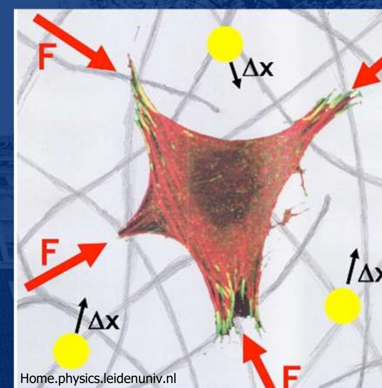
Cell and Tissue Engineering

Basic Solid Mechanics

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING



Welcome to cell and tissue engineering – Basic Solid Mechanics

We’ll talk about solid mechanics as they apply to biological systems.

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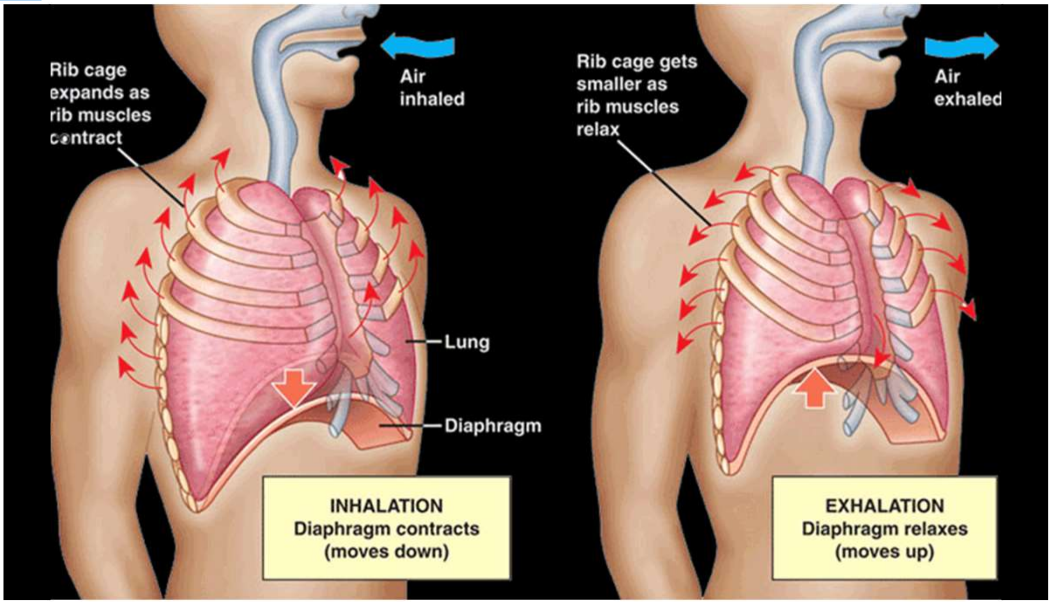
Example adapted from J. Fredberg, Harvard; Mindfulmum.co.uk

Let’s start todays lecture by taking a deep breath, in and out.

Make breath noises

Breathing dilates and stretches cells in your lungs



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Xtremepapers.com

I think we can all agree that breath felt pretty good.

You actually do that deep breath about once every 6 minutes, you just don't realize it.

What you're doing when you take a deep breath is stretching every adherent cell in your lungs by about 20%,. This is roughly 5x the stretch that these cell feel with a normal resting inspiration. This type of stretch is big enough to effect the cytoskeleton of those cells.

