Mechanism Design with Ambiguity: IMSI Tutorial (Module 2 of Decision Making and Uncertainty)

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22 Jul 2021

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- Principal may simply ask for the information, but agents may not report it truthfully unless the principal gives them incentives to do so by monetary payments or some other instrument that she controls.
- But provision of incentives involve costly trade-offs... hence the question designing game forms that provide incentives to elicit private information *optimally*, e.g., maximizing expected revenue in an auction.

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- We start by explaining the basic idea the design optimal auction in the regular case and then discuss how the design has to be adjusted to take into account ambiguity averse agents.

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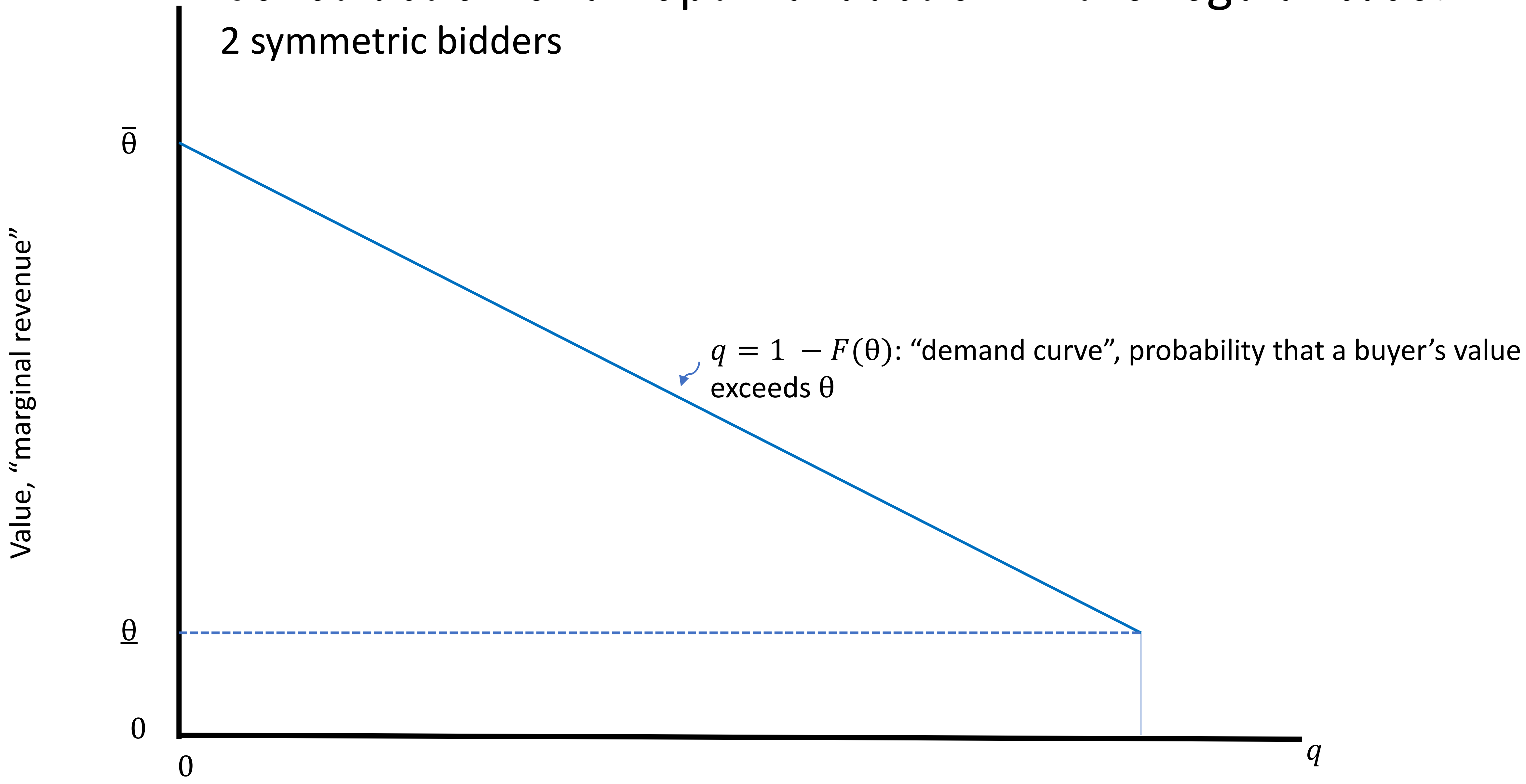
- The seller has a value known to be zero.
- What sales mechanism will maximize the seller's expected profits?

