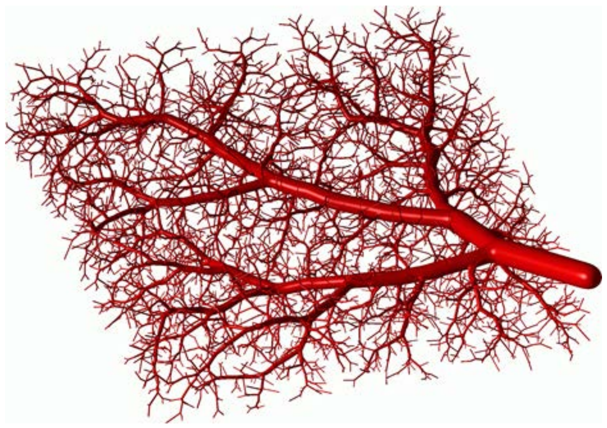
1. No-slip $v_z = 0$ when r = R

   And $\frac{\delta v_z}{\delta r} = 0$ when r = 0

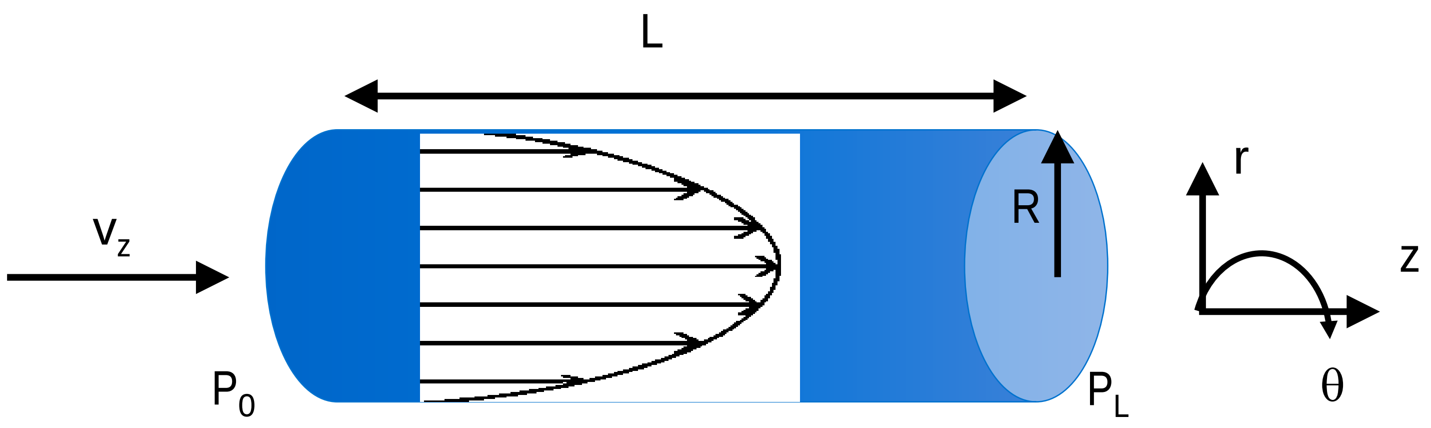
2. Known pressures at the inlet and outlet

   $p = P_0$ when z = 0

   And $p = P_L$ when z = L



Consider flow in a cylindrical tube (cont.)



