YVES_GREATTI_yg390_lab-week4-student

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1 DS-GA 3001.009 Modeling Time Series Data

2 Week 3 Kalman Filter

[1]:
```
# Install PyKalman
# pip install pykalman
import numpy as np
import matplotlib.pyplot as plt
from pykalman import KalmanFilter
from scipy.stats import multivariate_normal

# Data Visualiztion
def plot_kalman(x,y,nx,ny,kx=None,ky=None, plot_type="r-", label=None,␣
 ↪title='Parabola'):
    """
    Plot the trajectory
    """
    fig, ax = plt.subplots(1,2, figsize=(15,4))
    if kx is not None and ky is not None:
        ax[0].plot(x,y,'g-',nx,ny,'b.',kx,ky, plot_type)
        ax[0].plot(kx[0], ky[0], 'or')
        ax[0].plot(kx[-1], ky[-1], 'xr')

        ax[1].plot(x, kx, '.k', label='latent dim 1')
        ax[1].plot(y, ky, '.', color='grey', label='latent dim 2')
        ax[1].set_xlabel('real latent')
        ax[1].set_ylabel('estimated latent')
        ax[1].legend()
    else:
        ax[0].plot(x,y,'g-',nx,ny,'b.')

        ax[1].plot(x, nx, '.k', label='dim 1')
        ax[1].plot(y, ny, '.', color='grey', label='dim 2')
        ax[1].set_xlabel('latent')
        ax[1].set_ylabel('observed')
        ax[1].legend()
```

```
    ax[0].set_xlabel('X position')
    ax[0].set_ylabel('Y position')
    ax[0].set_title(title)
    ax[0].set_aspect(1)
    ax[1].set_aspect(1)

    if kx is not None and ky is not None and label is not None:
        ax[0].legend(('true','measured', label))
    else:
        ax[0].legend(('true','measured'))

    return fig

def visualize_line_plot(data, xlabel, ylabel, title):
    """
    Function that visualizes a line plot
    """
    plt.plot(data)
    plt.xlabel(xlabel)
    plt.ylabel(ylabel)
    plt.title(title)
    plt.show()

def print_parameters(kf_model, need_params=None, evals=False):
    """
    Function that prints out the parameters for a Kalman Filter
    @param - kf_model : the model object
    @param - need_params : a list of string
    """
    if evals:
        if need_params is None:
            need_params1 = ['transition_matrices', 'transition_covariance',␣
 ↪'observation_covariance', 'initial_state_covariance']
            need_params2 = ['observation_matrices', 'initial_state_mean']
        for param in need_params1:
            tmp = np.linalg.eig(getattr(kf_model, param))[0]
            print("{0} = {1}, shape = {2}\n".format(param, tmp, tmp.shape))
        for param in need_params2:
            print("{0} = {1}, shape = {2}\n".format(param, getattr(kf_model,␣
 ↪param), getattr(kf_model, param).shape))
    else:
        if need_params is None:
            need_params = ['transition_matrices', 'observation_matrices',␣
 ↪'transition_covariance', 'observation_covariance',
                          'initial_state_mean', 'initial_state_covariance']
        for param in need_params:
```

```
        print("{0} = {1}, shape = {2}\n".format(param, getattr(kf_model,␣
↪param), getattr(kf_model, param).shape))
```

2.1 Kalman

We want to infer the latent variable z_n given the observed variable x_n.

$$P(z_n|x_1, ..., x_n, x_{n+1}, ..., x_N) \sim N(\hat{\mu_n}, \hat{V_n})$$

2.1.1 Forward: Filtering

obtain estimates of latent by running the filtering from $n = 0, ...N$

prediction given latent space parameters <img src='./img/LDS_latent.svg', width = 110, height=90>

$$z_n^{pred} \sim N(\mu_n^{pred}, V_n^{pred})$$

$$\mu_n^{pred} = A\mu_{n-1}$$

this is the prediction for z_n obtained simply by taking the expected value of z_{n-1} and projecting it forward one step using the transition probability matrix A

$$V_n^{pred} = AV_{n-1}A^T + \Gamma$$

same for the covariance taking into account the noise covariance Γ

correction (innovation) from observation <img src='img/LDS_observed.svg', width = 40, height=80>

project to observational space:

$$x_n^{pred} \sim N(C\mu_n^{pred}, CV_n^{pred}C^T + \Sigma)$$

correct prediction by actual data:

$$z_n^{innov} \sim N(\mu_n^{innov}, V_n^{innov})$$

$$\mu_n^{innov} = \mu_n^{pred} + K_n(x_n - C\mu_n^{pred})$$

$$V_n^{innov} = (I - K_nC)V_n^{pred}$$

Kalman gain matrix:

$$K_n = V_n^{pred}C^T(CV_n^{pred}C^T + \Sigma)^{-1}$$

we use the latent-only prediction to project it to the observational space and compute a correction proportional to the error $x_n - CAz_{n-1}$ between prediction and data, coefficient of this correction is the Kalman gain matrix

<img src='img/Kfilter_Bishop.png', width = 600, height=600> from Bishop (2006), chapter 13.3

if measurement noise is small and dynamics are fast -> estimation will depend mostly on observed data

2.1.2 Backward: Smoothing

<img src='./img/LDS_smooth.svg', width = 110, height=100>
obtain estimates by propagating from x_n back to x_1 using results of forward pass ($\mu_n^{innov}, V_n^{innov}, V_n^{pred}$)

$$N(z_n|\mu_n^{smooth}, V_n^{smooth})$$

$$\mu_n^{smooth} = \mu_n^{innov} + J_n(\mu_{n+1}^{smooth} - A\mu_n^{innov})$$

$$V_n^{smooth} = V_n^{innov} + J_n(V_{n+1}^{smooth} - V_{n+1}^{pred})J_n^T$$

$$J_N = V_n^{innov} A^T (V_{n+1}^{pred})^{-1}$$

This gives us the final estimate for z_n.

$$\hat{\mu_n} = \mu_n^{smooth}$$
$$\hat{V_n} = V_n^{smooth}$$

2.2 EM algorithm

- want to maximize $logp(x|\theta)$
- need to marginalize out latent *(which is not tractable)*

$$log\left(p(x|\theta)\right) = log\left(\int p(x,z|\theta)dz\right)$$

- add a probability distribution $q(z)$ which will approximate the latent distribution

$$= \int_z q(z) logp(x|\theta)dz$$

- can be rewritten as

$$= \mathcal{L}(q,\theta) + KL\left(q(z)||p(z|x),\theta\right)$$

- $\mathcal{L}(q,\theta)$ contains the joint distribution of x and z
- $KL(q||p)$ contains the conditional distribution of $z|x$

Expectation step

- parameters are kept fixed
- find a good approximation $q(z)$: maximize lower bound $\mathcal{L}(q,\theta)$ with respect to $q(z)$
- (already implemented Kalman filter+smoother)

Maximization step

- keep distribution $q(z)$ fixed
- change parameters to maximize the lower bound $\mathcal{L}(q, \theta)$

