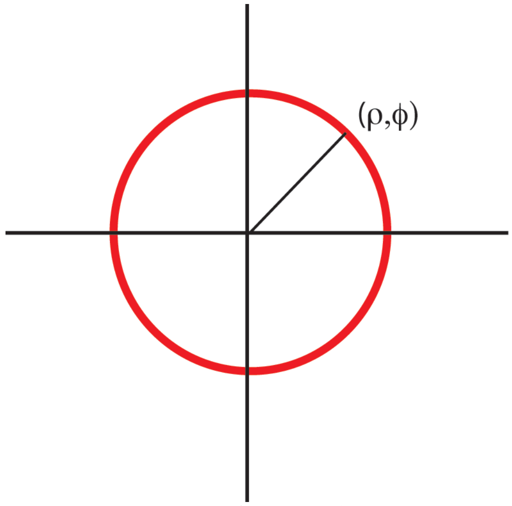
11.9



$u(\rho,\phi,t) \rightarrow u(\rho,\phi)$ for steady state, no t dependence

Note $u(\rho,\phi) \equiv T(\rho,\phi)$ in book

Boundary condition $u(a,\phi) = A + B\cos^2\phi$

Find the solution inside the circular membrane. From page 435.

$$u(\rho,\phi) = C_0 \ln\rho + D_0 + \sum_{n=1}^{\infty} (A_n \cos n\phi + B_n \sin n\phi)(C_n \rho^n + D_n \rho^{-n})$$

Apply boundary condition $u(a,\phi) = A + B\cos^2\phi$

Since the solution includes the point $\rho = 0$ and $\ln\rho$ undefind at this point we set $C_0 = 0$. Also the term $\rho^{-n} = \dfrac{1}{\rho^n}$ is also undefined at this point set $D_n = 0$. We are left with

$$u(\rho,\phi) = D_0 + \sum_{n=1}^{\infty} (A_n \cos n\phi + B_n \sin n\phi)(C_n \rho^n)$$

Applying the given boundary condition at $\rho = a$ gives

$$u(a,\phi) = D_0 + \sum_{n=1}^{\infty} (A_n \cos n\phi + B_n \sin n\phi)(C_n a^n) = A + B\cos^2\phi$$

$$= A + B(\frac{1}{2}\cos 2\phi + 1) = (A + \frac{B}{2}) + \frac{B}{2}\cos 2\phi$$

Equating the expression on the left and right hand side first gives $D_0 = A + \frac{B}{2}$

Then using the summation expression with n=1 $(A_1\cos 1\phi + B_1\sin 1\phi)C_1a^1 = 0$

So we have $A_1C_1a = 0$ and $B_1C_1a = 0$ therefore take $A_1 = B_1 = C_1 = 0$.

Finally for the summation expression with n=2 $(A_2\cos 2\phi + B_2\sin 2\phi)C_2a^2 = \frac{B}{2}\cos 2\phi$

In this case $A_2C_2a^2 = \frac{B}{2}$ and $B_2C_2a^2 = 0$.Therefore take $A_2C_2 = \frac{B}{2a^2}$ and $B_2 = 0$ (or $B_2C_2 = 0$)

Suibstitution for D_0, A_2C_2 and B_2C_2 in the expression for $u(\rho,\phi)$ gives

$$u(\rho,\phi) = A + \frac{B}{2} + \left(\frac{B}{2a^2}\cos 2\phi\right)\rho^2$$