5.21

(a) $L\{t^{5/2}\} = (\text{use equation 5.56}) = L\{t^2 t^{1/2}\} = (-1)^2 \frac{d^2}{ds^2} L\{t^{1/2}\} =$

$L\{t^{1/2}\} = (\text{ Table}) = \frac{1}{2}\left(\frac{\pi}{s^3}\right)^{1/2}$

Subst. gives $L\{t^{5/2}\} = (-1)^2 \frac{d^2}{ds^2} \frac{1}{2}\left(\frac{\pi}{s^3}\right)^{1/2} = (-1)^2 \frac{\sqrt{\pi}}{2} \frac{d^2}{ds^2} s^{-3/2} =$

$\frac{\sqrt{\pi}}{2} \frac{d}{ds}\left(\frac{-3}{2}\right) s^{-5/2} = \frac{\sqrt{\pi}}{2}\left(\frac{-3}{2}\right) \frac{d}{ds} s^{-5/2} = \frac{\sqrt{\pi}}{2}\left(\frac{-3}{2}\right)\left(\frac{-5}{2}\right) s^{-7/2} =$

$\frac{15\sqrt{\pi}}{8} s^{-7/2}$

(b) $L\left\{\frac{\sinh(at)}{t}\right\} = (\text{use equation 5.57}) = \int_s^\infty \tilde{f}(u)du$

where $\tilde{f}(u) = L[\sinh(at)] = \frac{a}{u^2 - a^2}$

$L\left\{\frac{\sinh(at)}{t}\right\} = \int_s^\infty \frac{a}{u^2 - a^2} du = (\text{use partial fractions to get answer in book}) =$

$\frac{a}{u^2 - a^2} = \frac{1/2}{u - a} + \frac{-1/2}{u + a}$

Therefore $\int_s^\infty \frac{a}{u^2 - a^2} du = \int_s^\infty \left[\frac{1/2}{u - a} + \frac{-1/2}{u + a}\right] du = \frac{1}{2}\left[\int_s^\infty \frac{1}{u - a} + \frac{-1}{u + a}\right] du =$

$\frac{1}{2}\int_s^\infty \left[\frac{1}{u - a} - \frac{1}{u + a}\right] du = \frac{1}{2}\left[\ln(u - a) - \ln(u + a)\right]_s^\infty = \frac{1}{2}\ln\left[\frac{(u - a)}{(u + a)}\right]\Bigg|_s^\infty =$

$\frac{1}{2}\left\{\lim_{u\to\infty} \ln\left[\frac{u - a}{u + a}\right] - \ln\left[\frac{(s - a)}{(s + a)}\right]\right\} \equiv \frac{1}{2}\left\{\lim_{u\to\infty} \ln\left[\frac{u}{u}\right] - \ln\left[\frac{(s - a)}{(s + a)}\right]\right\} = \frac{1}{2}\left\{\lim_{u\to\infty} \ln[1] - \ln\left[\frac{(s - a)}{(s + a)}\right]\right\} =$

$\frac{1}{2}\left\{\lim_{u\to\infty} 0 - \ln\left[\frac{(s - a)}{(s + a)}\right]\right\} = \frac{1}{2}\left\{-\ln\left[\frac{(s - a)}{(s + a)}\right]\right\} = \frac{1}{2}\ln\left[\frac{(s - a)}{(s + a)}\right]^{-1} = \frac{1}{2}\ln\left[\frac{(s + a)}{(s - a)}\right]$