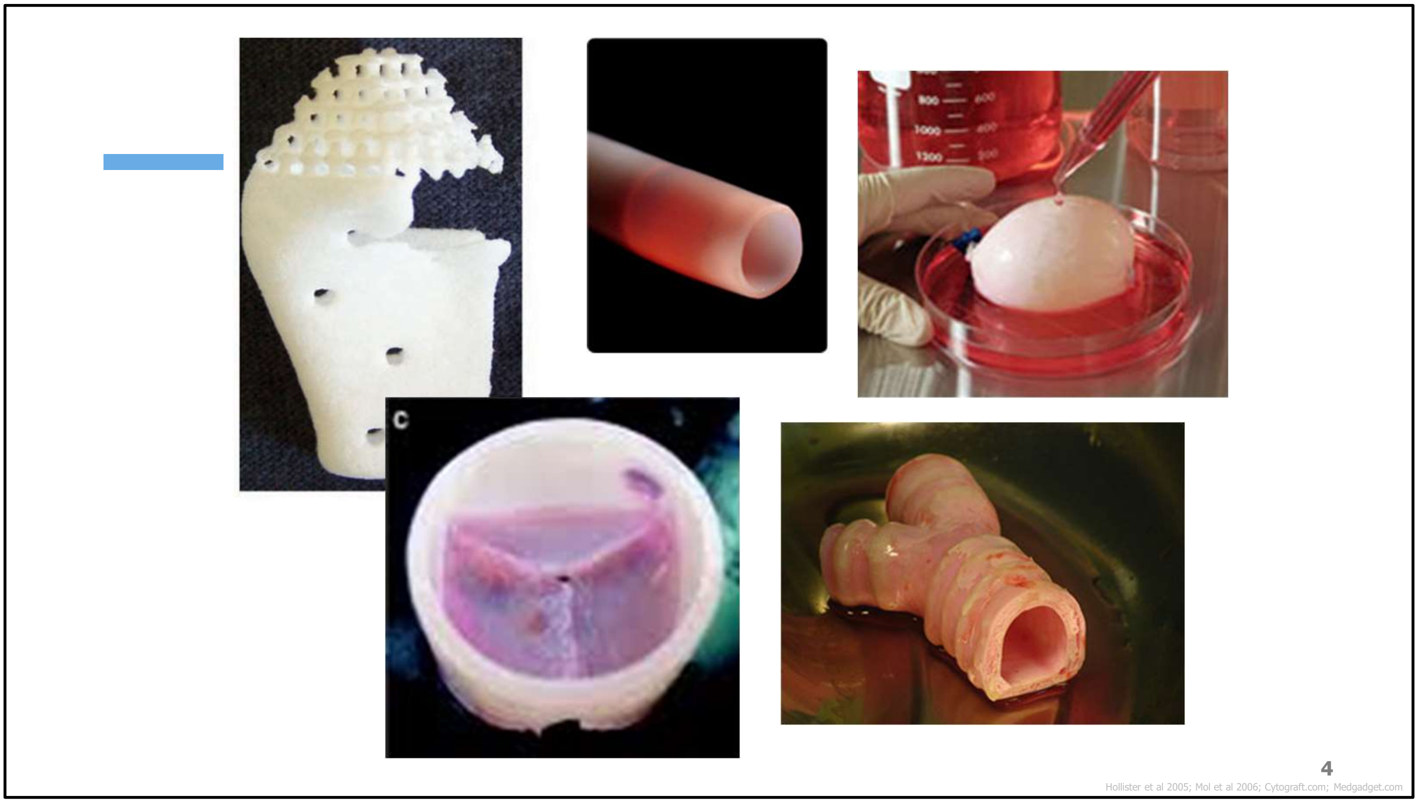
You may be imagining all of your cells as tiny elastics, As solid materials. But during stretch your cells actually behave more as fluid – they go through a process called **fluidization**, when the cytoskeleton is no longer in play. This apparent "phase change" in mechanical behavior is important to the function of your lungs.

Asthma sufferers have trouble breathing because their cells don't go through this mechanical phase change, they can't **fully** dilate their airways with a deep breath.

If you were designing an artificial lung would you want to design one with asthma? No, of course not – you would want your tissue to have the proper fully-functional mechanical capabilities of a healthy tissue.

We've talked about how critical mechanics are for morphogenesis, controlling cell migration, and even stem cell fate.

In this module we are going to get into the nitty gritty of analyzing cell mechanics.



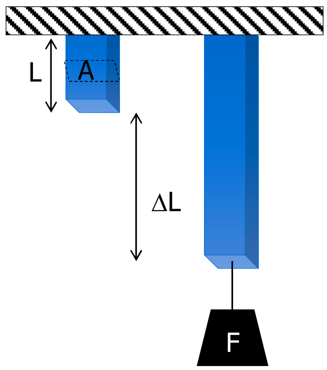
Let’s look at some examples other engineered tissues – here we can see

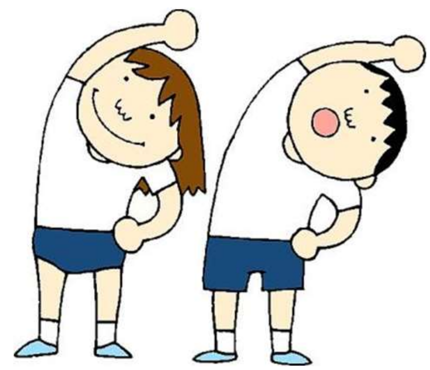
Engineered bone
Heart valve
Blood vessel
Bladder
Trachea

All of these tissues have different mechanical requirements, which are linked to their unique functions in the body.

When we design a cell or tissue engineered solution, we don’t’ just need the proper cell phenotypes – bladder smooth muscle or coronary endothelium – we need to pair that with the proper mechanical environment.