Regular case contd.

- For each bidder, graph the inverse of his cumulative distribution function, with value θ on the Y or "price axis" and the probability that the buyer's value exceeds a certain value, $1 - F(\theta) \equiv q$, on the X or "quantity axis."

Construction of an optimal auction in the regular case:

2 symmetric bidders



Regular case contd.

- For each bidder, graph the inverse of his cumulative distribution function, with value θ on the Y or "price axis" and the probability that the buyer's value exceeds a certain value, $1 - F(\theta) \equiv q$, on the X or "quantity axis."
- For each bidder, we then have something that looks like a demand curve, with the bidder's maximum possible value being the price at a quantity of zero and the bidder's minimum possible value being the price at a quantity of one.

Regular case contd.

- From the demand curve for each bidder draw a "marginal revenue" curve, calculated the way we always calculate marginal revenue curves from demand curves: Multiply "quantity," $q = 1 - F(\theta)$, times "price," $\theta = F^{-1}(1-q)$, and take the derivative with respect to "quantity":

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- Now express this as a function of θ:

$$MR(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)}$$

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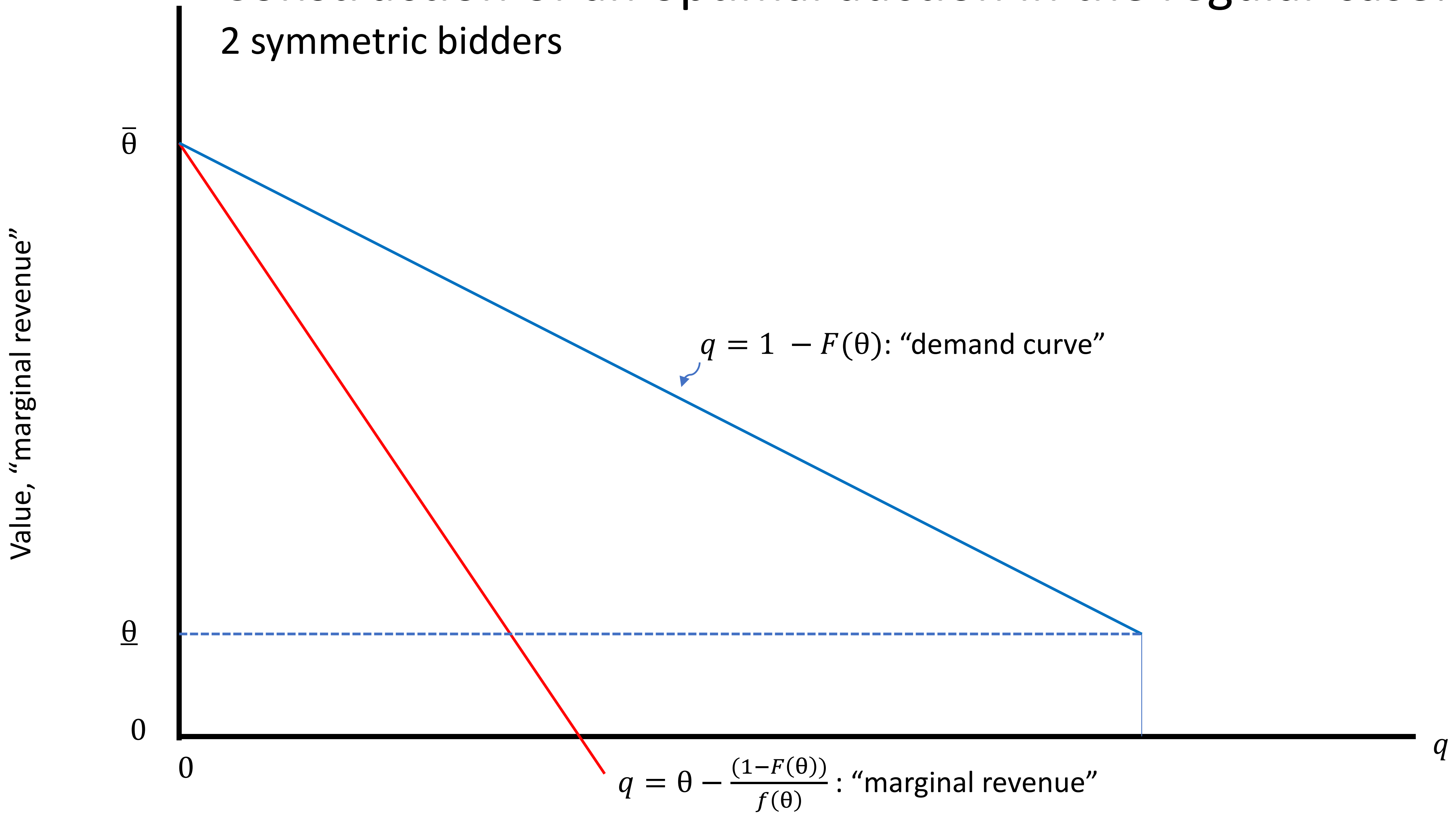

