

$$E_{\text{ion}} = \frac{61.56}{Z} \cdot \log_{10} \left(\frac{C_{\text{out}}}{C_{\text{in}}} \right)$$

where

E_{ion} = equilibrium potential or Ernst potential for a particular ion, in mV

C_{in} = intracellular concentration of the ion, in mM/L

C_{out} = extracellular concentration of the ion, in mM/L

Z = the valence of the ion

Using the concentration gradients from the table in the question, the Nersnt potentials for Na^+ , K^+ and Cl^- are:

$$\begin{aligned} E_{\text{Na}^+} &= \frac{61.56}{+1} \log_{10} \left(\frac{127}{10} \right) \approx 68\text{mV} \\ E_{\text{K}^+} &= \frac{61.56}{+1} \log_{10} \left(\frac{4}{120} \right) \approx -90\text{mV} \\ E_{\text{Cl}^-} &= \frac{61.56}{-1} \log_{10} \left(\frac{80}{2} \right) \approx -98\text{mV} \end{aligned}$$

First knowing that the plasma membrane behaves electrically like a capacitor of capacitance C_a F/m², the charge separation across the membrane is:

$$\begin{aligned} Q &= C_a \cdot (2\pi R^2 + 2\pi RL) \cdot V_m \\ &= C_a \cdot 2\pi R(R + L) \cdot V_m \end{aligned}$$

The amount of electricity that is carried by 1 mole of electrons is given by Faraday's constant: F (96,480 coulombs/mole). The amount of ion X moved to establish V_m is:

$$\begin{aligned} X_{i \rightarrow o} &= \frac{Q}{F} \\ &= \frac{C_a \cdot 2\pi R(R + L) \cdot V_m}{F} \end{aligned}$$

With a cylindrical cell of volume $\pi R^2 L$, the initial amount of ion X with an intracellular concentration of $[X]_i$ is:

$$X_i = \pi R^2 L [X]_i$$

Thus the ratio of the amount (in mol) of ion X moved to establish V_m to the initial amount of ion X is:

$$\frac{X_{i \rightarrow o}}{X_i} = 2 \cdot \frac{C_a}{F} \frac{R + L}{R \cdot L} \frac{V_m}{[X]_i}$$

GHK equation:

$$V_{\text{GHK, Na, K, Cl}} = \frac{R \cdot T}{F} \cdot \ln \left(\frac{P_{\text{Na}} \cdot [\text{Na}]_o + P_{\text{K}} [\text{K}]_o + P_{\text{Cl}} [\text{Cl}]_i}{P_{\text{Na}} \cdot [\text{Na}]_i + P_{\text{K}} [\text{K}]_i + P_{\text{Cl}} [\text{Cl}]_o} \right)$$

$$V_{\text{Cl}^-} = -\frac{R \cdot T}{F} \cdot \ln \left(\frac{[\text{Cl}^-]_o}{[\text{Cl}^-]_i} \right)$$

$$E_m = \frac{g_{\text{Na}^+}}{g_{\text{Na}^+} + g_{\text{K}^+}} E_{\text{Na}^+} + \frac{g_{\text{K}^+}}{g_{\text{Na}^+} + g_{\text{K}^+}} E_{\text{K}^+}$$

$$\text{let } r = \frac{g_{\text{Na}^+}}{g_{\text{K}^+}}$$

$$E_m = \frac{r}{1+r} E_{\text{Na}^+} + \frac{1}{1+r} E_{\text{K}^+}$$

multiplying on both sides by $(1+r)$

$$(1+r)E_m = rE_{\text{Na}^+} + E_{\text{K}^+}$$

$$r = \frac{E_{\text{K}^+} - E_m}{E_m - E_{\text{Na}^+}}$$

Using the concentrations of Na^+ and K^+ and their computed Nernst potentials as given in slide 8 of video 7, the ratio is:

$$\begin{aligned} r &= \frac{g_{\text{Na}^+}}{g_{\text{K}^+}} \\ &= \frac{E_{\text{K}^+} - E_m}{E_m - E_{\text{Na}^+}} \\ &= \frac{-94 - (-86)}{-86 - 61} \\ &= \frac{8}{147} \\ &= 0.054 \text{ or } 5.4\% \end{aligned}$$

Given:

$$\frac{X^2}{(U-X)(T-X)} = \frac{A^2}{(U-A)(R-A)}$$

Let $\frac{1}{\alpha} = \frac{A^2}{(U-A)(R-A)}$ then

$$\alpha X^2 = (U-X)(T-X)$$

$$(1-\alpha)X^2 - (U+T)X + UT = 0$$

Solving the quadratic assuming validity of the discriminant for real values:

$$X_{0,1} = \frac{U+T \pm \sqrt{(U-T)^2 + 4\alpha UT}}{2(1-\alpha)}$$

If we set $c = \frac{1}{\alpha}$ we obtain the reach curve white paper solution:

$$X_{0,1} = \frac{c(U+T) \pm \sqrt{c}\sqrt{c(T-U)^2 + 4UT}}{2(-1+c)}$$