- Given an history $[y_1, \cdots, y_T]$
- TASK: predict $[y_{T+1}, \cdots, y_{T+H}]$
- H: horizon, T: length of observations, m: periodicity of the data
- Standard scale-free metrics in the practice of forecasting:
  - sMAPE: symmetric Mean Absolute Percentage Error
  - MASE: Mean Absolute Scaled Error

$$\text{sMAPE} = \frac{2}{H}\sum_{i=1}^{H}\frac{|y_{T+i} - \hat{y}_{T+i}|}{|y_{T+i}| + |\hat{y}_{T+i}|} \text{ , MASE} = \frac{1}{H}\sum_{i=1}^{H}\frac{|y_{T+i} - \hat{y}_{T+i}|}{\frac{1}{T+H-m}\sum_{j=m+1}^{T+H}|y_j - y_{j-m}|}$$

Frequency	**Demographic**	**Finance**	**Industry**	**Macro**	**Micro**	**Other**	**Total**
Yearly	1,088	6,519	3,716	3,903	6,538	1,236	23,000
Quarterly	1,858	5,305	4,637	5,315	6,020	865	24,000
Monthly	5,728	10,987	10,017	10,016	10,975	277	48,000
Weekly	24	164	6	41	112	12	359
Daily	10	1,559	422	127	1,476	633	4,227
Hourly	0	0	0	0	0	414	414
Total	8,708	24,534	18,798	19,402	25,1212	3,437	100,000

$$l_t = \alpha\frac{y_t}{s_{t-m}} + (1-\alpha)l_{t-1}$$
$$s_t = \beta\frac{y_t}{l_{t-1}} + (1-\beta)s_{t-m}$$
$$\hat{y}_{\textbf{win}} = \textbf{ES-RNN}(\frac{y_{ti}}{s_{ti}l_{ti}})$$
$$y_{\textbf{truth}} = (\frac{y_{to}}{s_{to}l_{to}})$$

Time Series	Dilations
Quarterly	(1, 2), (4, 8)
Monthly	(1, 3), (6, 12)
Daily	(1, 7), (14, 28)
Yearly	(1, 2), (2, 6)
Weekly	(1, 14), (14, 28)
Hourly	(1, 24), (24, 48)

ES-RNN dilation parameters

$$\hat{\mathbf{y}}_{i,j} = \mathbf{W}_{i,j}\theta_{i,j} + \mathbf{b}_{i,j}$$

$$g_\theta(t) = \sum_{i=0}^{p} \theta_i t^i$$

where $\mathbf{t} = [0, 1, 2, \cdots, H-2, H-1]^T / H.$

$$g_\theta(t) = \sum_{i=0}^{\lfloor H/2-1 \rfloor} \theta_i \cos(2\pi i t) + \theta_{i+\lfloor H/2 \rfloor} \sin(2\pi i t)$$

where $\theta_{i,j}$ are Fourier coefficients predicted by a FC network of layer$j$ of stack i

	sMAPE by Frequency					
Model	**H344**	**D1**	**W246**	**M1**	**Q66**	**Y1**
ARIMA	23.24	0.28	5.51	30.40	5.96	11.20
GP	94.93	22.37	32.7	19.04	35.02	20
ES-RNN	36.68	6.48	23.44	10.19	10.2	1.99
N-BEATS	130.38	6.54	46.17	24.68	3.26	9.53

Comparison of results on specific Time Series

	sMAPE by Frequency					
Model	**Hourly**	**Daily**	**Weekly**	**Monthly**	**Quarterly**	**Yearly**
ES-RNN	30.26	**2.97**	14.84	**9.78**	10.51	14.58
N-BEATS	-	4.08	27.93	85.55	11.94	70.54

Performance on the M4 test set

Data Category	**Yearly**	**Quarterly**	**Monthly**
Demographic	**11.45**	11.67	**5.76**
Finance	**16.31**	**10.41**	**10.8**
Industry	21.98	8.74	**11.3**
Macro	**14.21**	10.13	**11.7**
Micro	**10.93**	11.96	**8.02**
Other	16.27	7.87	**7.76**
Overall	14.58	10.51	**9.78**

ES-RNN: breakdown of sMAPE by time period and category

Data Category	**Yearly**	**Quarterly**	**Monthly**
Demographic	57.62	11.74	86.67
Finance	77.52	13.90	87.68
Industry	71.86	10.85	89.2
Macro	71.86	11.35	84.48
Micro	65.01	12.22	80.2
Other	71.83	7.90	96.5
Overall	70.54	11.94	85.55

N-BEATS: breakdown of sMAPE by time period and category