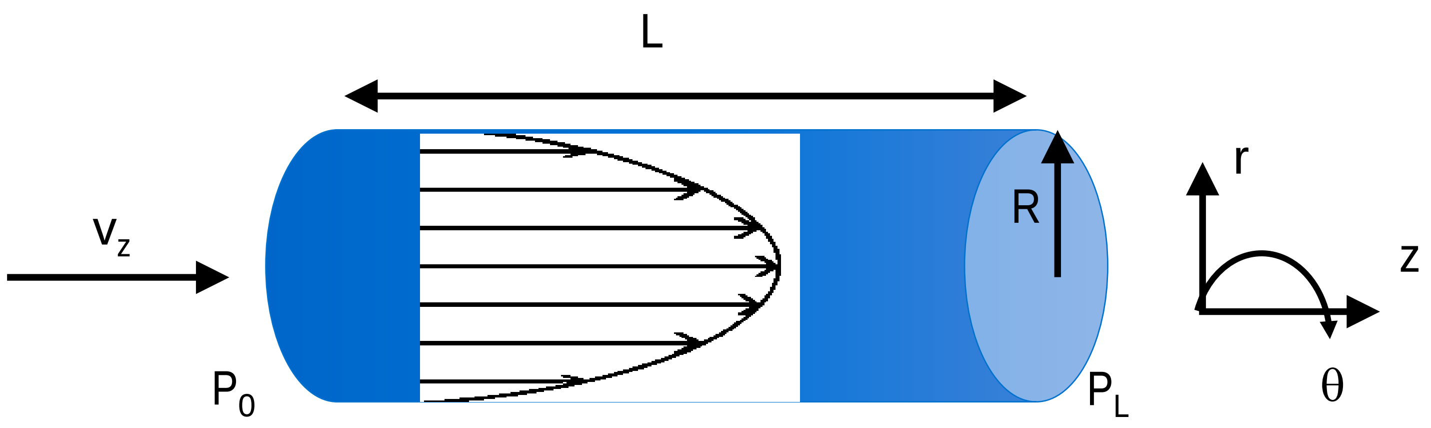
$$\frac{\delta p}{\delta z} = +\frac{\eta}{r}\frac{\delta}{\delta r}\left[r\frac{\delta v_z}{\delta r}\right]$$

Hagen-Poiseuille flow

$$v_z = \frac{(P_0 - P_L)R^2}{4\eta L}\left(1 - \frac{r^2}{R^2}\right)$$

ρ – density
υ – velocity
t – time
η – viscosity

Consider flow in a cylindrical tube (cont.)



ρ – density
υ – velocity
t – time
η – viscosity

$$\frac{\delta p}{\delta z} = +\frac{\eta}{r}\frac{\delta}{\delta r}\left[r\frac{\delta v_z}{\delta r}\right]$$

Hagen-Poiseuille flow

$$v_z = \frac{(P_0 - P_L)R^2}{4\eta L}\left(1 - \frac{r^2}{R^2}\right) \rightarrow Q = \int_0^{2\pi}\int_0^R v_z(r)\,r\,dr\,d\theta = \frac{(P_0 - P_L)\pi R^4}{8\eta L}$$

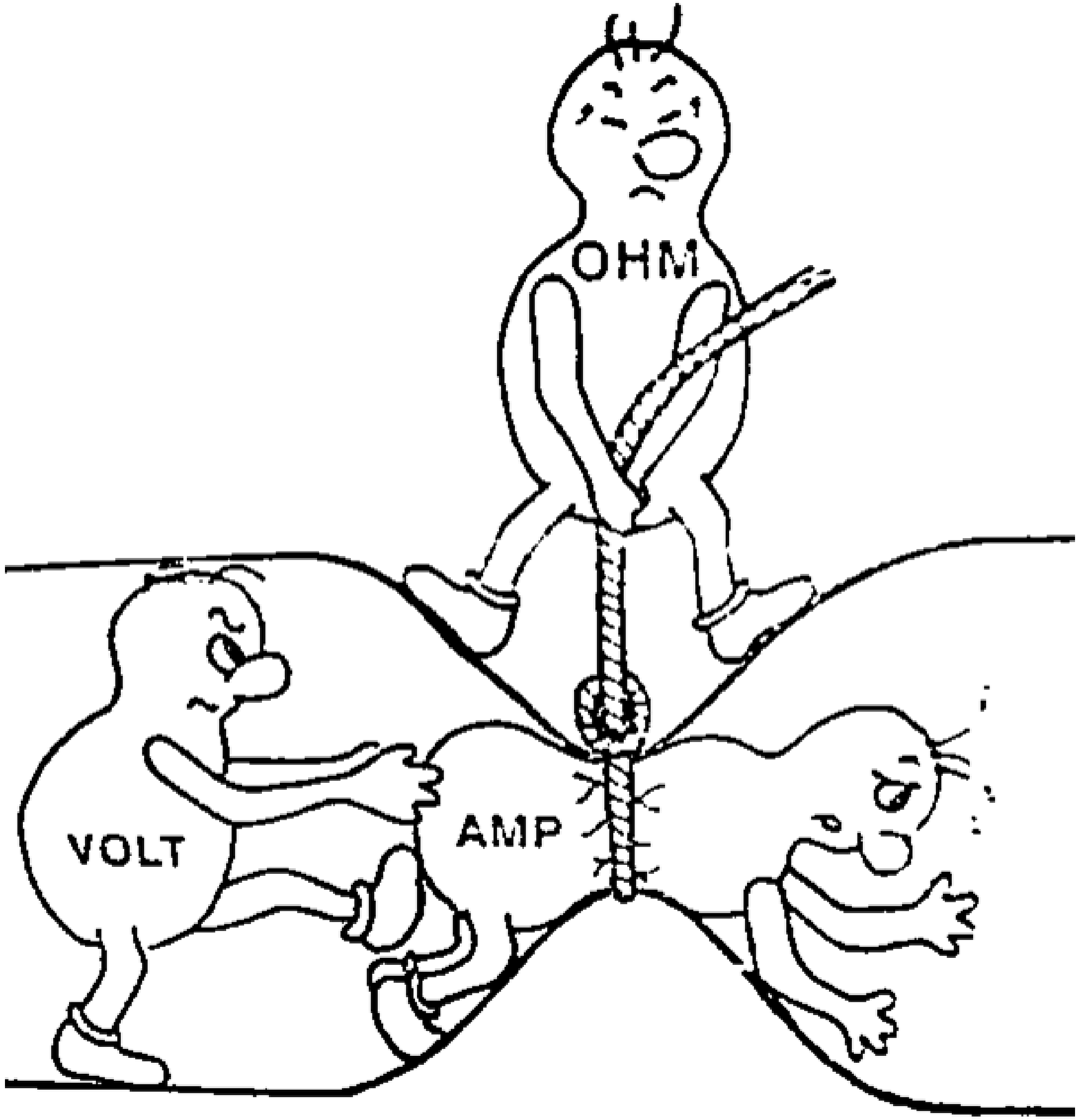
Consider flow in a cylindrical tube (cont.)

