(c)

There is no value of z such that $e^{\frac{1}{z}} = 0$ therefore there are no zeros. The function obviously has a singularity at z = 0

Now look at Taylor expansion of $e^{\xi} = \sum_{n=0}^{\infty} \frac{1}{n!} \xi^n$

Therefore
$$e^{\frac{1}{z}} = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{1}{z}\right)^n$$

Look at
$$\lim_{z\to 0} (z-0)^m \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{1}{z}\right)^n$$

There is no largest m such that this quantity is finite therefore this is an essential singularity