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Letter to the Editor

Comment on "Adomian Decomposition Method for a Class of Nonlinear Problems"

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Sánchez Cano in his paper "Adomian Decomposition Method for a Class of Nonlinear Problems" in application part pages 8, 9, and 10 had made some mistakes in context; in this paper we correct them.

1. Introduction

Adomian [1, 2] proposed a powerful method for solving nonlinear functional equation. The technique uses a decomposition of the nonlinear operator as a series of functions; each term of this series is a generalized polynomial called Adomian polynomial.

We will see that using the Adomian decomposition method together with some properties of the nested integral [3, 4] the solution of nonlinear ordinary differential equations system is obtained.

2. Correct Equations

In page 8, he showed that the solution $x_n(t)$ is given by

$$x(t) = x_0(t) + x_1(t) + x_2(t) + \dots + x_n(t) + \dots$$

$$= \infty + \int_0^t f(u) du + a\beta t + \left[-a^2 \propto \frac{t^2}{2!} - a^2 \frac{1}{\Gamma(3)} \right]$$

$$\times \int_0^t (t - u)^2 f(u) du$$

$$-a^{3}\beta \frac{t^{3}}{3!} + \left[a^{4} \propto \frac{t^{4}}{4!} + a^{4} \frac{1}{\Gamma(5)} \times \int_{0}^{t} (t - u)^{4} f(u) du\right] + \cdots$$
(1)

By rearranging he obtained

$$x(t) = \infty \left[1 - \frac{(at)^2}{2!} + \frac{(at)^4}{4!} + \dots (-1)^n \frac{(at)^{2n}}{2n!} + \dots \right]$$

$$+ \beta \left[(at) - \frac{(at)^3}{3!} + \frac{(at)^5}{5!} + \dots (-1)^n \frac{(at)^{2n+1}}{(2n+1)!} + \dots \right]$$

$$+ \int_0^t \left[\frac{(a(t-u))^0}{\Gamma(1)} - \frac{(a(t-u))^2}{\Gamma(3)} + \dots \right] f(u) du.$$

$$+ \dots (-1)^n \frac{(a(t-u))^{2n}}{\Gamma(2n+1)} + \dots \right] f(u) du.$$
(2)

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And similarly for $y_n(t)$ we will have

$$y(t) = y_0(t) + y_1(t) + y_2(t) + \cdots$$

$$+ y_n(t) + \cdots = \beta - \alpha at - a \frac{1}{\Gamma(2)}$$

$$\times \int_0^t (t - u) f(u) du$$

$$- a^2 \beta \frac{t^2}{2!} + a^3 \alpha \frac{t^3}{3!} + a^3 \frac{1}{\Gamma(4)}$$

$$\times \int_0^t (t - u)^3 f(u) du + a^4 \beta \frac{t^4}{4!} + \cdots$$
(3)

By rearranging he obtained

$$y(t) = \beta \left[1 - \frac{(at)^2}{2!} + \frac{(at)^4}{4!} + \dots + (-1)^n \frac{(at)^{2n}}{2n!} + \dots \right]$$

$$- \infty \left[(at) - \frac{(at)^3}{3!} + \frac{(at)^5}{5!} + \dots + (-1)^n \frac{(at)^{2n+1}}{(2n+1)!} + \dots \right]$$

$$- \int_0^t \left[\frac{(a(t-u))^1}{\Gamma(2)} - \frac{(a(t-u))^3}{\Gamma(4)} + \dots \right] f(u) du.$$

$$+ \dots + (-1)^n \frac{(a(t-u))^{2n+1}}{\Gamma(2n)} + \dots \right] f(u) du.$$
(4)

Continuing in this fashion, he concluded the following formulas:

$$x_{n}(t) = (-1)^{n} \left[a^{n} a \frac{t^{n}}{n!} + a^{n} \frac{1}{\Gamma(n+1)} \int_{0}^{t} (t-u)^{n} f(u) du \right],$$

$$y_{n}(t) = (-1)^{n+1} \left[a^{n} a \frac{t^{n}}{n!} + a^{n} \frac{1}{\Gamma(n+1)} \int_{0}^{t} (t-u)^{n} f(u) du \right].$$
(5)

