YG_390_lab-week5-student

October 17, 2019

1 DS-GA 3001.001 Special Topics in Data Science: Probabilistic Time Series Analysis

2 Week 5 particle filtering

[158]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import scipy.stats as stats
import math
from pykalman import KalmanFilter
import time

# Data Visualization
def plot_kalman(time, latent, data, ky=None, ky_var=None, plot_type="r-",␣
 ↪label=None, title='sample'):
    """
    Plot the trajectory
    """
    x, y = time, latent
    nx, ny = data[:, 0], data[:, 1]
    fig, ax = plt.subplots(1, 2, figsize=(18, 4))
    if ky is not None:
        ax[0].plot(x, y, 'g-', time, nx, 'b.', time, ny, 'b.', time, ky, 'r-',␣
 ↪plot_type)
        if ky_var is not None:
            ax[0].fill_between(time, ky - np.sqrt(ky_var),
                               ky + np.sqrt(ky_var), color='r', alpha=.5,␣
 ↪label='estimate')
        ax[0].legend()
        ax[1].plot(y, ky, '.', color='grey')
        ax[1].set_xlabel('real latent')
        ax[1].set_ylabel('estimated latent')
        ax[1].set_title('cc %.3f' %(np.corrcoef(y[:,0], ky)[0,1]))
    else:
        ax[0].plot(x, y, 'g-', x, nx, 'b.', x, ny, 'b.')
```

```
        ax[1].plot(y, nx, '.k', label='observed dim 1')
        ax[1].plot(y, ny, '.', color='grey', label='observed dim 2')
        ax[1].set_xlabel('latent')
        ax[1].set_ylabel('observed')
        ax[1].legend()

    ax[0].set_xlabel('time')
    ax[0].set_ylabel('latent')
    ax[0].set_title(title)
    ax[1].set_aspect(1)

    return fig


def print_parameters(kf_model, need_params=None, evals=False):
    """
    Function that prints out the parameters for a Kalman Filter
    @param - kf_model : the model object
    @param - need_params : a list of string
    """
    if need_params is None:
        need_params = ['transition_matrices', 'observation_matrices',␣
 ↪'transition_covariance', 'observation_covariance',
                      'initial_state_mean', 'initial_state_covariance']
    for param in need_params:
        print("{0} = {1}, shape = {2}\n".format(param, getattr(kf_model,␣
 ↪param), getattr(kf_model, param).shape))
```

2.0.1 example linear dynamical system

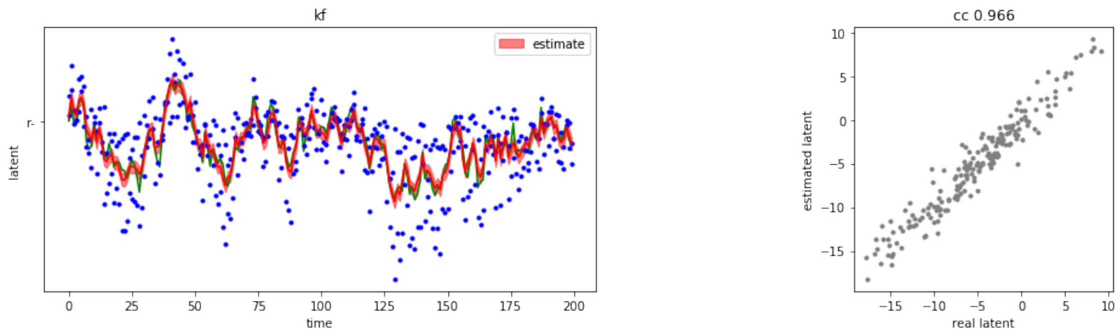
[434]:
```
np.random.seed(0)
# Sampling
n_dim_state = 1
n_dim_obs = 2
tt = np.arange(200)
kf_GT = KalmanFilter(n_dim_state=n_dim_state, n_dim_obs=n_dim_obs,
                  transition_matrices = np.eye(n_dim_state)*.9,
                  transition_covariance = np.eye(n_dim_state)*10,
                  observation_matrices = np.random.randn(n_dim_state*n_dim_obs).
 ↪reshape(n_dim_obs, n_dim_state),
                  observation_covariance = np.eye(n_dim_obs)*10,
                  initial_state_mean = np.zeros(n_dim_state),
                  initial_state_covariance = np.eye(n_dim_state))
```

