4.4

$$f(t)=\begin{cases} -1 & -T/2 \le t < 0 \\ 1 & 0 < t < T/2 \end{cases}$$

odd $a_r = 0$

From book page 174. $b_r = \dfrac{2}{\pi r}[1-(-1)^r] = \dfrac{4}{\pi r}, r-\text{odd}$

Therefore $f(t) = \sum_{r,odd}^{\infty} b_r \sin(2\pi rt/T) = \dfrac{4}{\pi}\sum_{r,odd}^{\infty}\dfrac{1}{r}\sin(2\pi rt/T)$

Now let $\tilde{t} = t - T/4 \rightarrow t = \tilde{t} + T/4$

FIXED >>> $f(\tilde{t})=\begin{cases} -1 & -T/2 \le \tilde{t} < -T/4 \\ 1 & -T/4 < \tilde{t} < T/4 \\ -1 & T/4 < \tilde{t} \le T/2 \end{cases}$

and even function, therefore symmetric from 0 to $-T/2$ period same as above

Replace t with $\tilde{t} + T/4$ from above and use trigonometry identity

$$f(\tilde{t}) = \frac{4}{\pi}\sum_{r,odd}^{\infty}\frac{1}{r}\sin(2\pi r(\tilde{t}+T/4)/T) = \frac{4}{\pi}\sum_{r,odd}^{\infty}\frac{1}{r}\sin\left(\frac{2\pi r\tilde{t}}{T}+\frac{\pi r}{2}\right) =$$

$$\frac{4}{\pi}\sum_{r,odd}^{\infty}\frac{1}{r}\left[\sin\left(\frac{2\pi r\tilde{t}}{T}\right)\cos\left(\frac{\pi r}{2}\right)+\cos\left(\frac{2\pi r\tilde{t}}{T}\right)\sin\left(\frac{\pi r}{2}\right)\right] =$$

$$\frac{4}{\pi}\sum_{r,odd}^{\infty}\frac{1}{r}\left[\sin\left(\frac{2\pi r\tilde{t}}{T}\right)0+\cos\left(\frac{2\pi r\tilde{t}}{T}\right)\sin\left(\frac{\pi r}{2}\right)\right] = \frac{4}{\pi}\sum_{r,odd}^{\infty}\frac{1}{r}\cos\left(\frac{2\pi r\tilde{t}}{T}\right)\sin\left(\frac{\pi r}{2}\right)$$

Also do it directly by calculating a_0 and a_r, Period is T,

using $f(\tilde{t})$. Note to finish this calculation I will replace $\tilde{t}$ with t, that is

$$f(t)=\begin{cases} -1 & -T/2\le t<-T/4 \\ 1 & -T/4<t<T/4 \\ -1 & T/4<t\le T/2 \end{cases}$$

nothing changes - this is still an even function (just makes typing easier)!!!

Note by inspection $a_0 =$ 0 (or you can do the integral)

FIXED -minor sign error for f(t) >>>> $a_0=\frac{2\cdot 2}{T}\int_0^{T/2} f(t)dt=\frac{4}{T}\left[\int_0^{T/4} 1dt+\int_{T/4}^{T/2}(-1)dt\right]=\cdots=0$

and $a_r=\frac{4}{T}\int_0^{T/2} f(t)\cos\left(\frac{2\pi rt}{T}\right)dt=\frac{4}{T}\left[\int_0^{T/4}(1)\cos\left(\frac{2\pi rt}{T}\right)dt+\int_{T/4}^{T/2}(-1)\cos\left(\frac{2\pi rt}{T}\right)dt\right]=$

$$\frac{4}{T}\left[\frac{1}{\frac{2\pi r}{T}}\sin\left(\frac{2\pi rt}{T}\right)\Bigg|_0^{T/4}-\frac{1}{\frac{2\pi r}{T}}\sin\left(\frac{2\pi rt}{T}\right)\Bigg|_{T/4}^{T/2}\right]=$$

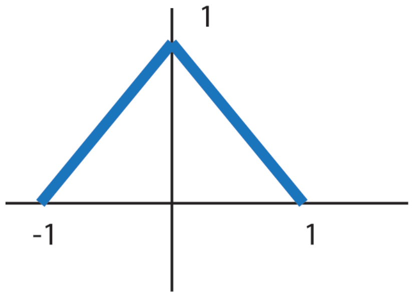
$$=\frac{2}{\pi r}\sin\left(\frac{2\pi rt}{T}\right)_0^{T/4}-\frac{2}{\pi r}\sin\left(\frac{2\pi rt}{T}\right)_{T/4}^{T/2}=\frac{2}{\pi r}\left[\sin\left(\frac{\pi r}{2}\right)-\sin 0\right]-\frac{2}{\pi r}\left[\sin\pi r-\sin\left(\frac{\pi r}{2}\right)\right]=$$

$$\frac{2}{\pi r}\left[\sin\left(\frac{\pi r}{2}\right)-s0\right]-\frac{2}{\pi r}\left[0-\sin\left(\frac{\pi r}{2}\right)\right]=\frac{4}{\pi r}\sin\left(\frac{\pi r}{2}\right)$$

