

5.3

$$\begin{aligned}\tilde{f}(\omega) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} H(x-a) e^{-bx} e^{-i\omega x} dx = \frac{1}{\sqrt{2\pi}} \int_a^{\infty} e^{-bx} e^{-i\omega x} dx = \\ &= \frac{1}{\sqrt{2\pi}} \int_a^{\infty} e^{-(b+i\omega)x} dx = \frac{1}{\sqrt{2\pi}} \frac{e^{-(b+i\omega)x}}{-(b+i\omega)} \Big|_a^{\infty} = \frac{1}{\sqrt{2\pi}} \frac{1}{-(b+i\omega)} \left[ e^{-(b+i\omega)\infty} - e^{-(b+i\omega)a} \right] = \\ &= \frac{1}{\sqrt{2\pi}} \frac{1}{-(b+i\omega)} \left[ 0 - e^{-(b+i\omega)a} \right] = \frac{1}{\sqrt{2\pi}} \frac{e^{-(b+i\omega)a}}{(b+i\omega)}\end{aligned}$$