De-convolution and De-nosing by Non-Local Mean Prior Regularization with Iterative Gradient Descent Method

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1 Deblurring by Total Variation Regularization

Blurry and Gaussian Noisy Image:

$$y = k * u + n = Ku + n \tag{1}$$

$$u^* \in \arg\min_u ||y - Ku||^2 + \lambda J(u) \tag{2}$$

$$E(u) = \lambda \int ||y - k * u||^2 + J(u) \tag{3}$$

Total Variation Regularization:

$$J(u) = \sum ||\nabla u|| \tag{4}$$

or

$$J(u) = \sum \sqrt{||\nabla u||^2 + \epsilon} \tag{5}$$

2 Solver: Iterative Gradient Descent Method

Gradient Descent:

$$u^{(i+1)} = u^{(i)} - \tau \left(k * (k * u^{(i)} - y) + \lambda GradJ(u^{(i)}) \right) \tag{6}$$

Where,

- The step size τ can be estimated by line search.
- The gradient of the TV term is:

$$GradJ(u) = -div \left(\frac{\nabla u}{\sqrt{||\nabla u||^2 + \epsilon}} \right) \tag{7}$$

  - The gradient of u can be computed as

$$\nabla u = [u_x, u_y] \tag{8}$$

$$u_x = u_{i+1,j} - u_{i,j}$$

$$u_y = u_{i,j+1} - u_{i,j}$$

– Euclidean Norm:

$$||\nabla u|| = \sqrt{u_x^2 + u_y^2} \tag{9}$$

– The divergence of the field is the following scheme:

$$div(u) = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \tag{10}$$

$$(div(u))_{i,j} = \begin{cases} u_x(i,j) - u_x(i-1,j), & \text{if 1<i<n;} \\ u_x(i,j), & \text{if i=1;} \\ -u_x(i-1,j), & \text{if i=n.} \end{cases}$$

$$+ \begin{cases} u_y(i,j) - u_y(i,j-1), & \text{if 1<j<n;} \\ u_y(i,j), & \text{if j=1;} \\ -u_y(i,j-1), & \text{if j=n.} \end{cases}$$

3 De-convolution and De-noising by Using Non-Local Mean Prior

$$J(x) = 2\lambda \sum w_{i,j}(x_i - x_j)^2 \tag{11}$$

where, weights $w_{i,j}$ is from non-local mean(NLM) algorithm.

4 Solver: Iterative Gradient Descent Method

Partial Derivatives:

$$\frac{\partial E}{\partial x_i} = 2\Big[k * \Big([k * x^{(n)}]_i - y_i\Big)\Big]_i + 2\lambda \sum_j w_{i,j}(x_i^{(n)} - x_j^{(n)}) \tag{12}$$

Gradient Descent:

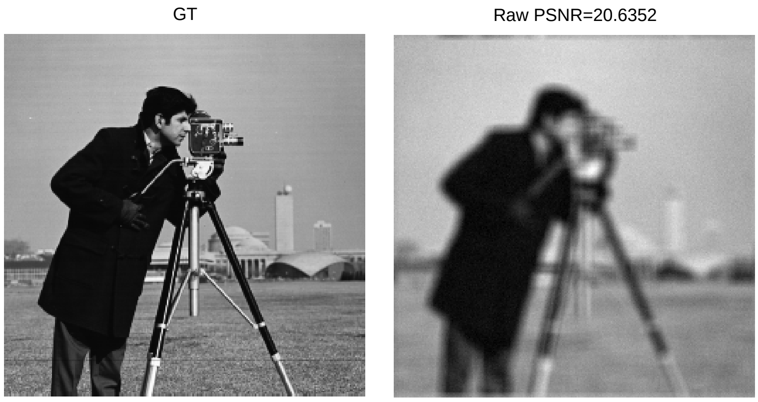
$$x_i^{(n+1)} = x_i^{(n)} - \tau \left(\Big[k * \Big([k * x^{(n)}]_i - y_i\Big)\Big]_i + \lambda \sum_j w_{i,j}(x_i^{(n)} - x_j^{(n)})\right) \tag{13}$$

5 Convergence

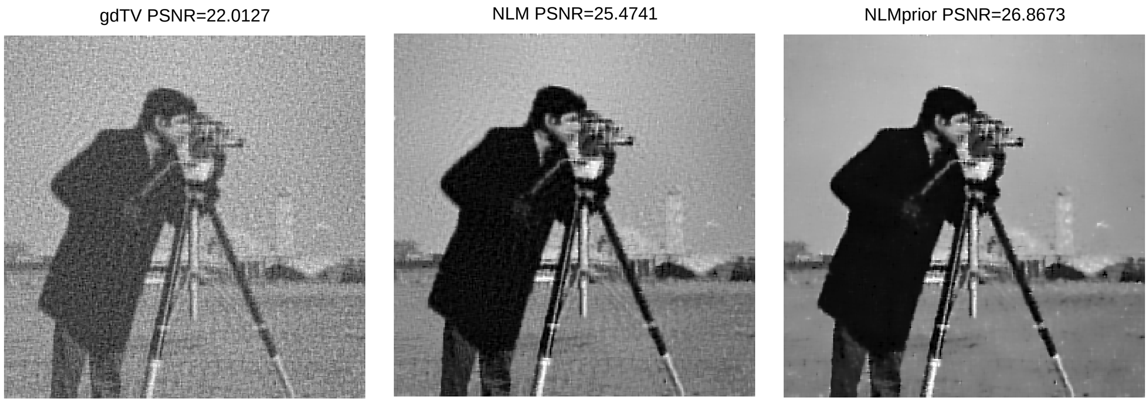
$$E(u) = \lambda \int ||y - k * u||^2 + J(u) \tag{14}$$

$$|E(u^{i+1}) - E(u^i)| < \epsilon \tag{15}$$

6 Results



(a) Ground Truth (b) Blurry and Noisy Image



(c) Initial image by TV prior (d) NLM (e) Restoration by NLM prior

Figure 1: Comparison: Blur kernel average(9x9), Gauss Noisy sigma=3