

Solutions to Homework Assignment – Module 1

1. [5 points] With reference to the fluid mosaic model of the cell membrane, which of the following statements is the most correct?
 - A. Only electrically charged molecules can move through the membrane.
 - B. The positions of various channels with respect to each other is fixed within the membrane.
 - C. A particular protein that at a given time is located at the outer surface of the membrane can migrate to the inner surface of the membrane.
 - D. The lipids that make up the outer membrane layer are always identical to those that make up the inner membrane layer.

2. [5 points] Which of the following statements about channels in mammalian cell membranes is the most correct?
 - A. Polar molecules do not pass through channels of any type.
 - B. Some channels are selective for a class of molecules (e.g., singly charged molecules); other channels are selective for only one molecular species (e.g., Na^+).
 - C. As a general rule, it is easier for larger, as compared to smaller, molecules to pass through channels.
 - D. VHF channels, due to their lower frequency, allow the passage of larger molecules than UHF channels (channel width is inversely proportional to frequency; $c = f\lambda$).

3. [15 points¹] Using the concentrations provided in the table (below) calculate the Nernst potential at $T = 24^\circ \text{C}$ for Na^+ , K^+ , and Cl^- . **SHOW YOUR WORK**

Ion	Intracellular, mM	Extracellular, mM
Cl^-	2.0	80
K^+	120	4.0
Na^+	10	127
Ca^{2+}	110 μM	2.4
HCO_3^-	12.4	29

Recall that the Nernst equation is ...

¹ 5 points for each ion in question.

Rev 0, 8/2/2018 – adapted from Rev 2 of Fall 2017. Content essentially identical to Rev 2 of Fall 2017; equations re-written for clarity and some additional detail added. Up-date to 601.

Rev 1, 9/2/2018 – correct typos; no content changes.

Rev 2, 8/13/19 – add SHOW YOUR WORK to text of Q4.

Rev 3, 1/28/20 – space holder; there is no Rev 3

Rev 4, 1/28/20 – to match Rev 4 of the Module 1 assignment

$$V_{\text{membrane}} = V_{1 \rightarrow 2} = \frac{RT}{zF} \cdot \ln \left(\frac{[S]_{\text{out}}}{[S]_{\text{in}}} \right) = 2.303 \cdot \frac{RT}{zF} \cdot \log_{10} \left(\frac{S_{\text{out}}}{S_{\text{in}}} \right)$$

Putting in numerical values for R (8.314 J °K⁻¹mol⁻¹), T (297 °K), z (±1) and F (96,485 Coul/mol) we get

RT/zF = ± 25.59 mV; multiplying by 2.303 gives ±58.94 mV, so that ...

$$V_{\text{membrane}} = \pm 58.94 \cdot \log_{10} \left(\frac{[S]_{\text{out}}}{[S]_{\text{in}}} \right); \text{ where } V_{\text{membrane}} \text{ is in mV}$$

A. For Na⁺

$$V_{\text{membrane}} = +58.94 \cdot \log_{10} \left(\frac{127}{10} \right) = \mathbf{65.06} (\approx \mathbf{65}) \text{ mV}$$

B. For K⁺

$$V_{\text{membrane}} = +58.94 \cdot \log_{10} \left(\frac{4.0}{120} \right) = \mathbf{-87.06} (\approx \mathbf{-87.1}) \text{ mV}$$

C. For Cl⁻

$$V_{\text{membrane}} = -58.94 \cdot \log_{10} \left(\frac{80}{2.0} \right) = \mathbf{-94.43} (\approx \mathbf{-94.4}) \text{ mV}$$

4. [20 points] In our derivation of the Nernst potential we made several assumptions, one of which was that the concentration of the permeant ion(s) on each side of the cell membrane did not change as the problem went to completion.

Assume a cylindrical cell of radius R meters and length L meters, having a membrane capacitance C_a farads/m², one singly-charged permeant species, X, with an intracellular concentration of [X]_i mol/L and an extracellular concentration of [X]_o mol/L (assume that [X]_i > [X]_o so that some X moves out of the cell to establish electrochemical equilibrium) and a membrane (= Nernst; assume no membrane ion pumps) potential of V_m volts.

Derive an expression for the ratio of the amount (in mol) of ion X moved (to establish the Nernst potential, V_m, of the cell) to the initial amount (in mol) of ion X inside the cell.

