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Zentrum Mathematik

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Structure preserving methods on staggered grids, SS 2019

http://www-m16.ma.tum.de/Allgemeines/StructPresStag19

Exercise sheet 1 (May 6, 2019)

Maxwell's equations on a periodic 1D grid of $[0, L]$

Consider the periodic 1D interval $[0, L]$. The primal grid consists of the nodes of a uniform grid of N cells and the dual grid consists of the midpoints of the cells.

1. Implement and test splitting methods of order 2 (Strang), 4 (triple jump) and 6 for a system of two ordinary differential equations. The splitting coefficients will be provided for the higher order methods.

   - Test 1: Check the order of convergence of the methods on the system

   $$\frac{\mathrm{d}x}{\mathrm{d}t} = y,$$
   $$\frac{\mathrm{d}y}{\mathrm{d}t} = -x,$$

   for which an exact solution should be computed
   - Propose another test and implement it.

2. Consider the homogeneous 1D Maxwell equations on our periodic domain

   $$\frac{\partial E}{\partial t} + \frac{\partial B}{\partial x} = 0,$$
   $$\frac{\partial B}{\partial t} + \frac{\partial E}{\partial x} = 0$$

   with initial conditions $E_0(x)$, $B_0(x)$.

   - Find an exact solution based on the form $\cos(\omega t + kx)$, $\sin(\omega t + kx)$ and write a test based on solutions of this form.
   - Check that the fully discrete scheme (2.18)-(2.19) corresponds to the second order semi-discrete scheme with a time discretisation based on the Strang splitting.
   - Implement and test the order of convergence of this second order scheme.
   - Implement and test in the same way the semi-discrete numerical schemes of order 4 and 6 given in the lecture, with different splitting methods and check the order in each case.