Table of Contents


```
% Description
% Compute the total concentration
% of FDG in the brain over time, Ci(t), by performing a
% numerical integration.
%   Ci(t) = Ce(t) + Cp(t)
% where
%   - Ce(t): FDG in the brain tissue
%   - Cp(t): FDG concentration in the arterial system
% However in the first part of the project, we showed that the
% Ci(t) can be expressed as a sum of two convolutions:
%   => A * conv(exp(-alpha_1* t), Cp(t))
%   => B * conv(exp(-alpha_2* t), Cp(t))
% The full expression of Ci(t) and values for
% alpha_1, alpha_1, Ci(t), A, and B: are given in Brooks paper in:
% equation (3), (4), (5) and (6).
% We then perform a numerical integration to compute the two sums
 above
% to finally compute Ci.
% After computings the values of Ci(t), we plot Cp(t) and Ci(t) versus
 time.

% Clean environment
clear all;
close all;
clc;
format long g
```

Read the data and parameters

```
[ts, cp, alpha_1, alpha_2, A, B] = get_parameters();
```

Compute first convolution between exp(-alpha_1 t) and Cp(t)

```
conv_alpha_1 = convolution_by_integration(cp, ts, alpha_1);
```

Compute second convolution between exp(-alpha_2 t) and Cp(t)

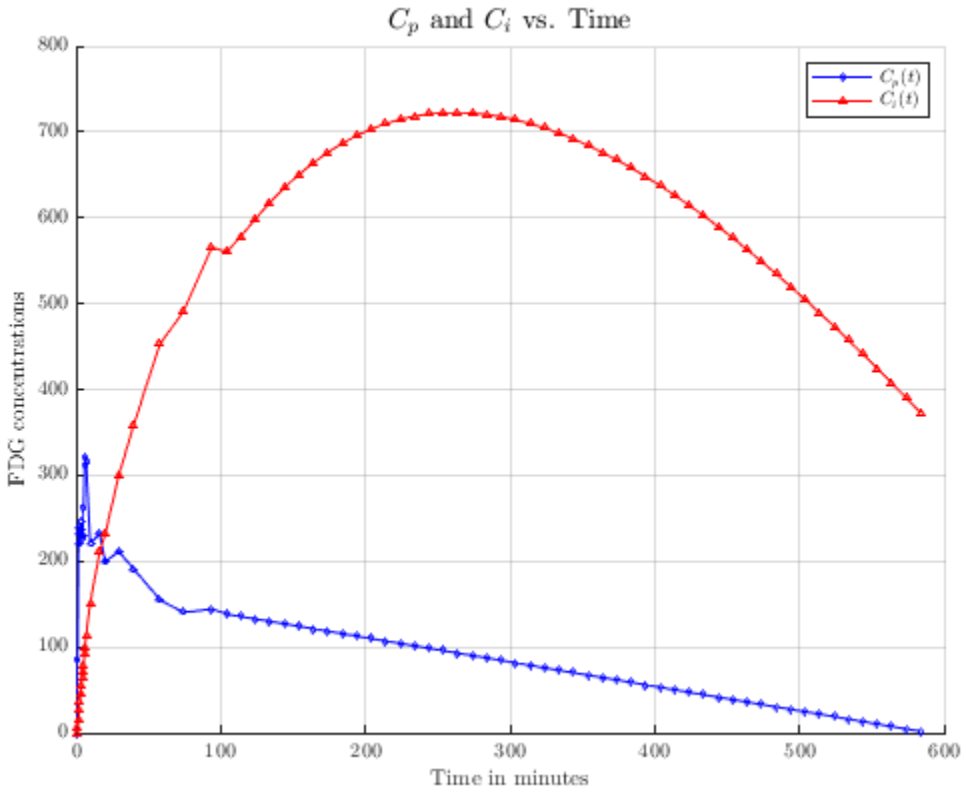
```
conv_alpha_2 = convolution_by_integration(cp, ts, alpha_2);
ci = A .* conv_alpha_1 + B .* conv_alpha_2;
```

Plot Cp and Ct versus time.

```
figure, hold on, grid on

plot(ts, cp, "b-o", 'LineWidth',1,'MarkerSize',3);
plot(ts, ci, "r-^", 'LineWidth',1,'MarkerSize',3);

title('$C_p$ and $C_i$ vs. Time','FontSize', 14);
xlabel('Time in minutes');
ylabel('FDG concentrations');
legend('$C_p(t)$','$C_i(t)$');
saveas(gcf,"cp_ci_vs_time",'pdf')
```



