14.5

(a)

$(z-2)^{-1} = \frac{1}{(z-2)^1}$ therefore z = 2 is isolated singularity, pole of order n = 1

at z = 0 $\frac{1}{(0-2)^1} = -\frac{1}{2}$ finite, not singular

at z = $\pm\infty$ $\frac{1}{(\pm\infty-2)^1} = -\frac{1}{\pm\infty} = 0$ finite, not singular

(c)

Use 14.24 $\lim_{z\to 0}[(z-0)^n \sinh\left(\frac{1}{z}\right)] = \lim_{z\to 0} z^n \sinh\left(\frac{1}{z}\right)$

Taylor expansion of $\sinh\left(\frac{1}{z}\right) = \sum_{m=0}^{\infty} \frac{\left(\frac{1}{z}\right)^{2m+1}}{(2m+1)!}$

Therefore $\lim_{z\to 0} z^n \sinh\left(\frac{1}{z}\right) = \lim_{z\to 0} z^n \sum_{m=0}^{\infty} \frac{\left(\frac{1}{z}\right)^{2m+1}}{(2m+1)!} = \lim_{z\to 0} \sum_{m=0}^{\infty} \frac{1}{(2m+1)!} \frac{z^n}{z^{2m+1}}$

For some large m 2m+1 > n and denominator would be 0 so 1/0 is undefined

and we have an esssential singularity for z = 0

For z = $\pm\infty$ we just have to look at $\lim_{z\to\infty} \sinh\left(\frac{1}{z}\right) = \sinh(0) = 0$ and not singular for z = $\pm\infty$

(d)

$$\frac{e^z}{z^3}$$

For z = 0 $e^z \to 1$

Therefore take at look at $\lim\limits_{z\to 0}\frac{e^z}{z^3} = \lim\limits_{z\to 0}\frac{1}{z^3}$

Therefore singularity, pole of order 3

For $\lim\limits_{z\to\infty}\frac{e^z}{z^3}$ {take Taylor expansion for e^z} $= \lim\limits_{z\to\infty}\frac{\sum_{n=0}^{\infty}\frac{z^n}{n!}}{z^3} = \lim\limits_{z\to\infty}\sum_{n=0}^{\infty}\frac{z^{n-3}}{n!}$

Therefore limit undefined and essential singularity at z = ∞