2.2.1 M-step

(see Bishop, chapter 13.3.2 Learning in LDS)

Update parameters of the probability distribution

Initial parameters

$$\mu_0^{new} = E(z_1)$$
$$\Gamma_0^{new} = E(z_1 z_1^T) - E(z_1)E(z_1^T)$$

Latent parameters

$$A^{new} = \left(\sum_{n=2}^{N} E(z_n z_{n-1}^T)\right)\left(\sum_{n=2}^{N} E(z_{n-1} z_{n-1}^T)\right)^{-1}$$

$$\Gamma^{new} = \frac{1}{N-1}\sum_{n=2}^{N} E(z_n z_n^T) - A^{new}E(z_{n-1}z_n^T) - E(z_n z_{n-1}^T)A^{new} + A^{new}E(z_{n-1}z_{n-1}^T)(A^{new})^T$$

Observable space parameters

$$C^{new} = \left(\sum_{n=1}^{N} x_n E(z_n^T)\right)\left(\sum_{n=1}^{N} E(z_n z_n^T)\right)^{-1}$$

$$\Sigma^{new} = \frac{1}{N}\sum_{n=1}^{N} x_n x_n^T - C^{new}E(z_n)x_n^T - x_n E(z_n^T)C^{new} + C^{new}E(z_n z_n^T)C_{new}$$

For the updates in the M-step we will need the following posterior marginals obtained from the Kalman smoothing results $\hat{\mu}_n, \hat{V}_n$

$$E(z_n) = \hat{\mu}_n$$
$$E(z_n z_{n-1}^T) = J_{n-1}\hat{V}_n + \hat{\mu}_n\hat{\mu}_{n-1}^T$$
$$E(z_n z_n^T) = \hat{V}_n + \hat{\mu}_n\hat{\mu}_n^T$$

3 Kalman + EM Implementation

In this part of the exercise, you will implement the EM algorithm, building up on the exercises from last week.

