10.4

(a)

$$y\frac{\partial u}{\partial x}-x\frac{\partial u}{\partial y}=0; \quad u(x,0)=1+\sin x$$

Using the more general technique from page 394-5

We have $A=y,\ B=-x$

Therefore

$$\frac{dx}{y}=\frac{dy}{-x}$$

Solving

$-xdx=ydy$ gives $-x^2=y^2+C$

or just set $C=y^2+x^2$ that is p(x,y) = C, that is

$p(x,y)=y^2+x^2$

Then using $u(x,0)=1+\sin x$

Therefore look at value of p on line y = 0, that is $p(x,0)=x^2$

Therefore $p=x^2$ or $x=p^{1/2}$ and we have $u(x,0)=1+\sin x\equiv 1+\sin(p^{1/2})$

which looks correct!

Therefore in general to match this answer for y = 0

and have general u(x,y) we take p above. That is $p(x,y)=y^2+x^2$

so the answer is

$u(x,y)=1+\sin(y^2+x^2)^{1/2}$

Of course this also satisfies the condition given for $y=0$

$u(x,0)=1+\sin(x^2)^{1/2}=1+\sin x$