

Input image I , filter K of dimensions $k_1 \times k_2$:

- Cross-correlation:

$$(I \otimes K)_{ij} = \sum_m \sum_n \sum_c K_{m,n} I_{i+m,j+n}$$

- Convolution

$$(I \star K)_{ij} = \sum_m \sum_n \sum_c K_{m,n} I_{i-m,j-n}$$

Convolution is the same as cross-correlation with a 180-degree rotated kernel.

- Forward equation:

$$y_{i,j} = \sum_m \sum_n w_{m,n} x_{i+m,j+n} + b$$

- Backward equations:

$$\begin{aligned} \frac{\partial E}{\partial w_{m',n'}} &= \sum_i \sum_j \frac{\partial E}{\partial y_{i,j}} \cdot \frac{\partial y_{i,j}}{\partial w_{m',n'}} \\ &= \sum_i \sum_j \delta_{i,j} \cdot \frac{\partial \sum_m \sum_n w_{m,n} x_{i+m,j+n} + b}{\partial w_{m',n'}} \\ &= \sum_i \sum_j \delta_{i,j} \cdot x_{i+m',j+n'} \\ \frac{\partial E}{\partial x_{i',j'}} &= \sum_m \sum_n \frac{\partial E}{\partial y_{i'-m,j'-n}} \cdot \frac{\partial y_{i'-m,j'-n}}{\partial x_{i',j'}} \\ &= \sum_m \sum_n \delta_{i'-m,j'-n} \cdot \frac{\partial \sum'_m \sum'_n w_{m',n'} x_{i'-m+j',j'-n+n'} + b}{\partial x_{i',j'}} \\ &= \sum_m \sum_n \delta_{i'-m,j'-n} \cdot w_{m,n} \end{aligned}$$