# Dynamic Decision Making Under Uncertainty:

## Applications

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IMSI Tutorial on Decision Making and Uncertainty

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# Growth rate uncertainty

Sources of uncertainty that are difficult to quantify probabilistically with full confidence:

- macroeconomists speculate about the potential permanence of secular stagnation,
- entrepreneurs and economists speculate about the impact of the digital economy on growth and development,
- epidemiologists and economists explore the short-term and long-term implications of pandemics,
- geo-scientists and environmental economists make conjectures about how climate change could alter economic opportunities.

# Where does uncertainty emerge?

Quantitative story-telling with multiple stories:

- risk: each model has random impulses and requires numerical inputs,
- ambiguity: multiple models give rise to multiple "stories" with different implications,
- misspecification: each model is an abstraction and is not intended to be a complete description of reality.

# Robustness concerns illustrated

- Initial model

$$dY_t = (.01)\left(\widehat{\alpha}_y + \widehat{\beta}_y Z_t\right) dt + (.01)\sigma_y \cdot dW_t$$
$$dZ_t = \widehat{\alpha}_z dt - \widehat{\beta}_z Z_t dt + \sigma_z \cdot dW_t$$

- $W$ a multivariate Brownian motion
- $Y$ is log capital, consumption, or output
- $Z$ generates "long-run risk" or growth rate uncertainty

The coefficient $\widehat{\beta}_y$ captures the exposure to growth rate uncertainty and $\widehat{\beta}_z$ captures persistence in the macro growth rate.

# Family of Restricted Models

- parameters: $\alpha_y, \beta_y, \alpha_z, \beta_z$
- evolution:

$$dY_t = .01\left(\alpha_y + \beta_y Z_t\right) dt + .01\sigma_y \cdot dW_t^S$$
$$dZ_t = \alpha_z dt - \beta_z Z_t dt + \sigma_z \cdot dW_t^S$$

- Construct drift distortion for the Brownian motion $dW_t = S_t dt + dW_t^S$ where $S_t = \eta(Z_t) \equiv \eta_0 + \eta_1 Z_t$ and where

$$\sigma = \begin{bmatrix} \sigma_y{}' \\ \sigma_z{}' \end{bmatrix},$$

and

$$\sigma\eta_0 = \begin{bmatrix} \alpha_y - \widehat{\alpha}_y \\ \alpha_z - \widehat{\alpha}_z \end{bmatrix} \quad \sigma\eta_1 = \begin{bmatrix} \beta_y - \widehat{\beta}_y \\ \widehat{\beta}_z - \beta_z \end{bmatrix}$$

# Restraining structured models

- Compute relative entropy, an exponentially weighted average for $S_t = \eta(Z_t)$, using recursive methods:

$$\rho(z) = \frac{1}{2}\int_0^\infty \exp(-\delta t) E\left( M_t^S |S_t|^2 \Big| Z_0 = z \right) dt.$$

- $\rho$ satisfies the Feynman-Kac equation

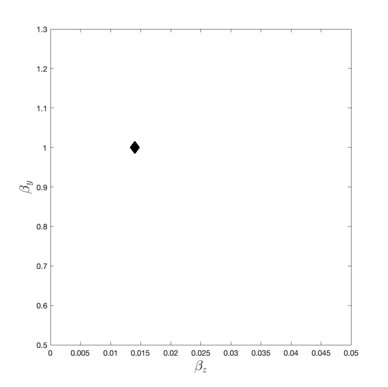
$$\frac{d\rho}{dz}(z) \cdot [\widehat{\mu}(z) + \sigma_z \cdot \eta] + \frac{1}{2}\left[\frac{d^2\rho}{dz^2}(z)|\sigma_z|^2\right] + \frac{|\eta|^2}{2} = \delta\rho(z)$$

- Use a prespecified $\rho$ to restrain structured models

$$\frac{d\rho}{dz}(z) \cdot [\widehat{\mu}(z) + \sigma_z \cdot s] + \frac{1}{2}\left[\frac{d^2\rho}{dz^2}(z)|\sigma_z|^2\right] + \frac{|s|^2}{2} \leq \delta\rho(z)$$

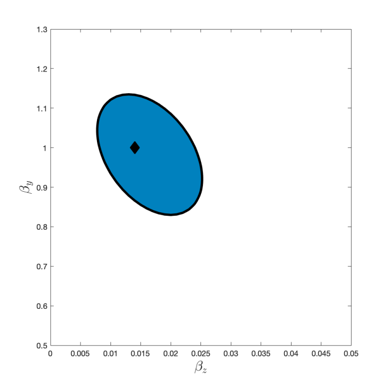
Constrain the local evolution of relative entropy.

