11.1a

$$\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} = 0$$

and for this p we have

Let u(x,y) = X(x)Y(y) Substitution into DE gives

$$Y\frac{\partial X}{\partial x} - xX\frac{\partial Y}{\partial y} = 0$$

$$\frac{1}{xX}\frac{\partial X}{\partial x} = \frac{1}{Y}\frac{\partial Y}{\partial y} = k$$

Then (and also changing partial to total derivatives) gives

$$\frac{1}{xX}\frac{dX}{dx} = k \text{ and } \frac{1}{Y}\frac{dY}{dy} = k$$

or

$$\frac{dX}{X} = kxdx \text{ and } \frac{dY}{Y} = kdy$$

Integrating (note: take constants as lnC, lnD as conviences)

$$\ln X = \frac{1}{2}kx^2 + \ln C \text{ and } \ln Y = ky + \ln D$$

or

$$X = Ce^{\frac{1}{2}kx^2} \text{ and } Y = De^{ky}$$

Therefore (with XD = A) gives u(x,y) = $Ae^{k(\frac{1}{2}x^2+y)}$

11.1(b)

$$x\frac{\partial u}{\partial x} - 2y\frac{\partial u}{\partial y} = 0$$

Similar to part (a) except we get the following seperated equations

$\frac{dX}{X} = \frac{k}{x}dx$ and $\frac{dY}{Y} = \frac{k}{2y}dy$

Solving

$\ln X = k\ln x + \ln C$ and $\ln Y = \frac{k}{2}\ln y + \ln D$

or

$X = Ce^{k\ln x} = C(e^{\ln x})^k = Cx^k$ and $Y = De^{\frac{k}{2}\ln y} = Dy^{\frac{k}{2}}$

$u(x,y) = Ax^k y^{\frac{k}{2}}$