4.26

$$E_m = \int_{-\pi}^{\pi} [f(x) - \sum_{n=1}^{m} b_n \sin nx]^2 dx$$

is the residual difference (or error) of the function over its primary interval

-π to π

We minimize this quantity with respet to a particular b_n in this case b_p

we take a derivative with respect to b_p (a particular p in sum of n = 1 to m <<< IMPORTANT)

The partial derivative is moved inside integral (assume integral converges) and applied!

FIXED >>>>> $\dfrac{\partial E_m}{\partial b_p} = \int_{-\pi}^{\pi} 2[f(x) - \sum_{n=1}^{m} b_n \sin nx][-\sin px]dx = 0$

$$-2\int_{-\pi}^{\pi} f(x)\sin px\,dx + 2\int_{-\pi}^{\pi} [\sum_{n=1}^{m} b_n \sin nx]\sin px\,dx = 0$$

The second integral has only a contribution when n = p!!!!

The first integral $-2\int_{-\pi}^{\pi} f(x)\sin px\,dx$ and the second integral reduces to $2b_p\int_{-\pi}^{\pi} \sin^2 px\,dx = 2b_n\pi$

$$-2\int_{-\pi}^{\pi} f(x)\sin pxdx + 2b_p\pi = 0 \rightarrow\ b_p = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin px\,dx$$

For the second part we have

A quick approximately way to calculate the error is as follows

And use the following form for error, b_r above and m = 3 and do integral

$$b_r = \frac{4}{\pi r^3}[(-1)^r - 1] = \begin{cases} 0 & \text{r even} \\ \dfrac{8}{\pi r^3} & \text{r odd} \end{cases}$$

$$E_m = \int_{-\pi}^{\pi} [\sum_{r=1}^{\infty} b_r \sin rx - \sum_{r=1}^{m} b_r \sin rx]^2 dx = \int_{-\pi}^{\pi} [\sum_{r=m+1}^{\infty} b_r \sin rx]^2 dx$$

Note since no contribution from even values of r lower limit starts not at 3+1 but r = 5!!

$$E_3 = \int_{-\pi}^{\pi} [\sum_{r=3+1}^{\infty} b_r \sin rx]^2 dx = \sum_{r=3+1}^{\infty} b_r^2 \int_{-\pi}^{\pi} \sin^2 rxdx \equiv \sum_{r=5,\text{ odd only}}^{\infty} b_r^2 \int_{-\pi}^{\pi} \sin^2 rxdx$$

Using $\int_{-\pi}^{\pi} \sin^2 rxdx = \pi$, for any r and substitute for b_r

$$E_3 = \sum_{r=5,\text{ odd only}}^{\infty} \left(\frac{8}{\pi r^3}\right)^2 \pi = \frac{8^2}{\pi} \sum_{r=5,\text{ odd only}}^{\infty} \frac{1}{r^6} \approx \frac{8^2}{\pi} \sum_{r=5,\text{ odd only}}^{9} \frac{1}{r^6} \rightarrow E_3 = \frac{64}{\pi}\sum_{r=5}^{\infty}\frac{1}{r^6} \cong .0013$$

which is a very good approximation since sum over $\frac{1}{r^6}$ converges very quickly