$$Q = \frac{(P_0 - P_L)\pi R^4}{8\eta L}$$

$$P_0 - P_L = \Delta P$$

$$\Omega = \frac{\Delta P}{Q} = \frac{8\eta L}{\pi R^4}$$

$$\text{Resistance} = \frac{\text{voltage}}{\text{current}}$$

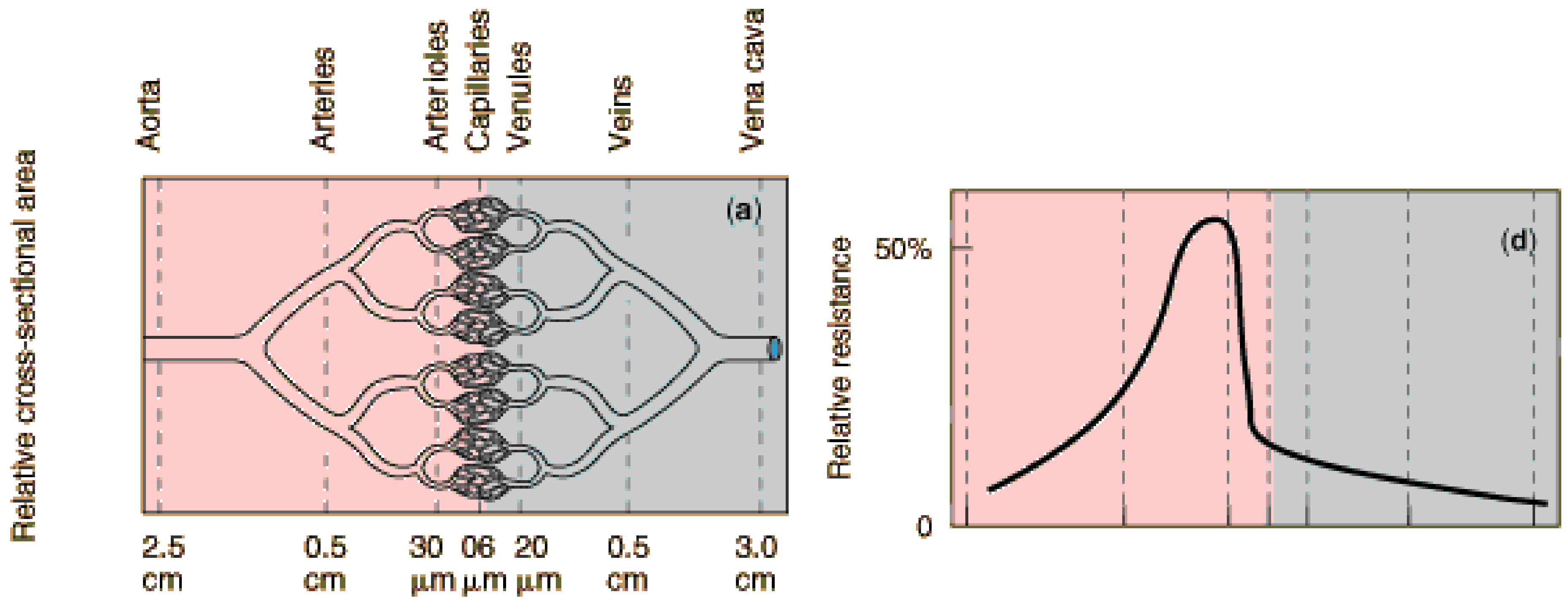


Why do we want to know about resistance?

$$\Omega = \frac{\Delta P}{Q} = \frac{8\eta L}{\pi R^4}$$

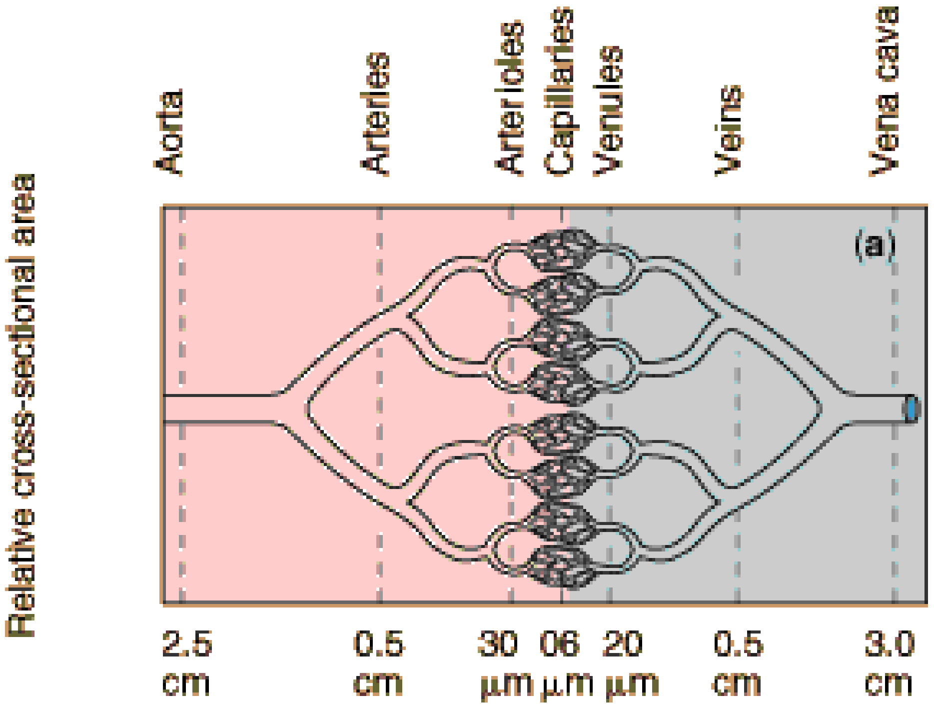
Why do we want to know about resistance? (cont.)

