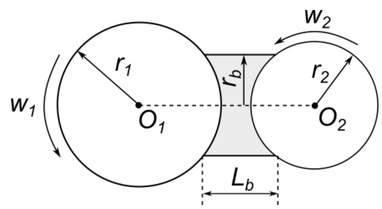
Solid Bond Model: Kelvin-Voigt model



Bond properties:

$$L_b = L_1 - \sqrt{r_1^2 - r_b^2} - \sqrt{r_2^2 - r_b^2} \quad \textit{(Recalculated in each step)}$$

$$I = \frac{\pi \cdot R_b^4}{4}$$

$$J = \frac{\pi \cdot R_b^4}{2}$$

$$A_b = \pi \cdot R_b^2$$

Contact vector:

$$\bar{r}_c = O_2 - O_1$$

$$\bar{r}_n = \frac{O_2 - O_1}{|O_2 - O_1|}$$

Velocities:

$$\bar{v}_{rel} = \bar{v}_2 - \bar{v}_1 - \frac{(\bar{\omega}_1 + \bar{\omega}_2) \times \bar{r}_c}{2}$$

$$\bar{\omega}_{rel} = \bar{\omega}_1 - \bar{\omega}_2$$

$$\bar{\omega}_{rel,n} = \bar{r}_n \cdot (\bar{r}_n \cdot \bar{\omega}_{rel})$$

$$\bar{v}_{rel,n} = \bar{r}_n \cdot (\bar{r}_n \cdot \bar{v}_{rel})$$

$$\bar{\omega}_{rel,t} = \bar{\omega}_{rel} - \bar{\omega}_{rel,n}$$

$$\Delta\bar{\delta}_{\omega n,b} = \bar{\omega}_{rel,n} \cdot \Delta t$$

$$\Delta\bar{\delta}_{\omega t,b} = \bar{\omega}_{rel,t} \cdot \Delta t$$

$$\Delta\bar{\delta}_t = \vec{v}_{rel,t} \cdot \Delta t$$

Force and moment in normal direction:

$$\bar{F}_{n,b} = \bar{r}_n \times (L_b - L_{init}) \cdot \frac{E}{L_{init}} \cdot A_b$$

$$\bar{F}_{n,diss} = \frac{\bar{v}_{rel,n}}{L_{init}} \cdot \eta \cdot A_b$$

$$\bar{M}_{n,b} = T \cdot \bar{M}_{n,b} + \Delta\vec{\delta}_{\omega n,b} \cdot \frac{E}{2L_{init}(1+\nu)} \cdot J$$

Force and moment in shear direction:

$$\bar{F}_{t,b} = T \cdot \bar{F}_{t,b} + \Delta\bar{\delta}_t \cdot \frac{E}{2L_{init}(1+\nu)} \cdot A_b$$

$$\bar{F}_{t,diss} = \frac{\bar{v}_{rel,t}}{L_{init}} \cdot \eta \cdot A_b$$

$$\bar{M}_{t,b} = T \cdot \bar{M}_{t,b} + \Delta\vec{\delta}_{\omega t,b} \cdot \frac{E}{L_{init}} \cdot I$$

Total force:

$$\bar{F}_{t,tot} = \bar{F}_{t,b} + \bar{F}_{t,diss}$$

$$\bar{F}_{n,tot} = \bar{F}_{n,b} + \bar{F}_{n,diss}$$

Breakage criteria:

$$\frac{F_{n,tot}}{A_b} + \frac{M_{t,b} \cdot R_b}{I} = \sigma_{max}$$

$$\frac{F_{t,tot}}{A_b} + \frac{M_{n,b} \cdot R_b}{J} = \tau_{max}$$

Literature

This model is a part of current research work – results will be published.

Symbol	Description
$\boldsymbol{A_b}$	Cross-cut surface of the bond [m^2]
$\Delta\overline{\boldsymbol{\delta}}_{\boldsymbol{\omega n,b}}, \Delta\overline{\boldsymbol{\delta}}_{\boldsymbol{\omega t,b}}$	Increment of displacement (in current step) between particles in contact point due to the rotational velocities [m]
$\Delta\overline{\boldsymbol{\delta}}_{\boldsymbol{t}}$	Tangential displacement in the current step [m]
$\boldsymbol{\eta}$	Dynamic viscosity [Pa s]
$\boldsymbol{E}$	Young modulus for particle or bond [Pa]
$\boldsymbol{I}, \boldsymbol{J}$	Moments of inertia of the bond [m^3]
$\boldsymbol{L_b}$	Current bond length
$\boldsymbol{L_{init}}$	Initial bond length [m]
$\boldsymbol{O_1}, \boldsymbol{O_2}$	Centers of contact partners [m]
$\boldsymbol{r_1}, \boldsymbol{r_2}$	Particle radii [m]
$\boldsymbol{r_b}$	Bond radius [m]
$\boldsymbol{T}$	Transformation matrix (to consider rotation of the bond in the global coordinate system) [-]
$\overline{\boldsymbol{\omega}}_{\boldsymbol{1}}, \overline{\boldsymbol{\omega}}_{\boldsymbol{2}}$	Rotation velocities of particles [rad/s]