```
latent, data = kf_GT.sample(len(tt), initial_state=kf_GT.initial_state_mean,
    random_state=np.random.RandomState(0))
#fig = plot_kalman(tt, latent, data, title='sample');
#print_parameters(kf_GT)
```

2.0.2 use KF to do inference

[435]:
```
filtered_mean, filtered_cov = kf_GT.filter(data)
smoothed_mean, smoothed_cov = kf_GT.smooth(data)

fig = plot_kalman(tt, latent, data, ky = smoothed_mean[:,0],
    ky_var=smoothed_cov[:,0,0], title='kf');
```



2.1 Particle Filtering: alternative inference

We know: data and parameters (A, C, Γ, Σ)

We assume: linear transformation in latent space, linear mapping from latent to observed space, Gaussian observations

<img src='img/LDS.svg', width = 300, height=300>

We want: approximation of the posterior marginals $P(z_n|x_{1:t})$

How: generate samples of $P(z_n^{(i)}|z_{n-1})$ through particle filtering, reweigh by observations, and average to obtain expected value

<img src='img/PF_illustration.svg', width = 200, height=200>

2.1.1 A) initial samples for z_1

1) draw N_{samp} samples (=particles) given initial condition μ_0 and Γ_0

$$P(z_0^{(i)}|\mu_0, \Gamma_0)$$

where $i = 1, ..., N_{samp}$

2) propagate samples forward one time step ($n = 1$) through linear transformation A and adding noise with covariance Γ

$$P(z_1^{(i)}|z_0^{(i)})$$

2.1.2 B) for loop:

1) weigh samples for $z_n^{(i)}$ given observational evidence from x_n

2) compute the probability for the data for each sampled $z_n^{(i)}$:

$$P(x_n|z_n^{(i)})$$

3) compute the weights $w_n^{(i)}$ given $P(x_n|z_n^{(i)})$:

$$w_n^{(i)} = \frac{P(x_n|z_n^{(i)})}{\sum_i P(x_n|z_n^{(i)})}$$

2) produce new samples at $n + 1$

4) draw from multinomial distribution with probabilities w_n, which will give you class assignments $c_{(i)}$ that indicate which samples $z_n^{(i)}$ to use

5) $z_n^{(c_{(i)})}$ become your new priors form which you sample $z_{n+1}^{(i)}$

$$P\left(z_{n+1}^{(i)}|z_n^{(c_{(i)})}, \Gamma\right)$$

6) keep going WITHIN THE LOOP

3 coding: implement the particle filter

[436]:
```
from scipy.stats import multivariate_normal

class myParticleFiltering:

    def __init__(self, time, transition_matrices, transition_covariance,␣
↪observation_matrices,
                 observation_covariance, initial_state_mean,␣
↪initial_state_covariance):
        self.transition_matrices = transition_matrices
        self.transition_covariance = transition_covariance
        self.observation_matrices = observation_matrices
        self.observation_covariance = observation_covariance
```

```
        self.initial_state_mean = initial_state_mean
        self.initial_state_covariance = initial_state_covariance
        self.time = time
        # placeholder
        self.est_z_mean = np.zeros(len(time)) * np.nan
        self.est_z_var = np.zeros(len(time)) * np.nan

