

Deconvolution by Total Variation Regularization and Iterative Gradient Descent Method

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1 Deblurring and Denoising by Total Variation Regularization

Blurry and Gaussian Noisy Image:

$$y = k * u + n = Ku + n \quad (1)$$

$$u^* \in \arg \min_u ||y - Ku||^2 + \lambda J(u) \quad (2)$$

$$E(u) = \lambda \int ||y - k * u||^2 + J(u) \quad (3)$$

Total Variation Regularization:

$$J(u) = \sum ||\nabla u|| \quad (4)$$

or

$$J(u) = \sum \sqrt{||\nabla u||^2 + \epsilon} \quad (5)$$

2 Solver: Iterative Gradient Descent Method

Gradient Descent:

$$u^{(i+1)} = u^{(i)} - \tau \left(k * (k * u^{(i)} - y) + \lambda \text{Grad}J(u^{(i)}) \right) \quad (6)$$

Where,

- The step size τ can be estimated by line search.
- The gradient of the TV term is:

$$\text{Grad}J(u) = -\text{div} \left(\frac{\nabla u}{\sqrt{||\nabla u||^2 + \epsilon}} \right) \quad (7)$$

– The gradient of u can be computed as

$$\nabla u = [u_x, u_y] \quad (8)$$

$$u_x = u_{i+1,j} - u_{i,j}$$

$$u_y = u_{i,j+1} - u_{i,j}$$

– Euclidean Norm:

$$||\nabla u|| = \sqrt{u_x^2 + u_y^2} \quad (9)$$

– The divergence of the field is the following scheme:

$$div(u) = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \quad (10)$$

$$(div(u))_{i,j} = \begin{cases} u_x(i,j) - u_x(i-1,j), & \text{if } 1 < i < n; \\ u_x(i,j), & \text{if } i=1; \\ -u_x(i-1,j), & \text{if } i=n. \end{cases}$$

$$+ \begin{cases} u_y(i,j) - u_y(i,j-1), & \text{if } 1 < j < n; \\ u_y(i,j), & \text{if } j=1; \\ -u_y(i,j-1), & \text{if } j=n. \end{cases}$$

3 Convergence

$$E(u) = \lambda \int ||y - k * u||^2 + J(u) \quad (11)$$

$$|E(u^{i+1}) - E(u^i)| < \epsilon \quad (12)$$