get_parameters function

```
function [ts, cp, alpha_1, alpha_2, A, B] = get_parameters(workbook,
 worksheet)
% Description
```

```
% Set the parameters needed to determine Ci(t).
% The parameters are given in PET Scan Brooks paper.
% Inputs:
% workbook: Excel workbook with the values points for Cp(t)
% extended pass 94 minutes.
% worksheet: Excel worksheet where the data is
% Outputs:
% ts: time in minutes when Cp data is sampled.
% Cp: FDG concentration in arterial system.
% alpha_1: computed value in equation (3) of Brooks paper.
% alpha_2: computed value in equation (3) of Brooks paper.
% A: computed value in equation (3) of Brooks paper.
% B: computed value in equation (3) of Brooks paper.

if ~exist('workbook','var')
    workbook = "./project_1_extended_data.xlsx";
    worksheet = "Sheet1";
end
data = import_data(workbook, worksheet);

% Cp(t) is given from an experiment running from 0 to 94 minutes
% It is then extended pass 94 minutes with a least-square regression
% using the the samples starting with 58 minutes upt to 584 minutes
% when the concentration Cp(t) is alomost zero.

cp = data.Concentration(2:end);
ts = data.Time(2:end);
% k1, k2 ,k3 , k4 rate constants
k1 = 0.102;
k2 = 0.130;
k3 = 0.062;
k4 = .0068;
d = sqrt((k2 + k3 + k4)^2 -4 * k2 *k4);
% Alpha_1, Alpha_2, A, B
alpha_1 = 0.5 * (k2 + k3 + k4  - d);
alpha_2 = 0.5 * (k2 + k3 + k4  + d);
A = (k1 * (k3 + k4 - alpha_1)) / (alpha_2 - alpha_1);
B = (k1 * (alpha_2 -k3 -k4)) / (alpha_2 - alpha_1);

end
```

convolution_by_integration function

```
function conv_res = convolution_by_integration(cp, ts, alpha)
% Description
% Performs a convolution between the exponential function
% parametrized by alpha with concentration Cp over time.
% Inputs:
% Cp: FDG concentration in arterial system.
% ts: time in minutes when Cp data is sampled.
% alpha: one of the computed values in equation (3) of Brooks paper.
% Ouput:
% conv_res: convolution between exp(-alpha t) and Cp(t).
```

```
% To compute the integral, we use the trapezoidal method as described
 here:
% https://www.mathworks.com/help/matlab/ref/trapz.html
% We use two loops:
%  - first loop is indexed by i, and use the time ts(i)
% for which we want to determine the value of the convolution
% conv_res(ts(i))
%  - second loop concerns the "running" or integration variable,
% indexed by ts(j).
% For each ts(i)
%       total_area = 0
%       For each ts(j)
%           area = compute integral  between ts(j), ts(j-1)
%           total_area = total_area + area
%       End
% End

n = size(ts,1);
conv_res = zeros(n,1);
for i=1: n
    total_area = 0;
    for j=2: i
        dt_j = ts(j) - ts(j-1);
        f_value_j_1 = alpha_function(i, j-1, alpha, cp, ts);
        f_value_j = alpha_function(i, j, alpha, cp, ts);
        area = dt_j * ((f_value_j_1 + f_value_j)/2);
        total_area = total_area + area;
    end
    conv_res(i) = total_area;
end
end
```

alpha_function function

```
function f_value = alpha_function(t1, t2, alpha, cp, ts)
% Description
% Compute the value of exp(-alpha * (t2-t1)) * Cp(t2)
% Inputs:
%   t1: index of time of sampling.
%   t2: index of "running "or integration time variable .
%   alpha: one of the computed values in equation (3) of Brooks paper.
%   Cp: FDG concentration in arterial system.
%   ts: time in minutes when Cp data is sampled.
% Ouput:
%   exp(-alpha * (ts(t1)-ts(t2))) * Cp(t2)
    diff_t = ts(t1) - ts(t2);
    exp_f = exp(-alpha * diff_t);
    f_value = exp_f * cp(t2);
end
```

import_data function

```
function project1extendeddata = import_data(workbookFile, sheetName,
 dataLines)
% import_data Import data from a spreadsheet
%  data = import_data(FILE) reads data from the first
%  worksheet in the Microsoft Excel spreadsheet file named
 workbookFile.
%  Returns the data as a table.
%
%  data = import_data(FILE, SHEET) reads from the
%  specified worksheet.
%
%  data = import_data(FILE, SHEET, DATALINES) reads from
%  the specified worksheet for the specified row interval(s). Specify
%  DATALINES as a positive scalar integer or a N-by-2 array of
 positive
%  scalar integers for dis-contiguous row intervals.
%
%  Example:
%  data = importfile("./project_1_extended_data.xlsx", "Sheet1", [1,
 71]);
%

% Input handling

% If no sheet is specified, read first sheet
if nargin == 1 || isempty(sheetName)
    sheetName = 1;
end

% If row start and end points are not specified, define defaults
if nargin <= 2
    dataLines = [1, 71];
end

% Set up the Import Options and import the data
opts = spreadsheetImportOptions("NumVariables", 3);

% Specify sheet and range
opts.Sheet = sheetName;
opts.DataRange = "A" + dataLines(1, 1) + ":C" + dataLines(1, 2);

% Specify column names and types
opts.VariableNames = ["VarName1", "Time", "Concentration"];
opts.VariableTypes = ["double", "double", "double"];

% Import the data
project1extendeddata = readtable(workbookFile, opts, "UseExcel",
 false);

for idx = 2:size(dataLines, 1)
```

```
    opts.DataRange = "A" + dataLines(idx, 1) + ":C" + dataLines(idx,
 2);
    tb = readtable(workbookFile, opts, "UseExcel", false);
    project1extendeddata = [project1extendeddata; tb]; %#ok<AGROW>
end

end
```

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