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```
% Description

% Compute the total concentration
% of FDG in the brain over time, Ci(t), by performing a
% numerical integration.
% From the first question of the project and Brooks paper:
%   Ci(t) = Ce(t) + Cm(t)
% where
%   - Ce(t): "free" FDG in the brain tissue
%   - Cm(t): FDG concentration "trapped" in brain tissue
%
 -------------------------------------------------------------------------
% In the first part of the project, we showed that the
% Ci(t) can be expressed as a sum of two convolutions:
%   => A * conv(exp(-alpha_1* t), Cp(t))
%   => B * conv(exp(-alpha_2* t), Cp(t))
% where Cp(t) is the FDG concentration in the arterial system.
% and conv is the convolution operator.
%
 -------------------------------------------------------------------------
% The full expression of Ci(t) and values for
% alpha_1, alpha_1, Ci(t), A, and B are given in Brooks paper in:
%       equation (3), (4), (5) and (6).
%
% To compute the convolution, we perform a numerical integration
% of the two sums above and we finally obtain Ci.
%
 -------------------------------------------------------------------------
% Lastly we plot Cp(t) and Ci(t) versus time, and  make some
% observations related to the plot.

% Clean environment
clear all;
close all;
clc;
format long g
```

Read the data and parameters

```
[ts, cp, alpha_1, alpha_2, A, B] = get_parameters();
```

Compute first convolution between exp(-alpha_1 t) and Cp(t)

```
conv_alpha_1 = convolution_by_integration(cp, ts, alpha_1);
```

Compute second convolution between exp(-alpha_2 t) and Cp(t)

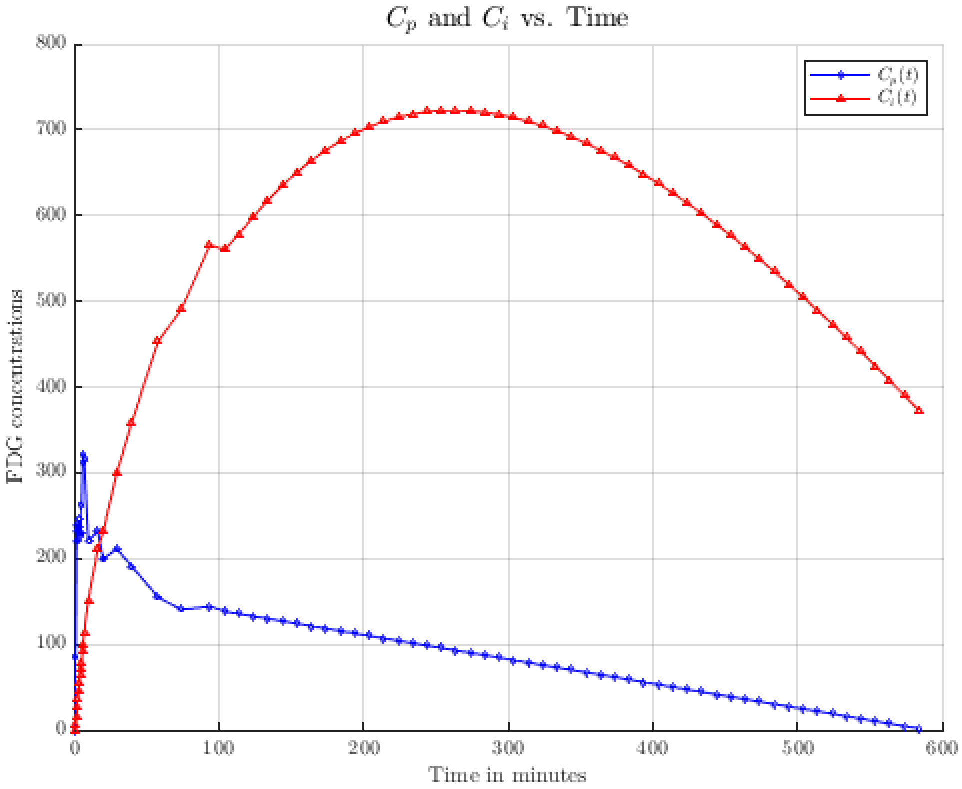
```
conv_alpha_2 = convolution_by_integration(cp, ts, alpha_2);
ci = A .* conv_alpha_1 + B .* conv_alpha_2;
```

Plot Cp and Ct versus time.

```
figure, hold on, grid on

plot(ts, cp, "b-o", 'LineWidth',1,'MarkerSize',3);
plot(ts, ci, "r-^", 'LineWidth',1,'MarkerSize',3);

title('$C_p$ and $C_i$ vs. Time','FontSize', 14);
xlabel('Time in minutes');
ylabel('FDG concentrations');
legend('$C_p(t)$','$C_i(t)$');
saveas(gcf,"cp_ci_vs_time",'pdf')
```



Interpretation of the plot

As the FDG is injected into the blood, the concentration of FDG into the brain increases rapidly to reach a peak after 250 minutes or four hours of the initial injection of FDG into the blood and then slowly decreases as the FDG is eliminated from the blood by urination. In order to have the best PET tracing (quality of the scanning), in this experiment, scanning has to be performed around 4 hours after the injection of FDG into the blood.

get_parameters

```
function [ts, cp, alpha_1, alpha_2, A, B] = get_parameters(workbook,
 worksheet)
% Description
% Set the parameters needed to determine Ci(t).
% The parameters are given in PET Scan Brooks paper.
%
% Parameters:
% ----------
% workbook: Excel workbook with the values points for Cp(t)
% extended pass 94 minutes.
% worksheet: Excel worksheet where the data is
% Outputs:
% -------
% ts: time in minutes when Cp data is sampled.
% Cp: FDG concentration in arterial system.
% alpha_1: computed value in equation (3) of Brooks paper.
% alpha_2: computed value in equation (3) of Brooks paper.
% A: computed value in equation (3) of Brooks paper.
% B: computed value in equation (3) of Brooks paper.

