(a)

Let $x(t)=A(1-e^{-\frac{t}{T}})H(t)\equiv A(1-e^{-\frac{t}{T}})H(t-0)$

$x(0)=A(1-e^{-\frac{0}{T}})H(0-0)=A(1-1)0=0$

Substitution gives $\frac{dx}{dt}=\frac{d}{dt}A(1-e^{-\frac{t}{T}})H(t)=$

$$=A(-\left(-\frac{1}{T}\right)e^{-\frac{t}{T}})H(t)+A(1-e^{-\frac{t}{T}})\delta(t-0)=\frac{A}{T}e^{-\frac{t}{T}}H(t)+A(1-e^{-\frac{0}{T}})=\frac{A}{T}e^{-\frac{t}{T}}H(t)+A(1-1)=\frac{A}{T}e^{-\frac{t}{T}}H(t)$$

Then substitution of x and $\frac{dx}{dt}$ into the original equation gives

$$T\frac{dx}{dt}+x=T\left[\frac{A}{T}e^{-\frac{t}{T}}H(t)\right]+A(1-e^{-\frac{t}{T}})H(t)=Ae^{-\frac{t}{T}}H(t)+A(1-e^{-\frac{t}{T}})H(t)=AH(t)$$

(b) Note the solution for $T\frac{dx}{dt}+x=AH(t-\tau)$ is $x_\tau(t)=A(1-e^{-\frac{t}{T}})H(t-\tau)$ following the

prescription in (a). Also our original solution form (a) we can write

$x_0(t)=A(1-e^{-\frac{t}{T}})H(t-0)$

Therefore define a block function from t = 0 to t = τ of a superposition of these two solutions. Note below (IMPORTANT)

step up at t = 0 with H(t-0) and step down at t = τ with $-H(t-\tau)$

therefore our solution is $A((1-e^{-t/T})H(t-0)-A((1-e^{-(t-\tau)/T})H(t-\tau)$

Aside: Note as motivation for (c)

$$\delta(t)=\lim_{\tau\to 0}=\frac{1}{\tau}[H(t-0)-H(t-\tau)]$$

To evaluate (c) the impulse

$$x(t)_{\text{IMPULSE}}=\lim_{\tau\to 0}\frac{1}{\tau}[(1-e^{-t/T})H(t-0)-(1-e^{-(t-\tau)/T})H(t-\tau)]$$

$$=\lim_{\tau\to 0}\frac{1}{\tau}[(1-e^{-t/T})H(t-0)-(1-e^{-t/T}e^{\tau/T})H(t-\tau)]$$

I had to use **Taylor series expansions for** $e^{\tau/T}$ for this exponential to handle tau going to 0 in the denominator. See if you can finish.

(d) is much easier, just take Laplace transform of the give DE with delta and solve in s space and inverse transform back to t!!!