Elementary solid mechanics…





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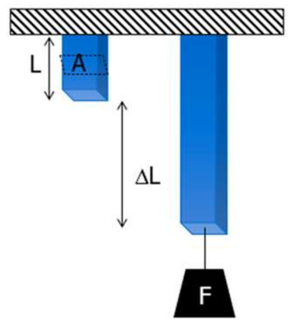
Examiner.com

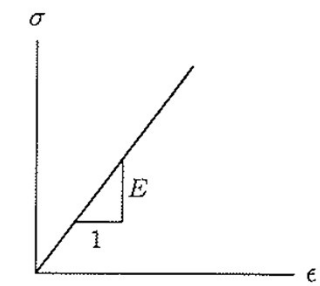
This bring us to elementary solid mechanics.

We'll start by considering a biological tissue that is fixed at one end and exposed to a constant applied force. --- this force here is unchanging.

As you would expect the tissue deforms under the load, stretching down by delta L.

Elastic deformation





$$\sigma = \frac{F}{A} = E \times \varepsilon = \frac{E \times \Delta L}{L}$$

Hooke's Law

σ = stress
F = force
A = area
E = elastic or Young's modulus
ε = strain

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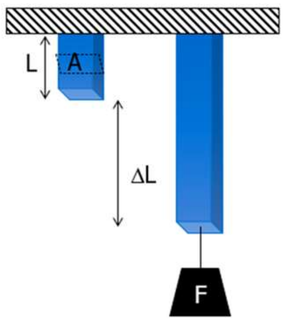
For a purely elastic material the stress would vary linearly with the strain, as you're seeing in this graph.

In this graph we have strain on the x axis and stress is on the y axis.

As a reminder stress is **force per area** (here you can see the force in black and the area it is applied to is the cross sectional area of this beam (aka tissue).

Strain is the **change in length per length**, that is delta L over the initial length, L

Young’s modulus is a material property



$$\sigma = \frac{F}{A} = E \times \varepsilon = \frac{E \times \Delta L}{L}$$

Hooke’s Law

E = Elastic/Young’s modulus

Material	Modulus (MPa)
Long bone	15-30,000
Skull bone	6,500
cartilage	1-10
tendon	1-2,000
skin	0.1-2
brain	0.067
polystyrene	2,300-3,300
Stainless steel	210,000

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Hooke’s law tells us that stress is equal to strain times young’s modulus. for an elastic material.

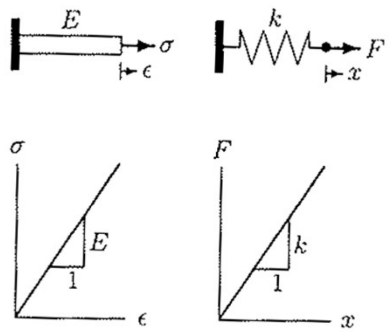
The elastic or young’s modulus is a material property. Your book has an excellent table showing you common values for elastic modulus for biological materials including tissues, polymers, and metals.

I’ve highlighted a few here so that we can get an idea of scale.

Long bones having an impressive modulus of 30,000 megapascals, compared to skin at just 2 MPA, and stainless steel at over 200,000 MPA

Elastic materials behave as springs

$$\sigma = E\varepsilon \qquad F = kx$$



σ = stress
E = elastic or Young's modulus
ε = strain
F = force
x = displacement
k = spring constant

$$\sigma = \frac{F}{A} = E \times \varepsilon = \frac{E \times \Delta L}{L}$$

Hooke's Law

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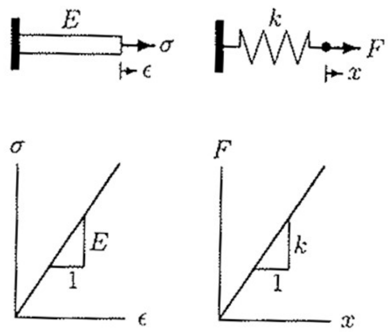
Hooke's law is named for the physicist that first described this behavior with elastic springs. He applied loads to springs **–that is F**, measured the deformation **x,** and calculated the spring constant **k**.

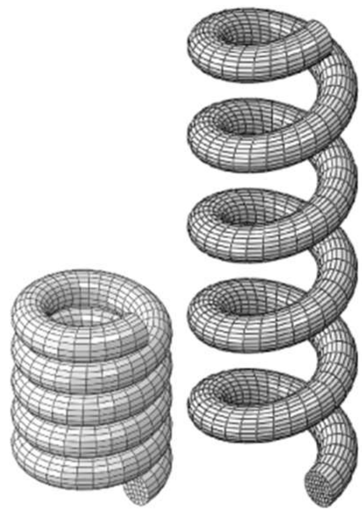
This is analogous to stress = strain times modulus relationship.