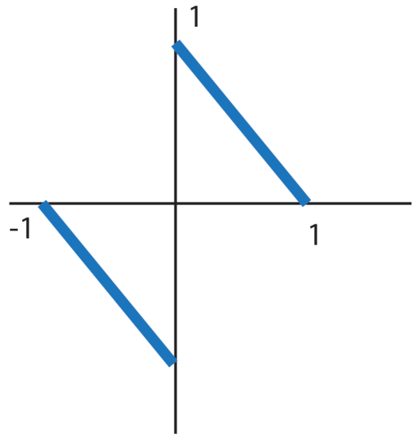
Therefore f(t) = $\frac{4}{\pi}\sum_{r,odd}^{\infty}\frac{1}{r}\sin\left(\frac{\pi r}{2}\right)\cos\left(\frac{2\pi rt}{T}\right)$

4.6

EVEN



ODD



Even extension $a_r = \frac{2 \cdot 2}{2}\int_0^1 f(x)\cos(\frac{2\pi r x}{2})dx$ with period = 2, $b_r = 0$

For $a_0 = 1$ by inspection since $\frac{a_0}{2} = \langle f(x) \rangle = \frac{1}{2}$ (or do the integral)

$$a_r = 2\int_0^1 (1-x)\cos \pi r x\, dx = \cdots = \frac{-2}{\pi^2 r^2}[\cos(\pi r) - 1] =$$

$$\frac{-2}{\pi^2 r^2}[(-1)^r - 1] = \begin{cases} 0 & \text{even} \\ \dfrac{4}{\pi^2 r^2} & \text{odd} \end{cases}$$

$$f(x) = \frac{1}{2} + \frac{4}{\pi^2}\sum_{r,\text{odd}}^{\infty} \frac{1}{r^2}\cos \pi r x$$

Odd extension, $a_r = 0$, $b_r = 2\int_0^1 f(x)\sin \pi r x\, dx = 2\int_0^1 (1-x)\sin \pi r x\, dx = \cdots = \frac{2}{\pi r}$

$$f(x) = \frac{2}{\pi}\sum_{r=1}^{\infty} \frac{1}{r}\sin \pi r x$$

Therefore terms with $\frac{1}{r^2}$(and just odd indicies) in even extension converge faster as a function of r

Also note discontinuity for odd extension at x = 0!!, Use equation on page 176
to see that the value of the Fourier series is 0 at this point for the odd extension.

4.6 (by parts for even extension)

$$a_r = 2\int_0^1 (1-x)\cos\pi rx\,dx = 2\int_0^1 \cos\pi rx\,dx - 2\int_0^1 x\cos\pi rx\,dx =$$

$$2\frac{\sin\pi rx}{\pi r}\bigg|_0^1 - 2\int_0^1 x\cos\pi rx\,dx =$$

First term is 0, that is $2\frac{\sin\pi rx}{\pi r}\bigg|_0^1 = 0$

For integral let $u = x \to du = dx,\ dv = \cos\pi rx dx \to v = \frac{1}{\pi r}\sin\pi rx$

$$\int_0^1 x\cos\pi rx\,dx \to x\frac{1}{\pi r}\sin\pi rx\bigg|_0^1 - \frac{1}{\pi r}\int_0^1 \sin\pi rx\,dx = x\frac{1}{\pi r}\sin\pi rx\bigg|_0^1 - \frac{1}{\pi r}\left(\frac{-\cos\pi rx}{\pi r}\right)\bigg|_0^1 =$$

$$x\frac{1}{\pi r}\sin\pi rx\bigg|_0^1 + \frac{\cos\pi rx}{\pi^2 r^2}\bigg|_0^1 = (\text{first term } 0) + \frac{\cos\pi rx}{\pi^2 r^2}\bigg|_0^1 = \frac{\cos\pi r}{\pi^2 r^2} - \frac{\cos 0}{\pi^2 r^2} = \frac{1}{\pi^2 r^2}[\cos(\pi r) - 1]$$

Therefore $\int_0^1 x\cos\pi rx\,dx = \frac{1}{\pi^2 r^2}[\cos(\pi r) - 1]$ and

$$a_r = -2\int_0^1 x\cos\pi rx\,dx = \frac{-2}{\pi^2 r^2}[(-1)^r - 1] = \begin{cases} 0 & \text{even} \\ \frac{4}{\pi^2 r^2} & \text{odd} \end{cases}$$