    def plot_particle_update(self, z_samp, w, Nbins=100, seed=0):
        np.random.seed(seed)
        plt.figure(figsize=(8, 4))
        htmp = np.histogram(z_samp, Nbins)
        plt.fill_between(htmp[1][1:], np.zeros(Nbins), htmp[0], color='blue',␣
↪alpha=.4, label='sample distribution')
        plt.plot([np.mean(z_samp), np.mean(z_samp)], [0, z_samp.shape[0]/
↪Nbins], 'b-', label='initial mean')
        # create new particles
        k = np.random.multinomial(z_samp.shape[0], w)
        z_samp_new = np.repeat(z_samp, k)
        htmp = np.histogram(z_samp_new, Nbins)
        plt.fill_between(htmp[1][1:], np.zeros(Nbins), htmp[0], color='red',␣
↪alpha=.4, label='reweighted sample distribution')
        plt.plot([np.mean(z_samp_new), np.mean(z_samp_new)], [0, z_samp.
↪shape[0]/Nbins], 'r-', label='mean of weighted samples')
        plt.legend()
        plt.xlabel('latent $z_n^{(i)}$')
        plt.ylabel('counts')

    def particle_filter(self, data, Nsamp, seed=0):
        ###############################################
        # TODO: implementation of the particle filter #
        ###############################################
        np.random.seed(seed)
        # initial conditions:
        self.est_z_mean[0] = self.initial_state_mean.copy()
        self.est_z_var[0] = self.initial_state_covariance.copy()

        # placeholder
        self.z_samp = np.zeros([Nsamp, len(self.time)])
        self.w = np.zeros([Nsamp, len(self.time)])

        ### create samples from distribution with initial conditions
        # TODO: your code here!
        #z_samp0 = np.repeat(0, Nsamp)
        z_samp0 = np.random.normal(pf.initial_state_mean[0], pf.
↪initial_state_covariance[:,0][0], Nsamp)

        ### propagate and create samples at time point n=1
```

```
        # TODO: your code here!
        #z_samp = np.repeat(0, Nsamp)
        z_samp = np.squeeze(np.random.normal(self.transition_matrices *␣
↪z_samp0, self.transition_covariance))

        ### save those samples from n=1
        self.z_samp[:,0] = z_samp.copy()

        for nn in range(1, len(self.time)):

            ### compute the weights (implement function below)
            w = self.compute_w(data[nn-1, :], z_samp)

            ### keep track of mean and variance of the weighted samples
            # TODO: your code here:
            weighted_samples = [z_samp[i]* w[i] for i in range(len(w))]
            self.est_z_mean[nn-1] = np.sum(weighted_samples)
            self.est_z_var[nn-1] = np.var(weighted_samples)

            ### compute class assignments
            # TODO: your code here:
            # k = np.ones(Nsamp)
            k =  np.random.multinomial(Nsamp, w)

            ### particles according to class assignments (=reweighted particles)
            # TODO: your code here:
            # z_samp_new = np.zeros(Nsamp)
            z_samp_new = np.repeat(z_samp, k)

            ### propagate and create samples at time point n+1 (using the␣
↪reweighted particles)
            # TODO: your code here:
            #z_samp = np.zeros(Nsamp) * np.nan
            z_samp =  np.squeeze(np.random.normal(self.transition_matrices *␣
↪z_samp_new, self.transition_covariance))

            # save particles and weights
            self.w[:, nn-1] = w
            self.z_samp[:, nn] = z_samp

        # track for last sample:
        last_z_samp = self.z_samp[:, -1]
        w = self.compute_w(data[-1, :], z_samp)
        weighted_samples = [last_z_samp[i] * w[i] for i in range(len(w))]
        self.est_z_mean[-1] = np.sum(weighted_samples)
        self.est_z_var[-1] = np.var(weighted_samples)
```