When we use Hooke's law to describe stress-strain relationships for elastic materials, we are using a **spring model.**

Elastic materials behave as springs (cont.)

$$\sigma = E\varepsilon \qquad F = kx$$





$$\sigma = \frac{F}{A} = E \times \varepsilon = \frac{E \times \Delta L}{L}$$

Hooke's Law

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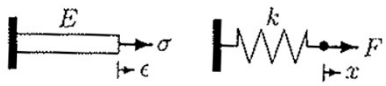
Some materials will recover their original shape after deformation. So we can extend the spring, release the load, and it would return to it's initial shape.

If we looked at these on the stress-strain curve, they would load and unload along the same line. (draw this on the force and extension plot)

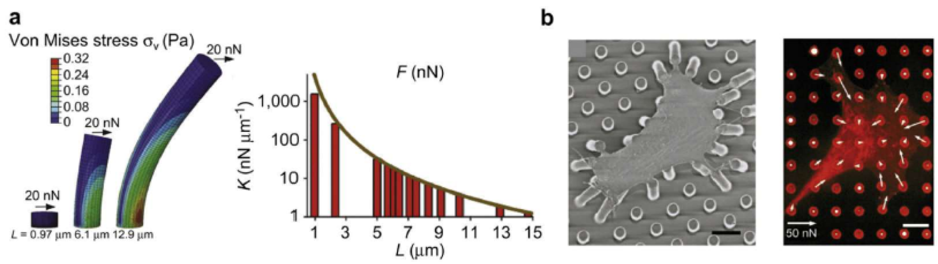
Materials with overlaid loading and unloading curves like this are called **ideal elastic materials.**

Elastic materials behave as springs (cont.)

$$\sigma = E\varepsilon \qquad F = kx$$



$$F = k \cdot \Delta x = \left(\frac{3}{4} \pi E \frac{r^4}{L^3} \right) \cdot \Delta x$$



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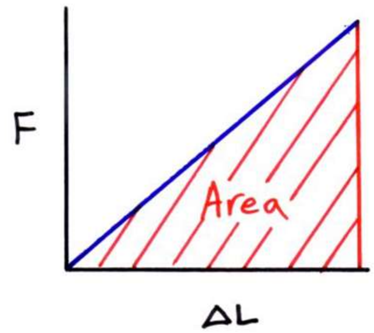
Nikkhah et al, 2013

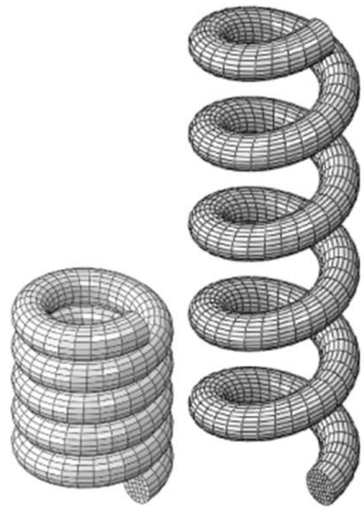
In the assigned article this week you'll see the spring equations are used to model microtopographies, these are used to study traction forces. We looked at this assay earlier this semester - this is an elastomeric substrate – with flexible posts.

By measuring the displacement of the posts (and knowing the geometry and mechanical properties of the post), researchers can determine the traction forces that the cell is applying to each post.

In this case, we know **k** and **x** and are solving for **F**.

Elastic materials behave as springs (cont.)





Strain Energy $\frac{F \cdot \Delta L}{2} = \frac{\sigma \cdot \varepsilon}{2}$

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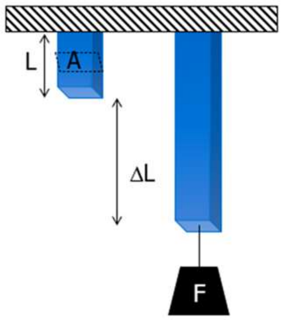
So what's happening when a material is stressed, when a load is applied to stretch a spring, or when a post is deflected by a cell.

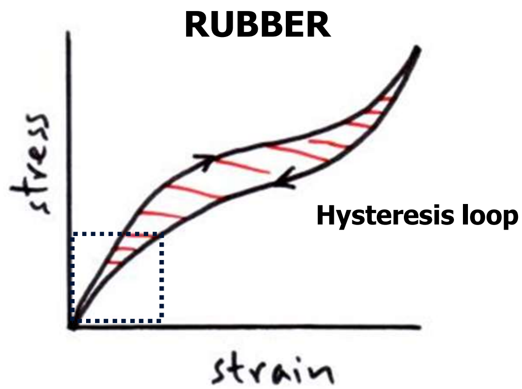
When forces are applied, energy is added - this mechanical energy is called **strain energy.**

You can graphically determine strain energy as the area under the stress strain or force displacement curve – here is the mathematical representation of that area using simple geometry.

