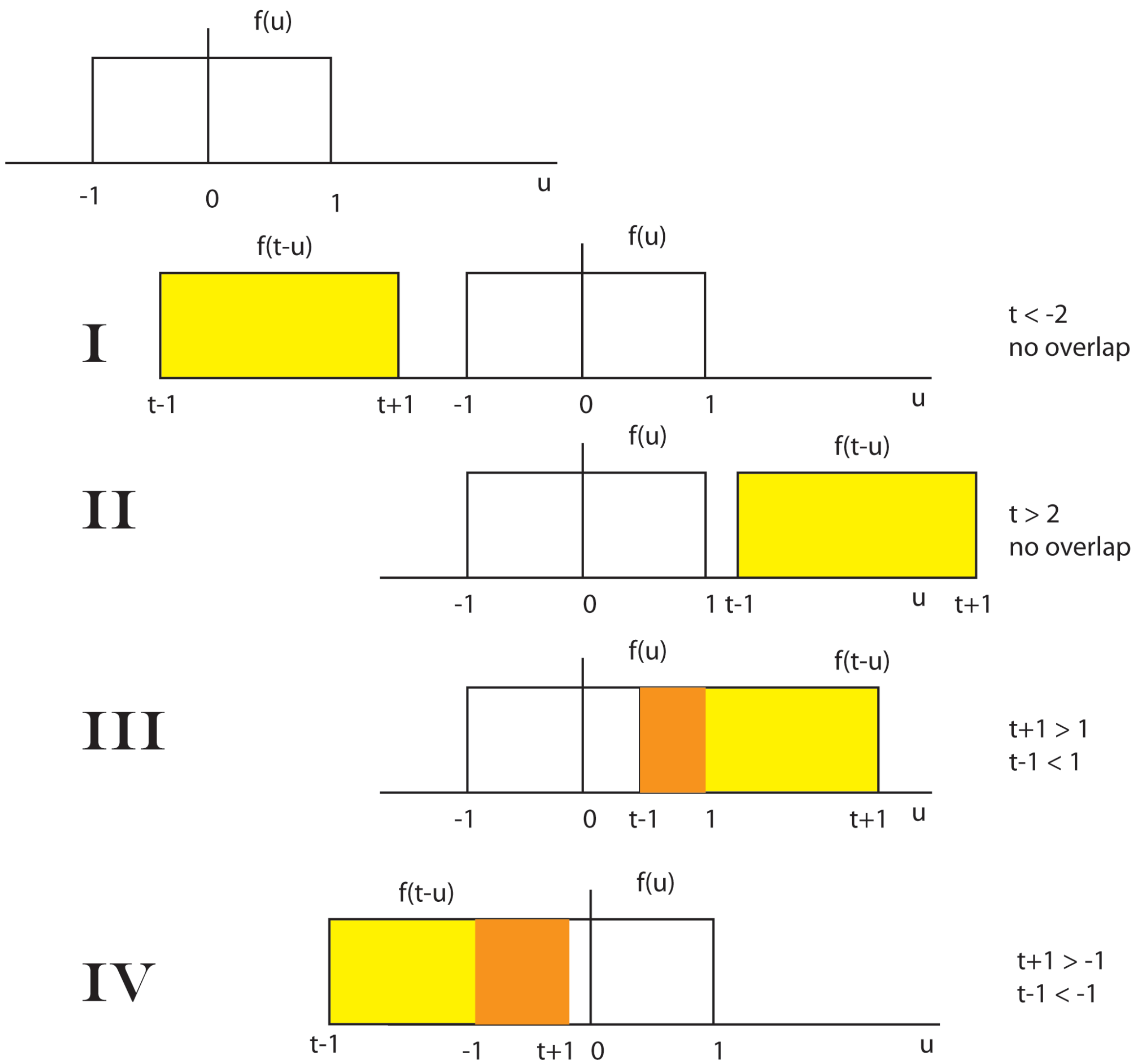
5.7 See calculation below to see how diagram applies



We have fot the Fourier transform of the function

$$f(t) = \begin{cases} 1 & -1<t<1 \\ 0 & \text{otherwise} \end{cases} \quad \rightarrow \tilde{f}(w) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{-iwt}\,dt = \frac{1}{\sqrt{2\pi}}\int_{-1}^{1} 1e^{-iwt}\,dt = \frac{1}{\sqrt{2\pi}}\frac{2\sin w}{w}$$