```
    def compute_w(self, data_nn, z_samp, seed=0):
        np.random.seed(seed)
        ###############################################
        ####### function to compute weights ###########
        ###############################################
        # TODO: your code here:
        n_samples = z_samp.shape[0]
        weights = np.ones(n_samples)
        for i in range(n_samples):
            projected_mean = np.squeeze(np.dot(self.observation_matrices,␣
 ↪z_samp[i]))
            projected_covar = self.observation_covariance
            weights[i] = multivariate_normal.pdf(data_nn, projected_mean,␣
 ↪projected_covar)
        return weights/weights.sum()
#        return np.ones(z_samp.shape[0])/z_samp.shape[0]
```

[437]:
```
# create particle filter class with true parameters
pf = myParticleFiltering(tt, kf_GT.transition_matrices[0], kf_GT.
 ↪transition_covariance,
                         kf_GT.observation_matrices, kf_GT.
 ↪observation_covariance,
                         kf_GT.initial_state_mean, kf_GT.
 ↪initial_state_covariance)
```

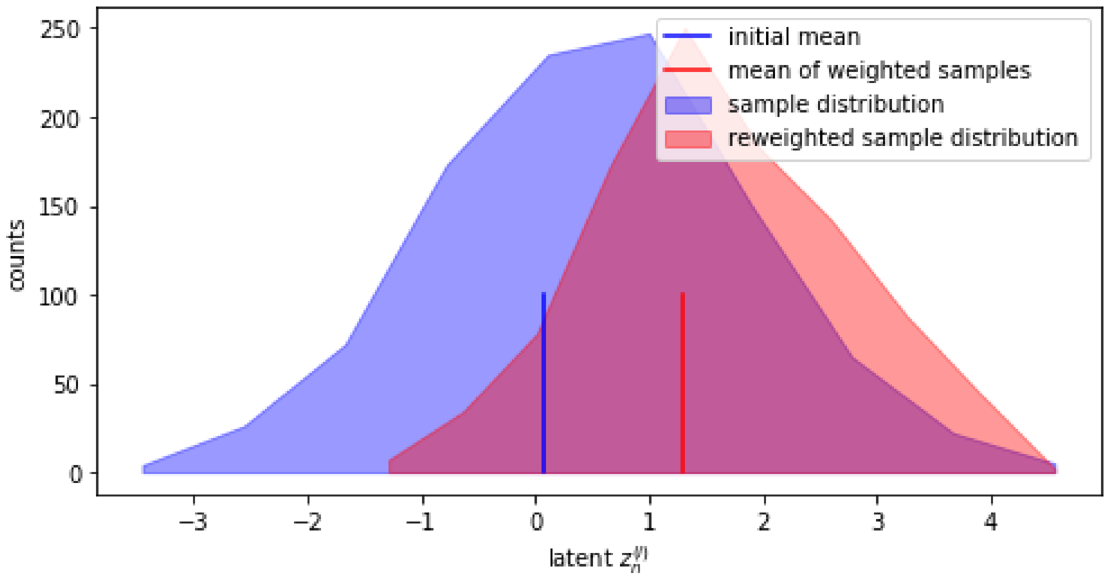
run the particle filter with 100 particles

[438]:
```
start = time.time()
pf.particle_filter(data, Nsamp=1000, seed=1)
end = time.time()
print('time required for particle filter: ', np.round(end-start,3))
```