if ~exist('workbook','var')
    workbook = "./project_1_extended_data.xlsx";
    worksheet = "Sheet1";
end
data = import_data(workbook, worksheet);

% Cp(t) is given from an experiment running from 0 to 94 minutes
% It is then extended pass 94 minutes with a least-square regression
% using the the samples starting at 58 minutes and extended up to 584
 minutes
% when the concentration Cp(t) is almost zero.

cp = data.Concentration(2:end);
ts = data.Time(2:end);
% k1, k2 ,k3 , k4 rate constants
k1 = 0.102;
k2 = 0.130;
k3 = 0.062;
k4 = .0068;
d = sqrt((k2 + k3 + k4)^2 -4 * k2 *k4);
% Alpha_1, Alpha_2, A, B
alpha_1 = 0.5 * (k2 + k3 + k4  - d);
alpha_2 = 0.5 * (k2 + k3 + k4  + d);
A = (k1 * (k3 + k4 - alpha_1)) / (alpha_2 - alpha_1);
B = (k1 * (alpha_2 -k3 -k4)) / (alpha_2 - alpha_1);

end
```

convolution_by_integration

```
function conv_res = convolution_by_integration(cp, ts, alpha)
% Description
% Performs a convolution between the exponential function
% parametrized by alpha with concentration Cp over time.
%
% Parameters:
% ----------
% Cp: FDG concentration in arterial system.
% ts: time in minutes when Cp data is sampled.
% alpha: one of the computed values in equation (3) of Brooks paper.
% Output:
% ------
% conv_res: convolution between exp(-alpha * t) and Cp(t).

% To compute the integral, we use the trapezoidal method as described
 here:
% https://www.mathworks.com/help/matlab/ref/trapz.html
% We use two loops:
```

```
%  - first loop is indexed by i, and use the time ts(i)
%    for which we want to determine the value of the convolution
% conv_res(ts(i))
%  - second loop uses the "running" or integration variable,
%   indexed by ts(j).

% Algorithm is:
% For each ts(i)
%       total_area = 0
%       For each ts(j)
%           area = compute integral  between ts(j), ts(j-1) using
%           trapezoid method.
%           total_area = total_area + area
%       End
% End

n = size(ts,1);
conv_res = zeros(n,1);
for i=1: n
    total_area = 0;
    for j=2: i
        f_value_j_1 = alpha_function(i, j-1, alpha, cp, ts);
        f_value_j = alpha_function(i, j, alpha, cp, ts);
        dt = ts(j) - ts(j-1);
        area = dt * ((f_value_j_1 + f_value_j)/2);
        total_area = total_area + area;
    end
    conv_res(i) = total_area;
end
end
```

alpha_function

```
function f_value = alpha_function(t1, t2, alpha, cp, ts)
% Description
% Compute the value of exp(-alpha * (t1-t2)) * Cp(t2)
% Inputs:
%   t1: index of time of sampling.
%   t2: index of "running "or integration time variable .
%   alpha: one of the computed values in equation (3) of Brooks paper.
%   Cp: FDG concentration in arterial system.
%   ts: time in minutes when Cp data is sampled.
% Output:
%   exp(-alpha * (ts(t1)-ts(t2))) * Cp(t2)
diff_t = ts(t1) - ts(t2);
exp_f = exp(-alpha * diff_t);
f_value = exp_f * cp(t2);
end
```

import_data

```
function project1extendeddata = import_data(workbookFile, sheetName,
 dataLines)
```

```
% import_data Import data from a spreadsheet
%  data = import_data(FILE) reads data from the first
%  worksheet in the Microsoft Excel spreadsheet file named
 workbookFile.
%  Returns the data as a table.
%
%  data = import_data(FILE, SHEET) reads from the
%  specified worksheet.
%
%  data = import_data(FILE, SHEET, DATALINES) reads from
%  the specified worksheet for the specified row interval(s). Specify
%  DATALINES as a positive scalar integer or a N-by-2 array of
 positive
%  scalar integers for dis-contiguous row intervals.
%
%  Example:
%  data = importfile("./project_1_extended_data.xlsx", "Sheet1", [1,
 71]);
%

% Input handling

% If no sheet is specified, read first sheet
if nargin == 1 || isempty(sheetName)
    sheetName = 1;
end

% If row start and end points are not specified, define defaults
if nargin <= 2
    dataLines = [1, 71];
end

% Set up the Import Options and import the data
opts = spreadsheetImportOptions("NumVariables", 3);

% Specify sheet and range
opts.Sheet = sheetName;
opts.DataRange = "A" + dataLines(1, 1) + ":C" + dataLines(1, 2);

% Specify column names and types
opts.VariableNames = ["VarName1", "Time", "Concentration"];
opts.VariableTypes = ["double", "double", "double"];

% Import the data
project1extendeddata = readtable(workbookFile, opts, "UseExcel",
 false);

for idx = 2:size(dataLines, 1)
    opts.DataRange = "A" + dataLines(idx, 1) + ":C" + dataLines(idx,
 2);
    tb = readtable(workbookFile, opts, "UseExcel", false);
    project1extendeddata = [project1extendeddata; tb]; %#ok<AGROW>
end
```

```
end
```

Published with MATLAB® R2021a