With ideal elastic solids, no energy is lost or gained during deformation and relaxation cycles – The traces of the curves fall on top of each other- all of the energy stored during stretch or elongation is used during relaxation

Materials can be elastic and NOT obey Hooke's law!





$$\sigma = \frac{F}{A} = E \times \varepsilon = \frac{E \times \Delta L}{L}$$

Hooke's Law

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Now, some materials are bad a storing energy – take rubber for example.

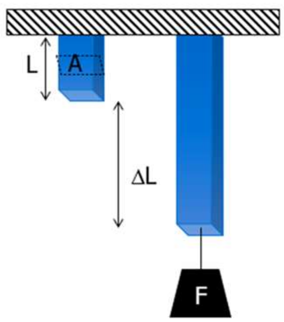
You can see here on the right that as this the rubber material is unloaded, it returns to the same shape - it gets back to the same point, but there is a gap between the loading and relaxation curves. The area of this gap is equal to the **loss of energy**. The shape of this loop is called a **Hysteresis loop**

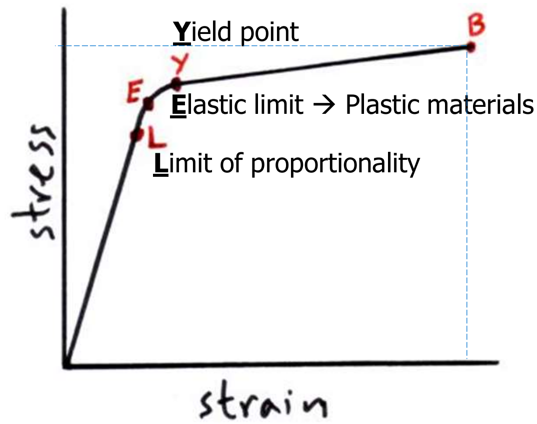
Something else you may notice here is that the **lines are curved** instead of straight – this is indicative of a **nonlinear elastic solid**.

Although these materials don't strictly follow Hooke's law for their entire stress strain curve, we can typically still use this relationship for very small deformations, for example in the zone inside the dotted box.

Let's zoom in on this box

Not all materials are elastic!





$$\sigma = \frac{F}{A} = E \times \varepsilon = \frac{E \times \Delta L}{L}$$

Hooke's Law

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Physicsnet.co.uk

Ok so if we look at this zoomed in region, we can better see where we can apply Hooke's law.

L here is the limit of proportionality – this is our stopping point for using Hooke's Law.

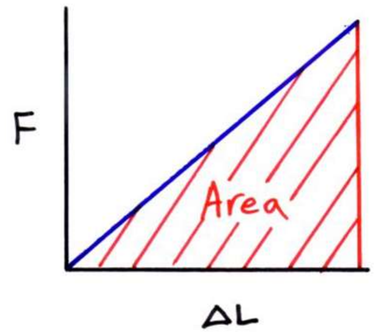
Just beyond that is called the elastic limit – beyond this point the material will not recover to its original length. Materials will become permanently stretched or deformed, and are called **plastic** materials.

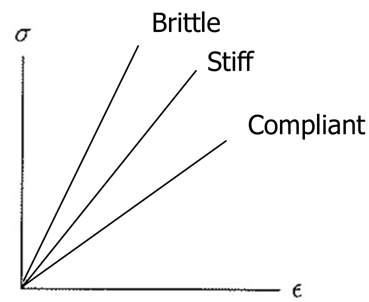
Beyond that is the **yield point** – beyond this point, a very **small** increase in force will grant a very **large** increase in length

And then way down here at the end of this curve is **B – the breaking point**, when the material cannot stretch any more and breaks or fractures (Draw this on the diagram of the blue pillar).