```
time required for particle filter:  16.084
```

look at an example distribution of samples and their corresponding reweighted samples

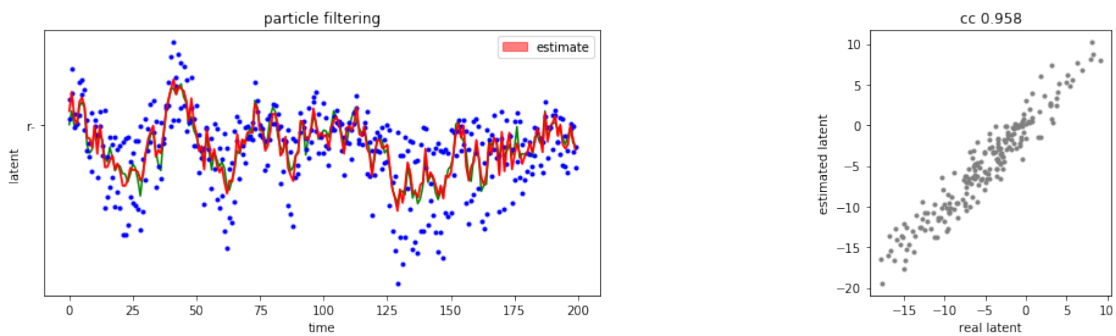
[421]:
```
pf.plot_particle_update(pf.z_samp[:,0], pf.w[:,0],Nbins=10)
```



should look something like this … <img src='img/PF_update.svg', width = 500, height=500>

particle-filter estimated latent trajectory

[439]:
```
fig = plot_kalman(tt, latent, data, ky = pf.est_z_mean, ky_var=pf.est_z_var,␣
 ↪title='particle filtering');
```

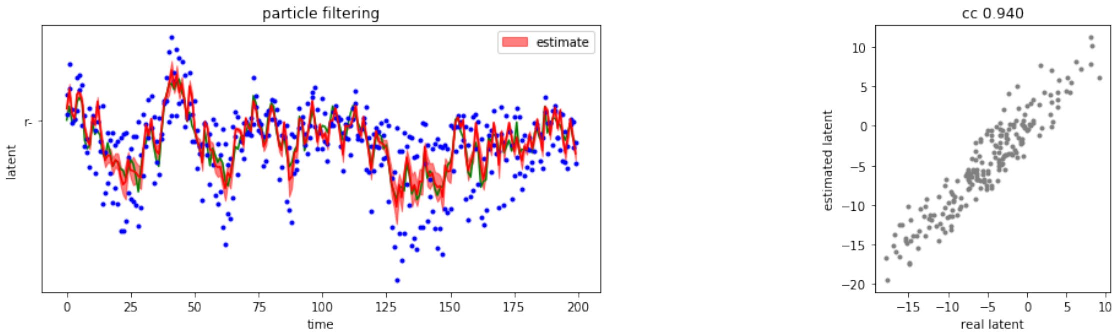


should look something like this … <img src='img/PF_1000.svg', width = 1000, height=1000>

decrease the number of particles you produce

[440]:
```
start = time.time()
pf.particle_filter(data, Nsamp=10, seed=1)
end = time.time()
print('time required for particle filter: ', np.round(end-start,3), ' sec')
fig = plot_kalman(tt, latent, data, ky = pf.est_z_mean, ky_var=pf.est_z_var,␣
 ↪title='particle filtering');
```

```
time required for particle filter:  0.219  sec
```

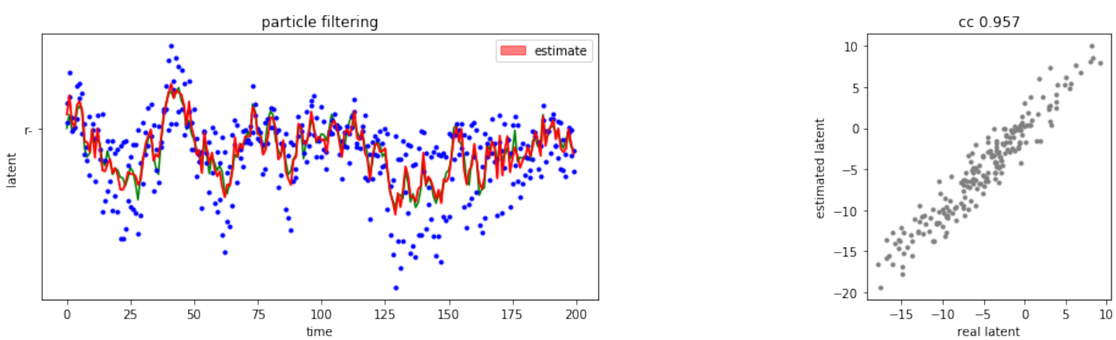


should look something like this ... <img src=‘img/PF_10.svg’, width = 1000, height=1000>

increase the number of particles you produce

[441]:
```
start = time.time()
pf.particle_filter(data, Nsamp=10000, seed=1)
end = time.time()
print('time required for particle filter: ', np.round(end-start,3))
fig = plot_kalman(tt, latent, data, ky = pf.est_z_mean, ky_var=pf.est_z_var,␣
 ↪title='particle filtering');
```

```
time required for particle filter:  160.687
```



should look something like this ... <img src='img/PF_10000.svg', width = 1000, height=1000>

3.0.1 Please turn in the code as a notebook AND as a pdf before 10/16/2019 3:00 pm. Please name your notebook netid.ipynb.

3.0.2 Your work will be evaluated based on the code and plots. You don't need to write down your answers to these questions in the text blocks.

[ ]:

```

```