[2]:
```
class MyKalmanFilter:
    """
    Class that implements the Kalman Filter
    """
    def __init__(self, n_dim_state=2, n_dim_obs=2):
        """
        @param n_dim_state: dimension of the laten variables
```

```
        @param n_dim_obs: dimension of the observed variables
        """
        self.n_dim_state = n_dim_state
        self.n_dim_obs = n_dim_obs
        self.transition_matrices = np.eye(n_dim_state)
        self.transition_covariance = np.eye(n_dim_state)
        self.observation_matrices = np.eye(n_dim_obs, n_dim_state)
        self.observation_covariance = np.eye(n_dim_obs)
        self.initial_state_mean = np.zeros(n_dim_state)
        self.initial_state_covariance = np.eye(n_dim_state)


    def sample(self, n_timesteps, initial_state=None, random_seed=None):
        """
        Method that gives samples
        @param initial_state: numpy array whose length == self.n_dim_state
        @param random_seed: an integer, for test purpose
        @output state: a 2d numpy array with dimension [n_timesteps, self.
↪n_dim_state]
        @output observation: a 2d numpy array with dimension [n_timesteps, self.
↪n_dim_obs]
        """

        if random_seed is not None:
            np.random.seed(random_seed)

        latent_state = np.zeros([n_timesteps, self.n_dim_state])
        observed_state = np.zeros([n_timesteps, self.n_dim_obs])

        ################
        ##### TODO #####
        ################
        # produce samples

        latent_state[0] = initial_state
        observed_state[0] = np.matmul(self.observation_matrices,␣
↪latent_state[0]) + \
                np.random.multivariate_normal(np.zeros(self.n_dim_obs), self.
↪observation_covariance)

        for i in range(1, n_timesteps):
            latent_state[i] = np.matmul(self.transition_matrices,␣
↪latent_state[i-1]) + \
                np.random.multivariate_normal(np.zeros(self.n_dim_state), self.
↪transition_covariance)
```

```
            observed_state[i] = np.matmul(self.observation_matrices,␣
↪latent_state[i]) + \
                np.random.multivariate_normal(np.zeros(self.n_dim_obs), self.
↪observation_covariance)

        return latent_state, observed_state


    def filter(self, X):
        """
        Method that performs Kalman filtering
        @param X: a numpy 2D array whose dimension is [n_example, self.
↪n_dim_obs]
        @output: filtered_state_means: a numpy 2D array whose dimension is␣
↪[n_example, self.n_dim_state]
        @output: filtered_state_covariances: a numpy 3D array whose dimension␣
↪is [n_example, self.n_dim_state, self.n_dim_state]
        """

        # validate inputs
        n_example, observed_dim = X.shape
        assert observed_dim==self.n_dim_obs

        # create holders for outputs
        filtered_state_means = np.zeros( (n_example, self.n_dim_state) )
        filtered_state_covariances = np.zeros( (n_example, self.n_dim_state,␣
↪self.n_dim_state) )

        # TODO: implement filtering

        # the first state mean and state covar is the initial epectation
        filtered_state_means[0] = self.initial_state_mean
        filtered_state_covariances[0] = self.initial_state_covariance

        # initialize internal variables
        current_state_mean = self.initial_state_mean.copy()
        current_state_covar = self.initial_state_covariance.copy()
        self.p_n_list = np.zeros((n_example, self.n_dim_obs, self.n_dim_obs))
        for i in range(1, n_example):
            current_observed_data = X[i,:]
            # run a single step forward filter
            # prediction step
            predicted_state_mean = np.dot(self.transition_matrices,␣
↪current_state_mean)
```

```
            predicted_state_cov = np.matmul(np.matmul(self.transition_matrices,␣
↪current_state_covar),np.transpose(self.transition_matrices)) + self.
↪transition_covariance
            # observation step
            innovation = current_observed_data - np.dot(self.
↪observation_matrices, predicted_state_mean)
            innovation_covariance = np.matmul(np.matmul(self.
↪observation_matrices, predicted_state_cov), np.transpose(self.
↪observation_matrices)) + self.observation_covariance
            # update step
            kalman_gain = np.matmul(np.matmul(predicted_state_cov, np.
↪transpose(self.observation_matrices)), np.linalg.inv(innovation_covariance))
            current_state_mean = predicted_state_mean + np.dot(kalman_gain,␣
↪innovation)
            current_state_covar = np.matmul( (np.eye(current_state_covar.
↪shape[0])-
                                         np.matmul(kalman_gain,self.
↪observation_matrices)), predicted_state_cov)
            # populate holders
            filtered_state_means[i, :] = current_state_mean
            filtered_state_covariances[i, :, :] = current_state_covar
            self.p_n_list[i, :, :] = predicted_state_cov
        return filtered_state_means, filtered_state_covariances

    def smooth(self, X):
        """
        Method that performs the Kalman Smoothing
        @param X: a numpy 2D array whose dimension is [n_example, self.
↪n_dim_obs]
        @output: smoothed_state_means: a numpy 2D array whose dimension is␣
↪[n_example, self.n_dim_state]
        @output: smoothed_state_covariances: a numpy 3D array whose dimension␣
↪is [n_example, self.n_dim_state, self.n_dim_state]
        """
        # validate inputs
        n_example, observed_dim = X.shape
        assert observed_dim==self.n_dim_obs

        # init for EM
        self.j_n = []

        # run the forward path
        mu_list, v_list = self.filter(X)

        # create holders for outputs
        smoothed_state_means = np.zeros( (n_example, self.n_dim_state) )
```

```
        smoothed_state_covariances = np.zeros( (n_example, self.n_dim_state,␣
↪self.n_dim_state) )

        # last time step doesn't need to be updated
        smoothed_state_means[-1, :] = mu_list[-1, :]
        smoothed_state_covariances[-1, :, :] = v_list[-1, :, :]

        # run the backward path
        # it's zero-indexed and we don't need to update the last elements
        for i in range(n_example-2, -1, -1):
            # used to store intermediate results
            p_i = np.copy(self.p_n_list[i+1, :, :]) # ALTERNATIVELY compute new:
↪ np.matmul(np.matmul(self.transition_matrices, v_list[i,:,:]), self.
↪transition_matrices.T) + self.transition_covariance
            j_i = np.matmul(np.matmul(v_list[i,:,:], self.transition_matrices.
↪T), np.linalg.inv(p_i))

            # calculate mu_bar and v_bar
            current_smoothed_mean = mu_list[i, :] + np.matmul(j_i,␣
↪(smoothed_state_means[i+1, :] - np.matmul(self.transition_matrices,␣
↪mu_list[i, :])))
            current_smoothed_covar = v_list[i,:] + np.matmul(np.matmul(j_i, (␣
↪smoothed_state_covariances[i+1, :, :] - p_i)), j_i.T)
            # propagate the holders
            smoothed_state_means[i, :] = current_smoothed_mean
            smoothed_state_covariances[i, :, :] = current_smoothed_covar
            # note that j_n is REVERSELY propagated from N-2 to 0␣
↪(zero-indexed)
            self.j_n.append(j_i)
        # add the last j_n
        p_N = np.matmul(np.matmul(self.transition_matrices, v_list[-1,:,:]), np.
↪linalg.inv(self.transition_matrices)) + self.transition_covariance
        j_N = np.matmul(np.matmul(v_list[-1,:,:], self.transition_matrices.T),␣
↪np.linalg.inv(p_N))
        self.j_n = list(reversed(self.j_n))
        self.j_n.append(j_N)

        return smoothed_state_means, smoothed_state_covariances

    def em(self, X, max_iter=10):
        """
        This part is OPTIONAL
        Method that perform the EM algorithm to update the model parameters
        Note that in this exercise we ignore offsets
        @param X: a numpy 2D array whose dimension is [n_example, self.
↪n_dim_obs]
```

```
        @param max_iter: an integer indicating how many iterations to run
        """
        # validate inputs have right dimensions
        n_example, observed_dim = X.shape
        assert observed_dim==self.n_dim_obs

        # keep track of log posterior (use function calculate_posterior below)
        self.avg_em_log_posterior = np.zeros(max_iter)*np.nan

        #############################
        #### TODO: EM iterations ####
        #############################

        for i_iter in range(max_iter):

            #1. Expectation Step
            # Smooth step

            smoothed_state_means, smoothed_state_covariances = self.smooth(X)
            self.avg_em_log_posterior[i_iter] = np.nanmean(self.
↪calculate_posterior(X, smoothed_state_means))


            # Update initial states and initial covariance
            self.initial_state_mean = smoothed_state_means[0]
            self.initial_state_covariance = smoothed_state_covariances[0]

            self.e_zn = []
            self.e_zn_znminus = []
            self.e_zn_zn = []
            # Compute E[z], E[zz] and E[zz_1]
            for i in range(n_example):
                self.e_zn.append(smoothed_state_means[i])
                self.e_zn_zn.append(smoothed_state_covariances[i] +
                                    np.outer(smoothed_state_means[i],␣
↪smoothed_state_means[i]))
                if i != 0:
                    self.e_zn_znminus.append(np.matmul(self.j_n[i-1],␣
↪smoothed_state_covariances[i]) + \
                                            np.outer(smoothed_state_means[i],␣
↪smoothed_state_means[i-1]))

            # Maximization Step - Latent Dynamics

            # Compute Sum E[zz] and Sum E[zz_1]
            ezzminus = np.zeros((self.n_dim_state, self.n_dim_state))
            ezz = np.zeros((self.n_dim_state, self.n_dim_state))
```

```
            for t in range(n_example-1):
                ezzminus += self.e_zn_znminus[t]
                ezz += self.e_zn_zn[t]
            ezz += self.e_zn_zn[-1]

            # Compute Sum_{2:N}E[zz] and Sum_{1:N-1}E[zz]
            ezz_minus_n = ezz - self.e_zn_zn[-1]
            ezz_minus_1 = ezz - self.e_zn_zn[0]

            self.transition_matrices = np.matmul(ezzminus, np.linalg.
↪pinv(ezz_minus_n))
            ezzminus_tmp = np.matmul(ezz_minus_n, self.transition_matrices.T)
            self.transition_covariance = ezz_minus_1 - np.matmul(self.
↪transition_matrices, ezzminus)
            - np.matmul(ezzminus, self.transition_matrices) + np.matmul(self.
↪transition_matrices, ezzminus_tmp)
            self.transition_covariance /= (n_example - 1)


            #2. Maximization Step - Observations
            # Compute observation and observation covariance matrices
            x_zn = np.zeros((self.n_dim_obs, self.n_dim_obs))
            for t in range(n_example):
                x_zn += np.outer(X[t], self.e_zn[t])

            self.observation_matrices = np.matmul(x_zn, np.linalg.pinv(ezz.T))

            self.observation_covariance = np.zeros((self.n_dim_obs, self.
↪n_dim_obs))
            for t in range(n_example):
                t1 = np.outer(X[t], X[t])
                t2 = np.outer(self.e_zn[t], X[t])
                t2 = np.matmul(self.observation_matrices, t2)
                t3 = np.outer(X[t], self.e_zn[t])
                t3 = np.matmul(t3, self.observation_matrices)
                t4 = np.matmul(self.observation_matrices,
                         np.matmul(self.e_zn_zn[t],
                                self.observation_matrices.T))
                self.observation_covariance += t1 -t2 -t3 + t4

            self.observation_covariance /= n_example

    def import_param(self, kf_model):
        """
        Method that copies parameters from a trained Kalman Model
        @param kf_model: a Pykalman object
        """
```

```
        need_params = ['transition_matrices', 'observation_matrices',␣
↪'transition_covariance',
                       'observation_covariance', 'initial_state_mean',␣
↪'initial_state_covariance']
        for param in need_params:
            setattr(self, param, getattr(kf_model, param))

    def calculate_posterior(self, X, state_mean, v_n=None):
        """
        Method that calculates the log posterior
        @param X: a numpy 2D array whose dimension is [n_example, self.
↪n_dim_obs]
        @param state_mean: a numpy 2D array whose dimension is [n_example, self.
↪n_dim_state]
        @output: a numpy 1D array whose dimension is [n_example]
        """

        if v_n is None:
            _, v_n = self.filter(X)
        llh = []
        for i in range(1,len(state_mean)):
            normal_mean = np.dot(self.observation_matrices, np.dot(self.
↪transition_matrices, state_mean[i-1]))
            p_n = self.transition_matrices.dot(v_n[i].dot(self.
↪transition_matrices))+self.transition_covariance
            #normal_cov = np.matmul(self.observation_matrices, np.matmul(self.
↪p_n_list[i], self.observation_matrices.T)) + self.observation_covariance
            normal_cov = np.matmul(self.observation_matrices, np.matmul(p_n,␣
↪self.observation_matrices.T)) + self.observation_covariance
            pdf_val = multivariate_normal.pdf(X[i], normal_mean, normal_cov)
            # replace 0 to prevent numerical underflow
            if pdf_val < 1e-10:
                pdf_val = 1e-10
            llh.append(np.log(pdf_val))
        return np.array(llh)
```

