# Lazy evaluation
# Review

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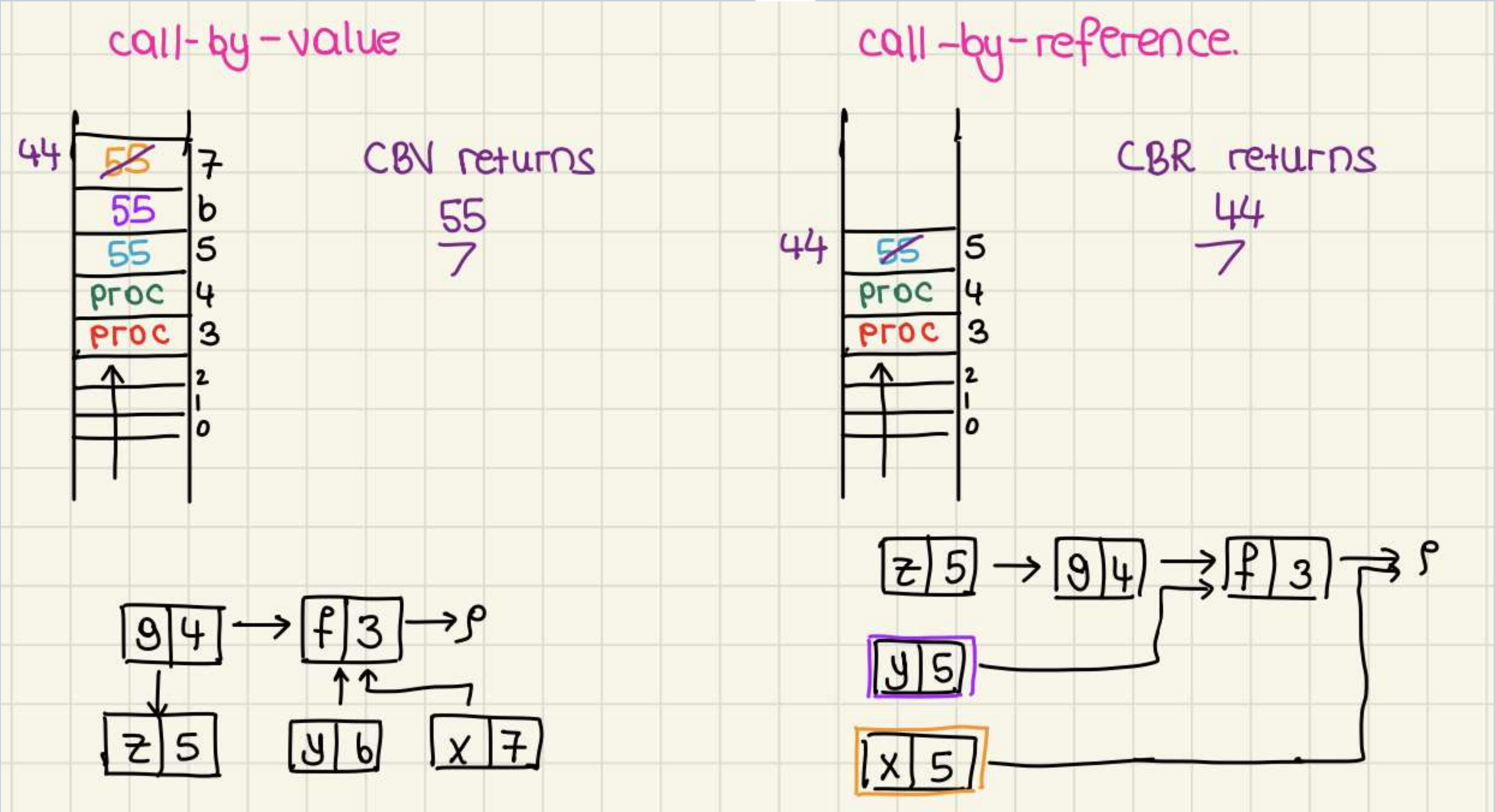
Parameter Passing Variations

- Natural (Proc)
- Call-by-value
- call-by-reference.
- call-by-name ⎤ lazy
- call-by-need ⎦ evaluation.

```
let p = proc (x) set x = 4
in let a = 3
   in begin (p a); a end
```

call by reference : 4
call by value : 3


call-by-value
call-by-reference.
CBV returns
55
7
CBR returns
44
7
proc
proc
z 5
g 4
f 3
y 6
x 7
y 5
x 5

call-by-name, call-by-need

```
letrec infinite-loop (x) = infinite-loop(-(x,-1))
in let f = proc (z) 11
   in (f (infinite-loop 0))
```

frozen.

An operand is not evaluated until we need it.
We can avoid non-termination.

```
(define-datatype thunk thunk?
  (a-thunk
    (exp1 expression?)
    (env environment?)))
```

- a new datatype
- will be evaluated once we need it.

**value-of-operand** : *Exp* × *Env* → *Ref*

```
(define value-of-operand
  (lambda (exp env)
    (cases expression exp
      (var-exp (var) (apply-env env var))
      (else
        (newref (a-thunk exp env))))))
```

★ same concept with call by reference

if var → get the reference ★

if not → create new thunk, ★ get its reference.

**value-of-thunk** : *Thunk* → *ExpVal*

```
(define value-of-thunk
  (lambda (th)
    (cases thunk th
      (a-thunk (exp1 saved-env)
        (value-of exp1 saved-env)))))
```

call by name

```
(var-exp (var)
  (let ((ref1 (apply-env env var)))
    (let ((w (deref ref1)))
      (if (expval? w)
        w
        (value-of-thunk w)))))
```

call by need

reference to the location of thunk

returns a reference

```
(var-exp (var)
  (let ((ref1 (apply-env env var)))
    (let ((w (deref ref1)))
      (if (expval? w)
        w
        (let ((val1 (value-of-thunk w)))
          (begin
            (setref! ref1 val1)
            val1)))))))
```

if expval

if thunk

memoization step

val1 is stored back in the original location of thunk.

# CPS

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# Recursive vs. