6.10

(a)

$$\frac{d^2f}{dt^2}+5\frac{df}{dt}+6f=0;\ f(0)=1,\ f'(0)=-4$$

Take Laplace transform

$$L\left\{\frac{d^2f}{dt^2}+5\frac{df}{dt}+6f=0\right\}\rightarrow[s^2\tilde{f}(s)-sf(0)-f'(0)]+5[s\tilde{f}(s)-f(0)]+6\tilde{f}(s)=0$$

Subst. for f(0) and f'(0) and solve for $\tilde{f}(s)=\frac{s+1}{s^2+5s+6}=\frac{s+1}{(s+3)(s+2)}$

Using partial fractions $\tilde{f}(s)=\frac{A}{s+3}+\frac{B}{s+2}$ gives A = 2, B = -1

$$\tilde{f}(s)=\frac{2}{s+3}+\frac{-1}{s+2}$$

Take inverse Laplace $L^{-1}\{\tilde{f}(s)\}=2L^{-1}\left\{\frac{1}{s+3}\right\}-L^{-1}\left\{\frac{1}{s+2}\right\}\rightarrow \text{Table}\rightarrow$

$$f(t)=2e^{-3t}-e^{-2t}$$

(b)

$$\frac{d^2f}{dt^2}+2\frac{df}{dt}+5f=0\ f(0)=1,\ f'(0)=0$$

Take Laplace transform

Solve for $\tilde{f}(s)=\frac{s+2}{s^2+2s+5}$

We will do this one a little different - complete the square!! and put into form for Table lookup!!!

$$\tilde{f}(s)=\frac{s+2}{s^2+2s+5}=\frac{s+2}{(s^2+2s+1)+(5-1)}=\frac{s+2}{(s+1)^2+2^2}=\frac{s+1}{(s+1)^2+2^2}+\frac{1}{(s+1)^2+2^2}=\frac{s+1}{(s+1)^2+2^2}+\frac{1}{2}\frac{2}{(s+1)^2+2^2}$$

Take inverse Laplace $L^{-1}\{\tilde{f}(s)\}=L^{-1}\left\{\frac{s+1}{(s+1)^2+2^2}\right\}+\frac{1}{2}L^{-1}\left\{\frac{2}{(s+1)^2+2^2}\right\}\rightarrow \text{Table}\rightarrow$

$$f(t)=e^{-t}\cos 2t+\frac{1}{2}e^{-t}\sin 2t$$