14.1

Find an analytic function where $v(x,y) = (y\cos y + x\sin y)e^x$

Using Cauchy conditions $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

$$\frac{\partial (y\cos y + x\sin y)e^x}{\partial x} = \frac{\partial (y\cos y)e^x}{\partial x} + \frac{(x\sin y)e^x}{\partial x} = (y\cos y + \sin y + x\sin y)e^x = -\frac{\partial u}{\partial y}$$

Therefore $\frac{\partial u}{\partial y} = -(y\cos y + \sin y + x\sin y)e^x$

Integration gives

$$u(x,y) = \int -(y\cos y + \sin y + x\sin y)e^x dy = -e^x \int y\cos y dy - e^x \int \sin y dy - e^x x \int \cos y dy$$

After simplification

$u(x,y) = e^x(-y\sin y + x\cos y) + f(x)$ Note $f(x)$ is our integration constant with respect to y

Next Cauchy conditions $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

$$\frac{\partial u}{\partial x} = \frac{\partial e^x(-y\sin y + x\cos y) + f(x)}{\partial x} = \frac{\partial}{\partial x} e^x(-y\sin y) + \frac{\partial}{\partial x}(e^x x)\cos y) + f'(x)$$

$$\frac{\partial u}{\partial x} = e^x(-y\sin y) + \cos y(e^x x + x) + f'(x)$$

Also similarly $\frac{\partial v}{\partial y} = \frac{\partial (y\cos y + x\sin y)e^x}{\partial y} = e^x(-y\sin y + \cos y + x\cos y)$

Substitution into $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ gives $f'(x) = 0$ that is $f)x) = C$

Therefore

$$f(x,y) = u(x,y) + iv(x,y) = e^x(-y\sin y + x\cos y) + C + i(y\cos y + x\sin y)e^x$$