3.1 Sampling

[3]:
```
# Sampling
n_dim_state = 2
n_dim_obs = 2
kf = KalmanFilter(n_dim_state=n_dim_state, n_dim_obs=n_dim_obs)
# set paramters
kf.transition_matrices = np.eye(kf.n_dim_state)*.5
kf.transition_covariance = np.eye(kf.n_dim_obs)
kf.observation_matrices = np.eye(kf.n_dim_state)
```

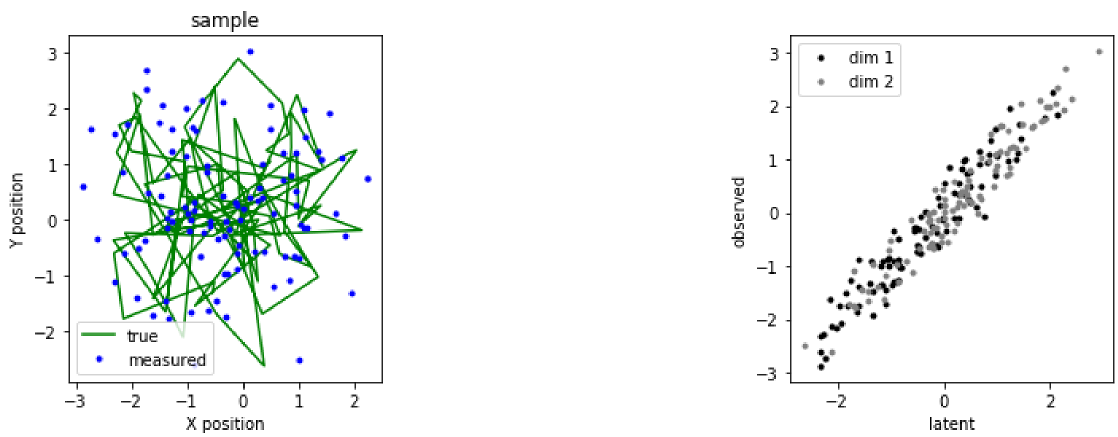
```
kf.observation_covariance = np.eye(kf.n_dim_obs)*.1
kf.initial_state_mean = np.zeros(kf.n_dim_state)
kf.initial_state_covariance = np.eye(kf.n_dim_state)*.1
# import to your own kalman object
my_kf = MyKalmanFilter(n_dim_state=n_dim_state, n_dim_obs=n_dim_obs)
my_kf.import_param(kf)
# print the parameters
print_parameters(my_kf, evals=True)
```

```
transition_matrices = [0.5 0.5], shape = (2,)

transition_covariance = [1. 1.], shape = (2,)

observation_covariance = [0.1 0.1], shape = (2,)

initial_state_covariance = [0.1 0.1], shape = (2,)

observation_matrices = [[1. 0.]
 [0. 1.]], shape = (2, 2)

initial_state_mean = [0. 0.], shape = (2,)
```

3.1.1 test that your sampling works:

[4]:
```
sampled_states, sampled_observations = kf.sample(100, initial_state=kf.
 ↪initial_state_mean, random_state=np.random.RandomState(0))
sampled_states_impl, sampled_observations_impl = my_kf.sample(100,␣
 ↪initial_state=kf.initial_state_mean, random_seed=0)
print('sampled states pykalman at t=2: ', sampled_states[2,:])
print('sampled states own implementation at t=2: ', sampled_states_impl[2,:])
fig = plot_kalman(sampled_states_impl[:,0],sampled_states_impl[:
 ↪,1],sampled_observations_impl[:,0],sampled_observations_impl[:,1],␣
 ↪title='sample');
```

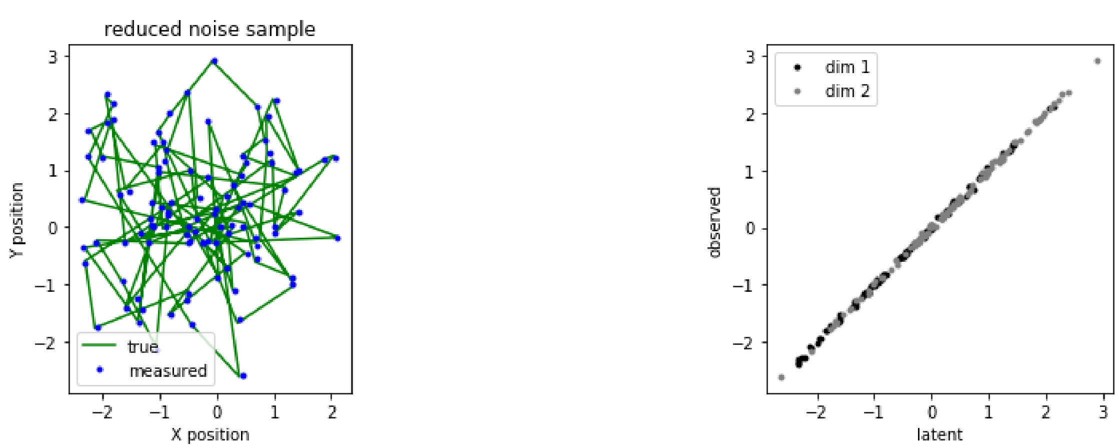
```
sampled states pykalman at t=2:  [1.43945741 0.96908939]
sampled states own implementation at t=2:  [1.43945741 0.96908939]
```