Next calculate the convolution in t space

$$f(t) = \begin{cases} 1 & -1<t<1 \\ 0 & \text{otherwise} \end{cases}$$

Convolution $f * f = \int_{-\infty}^{\infty} f(u)f(t-u)\,du$

For (diagram I) $t \le -2$ If f(u) has a value then f(t-u) is 0 or the opposite therefore $\int_{-\infty}^{\infty} 0\,du = 0$

For (diagram II) $t \ge 2$ If f(u) has a value then f(t-u) is 0 or the opposite therefore $\int_{-\infty}^{\infty} 0\,du = 0$

For (diagram III) In this case f(u) and f(t-u) overlap
when $t+1>1 \rightarrow t>0$ and $t-1<1 \rightarrow t<2$ and together $0<t<2$
and $\int_{-\infty}^{\infty} f(u)f(t-u)\,du$ = Area in yellow that varies with t, that is width·height=[1-(t-1)]·1 =2-t

For (diagram IV) In this case f(u) and f(t-u) overlap
when $t+1>-1 \rightarrow t>-2$ and $t-1<-1 \rightarrow t<0$ and together $-2<t<0$
and $\int_{-\infty}^{\infty} f(u)f(t-u)\,du$ = Area in yellow that varies with t, that is width·height=[(t+1)-(-1)]·1 =2+t

Finally putting this all together

$$f * f = \begin{cases} 0 & t \le -2 \\ 2+t & -2<t<0 \\ 2-t & 0<t<2 \\ 0 & t \ge 2 \end{cases}$$

Finally apply Parseval's Theorem

Parseval's Th $\int_{-\infty}^{\infty}\left|\tilde{f}(w)\right|^2 dw = \int_{-\infty}^{\infty}\left|f(t)\right|^2 dt$

Using our specific functions can be written

$$\int_{-\infty}^{\infty}\left|\tilde{f}(w)\right|^2 dw = \int_{-\infty}^{\infty}\left|\frac{1}{\sqrt{2\pi}}\frac{2\sin w}{w}\right|^2 dw = \int_{-\infty}^{\infty}\left|f(t)\right|^2 dt = \int_{-1}^{1} 1^2 dt = 2$$

Therefore

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4\sin^2 w}{w^2}dw = 2 \text{ and therefore} \int_{-\infty}^{\infty}\frac{\sin^2 w}{w^2}dw = \pi$$

Similar for other result but use convolutions in place of functions f(t) and $\tilde{f}(w)$

Note convolution is $F\{f*f\} = \sqrt{2\pi}\tilde{f}(w)\tilde{f}(w) = \sqrt{2\pi}\left(\frac{2\sin w}{\sqrt{2\pi}w}\right)^2 = \frac{4\sin^2 w}{\sqrt{2\pi}w^2}$

$$\int_{-\infty}^{\infty}\left|f*f(t)\right|^2 dt = \int_{-\infty}^{\infty}\left|F\{f*f\}\right|^2 dw$$

Now $\int_{-\infty}^{\infty}\left|F\{f*f\}\right|^2 dw = \int_{-\infty}^{\infty}\left|\frac{4\sin^2 w}{\sqrt{2\pi}w^2}\right|^2 dw = \int_{-\infty}^{\infty}\frac{16\sin^4 w}{2\pi w^4}dw = \frac{8}{\pi}\int_{-\infty}^{\infty}\frac{\sin^4 w}{w^4}dw$

and $\int_{-\infty}^{\infty}\left|f*f(t)\right|^2 dt = \int_{-2}^{2}\left|f*f(t)\right|^2 dt = \int_{-2}^{0}(2+t)^2 dt + \int_{0}^{2}(2-t)^2 dt = \cdots = \frac{16}{3}$

Therefore

$$\frac{16}{3} = \frac{8}{\pi}\int_{-\infty}^{\infty}\frac{\sin^4 w}{w^4}dw \text{ or } \int_{-\infty}^{\infty}\frac{\sin^4 w}{w^4}dw = \frac{16}{3}\frac{\pi}{8} = \frac{2}{3}\pi$$