# Growth rate ambiguity



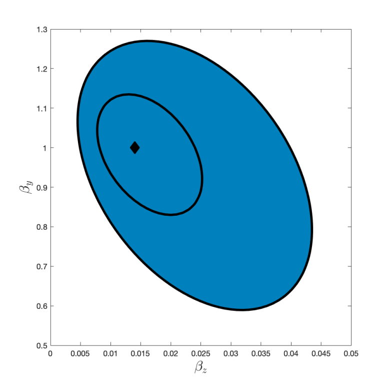

Ambiguity sets: parameter values constrained by relative entropy where $\beta_y$ quantifies exposure to the macro growth-rate process and $\beta_z$ quantifies the persistence of that process. The single point is the baseline.

# Growth rate ambiguity




Ambiguity sets: parameter values constrained by relative entropy where $\beta_y$ quantifies exposure to the macro growth-rate process and $\beta_z$ quantifies the persistence of that process. The single point is the baseline and the region is implied by a relatively tight relative entropy constraint.

# Growth rate ambiguity




Ambiguity sets: parameter values constrained by relative entropy where $\beta_y$ quantifies exposure to the macro growth-rate process and $\beta_z$ quantifies the persistence of that process. The single point is the baseline and the outer regions is implied by a relatively loose relative entropy constraints.

# Confronting misspecification

Use entropy of unstructured relative to alternative structured probability models

$$\Delta\left(M^U; M^S \mid \mathcal{F}_0\right) = \frac{1}{2}\int_0^\infty \exp(-\delta t) E\left(M_t^U \mid U_t - S_t \mid^2 \Big| \mathcal{F}_0\right) dt.$$

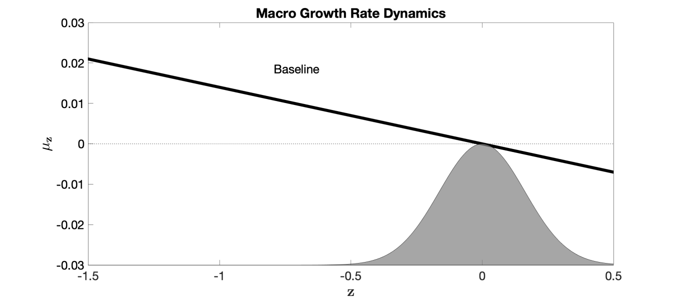
multiplied by a penalty parameter to "limit" the consequences of misspecification.

Observations:

- Add a penalty $\xi_u \frac{|u-s|^2}{2}$ to the HJB equation of a fictitious social planner.
- To accommodate misspecification , we minimize over $u$ given $s$ and to accommodate model ambiguity, we minimize over $s$ subject to a constraint.

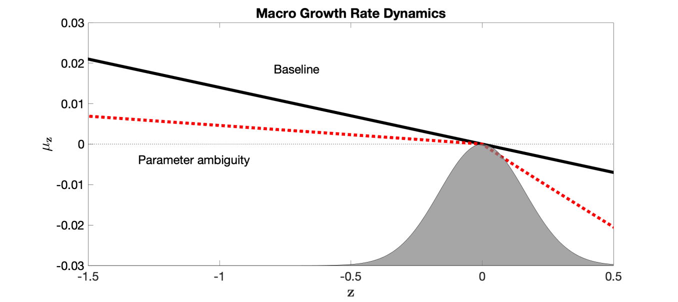
The resulting $M^{U^*}$ process is a martingale contribution to valuation.

# Growth-rate uncertainty



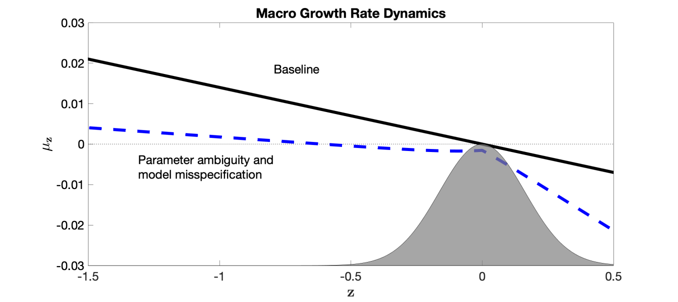

Conditional mean dynamics for the macro growth rate state under the baseline.

# Growth rate uncertainty



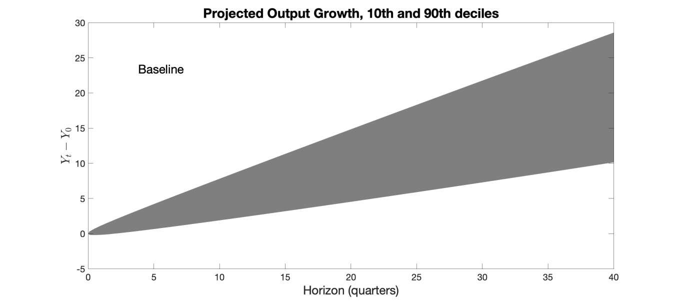

Conditional mean dynamics for the macro growth rate state under parameter ambiguity.

