$$b_r = \begin{cases} \dfrac{1}{\pi}\left[\dfrac{(-1)^{r-1}-1}{(r-1)^2} - \dfrac{(-1)^{r+1}-1}{(r+1)^2}\right] & r-\text{even} \\ 0 & r-\text{odd} \end{cases}$$

Take a look at this term and rearrange

$$\frac{(-1)^{r-1}-1}{(r-1)^2} - \frac{(-1)^{r+1}-1}{(r+1)^2} = \frac{(-1)^r(-1)^{-1}-1}{(r-1)^2} - \frac{(-1)^r(-1)^1-1}{(r+1)^2}$$

$$= \frac{-(-1)^r-1}{(r-1)^2} - \frac{-(-1)^r-1}{(r+1)^2} = [-(-1)^r-1]\left[\frac{1}{(r-1)^2} - \frac{1}{(r+1)^2}\right]$$

$$= [-(-1)^r-1]\left[\frac{(r+1)^2-(r-1)^2}{(r-1)^2(r+1)^2}\right] = [-(-1)^r-1]\left[\frac{4r}{(r-1)^2(r+1)^2}\right] = 4[-(-1)^r-1]\left[\frac{r}{r^4+\cdots}\right]$$

Therefore taking the largest power of r in the denom and that in the numerator we have $\dfrac{r}{r^4}$ or $\dfrac{1}{r^3}$

So we say this function is $O(r^{-3})$, that is big-O and in sum would converge as $\dfrac{1}{r^3}$ with reseect to only the even terms