The stress at this point is called the **ultimate tensile or failure stress**… similarly you would call this the **ultimate strain, yield stress, yield strain** and so on

Compliant materials store the most energy





Strain Energy $\dfrac{F \cdot \Delta L}{2} = \dfrac{\sigma \cdot \varepsilon}{2}$

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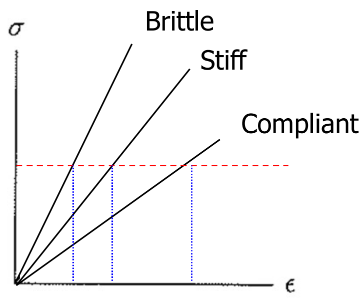
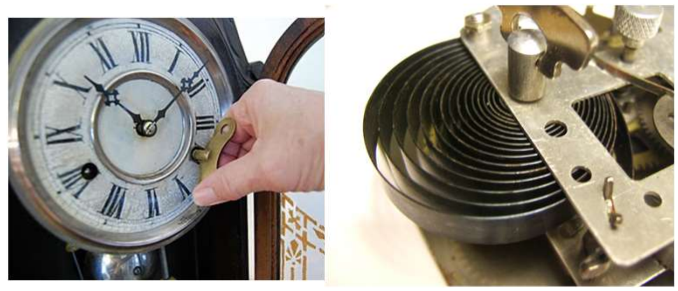
When you look at stress strain curves for different material types, you can see that brittle materials are have a steeper slope.

That is, they don't elongate very much with stress, while **compliant** materials will **elongate greatly** with applied stress.

Stiff materials fall somewhere in between

Compliant materials store the most energy (cont.)





Clock springs store energy through mechanical deformation

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Atomictoasters.com; Wikipedia.org

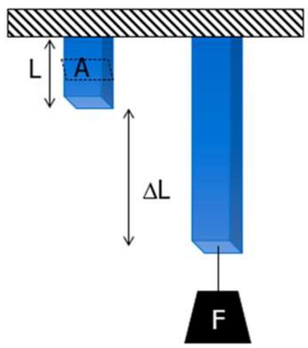
For a given stress denoted here by the red dotted line, you can calculate the strain energy in each material type, at the blue vertical lines. You can see from the resulting triangles under the curve at these points, that each material has different strain energy.

What you find is that compliant materials store more energy than brittle materials. This is important in biomechanics if you want the material you're using **to do work.**

Energy **stored** in a compliant material can be **used** when the material returns to its original state, when it relaxes.

Take a clock as an example. When you wind a clock you are deforming a large internal spring called the mainspring or torsion spring. As the spring unwinds and returns to its original shape, that energy is used to drive the wheels and gears inside the clock – moving clock hands and keeping time.

Materials elongate in the direction of force



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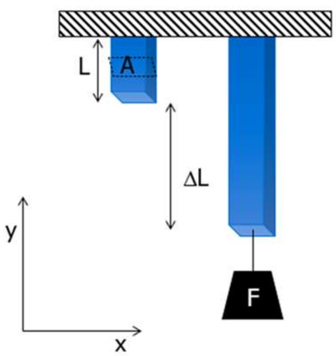
E-sunbear.com

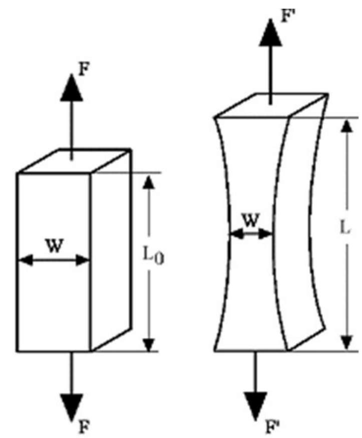
Now Let’s go back to our simple hanging beam/tissue example.

Most materials will elongate to **some degree** when we apply apply a force at one end.

They will elongate in the **direction** of the applied force.

Poisson’s ratio tells us about deformation perpendicular to force application





Positive Poisson’s ratio $\nu_{xy} = -\frac{\varepsilon_x}{\varepsilon_y}$ or z (transverse) (axial)

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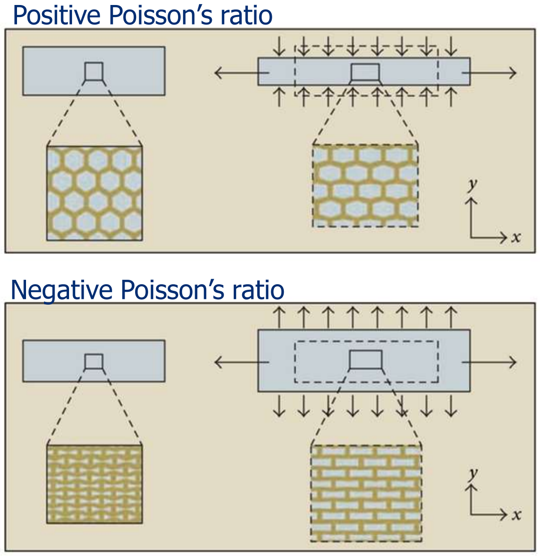
E-sunbear.com

But while elongating in that axial direction, the material will also decrease in size in the **perpendicular** direction.

