4.26 start

$$E_m = \int_{-\pi}^{\pi} [f(x) - \sum_{n=1}^{m} b_n \sin nx]^2 dx$$

is the residual difference (or error) of the function over its primary interval

-π to π

We minimize this quantity with respet to a particular b_n in this case b_p

we take a derivative with respect to b_p (a particular p in sum of n = 1 to m <<< IMPORTANT)

The partial derivative is moved inside integral (assume integral converges) and applied!

$$\frac{\partial E_m}{\partial b_p} = \int_{-\pi}^{\pi} 2[f(x) - \sum_{n=1}^{m} b_n \sin nx][-\sin px]dx = 0$$

So I just took the derivative of the sum above with respect to b_p

Note in one of the expanded terms in the sum n = p, thats why I only get $-\sin px$

with application of chain rule.

But it is actually something you have already done in your first calculus class!!!

Let's do a simple example for

$$\sum_{n=1}^{2} (2x)^n \equiv 2x + (2x)^2 = 2x + (2x)^2 \text{ and the derivative}$$

$$\frac{d}{dx} 2x + (2x)^2 = 2 + 2(2x)2 = 2 + 8x$$

Look closely I have used the chain rule for derivative and the distribution of derivative over sum!!!!

Now take derivative of sum

$$\frac{d}{dx} \sum_{n=1}^{2} (2x)^n = \sum_{n=1}^{2} \frac{d}{dx} (2x)^n = \sum_{n=1}^{2} n(2x)^{n-1}(2) = \sum_{n=1}^{2} 2n(2x)^{n-1}$$

I did the same thing again, used the chain rule and distribution of derivative over sum!!!!

Of course the last sum after I have taken the derivative can be expanded

$$\sum_{n=1}^{2} 2n(2x)^{n-1} \equiv 2 \cdot 1(2x)^{(1-1)} + 2 \cdot 2(2x)^{(2-1)} = 2(2x)^0 + 2 \cdot 2(2x)^{(1)} = 2(1) + 4(2x) = 2 + 8x$$