But the correct formulas are given by

$$x_{2n-1}(t) = (-1)^{n+1} \left(\frac{a^{2n-1}\beta t^{2n-1}}{(2n-1)!} \right),$$

$$x_{2n}(t) = (-1)^n \left[\frac{a^{2n} \propto t^{2n}}{(2n)!} + a^n \frac{1}{\Gamma(2n+1)} \right],$$

$$\times \int_0^t (t-u)^{2n} f(u) du,$$

$$y_{2n-1}(t) = (-1)^n \left[\frac{a^{2n-1} \propto t^{2n-1}}{(2n-1)!} + a^n \frac{1}{\Gamma(2n)} \right],$$

$$\times \int_0^t (t-u)^{2n-1} f(u) du,$$

$$y_{2n}(t) = (-1)^n \left(\frac{a^{2n}\beta t^{2n}}{(2n)!} \right).$$
(6)

Writing (2) and (4) as a single integral, he had

$$x(t) = \infty \cos(at) + \beta \sin(at) + \int_0^t \cos(a(t-u)) f(u) du$$
$$= \infty \cos(at) + \beta \sin(at) + f(t) * \cos(at).$$
 (7)

Similarly,

$$y(t) = \beta \cos(at) - \alpha \sin(at) - \int_0^t \sin(a(t-u)) f(u) du$$
$$= \beta \cos(at) - \alpha \sin(at) + f(t) * \sin(at).$$
 (8)

In page 9, he uses

$$\frac{dx}{dt} - ay(t) = f(t), \qquad \frac{dy}{at} + ax(t) = 0$$

$$(a \in R, a \neq 0).$$
(9)

With $x(0) = \alpha$ and $y(0) = \beta$, he obtained

$$\frac{d^2 y}{dt^2} + a^2 y(t) = f(t),$$

$$y(0) = \alpha, \qquad \acute{y}(0) = \beta.$$
(10)

In fact, the correct solution is given by

$$\frac{dx}{dt} - ay(t) = f(t) \longrightarrow \frac{dx}{dt} = ay(t) + f(t),$$

$$\frac{dy}{dt} + ax(t) = 0 \longrightarrow \frac{d^2y}{dt^2} + a\frac{dx}{dt} = 0,$$

$$\downarrow \qquad (11)$$

$$\frac{d^2y}{dt^2} + a^2y(t) = -af(t),$$

$$y(0) = \beta, \qquad \acute{y}(0) = \alpha.$$

In page 10, he showed two cases for the solutions x(t) and y(t) using (2), (4), and $f(t) = \cos(t)$.

Case 1 ($a \ne 1$). In this case, he obtained the solutions

$$x(t) = \alpha \cos(at) + \beta \sin(at) + \frac{1}{1 - a^2} \sin(t),$$

$$y(t) = \beta \cos(at) - \alpha \sin(at) + \frac{a}{1 + a^2} \cos(t).$$
(12)

But the correct method is the following:

$$x(t) = \alpha \cos(at) + \beta \sin(at) + \int_0^t \cos(a(t-u)) f(u) du$$
$$= \longrightarrow \int_0^t \cos(a(t-u)) f(u) du$$

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Similarly,

$$y(t) = \left(\beta - \frac{a}{1 - a^2}\right)\cos at - \alpha\sin at + \frac{a}{1 - a^2}\cos t.$$
(14)

Case 2 (a = 1). He arrived at the formulas x(t) and y(t) as follows:

$$x(t) = \alpha \cos(t) + \beta \sin(t) + \frac{1}{2}t \cos t,$$

$$y(t) = \left(\alpha - \frac{1}{2}\right) \cos t + \beta \sin t - \frac{1}{2}t \sin t.$$
(15)

But the correct formulas are given by

$$x(t) = \alpha \cos(t) + \beta \sin(t) + \int_0^t \cos((t - u)) f(u) du$$

$$= \longrightarrow \int_0^t \cos((t - u)) \cos u du$$

$$= \frac{1}{2} \left[\int_0^t \cos t + \cos(t - 2u) du \right]$$

$$= \frac{1}{2} \left[t \cos t - \frac{1}{2} \sin(t - 2u) \right]_0^t$$

$$= \frac{1}{2} t \cos t + \frac{1}{2} \sin t$$

$$\implies x(t) = \alpha \cos t + \beta \sin t + \frac{1}{2} t \cos t + \frac{1}{2} \sin t$$

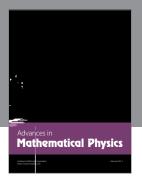
$$\implies x(t) = \alpha \cos t + \left(\beta + \frac{1}{2} \right) \sin t + \frac{1}{2} t \cos t.$$
(16)

Similarly,

$$y(t) = \beta \cos t - \alpha \sin t - \frac{1}{2}t \sin t.$$
 (17)

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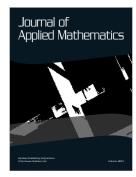


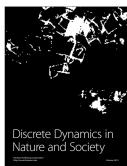














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