The charge moved by the permeant ion to establish the Nernst potential can be thought of as the charge needed to charge the membrane capacitor. So ...

$$Q = CV$$

Where C is the membrane capacitance and V is the membrane voltage. So ...

$$Q = C_a \cdot \{2 \cdot \pi \cdot R \cdot L + 2 \cdot \pi \cdot R^2\} \cdot V_{\text{membrane}}; \text{ where } Q \text{ is in Coulombs}$$

$$Q = C_a \cdot 2 \cdot \pi \cdot R \cdot (L + R) \cdot V_{\text{membrane}}$$

To convert from Coulombs to mols we need to divide by Faraday's number, F,

$$Q_{\text{mol}} = \frac{C_a \cdot 2 \cdot \pi \cdot R \cdot (L + R) \cdot V_{\text{membrane}}}{F}$$

Where F is Faraday's number ($=q_e N_A$), where q_e is the electron charge and N_A is Avogadro's number.

The amount, in mol, of permeant ion inside the cell at the beginning of the problem (time = 0) is ...

$$[X]_i \cdot \pi \cdot R^2 \cdot L \cdot 10^3 \text{ mol}$$

Where 10^3 accounts for the conversion from volume in liters to volume in cubic meters.

So, the ratio of the amount of permeant charge moved to the amount of permeant charge in the cell at time = 0 is ...

$$\frac{C_a \cdot 2 \cdot \pi \cdot R \cdot (L + R) \cdot V_{\text{membrane}}}{F} \bigg/ [X]_i \cdot \pi \cdot R^2 \cdot L \cdot 10^3$$

$$\frac{2 \cdot C_a \cdot (L + R) \cdot V_{\text{membrane}} \cdot 10^{-3}}{[X]_i \cdot R \cdot L \cdot F}$$

5. [10 points] With reference to slide 5 of video 7 explain why $[Cl]_i$ appears in the numerator and $[Cl]_o$ appears in the denominator of the GHK equation.

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One way to approach this is to remember that the GHK equation (slide 5, video 7) is a way of expressing the concept developed in slides 6 and 7 of video 7, that the membrane potential is a weighted average of the individual ion Nernst potentials. Accordingly, the GHK equation should reduce to the Nernst equation for an individual ion if the inside and outside concentrations of any two (of the three ions we considered in the GHK equation) are set at zero. So, if $[K]_i$, $[K]_o$, $[Cl]_i$, and $[Cl]_o$ are set at zero then

$$V = \frac{RT}{F} \cdot \ln \left(\frac{P_{Na} \cdot [Na]_o}{P_{Na} \cdot [Na]_i} \right) = \frac{RT}{F} \cdot \ln \left(\frac{[Na]_o}{[Na]_i} \right)$$

This equation is almost right, but the valence of the ion (z) is missing. Remember, when we developed the GHK equation we assumed that $z = \pm 1$; this equation does exactly match the sodium Nernst equation since for sodium $z = 1$, so (RT/F) in the equation above does match (RT/zF) in the Nernst equation for sodium because $z = 1$. This reasoning holds for potassium alone as well, since for potassium z also $= 1$. But chloride is different; $z = -1$. For the equation above (with Cl substituted for Na) to match the Nernst equation for chloride we have to multiply by -1 (to get $(RT/-F)$). We can do this by noting that multiplication of $\ln(x/y)$ by -1 gives $\ln(y/x)$, thus “flipping” $[Cl]_o$ into the denominator and $[Cl]_i$ into the numerator while allowing the factor $(RT/-F)$ to become (RT/F) for consistency with the sodium and potassium terms. So, the GHK equation is correctly shown in video 7, slide 5 - with $[Cl]_i$ in the numerator and $[Cl]_o$ in the denominator.

6. [20 points] Using the intracellular and extracellular concentrations of Na^+ and K^+ and their calculated Nernst potentials as given in slide 8 of video 7, calculate the ratio of g_{Na} to g_K . **SHOW YOUR WORK**

From slide 7 of video 7 ...

$$E_m = \frac{g_K}{g_K + g_{Na}} \cdot E_K + \frac{g_{Na}}{g_K + g_{Na}} E_{Na}$$

Let $g_{Na} = \alpha g_K$, then ...

$$E_m = \frac{g_K}{(1 + \alpha) \cdot g_K} \cdot E_K + \frac{\alpha \cdot g_K}{(1 + \alpha) \cdot g_K} \cdot E_{Na}$$

$$(1 + \alpha) \cdot E_m = E_K + \alpha \cdot E_{Na}$$

$$\alpha \cdot (E_m - E_{Na}) = E_K - E_m$$

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$$\alpha = \frac{E_K - E_m}{E_m - E_{Na}}$$
$$\alpha = \frac{-94 - (-86)}{-86 - 61} = \frac{-8}{-147} = 0.054 = \frac{g_{Na}}{g_K}$$

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