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- Now express this as a function of θ:
$$MR(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)}$$
- Define $MR(\theta) = -\infty$ for all $\theta < \underline{\theta}$.Assume, $MR(\theta)$ is monotonically increasing in θ.

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- If more than one bidder has a $+$ve MR, the highest bidder gets the item at a price $P = MR^{-1}(M_2)$, where M_2 is the MR of the 2nd highest bid: winner pays *the lowest value he could have had and still have had the maximal marginal revenue*.

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- The other buyer pays nothing.

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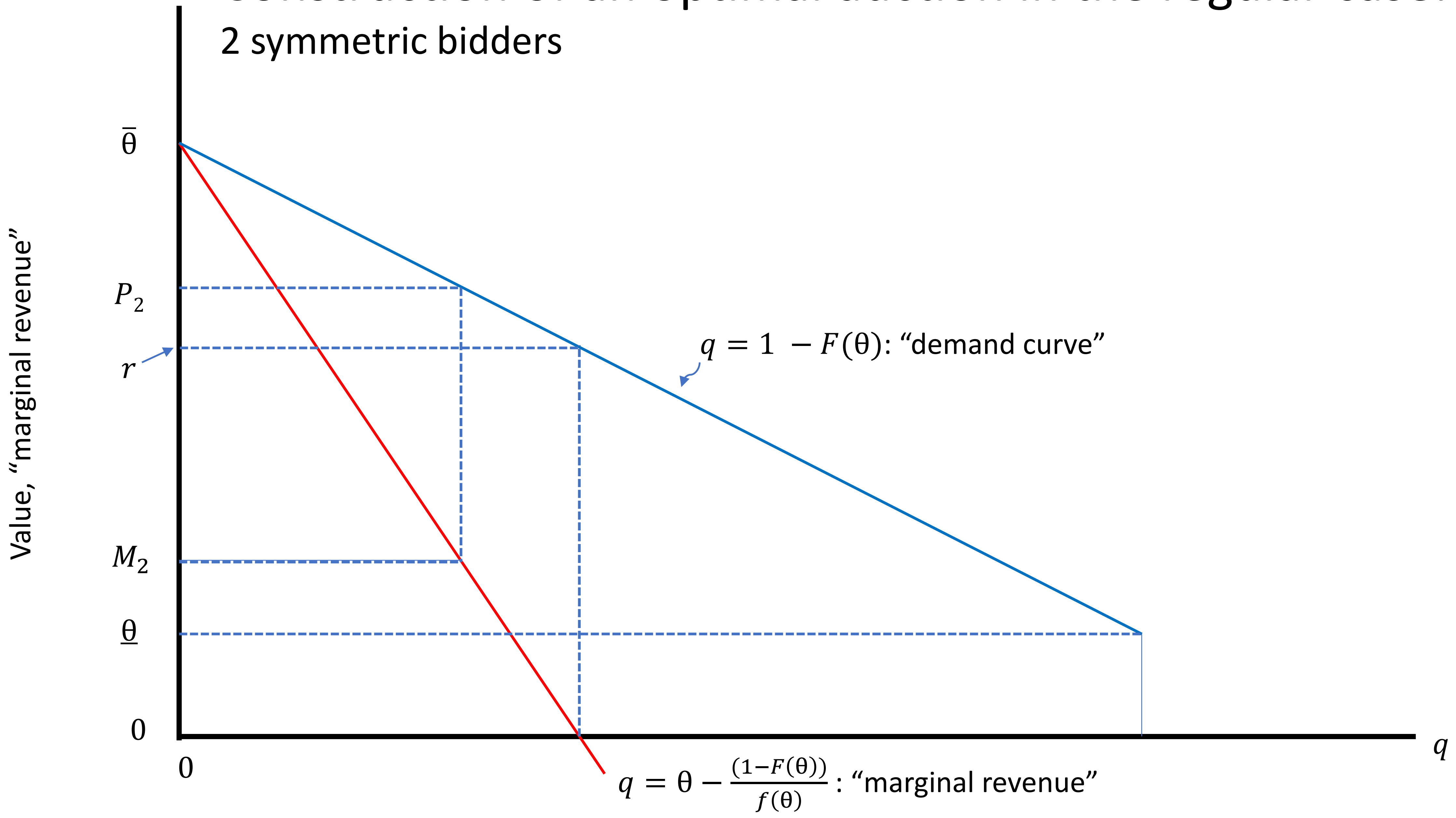

