



JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Modeling Approaches to Cell and Tissue Engineering

Modeling Cell Interaction with Viscoelastic Extracellular Matrix

Maxwell and Maxwell-Weichert Models of the Stress Relaxation Experiment for Four Alginate Hydrogels

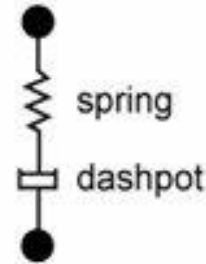
General Solution

$$\sigma(t) = \int_0^t \left[E_1 e^{-(t-u)/\tau_1} + E_2 e^{-(t-u)/\tau_2} \right] \dot{\epsilon}(u) du$$

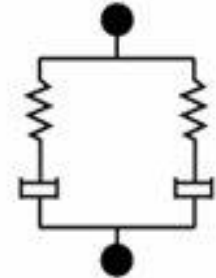
Solution for a Step Function

$$\epsilon(t) = \left[E_1 e^{-t/\tau_1} + E_2 e^{-t/\tau_2} \right] \epsilon_0$$

Maxwell model



2-element Maxwell-Weichert model



Overview

Alexander Spector (after Chaudhiri et al. , 2015)

- Modeling stress relaxation in viscoelastic ECM by using Maxwell and Maxwell-Weichert (2-element) models
- Modeling harmonic load experiment by using Maxwell and Maxwell-Weichert Models. Complex modulus
- Fitting the Results of Stress Relaxation Experiment with Maxwell-Weichert Model
- Estimation of the Material Parameters for Four Alginate Hydrogels
- Estimation of the Storage and Loss Modules for Four Alginate Hydrogels

Maxwell and Maxwell-Weichert Models for the Harmonic Loading Experiment

Stresses and Strains in the Harmonic Loading Case

$$\dot{\varepsilon} = \dot{\varepsilon}_1 = \frac{\dot{\sigma}_1}{E_1} + \frac{\dot{\sigma}_2}{E_2}, \quad \dot{\sigma} = \dot{\sigma}_1 + \dot{\sigma}_2, \quad \sigma = \sigma_1 + \sigma_2$$

Complex Modulus, Storage and Loss Modules

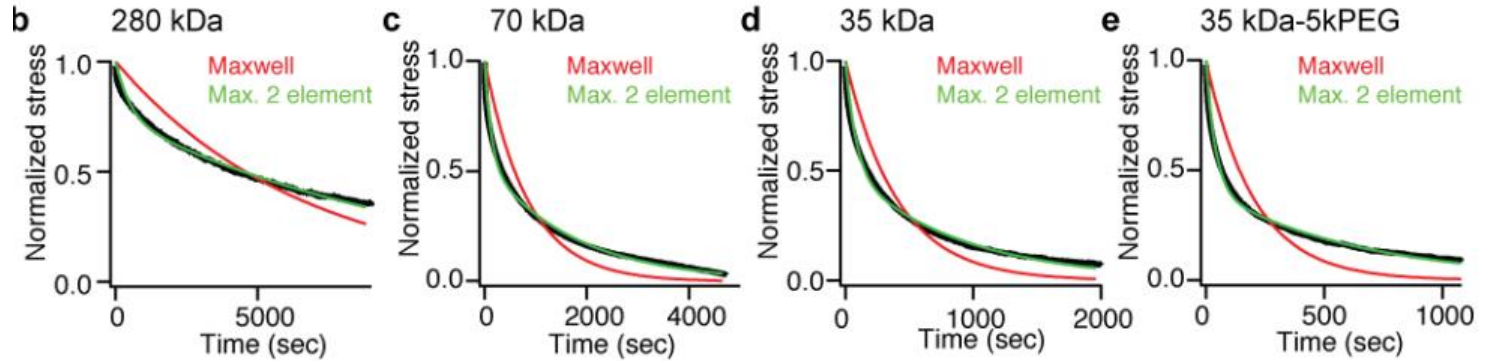
$$\sigma = G\varepsilon, G = G' + iG''$$

$$G' = E_1 \frac{\tau_1^2 \omega^2}{1 + \tau_1^2 \omega^2} + E_2 \frac{\tau_2^2 \omega^2}{1 + \tau_2^2 \omega^2}, \quad G'' = E_1 \frac{\tau_1 \omega}{1 + \tau_1^2 \omega^2} + E_2 \frac{\tau_2 \omega}{1 + \tau_2^2 \omega^2}$$

