CONTACT MODEL: HERTZ

Contact vector:

$$CP = O_1 + \frac{O_2 - O_1}{r_1 + r_2} \cdot r_1$$

$$\bar{r_c} = CP - O_1$$

$$\bar{r}_n = \frac{\bar{r_c}}{|\bar{r_c}|}$$

Relative velocity:

$$\bar{v}_{rel} = \bar{v}_2 - \bar{v}_1 + \bar{\omega}_1 \times \bar{r_c} - \bar{\omega}_2 \times \bar{r_c}_2$$

$$\bar{v}_{rel,n} = \bar{r}_n \cdot (\bar{r}_n \cdot \bar{v}_{rel})$$

$$\bar{v}_{rel,t} = \bar{v}_{rel} - \bar{v}_{rel,n}$$

Equivalent properties:

$$R^* = \frac{r_1 \cdot r_2}{r_1 + r_2}$$

$$E^* = \left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}\right)^{-1}$$

Normal force:

$$\xi_n = r_1 + r_2 - |O_2 - O_1|$$

$$k_n = 2E^* \sqrt{\xi_n \cdot R^*}$$

$$\bar{F_n} = -\bar{r}_n \cdot \frac{2}{3} \xi_n \cdot k_n$$

Tangential (shear) force:

$$\Delta\bar{\xi}_t = \bar{v}_{rel,t} \cdot \Delta t$$

$$k_t = 8 \cdot G^* \cdot \sqrt{R^* \cdot \xi_n}$$

$$\Delta\bar{F_t} = \left[k_t \cdot \Delta\bar{\xi}_t\right]$$

$$\bar{F}_{t,pr}^{cor} = \bar{F}_{t,pr} - \bar{r}_n \cdot \left(\bar{r}_n \cdot \bar{F}_{t,pr}\right)$$

$$\bar{F}_{t,pr}^{cor} = \bar{F}_{t,pr}^{cor} \cdot |\bar{F}_{t,pr}| / |\bar{F}_{t,pr}^{cor}|$$

$$\overline{F}_t = \overline{F}_{t,pr}^{cor} + \Delta\overline{F}_t$$

if

$$|\overline{F}_t| > \mu_{sl} \cdot |\overline{F}_n|$$

then

$$\overline{F}_t = \mu_{sl} \cdot |\overline{F}_n| \cdot \frac{\overline{F}_t}{|\overline{F}_t|}$$

Rolling friction:

$$\overline{M}_{ro,1} = -\mu_{ro} \cdot |\overline{F}_n| \cdot r_1 \cdot \frac{\overline{\omega}_1}{|\overline{\omega}_1|}$$

$$\overline{M}_{ro,2} = -\mu_{ro} \cdot |\overline{F}_n| \cdot r_2 \cdot \frac{\overline{\omega}_2}{|\overline{\omega}_2|}$$

Summarized forces and moments:

$$\overline{F}_{tot} = \overline{F}_n + \overline{F}_t$$

$$\overline{F}_1 = \overline{F}_n + \overline{F}_t$$

$$\overline{F}_2 = -\overline{F}_n - \overline{F}_t$$

$$\overline{M}_{tot,1} = \overline{r}_n \times \overline{F}_t \cdot r_1 + \overline{M}_{ro,1}$$

$$\overline{M}_{tot,2} = -\overline{r}_n \times \overline{F}_t \cdot r_2 + \overline{M}_{ro,2}$$

Literature

Hertz H. (**1882**). Über die Berührung fester elastischer Körper. *Journal die reine und angewandte Mathematik, 92, 156-171.*

Symbol	Description
$\Delta\bar{\xi}_t$	Increment of tangential displacement on the current step [m]
$\boldsymbol{CP}$	Contact point [m]
$\boldsymbol{E}^*$	Equivalent Young's modulus [Pa]
$\boldsymbol{E_1}, \boldsymbol{E_2}$	Young's moduli of contact partners [Pa]
$\bar{\boldsymbol{F}}_n, \bar{\boldsymbol{F}}_t$	Force in normal and tangential directions [N]
$\bar{\boldsymbol{F}}_{t,pr}$	Tangential force on previous iteration [N]
$\bar{\boldsymbol{F}}_{ro}$	Force due to the rolling friction [N]
$\bar{\boldsymbol{M}}_{ro}$	Moment due to the rolling friction [N]
$\boldsymbol{m_1}, \boldsymbol{m_2}$	Particle masses [kg]
$\boldsymbol{O_1}, \boldsymbol{O_2}$	Centers of contact partners [m]
$\boldsymbol{\mu}_{ro}, \boldsymbol{\mu}_{sl}$	Coefficient of rolling friction and sliding friction [-]
$\bar{\boldsymbol{v}}_{rel}$	Relative velocity [m/s]
$\bar{\boldsymbol{v}}_1, \bar{\boldsymbol{v}}_2$	Translational velocities of contact partners [m/s]
$\boldsymbol{r_1}, \boldsymbol{r_2}$	Particle radii [m]
$\boldsymbol{R}^*$	Equivalent radius [m]
$\bar{\boldsymbol{r}}_c$	Contact vector [m]
$\bar{\boldsymbol{r}}_n$	Normalized contact vector [-]
$\bar{\boldsymbol{\omega}}_1, \bar{\boldsymbol{\omega}}_2$	Rotation velocities of particles [rad/s]
$\boldsymbol{\xi}_n$	Normal overlap [m]