4.8

Plot even extension from –pi to pi and from pi to 3pi.

Note the derivative (slope) of this function is undefined at pi, since right and left hand derivatives do not match!!!!

However for odd extension the derivative (slope) is well behaved at pi. Therefore take

odd extension

$$f(x)=\begin{cases} x\sin(x) & x\geq 0 \\ -x\sin(x) & x<0 \end{cases}$$

period from -π to π, T = 2π

$$b_r=\frac{2\cdot 2}{2\pi}\int_0^\pi x\sin(x)\sin(rx)dx=(\text{use trig identity})=\frac{2}{\pi}\int_0^\pi x\frac{1}{2}[\cos(r-1)x-\cos(r+1)x]dx$$

$r=1$ case

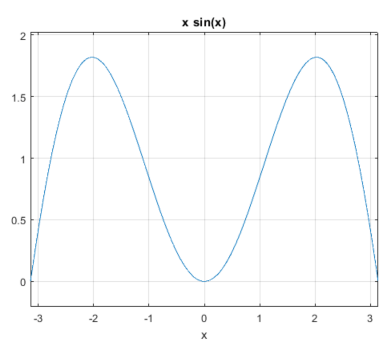
$$b_1=\frac{2}{\pi}\int_0^\pi x\frac{1}{2}[\cos(1-1)x-\cos(1+1)x]dx=\frac{1}{\pi}\int_0^\pi x[1-\cos 2x]dx=\frac{\pi}{2}$$

otherwise $(r\neq 1)$ use general b_r equation

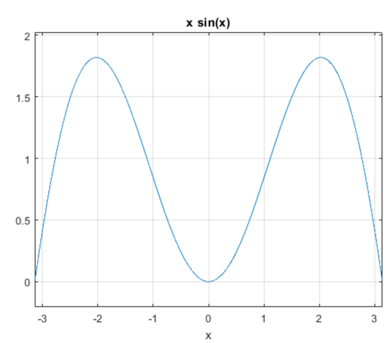
$$b_r=\frac{2}{\pi}\int_0^\pi x\frac{1}{2}[\cos(r-1)x-\cos(r+1)x]dx=\frac{1}{\pi}\left\{\int_0^\pi x[\cos(r-1)x]dx-\int_0^\pi \cos[(r+1)x]dx\right\}=$$

$$(\text{using table})\cdots=\begin{cases} \frac{1}{\pi}\left[\frac{(-1)^{r-1}-1}{(r-1)^2}-\frac{(-1)^{r+1}-1}{(r+1)^2}\right] & r-\text{even} \\ 0 & r-\text{odd} \end{cases}$$

even extension



odd extension



4.9 Neither even nor odd

Note period from -1 to 1, T = 2

$$a_0 = \frac{2}{2}\int_{-1}^{1} e^x\,dx = e^x\Big|_{-1}^{1} = e^x - e^{-x} = (\text{use def. for sinh}) = 2\sinh(1) \text{ and } \frac{a_0}{2} = \sinh(1)$$

$$a_r = \frac{2}{2}\int_{-1}^{1} e^x \cos\left(\frac{2\pi r}{2}\right)x\,dx = \int_{-1}^{1} e^x \cos\pi rx\,dx = \frac{1}{1+\pi^2r^2}e^x(\cos\pi rx + \pi r\sin\pi rx)\Big|_{-1}^{1} =$$

$= \cdots = \dfrac{2\sinh(1)(-1)^r}{1+\pi^2r^2}$, since well defined for r = 0 we could just use this, however often that is not the case and r = 0 case must be done on its own

Similarly $b_r = \int_{-1}^{1} e^x \sin\pi rx\,dx = \dfrac{2\pi r\sinh(1)(-1)^{r+1}}{1+\pi^2r^2}$

$$f(x) = e^x = \sinh(1) + 2\sinh(1)\sum_{r=1}^{\infty}\left[\frac{(-1)^r}{1+\pi^2r^2}\cos\pi rx + \frac{\pi r(-1)^{r+1}}{1+\pi^2r^2}\sin\pi rx\right]$$

Note at $x = 2$, since primary period -1 to 1 (and then repeats from 1 to 3) it must have the same value at x =2 as that at x = 0!!! which is $e^0 = 1$.