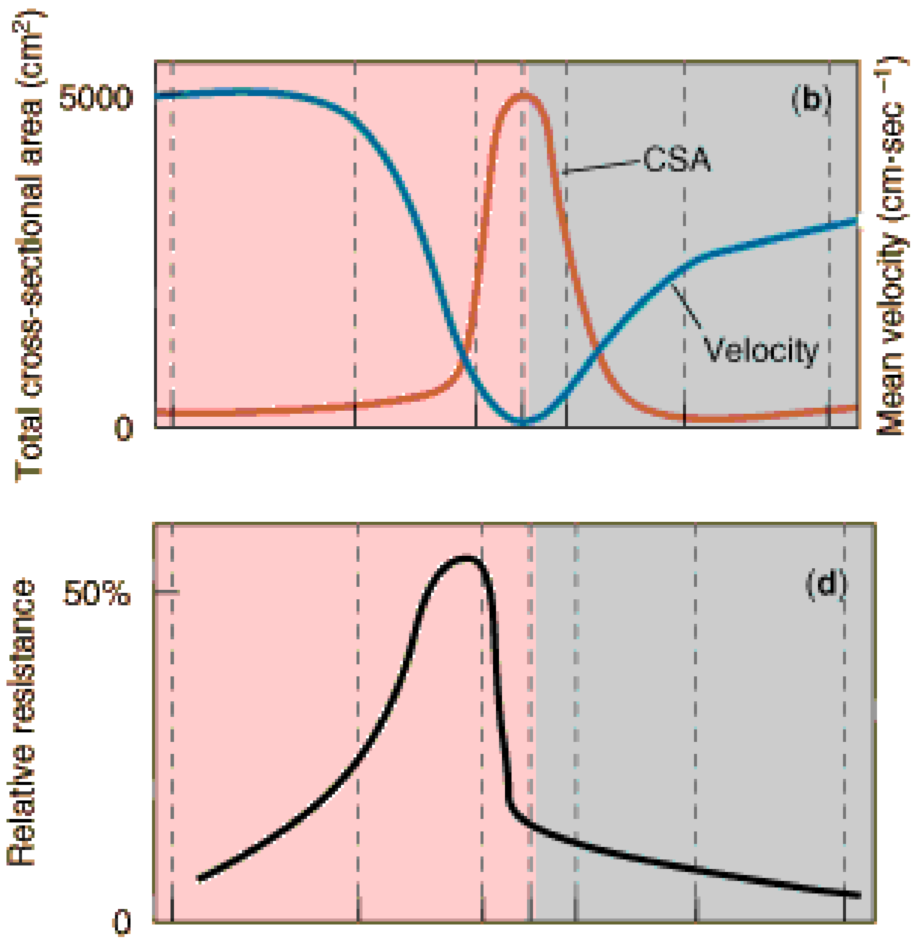
$$\Omega = \frac{\Delta P}{Q} = \frac{8\eta L}{\pi R^4}$$



Why do we want to know about resistance? (cont.)

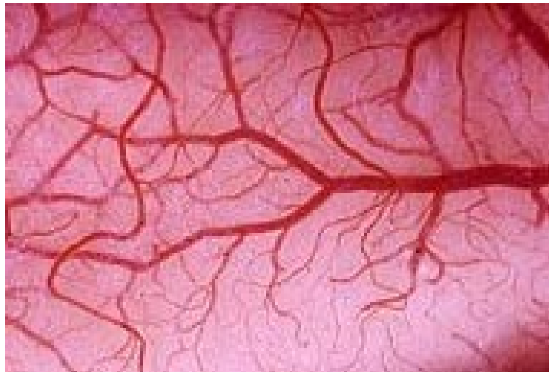
$$\Omega = \frac{\Delta P}{Q} = \frac{8\eta L}{\pi R^4}$$

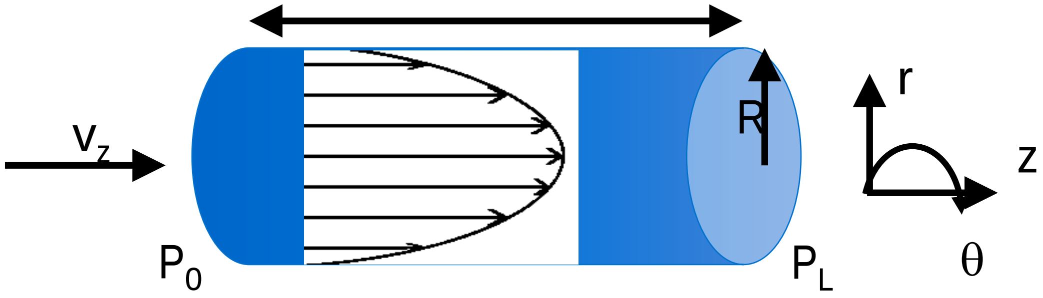




Review and rewind

Fluid mechanics in a cylindrical pipe





$$\mathrm{Re} = \frac{\rho v L}{\mu}$$

$$Q = \frac{(P_0 - P_L)\pi R^4}{8\eta L}$$

$$\Omega = \frac{\Delta P}{Q} = \frac{8\eta L}{\pi R^4}$$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