The ratio of the strains (that is the transverse strain and the axial strain) is given by **Poisson’s ratio.** This ratio was first published over 200 years ago in 1811.

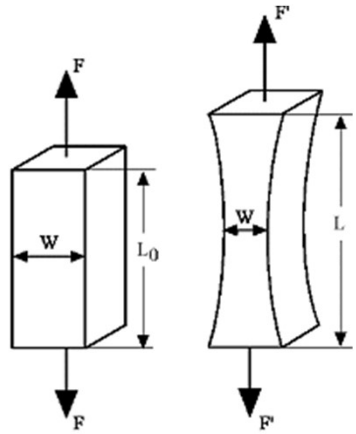
Poisson's ratio tells us about deformation perpendicular to force application (cont.)

Positive Poisson's ratio



Negative Poisson's ratio

auxetic structures



$$\nu_{xy} = -\frac{\varepsilon_x}{\varepsilon_y}$$

or z (transverse)

(axial)

Positive Poisson's ratio

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E-sunbear.com; Hindawi.com

As you would expect for most materials, Poisson's ratio is positive. The strain in the transverse direction will be negative due to **shrinking** – times the negative in the equation gives a positive. The denominator, strain in the axial or loading direction is positive, as we are growing or elongating in that direction.

Take this example on the **left**. ON the **top** you see a honeycomb material which is being compressed perpendicular to the direction of force.

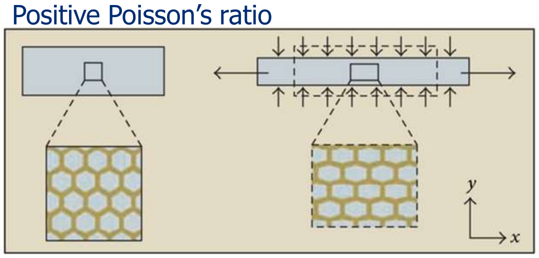
Some examples of biological materials with positive Poisson's ratios are cartilage and bone. Cartilage which readily narrows has a ratio of 0.16 while bone which doesn't narrow as easily has a ratio of 0.3

However, depending on the material structure, it is possible to have a **negative strain ratio.**

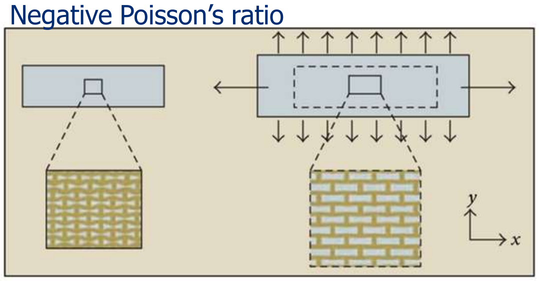
In the material on the bottom, the material is designed so that the pore expend both in the direction of force and perpendicular direction. These materials are described as having an **auxetic** structure – and are touted as having high energy absorption and indentation resistance.

Poisson's ratio tells us about deformation perpendicular to force application (cont.)