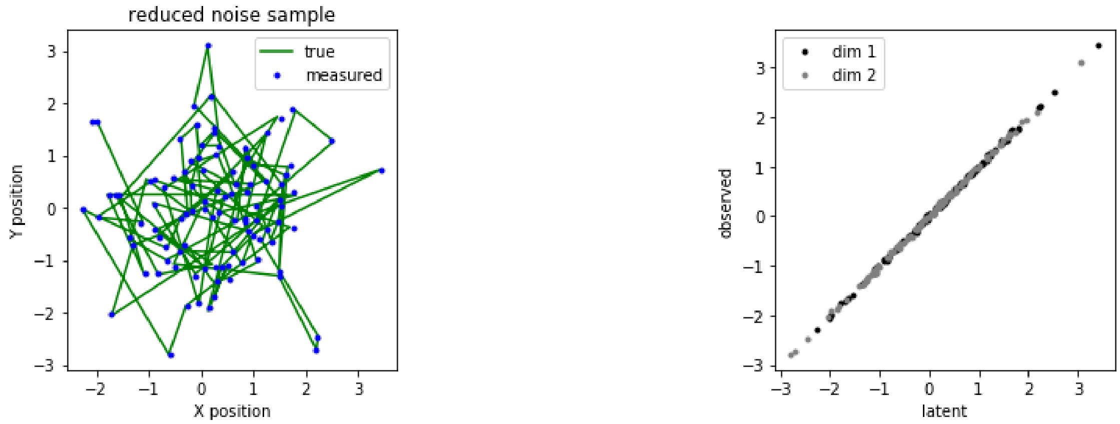


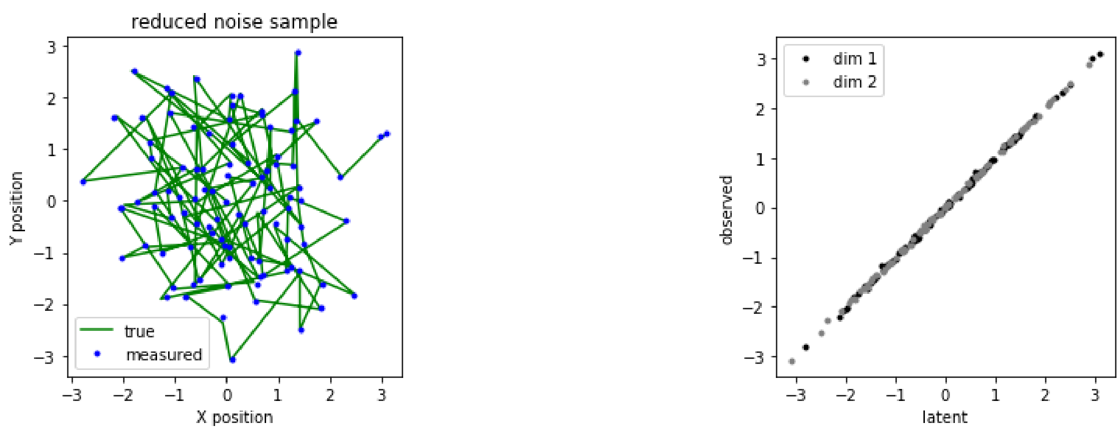
3.1.2 reduce observation noise

What do you expect should happen?

[5]:
```python
# TODO ####
#### reduce observation noise ####
observation_covariance = my_kf.observation_covariance.copy()
my_kf.observation_covariance  = my_kf.observation_covariance * .01

# plot
for nn in range(3):
    sampled_states_impl, sampled_observations_impl = my_kf.sample(100, initial_state=kf.initial_state_mean, random_seed=nn)
    fig = plot_kalman(sampled_states_impl[:,0],sampled_states_impl[:,1],sampled_observations_impl[:,0],sampled_observations_impl[:,1], title='reduced noise sample');
    plt.axis('square');
```





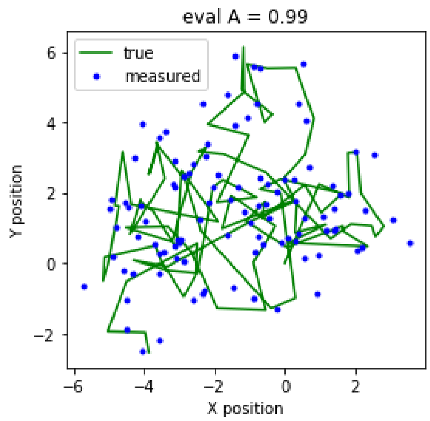


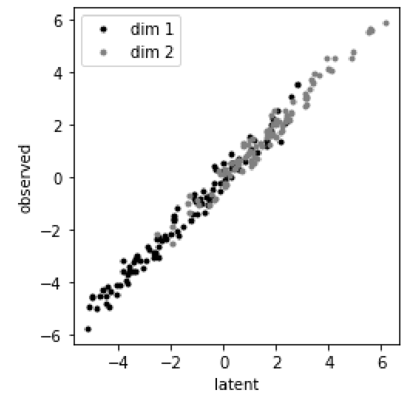
3.1.3 increase the respective temporal dynamics

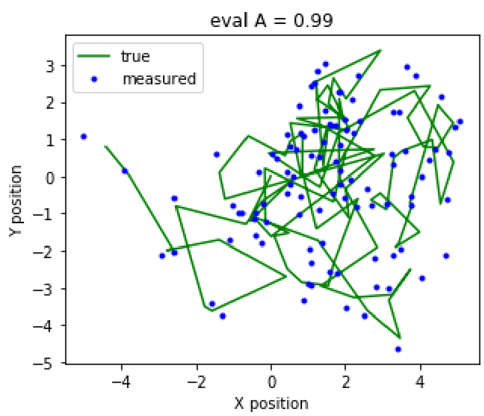
What do you expect should happen?

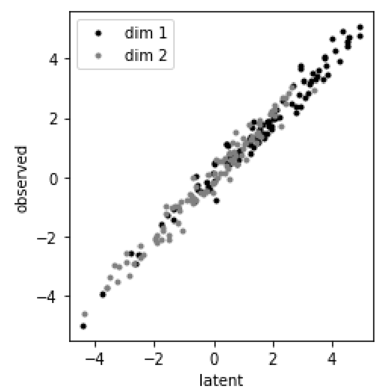
[6]:
```
#### TODO ####
#### increase latent temporal dependency ####

my_kf.observation_covariance = observation_covariance
transition_matrices = my_kf.transition_matrices.copy()
my_kf.transition_matrices = np.eye(my_kf.n_dim_state) * .9

# plot
for nn in range(3):
    sampled_states_impl, sampled_observations_impl = my_kf.sample(100,␣
 ↪initial_state=kf.initial_state_mean, random_seed=nn)
    fig = plot_kalman(sampled_states_impl[:,0],sampled_states_impl[:,1],
```

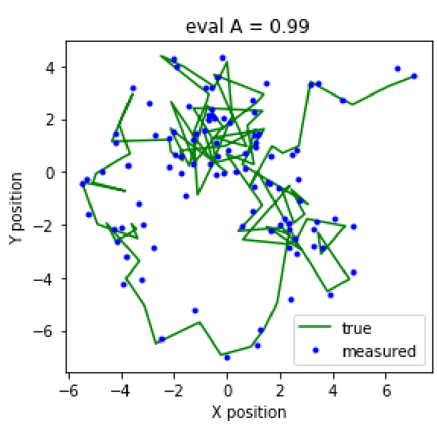
```
                        sampled_observations_impl[:,0],sampled_observations_impl[:
↪,1], title='eval A = '+np.str(.99));
```