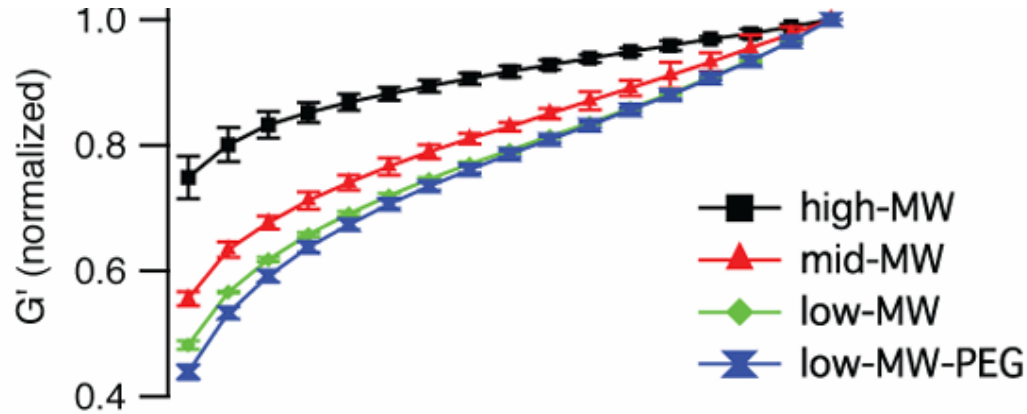
Parameters of Fitting the Results of Stress Relaxation Experiment with Maxwell-Weichert Model for Four Alginate Hydrogels

Alginate Type:	280 kDa	70 kDa	35 kDa	35 kDa – 5k PEG
τ_1 (s)	510	125	81	39
E1 (norm.)	0.25	0.46	0.5	0.58
τ_2	11111	1734	900	637
E2	0.75	0.54	0.5	0.42

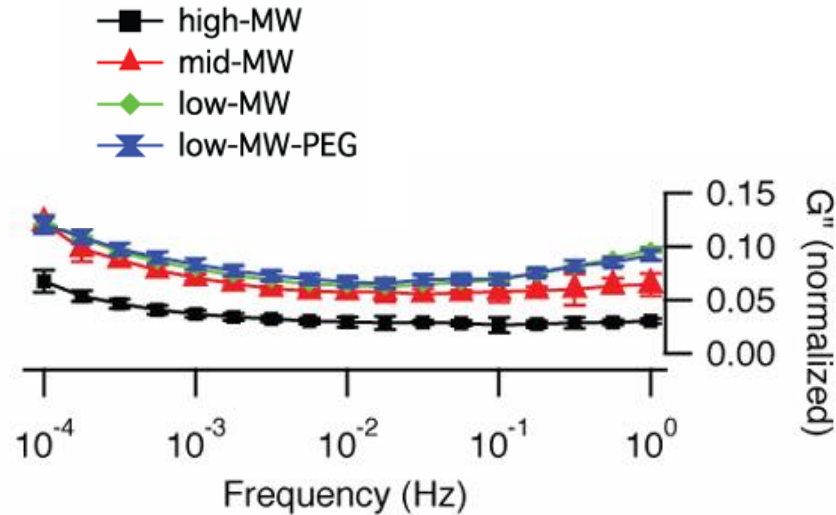
Fitting the Results of Stress Relaxation Experiment with Maxwell and Maxwell-Weichert Models



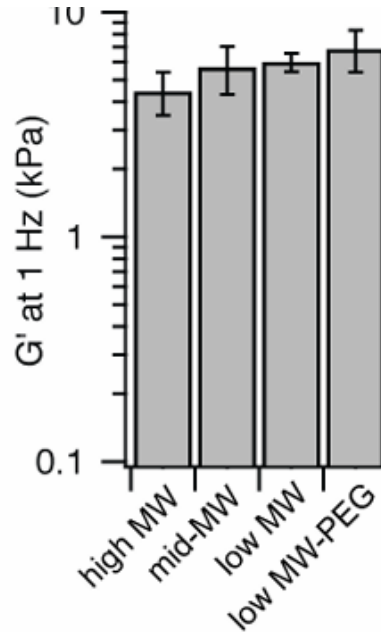
Storage Modulus vs. Frequency for Four Alginate Hydrogels



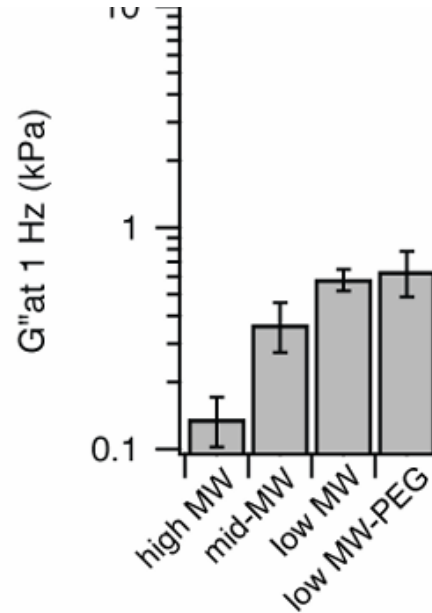
Loss Modulus vs. Frequency for Four Alginate Hydrogels



Storage Modulus at Frequency of 1Hz for Four Alginate Hydrogels



Loss Modulus at Frequency of 1Hz for Four Alginate Hydrogels





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