



Exercise sheet 2 (May 13, 2019)

**Construction of Reduction, Interpolation and Hodge operators
on a periodic 1D grid of $[0, 1]$**

Consider a periodic 1D interval $[0, 1]$. The primal grid consists of the nodes of a uniform grid of N cells and the dual grid consists of the midpoints of the cells.

We aim at constructing interpolation operators for point values and cell integrals on the primal and dual grids, with the help of centred Lagrange interpolation.

This exercise can be done with the class `lagrange` from `scipy.interpolate`, in particular the `deriv` and `integ` methods.

1. Interpolation of point values on the primal grid ($\mathcal{C}_0 \rightarrow V_0$) and on the dual grid ($\tilde{\mathcal{C}}_0 \rightarrow \tilde{V}_0$).
2. Interpolation of cell integrals on the primal grid ($\mathcal{C}_1 \rightarrow V_1$) and on the dual grid ($\tilde{\mathcal{C}}_1 \rightarrow \tilde{V}_1$).
3. Using this interpolations and the adequate reduction operators, construct the Hodge operators \mathbb{H}
4. Test the reduction, interpolation and Hodge operators on some examples, eg. polynomials that should be exactly preserved. Smooth functions, check the order.