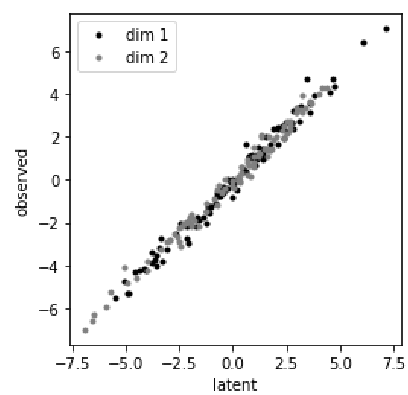










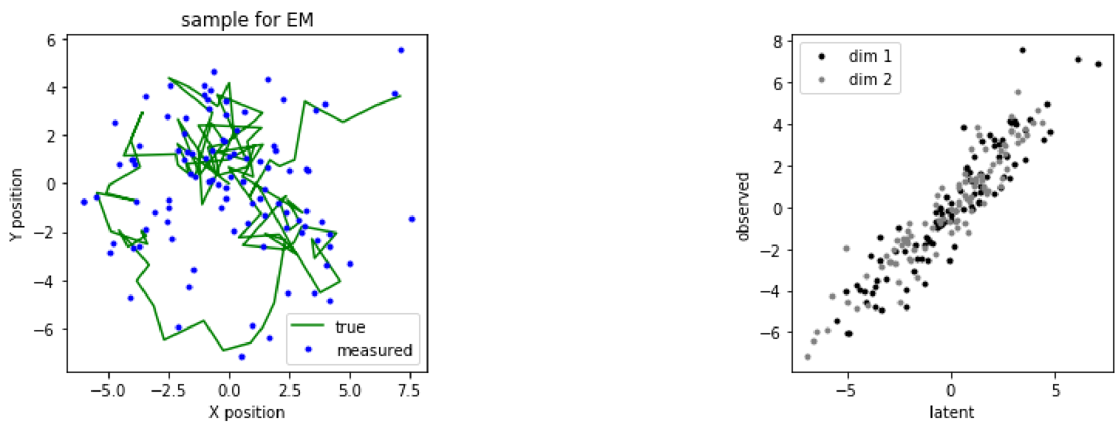


4 EM

4.0.1 data to use

[7]:
```
kf_GT = KalmanFilter(n_dim_state=n_dim_state, n_dim_obs=n_dim_obs)
# set paramters
kf_GT.transition_matrices = np.eye(n_dim_state)*.9
kf_GT.transition_covariance = np.eye(n_dim_obs)
kf_GT.observation_matrices = np.eye(n_dim_state)
kf_GT.observation_covariance = np.eye(n_dim_obs)
kf_GT.initial_state_mean = np.zeros(n_dim_state)
kf_GT.initial_state_covariance = np.eye(n_dim_state)*.1
# import to your own kalman object
my_kf_GT = MyKalmanFilter(n_dim_state=n_dim_state, n_dim_obs=n_dim_obs)
my_kf_GT.import_param(kf_GT)
# print the parameters
print_parameters(my_kf_GT, evals=True)



# sample
latent, data = kf_GT.sample(100, initial_state=kf_GT.initial_state_mean,␣
 ↪random_state=np.random.RandomState(2))
_, _ = kf_GT.filter(data)
estlat,_ = kf_GT.smooth(data)
fig = plot_kalman(latent[:,0],latent[:,1],data[:,0],data[:,1], title='sample␣
 ↪for EM');
```

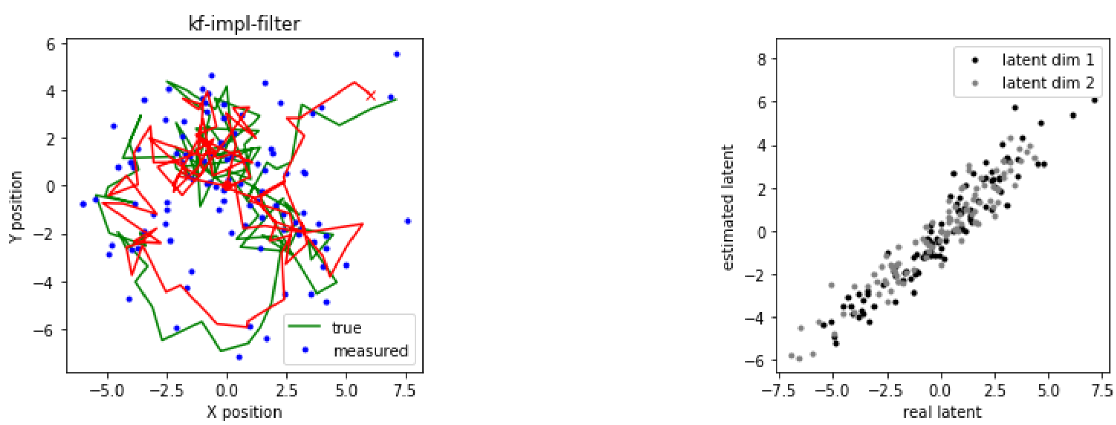
```
transition_matrices = [0.9 0.9], shape = (2,)

transition_covariance = [1. 1.], shape = (2,)

observation_covariance = [1. 1.], shape = (2,)

initial_state_covariance = [0.1 0.1], shape = (2,)

observation_matrices = [[1. 0.]
 [0. 1.]], shape = (2, 2)

initial_state_mean = [0. 0.], shape = (2,)
```



4.0.2 Filtering

with known parameters

[8]:
```
filtered_state_means_impl, filtered_state_covariances_impl = my_kf_GT.filter(data)
fig = plot_kalman(latent[:,0],latent[:,1],data[:,0],data[:,1], filtered_state_means_impl[:,0], filtered_state_means_impl[:,1], "r-", title ="kf-impl-filter")
plt.axis('square');
```

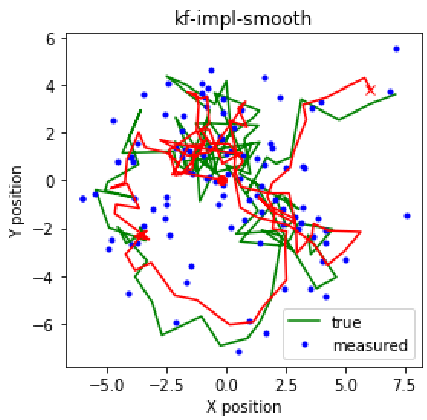


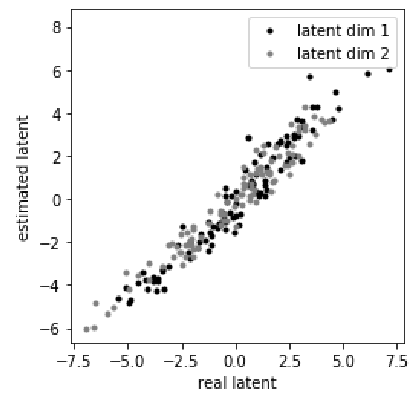
4.0.3 Smoothing

with known parameters

[9]:
```
smoothed_state_means_impl, smoothed_state_covariances_impl = my_kf_GT.smooth(data)
fig = plot_kalman(latent[:,0],latent[:,1],data[:,0],data[:,1],
```

```
                    smoothed_state_means_impl[:,0], smoothed_state_means_impl[:
↪,1], "r-", title="kf-impl-smooth")
plt.axis('square');
```