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  - while any lie that does not affect the outcome also does not affect the amount paid by the liar and so does not change the liar's utility.

The case with ambiguity averse bidders

Bose, Ozdenoren and Pape (2006)

- 2 risk-neutral potential buyers, as before, except now they have maxmin expected utility preferences:

$$\triangle_B = \{G : G = (1-\varepsilon)F + \varepsilon H \text{ for any distribution } H \text{ on } [\underline{\theta}, \overline{\theta}]\}$$

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  - The set $\triangle_B$ represents each bidder's belief about the other bidder's valuation.
- The seller is ambiguity neutral, i.e., $\triangle_S = \{F\}$.
- We assume that the seller's distribution F is a focal point, and bidders allow for an ε-order amount of noise around this focal distribution.

Case with ambiguity averse bidders contd.

- We consider a direct mechanism where bidders simultaneously report their types. The mechanism stipulates a probability $x_i(\widetilde{\theta}, \theta')$ for assigning the item and a transfer rule $t_i(\widetilde{\theta}, \theta')$ as a function of reported types.

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- Let $x_i(\widetilde{\theta}, \theta')$ be the item assignment probability function and $t_i(\widetilde{\theta}, \theta')$ the transfer rule for bidder $i \in \{1, 2\}$.
- We assume that type θ of bidder i chooses a report $\widetilde{\theta}$ to maximize

$$\min_{G \in \triangle_B} \int \left(x_i(\widetilde{\theta}, \theta')\theta - t_i(\widetilde{\theta}, \theta') \right) dG\left(\theta'\right)$$

Case with ambiguity averse bidders contd.

Seller finds a mechanism $\left\{(x_i, t_i)_{i=1,2}\right\}$ that solves,

$$\max_{\left\{(x_i,t_i)_{i=1,2}\right\}} \int \left(t_1(\theta,\theta') + t_2(\theta,\theta')\right) dF(\theta) dF\left(\theta'\right) \text{ s.t.}$$

$$\begin{aligned} \text{(IC)} \quad &: \quad \min_{G \in \triangle_B} \int \left(x_i(\theta,\theta')\theta - t_i(\theta,\theta')\right) dG\left(\theta'\right) \\ &\geq \quad \min_{G \in \triangle_B} \int \left(x_i(\widetilde{\theta},\theta')\theta - t_i(\widetilde{\theta},\theta')\right) dG\left(\theta'\right) \\ \text{(IR)} \quad &: \quad \min_{G \in \triangle_B} \int \left(x_i(\theta,\theta')\theta - t_i(\theta,\theta')\right) dG\left(\theta'\right) \geq 0 \end{aligned}$$

where $\left(x_1(\theta,\theta') + x_2(\theta',\theta)\right) \leq 1$, for all $\theta, \theta' \in \left[\underline{\theta}, \overline{\theta}\right]$.

Case with ambiguity averse bidders contd.

- **DEF** A *full insurance mechanism* is one where the (ex post) payoff of a given type of a bidder does not vary with the report of the competing bidder. That is, $\left\{(x_i, t_i)_{i=1,2}\right\}$ is a full insurance mechanism if, for almost all θ, $\left(x_i(\theta, \theta')\theta - t_i(\theta, \theta')\right)$ is constant as a function of θ'.

