10.8

$$2\frac{\partial u}{\partial x}+3\frac{\partial u}{\partial y}=10; \quad u(x,4x)=3$$

From page 394-5 we have $A=2$, $B=3$ and therefore $\frac{dx}{2}=\frac{dy}{3}$

Solving $dy=\frac{3}{2}dx$ gives $y=\frac{3}{2}x+C$

or $C=2y-3x$, then set it to p, that is $p=2y-3x$

Take the homogeneous solution

$u(x,y)=f(p)=f(2y-3x)$

For particular solution guess (or pick) $u_p(x,y)=5x$ then check it out

$$2\frac{\partial u_p}{\partial x}+3\frac{\partial u_p}{\partial y}=2\frac{\partial(5x)}{\partial x}+3\frac{\partial(5x)}{\partial y}=2\cdot 5+3\cdot 0=10$$

Therefore it works as it should! So add it to the homogeneous solution

Therefore $u(x,y)=f(p)+5x=f(2y-3x)+5x$

Now look at the boundary condition $u(x,4x)=3$

Therefore $u(x,4x)=f(p)+5x=f(p)+5x=3$

and since $p=2(4x)-3x=5x$ substitution into $f(p)+5x=3 \;\rightarrow\; f(p)+p=3$ or $f(p)=3-p$

Gives us the form of f(p) on the line (x,4x)

Therefore in general $f(p)=3-p=3-(2y-3x)$ in terms of x,y gives

$u(x,y)=f(p)+5x=3-(2y-3x)+5x=8x-2y+3$

Finally evaluate $u(2,4)=8\cdot 2-2\cdot 4+3=16-8+3=11$

Also as a last check make sure this u(x,y) satisfies the D.E. as it should

Substitute $u(x,y)=8x-2y+3$ in $2\frac{\partial u}{\partial x}+3\frac{\partial u}{\partial y}=10$

gives $2\frac{\partial u}{\partial x}+3\frac{\partial u}{\partial y}=2\cdot 8+3\cdot -2=10$ Therefore done!