# Growth rate uncertainty



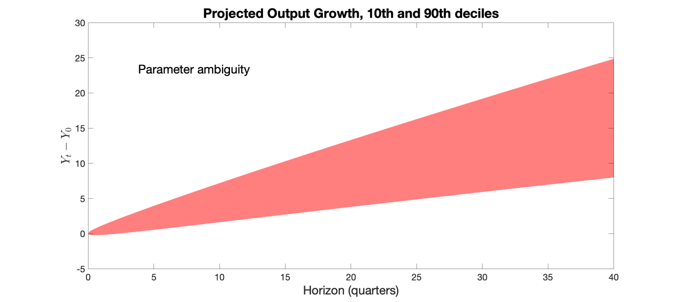

Conditional mean dynamics for the macro growth rate state under parameter ambiguity and model misspecification.

# Compounding uncertainty



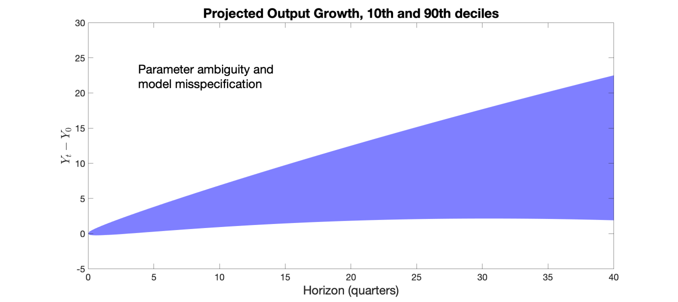

The gray shaded region gives the .1 and .9 deciles for projected consumption growth under the baseline specification.

# Compounding uncertainty




The red shaded region gives the .1 and .9 deciles for projected consumption growth adjusted for parameter ambiguity.

# Compounding uncertainty




The blue shaded region gives the .1 and .9 deciles for projected consumption growth adjusted for ambiguity and misspecification.

# What we have achieved

- tractable approach for confronting uncertainty
- a mechanism for inducing fluctuations in asset values
- investors fear persistence in bad times and fear the lack of persistence in good times

# Haunted by Hayek's forewarning

"Even if true scientists should recognize the limits of studying human behaviour, as long as the public has expectations, there will be people who *pretend* or *believe* that they can do more to meet popular demand than what is really in their power."
(From Hayek's Nobel address, 1974)

For quantitative policy analysis, how should we acknowledge the limits to our understanding?

# Uncertain Implications of Climate Change

Still to come:

- William Brock discussion of geo-scientific models of climate change
- Michael Barnett discussion of the inputs to a social planner's problem with uncertainty along with quantitative analysis of the solution

In what follows, I will highlight results from our research in this area.

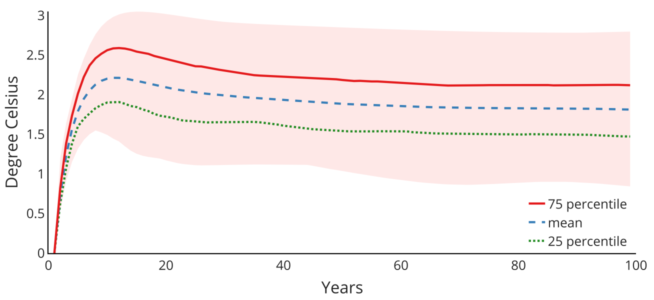
# Social Cost of Carbon (SCC)

Commonly referred to in policy discussions but meanings and targets of measurement *differ* across applications.

We use a well-posed version as an analytical tool to assess the impact of uncertainty.

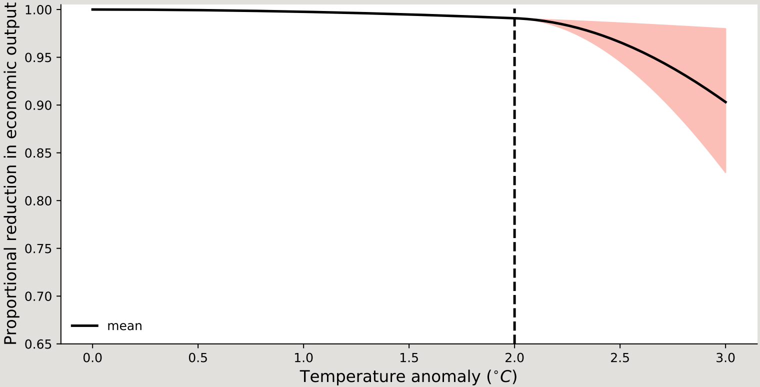
- externality - carbon emissions alter the climate, which in turn impacts economic opportunities and social well-being in the future
- social cost of carbon includes the socially efficient (Pigouvian) tax on carbon emissions that "corrects" this "externality"

