CONTACT MODEL: JKR

Velocities:

$$\bar{v}_{rel} = \bar{v}_2 - \bar{v}_1 + \bar{\omega}_1 \times \bar{r_c} - \bar{\omega}_2 \times \bar{r_c}_2$$

$$\bar{v}_{rel,n} = \bar{r}_n \cdot (\bar{r}_n \cdot \bar{v}_{rel})$$

$$\bar{v}_{rel,t} = \bar{v}_{rel} - \bar{v}_{rel,n}$$

Additional parameters:

$$\gamma_{12} = \sqrt{\gamma_1 \gamma_2}$$

$$R^* = \frac{r_1 \cdot r_2}{r_1 + r_2}$$

$$M^* = \frac{m_1 \cdot m_2}{m_1 + m_2}$$

Normal force:

$$R_c = \sqrt{\xi_n R^*}$$

$$F_n^{JKR} = \frac{4R_c^3 E^*}{3R^*} - \sqrt{8\pi E^* \gamma_{12} R_c^3}$$
$$-\bar{r}_n \cdot sgn\left(\bar{v}_{rel,n} \cdot \bar{r}_n\right) \cdot 1.8257 \cdot \alpha \cdot |\bar{v}_{rel,n}| \cdot \sqrt{k_n \cdot M^*}$$

Tangential force:

$$\Delta\bar{\xi}_t = \bar{v}_{rel,t} \cdot \Delta t$$

$$k_t = 8 \cdot G^* \cdot \sqrt{R^* \cdot \xi_n}$$

$$\Delta\bar{F}_t = \left[k_t \cdot \Delta\bar{\xi}_t - 1.8257 \cdot \alpha \cdot \bar{v}_{rel.t} \cdot \sqrt{k_t \cdot M^*}\right]$$

$$\bar{F}_{t,pr}^{cor} = \bar{F}_{t,pr} - \bar{r}_n \cdot \left(\bar{r}_n \cdot \bar{F}_{t,pr}\right)$$

$$\bar{F}_{t,pr}^{cor} = \bar{F}_{t,pr}^{cor} \cdot |\bar{F}_{t,pr}| / |\bar{F}_{t,pr}^{cor}|$$

$$\bar{F}_t = \bar{F}_{t,pr}^{cor} + \Delta\bar{F}_t$$

if

$$|\bar{F}_t| > \mu_{sl} \cdot |\bar{F}_n|$$

then

$$\bar{F}_t = \mu_{sl} \cdot |\bar{F}_n| \cdot \frac{\bar{F}_t}{|\bar{F}_t|}$$

Rolling friction:

$$\bar{M}_{ro,1} = -\mu_{ro} \cdot |\bar{F}_n| \cdot r_1 \cdot \frac{\bar{\omega}_1}{|\bar{\omega}_1|}$$

$$\bar{M}_{ro,2} = -\mu_{ro} \cdot |\bar{F}_n| \cdot r_2 \cdot \frac{\bar{\omega}_2}{|\bar{\omega}_2|}$$

Summarized forces and moments acting on particle:

$$\bar{F}_{tot} = \bar{F}_n + \bar{F}_t$$

$$\bar{F}_1 = \bar{F}_n + \bar{F}_t$$

$$\bar{F}_2 = -\bar{F}_n - \bar{F}_t$$

$$\bar{M}_{tot,1} = \bar{r}_n \times \bar{F}_t \cdot r_1 + \bar{M}_{ro,1}$$

$$\bar{M}_{tot,2} = -\bar{r}_n \times \bar{F}_t \cdot r_2 + \bar{M}_{ro,2}$$

Literature

Johnson K.L., Kendall K., Roberts A.D. (1971). Surface energy and the contact of elastic solids, A. Math. Phys. Sci. 324.

Symbol	Description
$\Delta\bar{\xi}_t$	Tangential displacement in the current step [m]
$\boldsymbol{CP}$	Contact point [m]
γ_{surf}	Surface energy [J/m^2]
$\bar{F}_n, \bar{F}_t$	Force in normal and tangential directions [N]
$\bar{F}_{t,pr}$	Tangential force on previous iteration [N]
G^*	Equivalent shear modulus [Pa]
$\bar{M}_{ro}$	Moment due to the rolling friction [N]
m_1, m_2	Particle masses [kg]
M^*	Equivalent mass [kg]
O_1, O_2	Centers of contact partners [m]
μ_{ro}, μ_{sl}	Coefficient of rolling friction and sliding friction [-]
$\bar{v}_{rel}$	Relative velocity [m/s]
$\bar{v}_1, \bar{v}_2$	Velocities of contact partners [m/s]
r_1, r_2	Particle radii [m]
R^*	Equivalent radius [m]
$\bar{r}_c$	Contact vector [m]
$\bar{r}_n$	Normalized contact vector [-]
$\bar{\omega}_1, \bar{\omega}_2$	Rotation velocities of particles [rad/s]
ξ_n	Normal overlap [m]