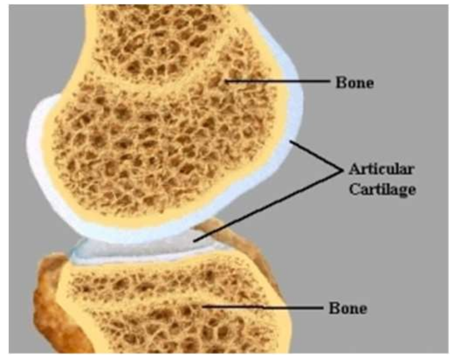
Positive Poisson's ratio



Negative Poisson's ratio



auxetic structures



$$\nu_{xy} = -\frac{\varepsilon_x}{\varepsilon_y}$$

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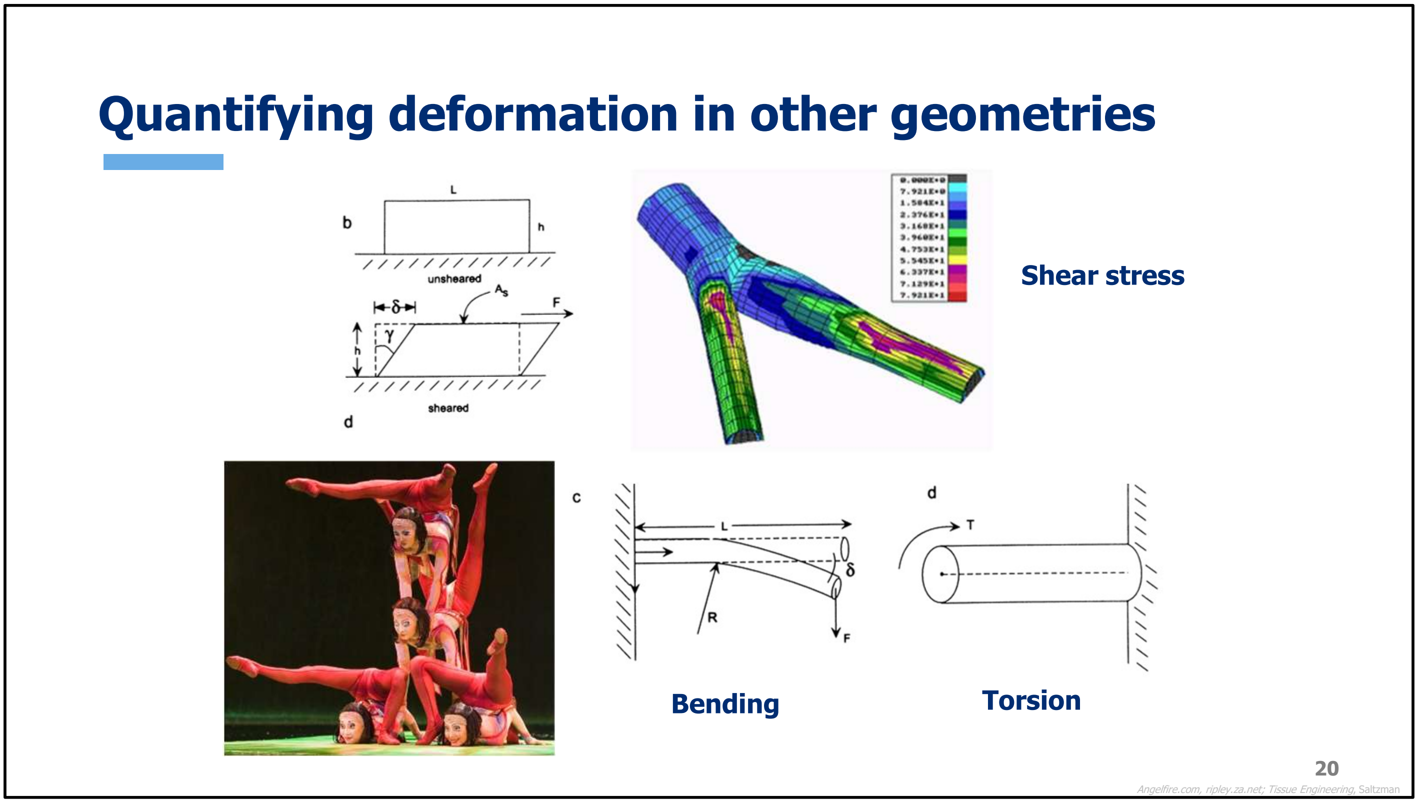
Glorysurgery.com; Hindawi.com

These types of materials are currently being investigated for tissue engineering purposes – and one example is a polymeric scaffolds for engineered articular cartilage.

One advantage of material with a negative Poisson's ratio is that when you apply a load, you won't crush the cells within the scaffold. Instead the scaffold expands to make room for the cells.

This change in strain distribution has been shown to alter properties like collagen production and cell proliferation.

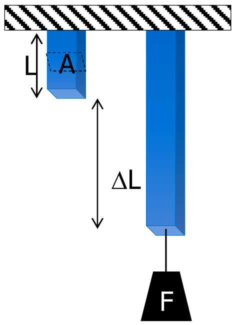


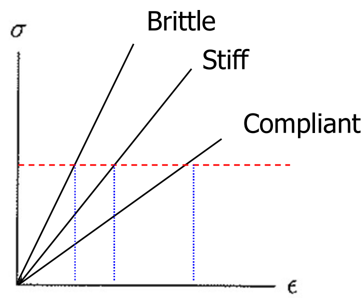
There are many other types of loads that materials need to withstand. We have only talked about compression and tension, but there's also bending and torsion. We have also covered how shear stresses alter the behavior of endothelial cells in blood vessels.

In this lecture we won't go over deformations in other geometries. Please see your text for information shear forces – like those felt by endothelial cells in the vasculature, bending forces (exemplified here by cirque du soil) and torsional forces.

Review and rewind

Elastic solid mechanics





$$\sigma = \frac{F}{A} = E \times \varepsilon = \frac{E \times \Delta L}{L}$$

Hooke's Law

$$\nu_{xy} = -\frac{\varepsilon_x}{\varepsilon_y}$$

Poisson's ratio

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In this lecture we talked about the basics of solid mechanics.

We covered Elastic solids, the stress strain curves, Hooke's law, strain energy, and Poisson's ratio


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