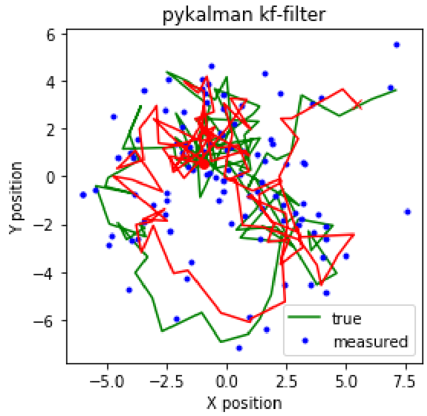
4.0.4 run EM

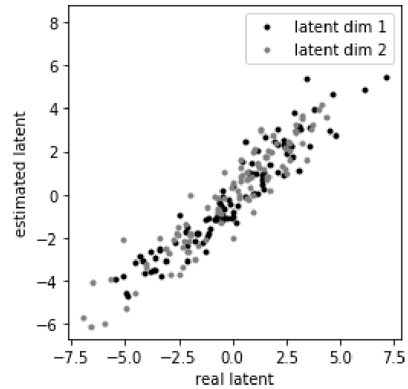
to learn parameters (M-step)

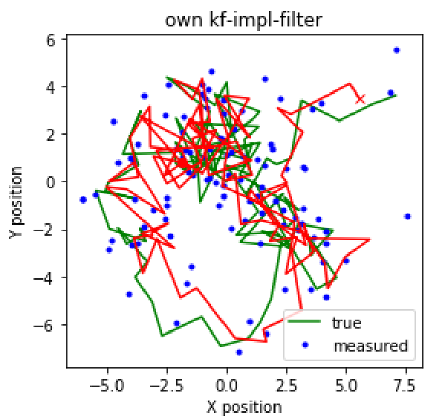
[10]:
```
np.random.seed(0)

iters = 5
# perturb starting parameters
kf  = KalmanFilter(n_dim_state=data.shape[1], n_dim_obs=data.shape[1],
                   transition_matrices= np.eye(data.shape[1]),
                   observation_matrices= np.eye(data.shape[1])+np.random.
↪randn(data.shape[1])*.1,
                   transition_covariance= np.eye(data.shape[1]),
                   observation_covariance = np.eye(data.shape[1]),
                   initial_state_mean=np.random.randn(data.shape[1]),
                   initial_state_covariance = np.eye(data.shape[1]))

my_kf  = MyKalmanFilter(n_dim_state=data.shape[1], n_dim_obs=data.shape[1])
my_kf.import_param(kf)

kf.em(data, n_iter=iters)
my_kf.em(data, max_iter=iters)

print('            pykalman EM:')
print(' ')
print_parameters(kf, evals=True)
print('            own implementation EM:')
print(' ')
print_parameters(my_kf, evals=True)
```

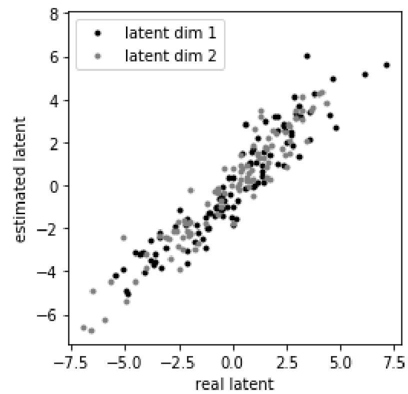
```
          pykalman EM:

transition_matrices = [1. 1.], shape = (2,)

transition_covariance = [0.72625394 1.01738674], shape = (2,)

observation_covariance = [0.80786144 1.16062478], shape = (2,)

initial_state_covariance = [0.0719822  0.11967453], shape = (2,)

observation_matrices = [[1.17640523 0.04001572]
 [0.17640523 1.04001572]], shape = (2, 2)

initial_state_mean = [-1.051357    0.59043655], shape = (2,)

          own implementation EM:

transition_matrices = [0.91603643+0.0620798j 0.91603643-0.0620798j], shape =
(2,)

transition_covariance = [1.35201748+0.74078334j 1.35201748-0.74078334j], shape =
(2,)

observation_covariance = [0.94950739+0.21281532j 0.94950739-0.21281532j], shape
= (2,)

initial_state_covariance = [0.30551733+0.13380899j 0.30551733-0.13380899j],
shape = (2,)

observation_matrices = [[1.08756767 0.09631472]
 [0.10880149 0.95468203]], shape = (2, 2)

initial_state_mean = [-1.87022703  0.79009479], shape = (2,)
```

[11]:
```
# compare the filter results
filtered_state_means, filtered_state_covariances = kf.filter(data)
filtered_state_means_impl, filtered_state_covariances_impl = my_kf.filter(data)
_ = plot_kalman(latent[:,0],latent[:,1],data[:,0],data[:,1],
                filtered_state_means[:,0], filtered_state_means[:,1], "r-",␣
 ↪title="pykalman kf-filter")
plt.axis('square');
_ = plot_kalman(latent[:,0],latent[:,1],data[:,0],data[:,1],
                filtered_state_means_impl[:,0], filtered_state_means_impl[:,1],␣
 ↪"r-", title="own kf-impl-filter")
plt.axis('square');
```