# Climate Change Uncertainty



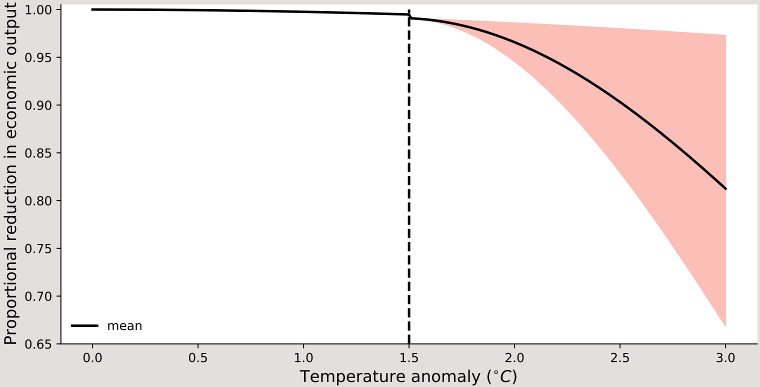

Baseline probabilities assign equal weight to all outcomes.

# Damage Function Uncertainty



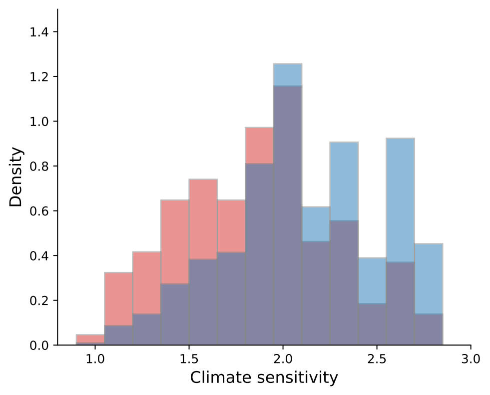

Range of possible damage functions when jump occurs at temperature anomaly of two degrees.

# Damage Function Uncertainty



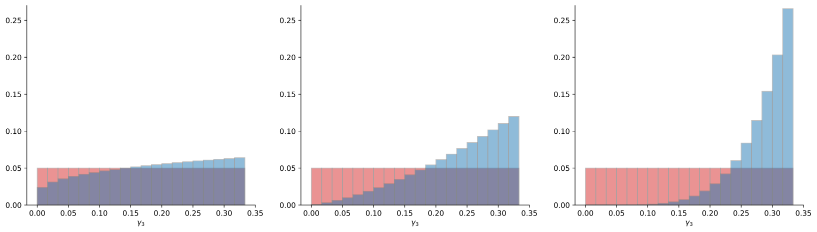

Range of possible damage functions when jump occurs at temperature anomaly of 1.5 degrees.

# Ambiguity Adjusted Climate Model Probabilities



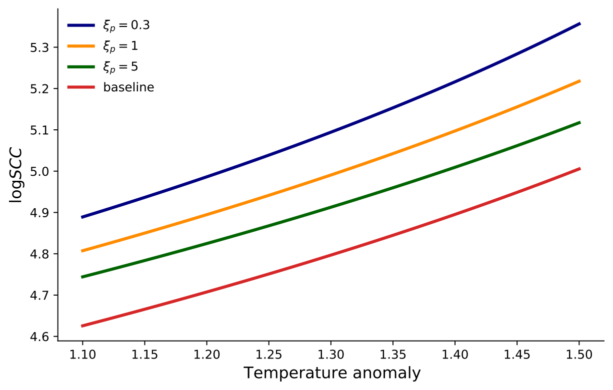

The red histogram is the outcome of equally weighting all 144 climate models. The blue histogram is the outcome of the minimization in the social planner's problem.

# Robust Adjusted Damage Function Probabilities




Blue bars are the baseline probabilities and the red bars are robust adjustments to the probabilities induced by model misspecification concerns. Left panel: $\xi_u = 5$, center panel: $\xi_u = 1$, right panel: $\xi_u = 0.3$.

# Social Cost of Carbon with Uncertainty




The logarithm of the social cost of carbon as a function of the temperature anomaly for different misspecification concerns.

# Taking inventory

- The SCC can be magnified by a prudent social planner who does not have full confidence in assigning probabilities to alternative geo-scientific and economic specifications
- The social planner implements the uncertainty aversion by replacing baseline probabilities with their adjusted counterparts
- The adjusted probabilities provide a revealing assessment of the magnitude of the aversion