10.1a

See page 392

You are given the following functional relations

$u(x,y) = x^2(x^2 - 4) + 4y(x^2 - 2) + 4(y^2 - 1)$

and $p = x^2 + 2y$

See page 392 for $\frac{\partial p}{\partial y}\frac{\partial u}{\partial x} = \frac{\partial p}{\partial x}\frac{\partial u}{\partial y}$

and for this p we have

$(2)\frac{\partial p}{\partial x} = 2x$ and $\frac{\partial p}{\partial y} = 2$ so we have $2\frac{\partial u}{\partial x} = 2x\frac{\partial u}{\partial y}$ or

$\frac{\partial u}{\partial x} = x\frac{\partial u}{\partial y}$

Then using the solution u(x,y) given above we have

$\frac{\partial u}{\partial x} = 4x^3 - 8x + 8yx$

$\frac{\partial u}{\partial y} = 4x^2 - 8 + 8y$

Substitution into the equation from page 392 gives

$\frac{\partial u}{\partial x} = (4x^3 - 8x + 8yx) = x(4x^2 - 8 + 8y) = x\frac{\partial u}{\partial y}$

Verifying it is a solution.

Next lets convert $u(x,y) \to u(p)$

therefore verifying it can be solved using the method on page 392.

Solving $p = x^2 + 2y$ for x^2 gives

$x^2 = p - 2y$ and substitute into u(x,y)

$u(x,y) = x^2(x^2 - 4) + 4y(x^2 - 2) + 4(y^2 - 1)$

$= (p - 2y)((p - 2y) - 4) - 4) + 4y((p - 2y) - 2) + 4(y^2 - 1)$

$= (p - 2y)((p - 2y - 4) + 4y(p - 2y - 2) + 4(y^2 - 1)$

$= (p^2 - 2py - 4p - 2yp + 4y^2 + 8y) + (4yp - 8y^2 - 8y) + 4y^2 - 4$

$= p^2 - 4p - 4 = f(p)$

Therefore u can be written as a function of p!