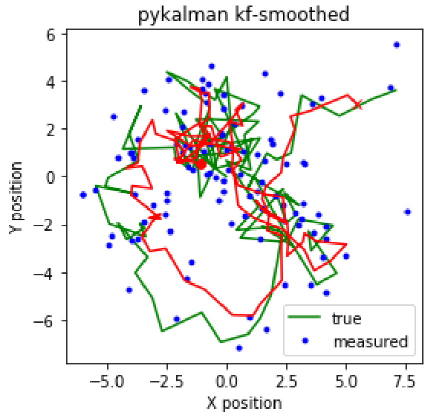


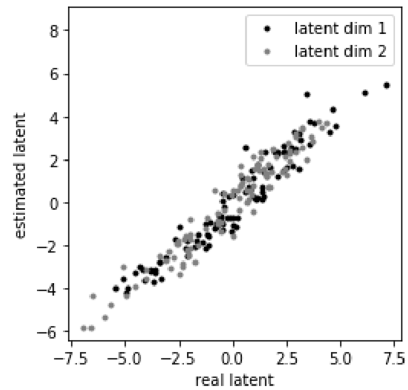


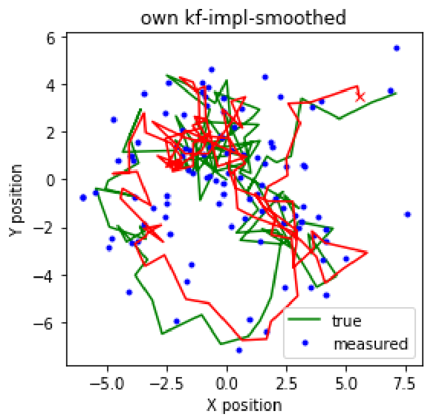


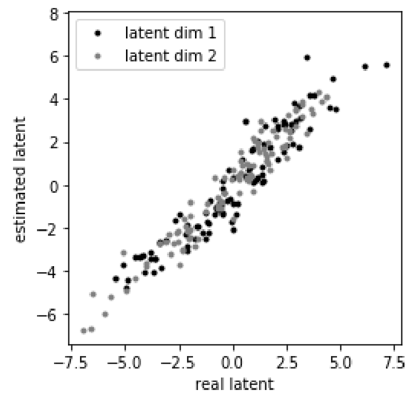


[12]:
```
# compare the smooth results
smoothed_state_means, smoothed_state_covariances = kf.smooth(data)
smoothed_state_means_impl, smoothed_state_covariances_impl = my_kf.smooth(data)
_ = plot_kalman(latent[:,0],latent[:,1],data[:,0],data[:,1],
                smoothed_state_means[:,0], smoothed_state_means[:,1], "r-",␣
 ↪title="pykalman kf-smoothed")
plt.axis('square');
_ = plot_kalman(latent[:,0],latent[:,1],data[:,0],data[:,1],
                smoothed_state_means_impl[:,0], smoothed_state_means_impl[:,1],␣
 ↪"r-", title="own kf-impl-smoothed")
plt.axis('square');
```

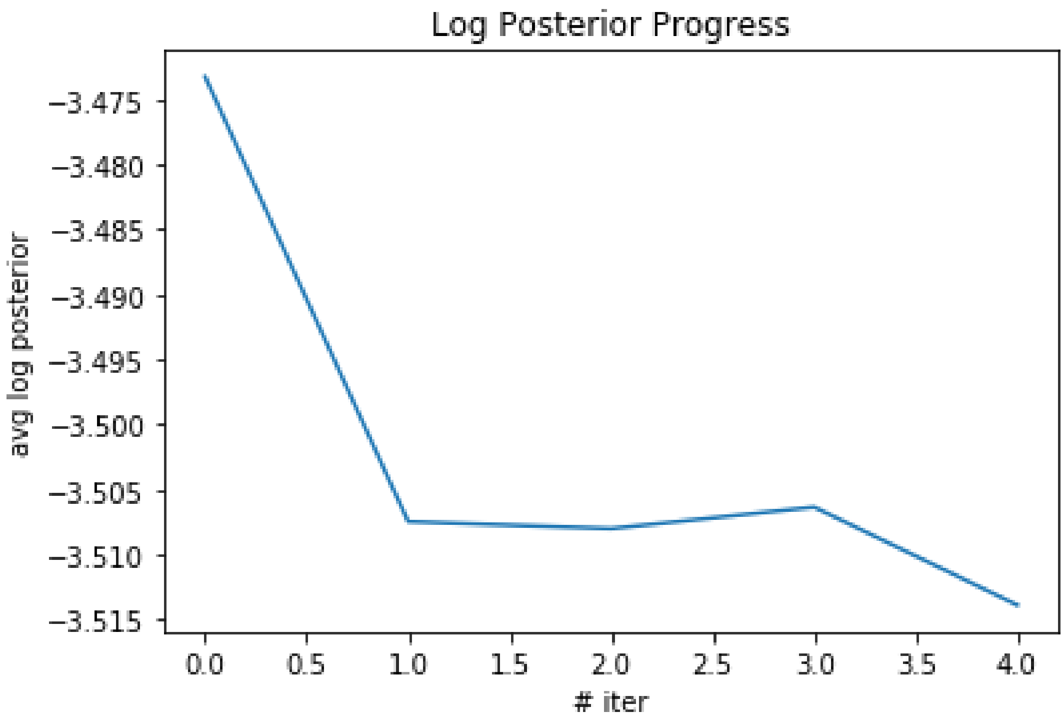






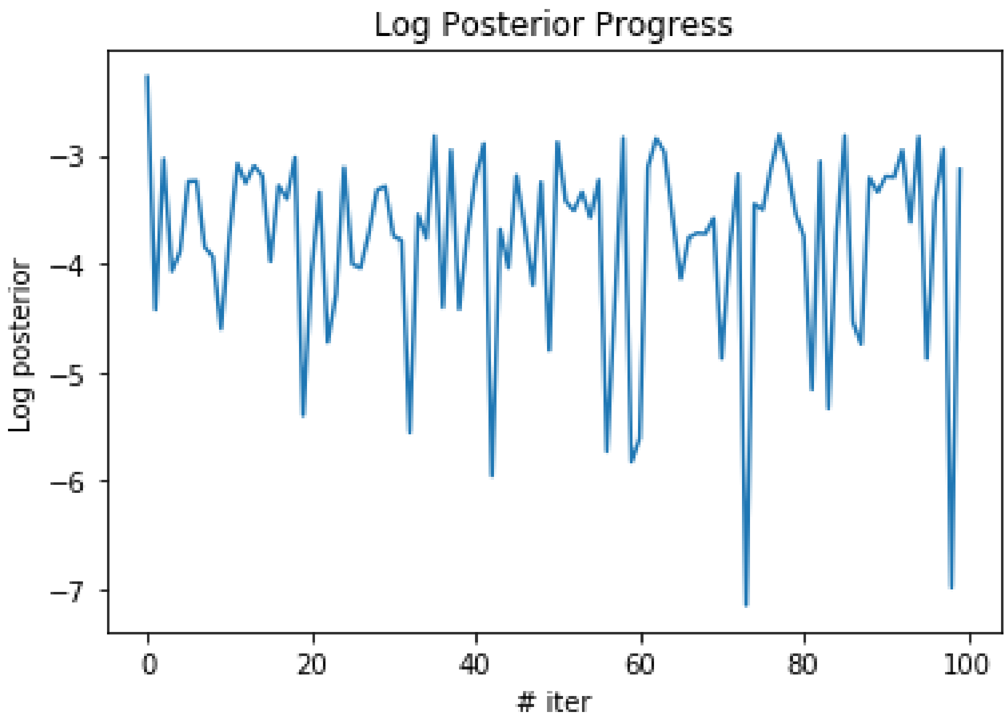


[13]:
```
# visualize the change of avg log posterior
visualize_line_plot(my_kf.avg_em_log_posterior, "# iter", "avg log posterior",␣
 ↪"Log Posterior Progress")
```



5 Comparing to the loglikelhood of pykalman

[14]:
```
logprobs,_ = kf.loglikelihood(data)
visualize_line_plot(logprobs, "# iter", "Log posterior", "Log Posterior␣
↪Progress")
```



5.0.1 Please turn in the code as a notebook AND as a pdf before 09/10/2019 3:00 pm. Please name your notebook netid.ipynb.

5.0.2 Your work will be evaluated based on the code and plots. You don't need to write down your answers to these questions in the text blocks.

[ ]:

```
```