

Learning to regularize, Experiments Using Neumann Networks on fastRMI datasets

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1 Purpose

To use Neumann networks and set up experiments on the fastRMI open datasets, to establish baselines in term of NMSE (normalized mean square error), PSNR (peak signal-to-noise ratio), SSIM(structural similarity) metrics and reconstruction time which could be a critical factor compared to longer timers reported by more traditional approach such as TV (total variation regularized least squares). To define modified versions of the Neumann network, or investigate alternative methods for inverting nonlinear operators leading to new neural network architectures.

Introduction

Linear inverse problem in imaging can be formulated as: $\mathbf{y} = \mathbf{X}\beta^* + \epsilon$ where the objective in the context of MRI images is to recover the original image $\beta \in \mathbb{R}^p$ from a set of undersampled k-space measurements $\mathbf{y} \in \mathbb{R}^m$, which are corrupted by additive gaussian noise $\epsilon \in \mathbb{R}^p$. $\mathbf{X} \in \mathbb{R}^{m \times p}$ represent a linear forward operator modeling the MR physics. In MR imaging, cross-sectional images of a person's anatomy are produced from a strong magnetic field and radio-frequency signal created by various coils. A receiver coil detects the patient body electromagnetic response fields. The measurements are points in a Fourier-space known as *k-space*. The overall scanning process can exceed 30 minutes leading to low throughput of images, potential patient discomfort and artifacts from patient motion. To improve the processing time, less k-samples are captured within a given maximum frequency range producing lower resolution images. In order to recover the initial image β^* , the original image must be reconstructed which in the case of a Neumann network includes a regularization function $r(\beta)$ which enables to reconstruct an estimated image $\hat{\beta}$ as close as the original image β^* . The fastRMI dataset contains four types of data from various Siemens MR scanners of knees and brain scans. The data identifies two type of tasks: single-coil and multi-coil reconstruction images. With enough data, a traditional machine learning approach learns directly a mapping $\hat{\beta} = \mathcal{F}(\mathbf{y})$ without any knowledge of \mathbf{X} . A decoupled approach based on a generative model first learns the image space of interest β_i and then learns a mapping $\hat{\beta} = \mathcal{F}(\mathbf{y}, \mathbf{X})$. A standard approach for solving this inverse

problems is the regularized least-squares estimator:

$$\hat{\beta} = \arg \min_{\beta} \frac{1}{2} \|\mathbf{y} - \mathbf{X}\beta\|_2^2 + r(\beta)$$

The key feature of these approaches is that \mathbf{X} is never considered and the learning model can be used for other inverse problems with different operators \mathbf{X} . This comes at the cost of $\mathcal{O}(N^{\frac{2\alpha+p}{\alpha}})$ by considering the whole sampling space where N is the number of training samples, p the dimension of $\phi(\beta)$, the probability distribution of the images, α a smoothness term. Incorporating \mathbf{X} into the learning process reduces the dimension p to p' , where $p' \ll p$, which corresponds to the subset of the image to which the operator \mathbf{X} is applied. Neumann networks directly incorporate the forward operator \mathbf{X} and accomplishes an iterative optimization algorithm referred as unrolled gradient descent:

$$\beta^{k+1} = \beta^k - \eta[\mathbf{X}^T(\mathbf{X}\beta^k - \mathbf{y}) + R(\beta^{(k)})], \eta > 0$$

The number of iterations is fixed and is denoted B for Blocks.

2 Related work

3 Our contribution

4 Conclusion