Deconvolution and Denosing by Total Variation Regularization and Iterative Gradient Descent Method

Yingying Gu

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1 Deblurring and Denoising by Total Variation Regularization

Blurry and Noisy Image:

$$y = k * u + n = Ku + n \tag{1}$$

$$u^* \in \arg\min_u ||y - Ku||^2 + \lambda J(u) \tag{2}$$

$$E(u) = \lambda \int ||y - k * u||^2 + J(u) \tag{3}$$

Total Variation Regularization:

$$J(u) = \sum ||\nabla u|| \tag{4}$$

or

$$J(u) = \sum \sqrt{||\nabla u||^2 + \epsilon} \tag{5}$$

2 Solver: Iterative Gradient Descent Method

Gradient Descent:

$$u^{(i+1)} = u^{(i)} - \tau \left(k * (k * u^{(i)} - y) + \lambda GradJ(u^{(i)}) \right) \tag{6}$$

Where,

- The step size τ should be smaller than twice the Lipschitz constant of the Gradient of the functional to be minimized, hence:

$$\tau < \frac{2}{1 + \lambda * 8/\epsilon} \tag{7}$$

- The gradient of the TV term is:

$$GradJ(u) = -div \left(\frac{\nabla u}{\sqrt{||\nabla u||^2 + \epsilon}} \right) \tag{8}$$

– The gradient of u can be computed as

$$\nabla u = [u_x, u_y] \tag{9}$$

$$u_x = u_{i+1,j} - u_{i,j}$$

$$u_y = u_{i,j+1} - u_{i,j}$$

– Euclidean Norm:

$$||\nabla u|| = \sqrt{u_x^2 + u_y^2} \tag{10}$$

– The divergence of the field is the following scheme:

$$div(u) = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \tag{11}$$

$$(div(u))_{i,j} = \begin{cases} u_x(i,j) - u_x(i-1,j), & \text{if 1<i<n;} \\ u_x(i,j), & \text{if i=1;} \\ -u_x(i-1,j), & \text{if i=n.} \end{cases}$$

$$+ \begin{cases} u_y(i,j) - u_y(i,j-1), & \text{if 1<j<n;} \\ u_y(i,j), & \text{if j=1;} \\ -u_y(i,j-1), & \text{if j=n.} \end{cases}$$

3 Convergence

$$E(u) = \lambda \int ||y - k * u||^2 + J(u) \tag{12}$$

$$|E(u^{i+1}) - E(u^i)| < \epsilon \tag{13}$$