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Hindawi Publishing Corporation
ISRN Applied Mathematics
Volume 2013, Article ID 806056, 3 pages
http://dx.doi.org/10.1155/2013/806056

Letter to the Editor

Comment on "Adomian Decomposition Method for a Class of Nonlinear Problems"

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Received 29 May 2013; Accepted 19 August 2013

Academic Editors: Z. Huang and X.-S. Yang

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Sánchez Cano in his paper *"Adomian Decomposition Method for a Class of Nonlinear Problems"* in application part pages 8, 9, and 10 had made some mistakes in context; in this paper we correct them.

1. Introduction

Adomian [1, 2] proposed a powerful method for solving nonlinear functional equation. The technique uses a decomposition of the nonlinear operator as a series of functions; each term of this series is a generalized polynomial called Adomian polynomial.

We will see that using the Adomian decomposition method together with some properties of the nested integral [3, 4] the solution of nonlinear ordinary differential equations system is obtained.

2. Correct Equations

In page 8, he showed that the solution $x_n(t)$ is given by

$$\begin{aligned} x(t) &= x_0(t) + x_1(t) + x_2(t) + \cdots + x_n(t) + \cdots \\ &= \propto + \int_0^t f(u)\,du + a\beta t + \left[-a^2 \propto \frac{t^2}{2!} - a^2 \frac{1}{\Gamma(3)} \times \int_0^t (t-u)^2 f(u)\,du\right] \\ &\quad - a^3 \beta \frac{t^3}{3!} + \left[a^4 \propto \frac{t^4}{4!} + a^4 \frac{1}{\Gamma(5)} \times \int_0^t (t-u)^4 f(u)\,du\right] + \cdots . \end{aligned} \tag{1}$$

By rearranging he obtained

$$\begin{aligned} x(t) &= \propto \left[1 - \frac{(at)^2}{2!} + \frac{(at)^4}{4!} + \cdots (-1)^n \frac{(at)^{2n}}{2n!} + \cdots\right] \\ &\quad + \beta \left[(at) - \frac{(at)^3}{3!} + \frac{(at)^5}{5!} + \cdots (-1)^n \frac{(at)^{2n+1}}{(2n+1)!} + \cdots\right] \\ &\quad + \int_0^t \left[\frac{(a(t-u))^0}{\Gamma(1)} - \frac{(a(t-u))^2}{\Gamma(3)} + \cdots (-1)^n \frac{(a(t-u))^{2n}}{\Gamma(2n+1)} + \cdots\right] f(u)\,du. \end{aligned} \tag{2}$$

And similarly for $y_n(t)$ we will have

$$\begin{aligned} y(t) &= y_0(t) + y_1(t) + y_2(t) + \cdots \\ &\quad + y_n(t) + \cdots = \beta - \propto at - a\frac{1}{\Gamma(2)} \\ &\quad \times \int_0^t (t-u) f(u)\,du \\ &\quad - a^2\beta\frac{t^2}{2!} + a^3 \propto \frac{t^3}{3!} + a^3\frac{1}{\Gamma(4)} \\ &\quad \times \int_0^t (t-u)^3 f(u)\,du + a^4\beta\frac{t^4}{4!} + \cdots . \end{aligned} \tag{3}$$

By rearranging he obtained

$$\begin{aligned} y(t) &= \beta\left[1 - \frac{(at)^2}{2!} + \frac{(at)^4}{4!} + \cdots (-1)^n\frac{(at)^{2n}}{2n!} + \cdots\right] \\ &\quad - \propto\left[(at) - \frac{(at)^3}{3!} + \frac{(at)^5}{5!} + \cdots (-1)^n\frac{(at)^{2n+1}}{(2n+1)!} + \cdots\right] \\ &\quad - \int_0^t \left[\frac{(a(t-u))^1}{\Gamma(2)} - \frac{(a(t-u))^3}{\Gamma(4)} \right. \\ &\quad \left. + \cdots (-1)^n\frac{(a(t-u))^{2n+1}}{\Gamma(2n)} + \cdots\right] f(u)\,du. \end{aligned} \tag{4}$$

Continuing in this fashion, he concluded the following formulas:

$$\begin{aligned} x_n(t) &= (-1)^n\left[a^n a\frac{t^n}{n!} + a^n\frac{1}{\Gamma(n+1)}\int_0^t (t-u)^n f(u)\,du\right], \\ y_n(t) &= (-1)^{n+1}\left[a^n a\frac{t^n}{n!} + a^n\frac{1}{\Gamma(n+1)}\int_0^t (t-u)^n f(u)\,du\right]. \end{aligned} \tag{5}$$

But the correct formulas are given by

$$\begin{aligned} x_{2n-1}(t) &= (-1)^{n+1}\left(\frac{a^{2n-1}\beta t^{2n-1}}{(2n-1)!}\right), \\ x_{2n}(t) &= (-1)^n\left[\frac{a^{2n} \propto t^{2n}}{(2n)!} + a^n\frac{1}{\Gamma(2n+1)} \right. \\ &\quad \left. \times \int_0^t (t-u)^{2n} f(u)\,du\right], \\ y_{2n-1}(t) &= (-1)^n\left[\frac{a^{2n-1} \propto t^{2n-1}}{(2n-1)!} + a^n\frac{1}{\Gamma(2n)} \right. \\ &\quad \left. \times \int_0^t (t-u)^{2n-1} f(u)\,du\right], \\ y_{2n}(t) &= (-1)^n\left(\frac{a^{2n}\beta t^{2n}}{(2n)!}\right). \end{aligned} \tag{6}$$

