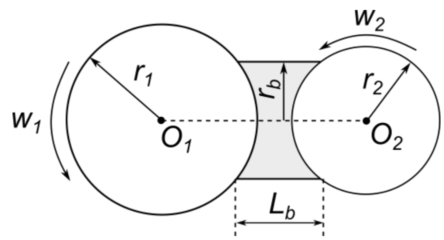
SOLID BOND MODEL: MAXWELL MODEL



Bond properties:

$$L_b = L_1 - \sqrt{r_1^2 - r_b^2} - \sqrt{r_2^2 - r_b^2}$$ *(Recalculated in each step)*

$$I = \frac{\pi \cdot R_b^4}{4}$$

$$J = \frac{\pi \cdot R_b^4}{2}$$

$$A_b = \pi \cdot R_b^2$$

$$k_n = E$$

$$k_t = \frac{E}{2(1+\nu)}$$

$$\tau_n = \eta / k_n$$

$$\tau_t = \eta / k_t$$

Contact vector:

$$\bar{r_c} = O_2 - O_1$$

$$\bar{r_n} = \frac{O_2 - O_1}{|O_2 - O_1|}$$

Velocities:

$$\bar{v}_{rel} = \bar{v}_2 - \bar{v}_1 - \frac{(\bar{\omega}_1 + \bar{\omega}_2) \times \bar{r_c}}{2}$$

$$\bar{\omega}_{rel} = \bar{\omega}_1 - \bar{\omega}_2$$

$$\bar{\omega}_{rel,n} = \bar{r_n} \cdot (\bar{r_n} \cdot \bar{\omega}_{rel})$$

$$\bar{v}_{rel,n} = \bar{r_n} \cdot (\bar{r_n} \cdot \bar{v}_{rel})$$

$$\overline{\omega}_{rel,t} = \overline{\omega}_{rel} - \overline{\omega}_{rel,n}$$

$$\Delta\bar{\delta}_{\omega n,b} = \overline{\omega}_{rel,n} \cdot \Delta t$$

$$\Delta\bar{\delta}_{\omega t,b} = \overline{\omega}_{rel,t} \cdot \Delta t$$

$$\Delta\bar{\delta}_{t} = \vec{v}_{rel,t} \cdot \Delta t$$

Forces and moments:

Forces and moments which are acting at the time point ($\mathrm{t} + \Delta t$) are calculated as increment to the values on the previous time step (t).

Normal direction

$$\bar{F}_{n,b}^{t+\Delta t} = T \cdot \bar{F}_{n,b}^{t} \cdot (1 - \Delta t/\tau_n) + \Delta\bar{\delta}_n \cdot k_n \cdot \frac{A_b}{L_{init}}$$

$$\overline{M}_{n,b}^{t+\Delta t} = T \cdot \overline{M}_{n,b}^{t} + \Delta\vec{\bar{\delta}}_{\omega n,b} \cdot \frac{k_t}{L_{init}} \cdot J$$

Tangential direction

$$\bar{F}_{t,b}^{t+\Delta t} = T \cdot \bar{F}_{t,b}^{t} \cdot (1 - \Delta t/\tau_t) + \Delta\bar{\delta}_t \cdot k_t \frac{A_b}{L_{init}} \quad ;$$

$$\overline{M}_{t,b}^{t+\Delta t} = T \cdot \overline{M}_{t,b}^{t} + \Delta\vec{\bar{\delta}}_{\omega t,b} \cdot \frac{k_n}{L_{init}} \cdot I$$

$$M_{tot} = M_{t,b} + M_{n,b} + \frac{r_c}{2} \cdot F_{t,b}$$

Breakage criteria

The model has parameter consider breakage (1 – yes, 0 – no). If 0 value is specified, then no breakage can occurs. This breakage analysis can be disabled to improve computational performance.

$$\frac{F_{n,b}}{A_b} + \frac{M_{t,b} \cdot R_b}{I} = \sigma_{max}$$

$$\frac{F_{t,b}}{A_b} + \frac{M_{n,b} \cdot R_b}{J} = \tau_{max}$$

Literature

This model is a part of current research work – results will be published.

Symbol	Description
$\boldsymbol{A_b}$	Cross-cut surface of the bond [m^2]
$\Delta\bar{\boldsymbol{\delta}}_{\boldsymbol{\omega n,b}}, \Delta\bar{\boldsymbol{\delta}}_{\boldsymbol{\omega t,b}}$	Increment of displacement (in current step) between particles in contact point due to the rotational velocities [m]
$\Delta\bar{\boldsymbol{\delta}}_{\boldsymbol{t}}$	Tangential displacement in the current step [m]
$\boldsymbol{\eta}$	Dynamic viscosity [Pa s]
$\boldsymbol{E}$	Young modulus for particle or bond [Pa]
$\boldsymbol{I}, \boldsymbol{J}$	Moments of inertia of the bond [m^3]
$\boldsymbol{k_n}, \boldsymbol{k_t}$	Spring constant [Pa]
$\boldsymbol{L_b}$	Current bond length
$\boldsymbol{L_{init}}$	Initial bond length [m]
$\boldsymbol{O_1}, \boldsymbol{O_2}$	Centers of contact partners [m]
$\boldsymbol{r_1}, \boldsymbol{r_2}$	Particle radii [m]
$\boldsymbol{r_b}$	Bond radius [m]
$\boldsymbol{T}$	Transformation matrix (to consider rotation of the bond) [-]
$\bar{\boldsymbol{\omega}}_{\boldsymbol{1}}, \bar{\boldsymbol{\omega}}_{\boldsymbol{2}}$	Rotation velocities of particles [rad/s]