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- The solution to the seller's problem is a full insurance mechanism.
- E.g. seller can make strict gains by switching to a full insurance auction from the optimal auction in the regular case.

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- The seller recognizes that he & bidders have different beliefs and will offer “side bets” using transfers.
- The seller can always adjust the transfers of type θ so that, under truth telling, θ gets the same minimum expected utility as he gets in the original mechanism in every state, and thus is fully insured against ambiguity in the new mechanism.

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- No other type wants to imitate θ in the new mechanism: the original mechanism is incentive compatible and imitation in the new mechanism is even worse given that the difference in transfers has weakly positive expected value under any distribution in $\triangle_B$.
- The seller's distribution is not in the minimizing set for the original mechanism, which means the additional transfers (to the seller) must have strictly positive expected value under the seller's distribution. Thus the seller is strictly better off.

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- A bidder who bids θ, pays $\theta - (1-\varepsilon)\int_r^\theta F(y)dy$ if he wins and $-(1-\varepsilon)\int_r^\theta F(y)dy$.
- r is s.t., $L^\varepsilon(r) = 0$, where $L^\varepsilon(r) \equiv r - (1-\varepsilon)\frac{1-F(r)}{f(r)}$

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Bose and Renou (2014)

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- Add an ambiguous mediated communication stage prior to the unambiguous allocation stage.
  - Classical: (a single) mapping from messages received by the designer to probabilities over messages sent by the designer
  - Here: *set* of mappings from messages received by the designer to probabilities over messages sent by the designer to probabilities over messages sent.

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- Even when a social choice function is not implementable given (possibly ambiguous) prior beliefs, the posterior beliefs resulting from the communication may be such that it becomes implementable.
- Main result shows, in a nutshell, for a social choice function to be implementable, it must be incentive compatible with respect to some set of (possibly ambiguous) beliefs.
  - Furthermore, the messages sent and received at the communication stage must be such that the required posterior belief sets are indeed generated.

A simple example (from Bose and Renou)

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- Assume that players have multiple-prior preferences (Gilboa and Schmeidler (1989)) and apply prior-by-prior updating (full Bayesian updating).
- Let P_i be the set of priors of player i about player j's types.

example contd.

- u_i :

	θ	θ'
x	0	1
y	1	0

,

f :

	θ	θ'
θ	x	y
θ'	y	x

example contd.

- $u_i:$ $\begin{array}{c|c|c|} & \theta & \theta' \\ \hline x & 0 & 1 \\ \hline y & 1 & 0 \\ \hline \end{array}$, $f:$ $\begin{array}{c|c|c|} & \theta & \theta' \\ \hline \theta & x & y \\ \hline \theta' & y & x \\ \hline \end{array}$
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- If $P_i = \Pi_i \equiv \{(0, 1), (1, 0)\}$ then f is implementable.

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- We now construct an ambiguous communication device that generates Π_i as posterior.
- Assume that each player can send message θ or θ' to the designer, can receive message ω or ω' from the designer, and that there are two possible probability systems λ and λ' .

example contd.

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- We now construct an ambiguous communication device that generates Π_i as posterior.
- Assume that each player can send message θ or θ' to the designer, can receive message ω or ω' from the designer, and that there are two possible probability systems λ and λ' .
- Define $\lambda\left(\left(\omega_1, \omega_2\right) \mid \left(\theta_1, \theta_2\right)\right) = \lambda_1\left(\omega_1 \mid \theta_2\right)\lambda_2\left(\omega_2 \mid \theta_1\right)$.

example contd.

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- $\lambda_i\left(\omega \mid \theta\right) = 1$, and $\lambda_i\left(\omega' \mid \theta'\right) = 1$, fully defines the first probability system λ.

example contd.

- Similarly, the second probability system λ' is fully specified by $\lambda_i\left(\omega \mid \theta'\right) = 1$ and $\lambda_i\left(\omega' \mid \theta\right) = 1$

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- Thus, regardless of the message received, player i's set of posteriors is $\Pi_i \equiv \{(0, 1), (1, 0)\}$, as required.
- Caveat: the mechanism exploits the fact that full Bayesian updating is dynamically inconsistent: effectively assumes that agents cannot commit to strategies, so it is not an implementation in (ex ante) Nash equilibrium.

References

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