Writing (2) and (4) as a single integral, he had

$$\begin{aligned} x(t) &= \propto \cos(at) + \beta\sin(at) + \int_0^t \cos(a(t-u)) f(u)\,du \\ &= \propto \cos(at) + \beta\sin(at) + f(t) * \cos(at). \end{aligned} \tag{7}$$

Similarly,

$$\begin{aligned} y(t) &= \beta\cos(at) - \propto \sin(at) - \int_0^t \sin(a(t-u)) f(u)\,du \\ &= \beta\cos(at) - \propto \sin(at) + f(t) * \sin(at). \end{aligned} \tag{8}$$

In page 9, he uses

$$\frac{dx}{dt} - ay(t) = f(t), \qquad \frac{dy}{at} + ax(t) = 0 \qquad (a \in R, a \neq 0). \tag{9}$$

With $x(0) = \alpha$ and $y(0) = \beta$, he obtained

$$\begin{aligned} &\frac{d^2y}{dt^2} + a^2y(t) = f(t), \\ &y(0) = \alpha, \qquad \acute{y}(0) = \beta. \end{aligned} \tag{10}$$

In fact, the correct solution is given by

$$\begin{aligned} &\frac{dx}{dt} - ay(t) = f(t) \longrightarrow \frac{dx}{dt} = ay(t) + f(t), \\ &\frac{dy}{dt} + ax(t) = 0 \longrightarrow \frac{d^2y}{dt^2} + a\frac{dx}{dt} = 0, \\ &\Downarrow \\ &\frac{d^2y}{dt^2} + a^2y(t) = -af(t), \\ &y(0) = \beta, \qquad \acute{y}(0) = \alpha. \end{aligned} \tag{11}$$

In page 10, he showed two cases for the solutions $x(t)$ and $y(t)$ using (2), (4), and $f(t) = \cos(t)$.

Case 1 ($a \neq 1$). In this case, he obtained the solutions

$$\begin{aligned} x(t) &= \alpha\cos(at) + \beta\sin(at) + \frac{1}{1-a^2}\sin(t), \\ y(t) &= \beta\cos(at) - \alpha\sin(at) + \frac{a}{1+a^2}\cos(t). \end{aligned} \tag{12}$$

But the correct method is the following:

$$\begin{aligned} x(t) &= \propto \cos(at) + \beta\sin(at) + \int_0^t \cos(a(t-u)) f(u)\,du \\ &\Longrightarrow \int_0^t \cos(a(t-u)) f(u)\,du \end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2}\left[\int_0^t \cos(at - au + u)\cos(at - au - u)\,du\right] \\
&= \left[\frac{1}{2(1-a)}\sin(at - au + u)\right. \\
&\qquad \left. + \frac{1}{2(-1-a)}\sin(at - au - u)\right]_0^t \\
&= \frac{1}{1-a^2}\sin t - \frac{a}{1-a^2}\sin at \\
&\longrightarrow x(t) = \propto \cos(at) + \beta\sin(at) + \frac{1}{1-a^2}\sin t \\
&\qquad - \frac{a}{1-a^2}\sin at, \\
&x(t) = \alpha\cos at + \left(\beta - \frac{a}{1-a^2}\right)\sin at + \frac{1}{1-a^2}\sin t.
\end{aligned}
\tag{13}
$$

Similarly,

$$
y(t) = \left(\beta - \frac{a}{1-a^2}\right)\cos at - \alpha\sin at + \frac{a}{1-a^2}\cos t. \tag{14}
$$

Case 2 ($a = 1$). He arrived at the formulas $x(t)$ and $y(t)$ as follows:

$$
\begin{aligned}
x(t) &= \propto \cos(t) + \beta\sin(t) + \frac{1}{2}t\cos t, \\
y(t) &= \left(\propto - \frac{1}{2}\right)\cos t + \beta\sin t - \frac{1}{2}t\sin t.
\end{aligned}
\tag{15}
$$

But the correct formulas are given by

$$
\begin{aligned}
&x(t) = \propto \cos(t) + \beta\sin(t) + \int_0^t \cos((t-u))\,f(u)\,du \\
&= \longrightarrow \int_0^t \cos((t-u))\cos u\,du \\
&= \frac{1}{2}\left[\int_0^t \cos\ t + \cos(t - 2u)\,du\right] \\
&= \frac{1}{2}\left[t\cos t - \frac{1}{2}\sin(t - 2u)\right]_0^t \\
&= \frac{1}{2}t\cos t + \frac{1}{2}\sin t \\
&\Longrightarrow x(t) = \alpha\cos t + \beta\sin t + \frac{1}{2}t\cos t + \frac{1}{2}\sin t \\
&\Longrightarrow x(t) = \alpha\cos t + \left(\beta + \frac{1}{2}\right)\sin t + \frac{1}{2}t\cos t.
\end{aligned}
\tag{16}
$$

Similarly,

$$
y(t) = \beta\cos t - \alpha\sin t - \frac{1}{2}t\sin t. \tag{17}
$$

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