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EN.585.743.81.SP25 Modeling Approaches to Cell and Tissue Engineering
Modeling Project
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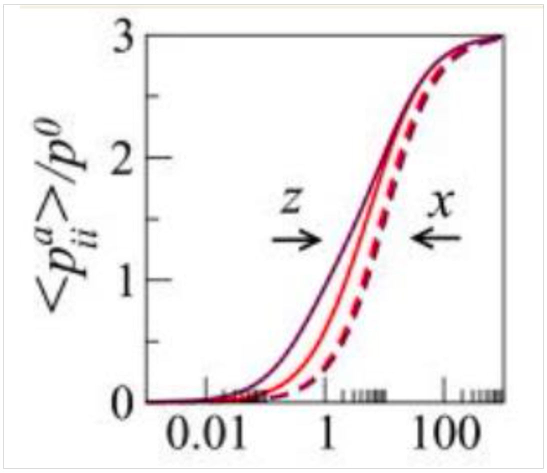
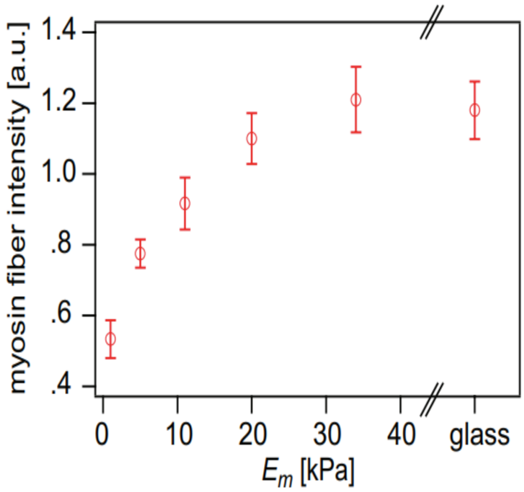
Introduction

In Lecture 12, the polarization of the stem cell cytoskeleton as a function of the stiffness of ECM is considered. A number of characteristics of cell/ECM interaction, such as the modeling-predicted order parameter [1] (Figure 1(a)) or experiment-estimated myosin fiber intensity [1] (Figure 1(b)) demonstrate an increase–saturation pattern as functions of ECM stiffness. Use the 1-D model where the cell and ECM are presented by the active and elastic springs, respectively [1] (Figure 2) and show that the active force generated by the cell, f_a, as a function of the ECM stiffness, has the same increase saturation pattern of behavior.



(a) Normalized order parameter vs. $\frac{E_m}{E_c}$.



(b) Myosin fibre intensity vs. E_m.

Figure 1: (a) Order–parameter saturation. (b) Actomyosin-intensity saturation.

Anisotropic Polarized Actomyosin Force

$$\frac{f^a}{f^0} = \alpha \frac{k_m}{\tilde{k}_c + k_m}$$

- α: polarizability factor
- $\tilde{k}_c = (1 + \alpha)\, k_c$: effective stiffness of the cell
- k_m: matrix rigidity

Interpretation

- For very soft matrices ($k_m \ll \tilde{k}_c$):

$$\frac{f^a}{f^0} = \alpha \frac{k_m}{\tilde{k}_c + k_m} \longrightarrow 0.$$

If $k_m \ll \tilde{k}_c$, then $\tilde{k}_c + k_m \approx \tilde{k}_c$ and

$$\frac{f^a}{f^0} \approx \alpha \frac{k_m}{\tilde{k}_c} \approx k_m \longrightarrow 0 \quad \text{as } k_m \to 0.$$

- For very stiff matrices ($k_m \gg \tilde{k}_c$):

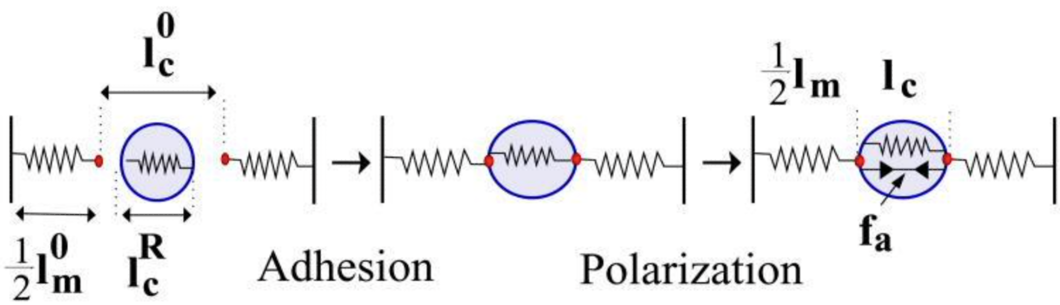
$$\frac{f^a}{f^0} = \alpha \frac{k_m}{\tilde{k}_c + k_m} \approx \alpha \frac{k_m}{k_m} \longrightarrow \alpha$$

Thus, the active force grows with matrix stiffness and saturates at a maximum value proportional to α.

Derivation.

Here we show, using the 1-D two-spring model of cell and matrix (Figure 2), that the cell's active force f_a vs. ECM stiffness k follows the same functional form:

Figure 3



(Zemel et al., 2010 and Lecture 12)

Figure 2: 1D model: active cell spring (stiffness k_c) in series with ECM spring (stiffness k_m).

Illustration

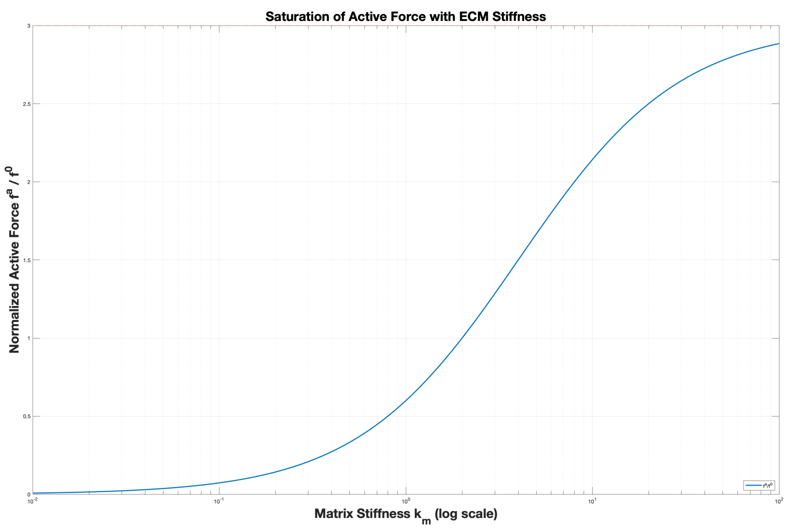


Figure 3: Active Force vs. Matrix stiffness.

References

[1] Assaf Zemel, Florian Rehfeldt, Alexander E.X. Brown, Dennis E. Discher, and Samuel A. Safran. Optimal matrix rigidity for stress-fibre polarization in stem cells. *Nature Physics*, 6(6):468–473, 2010.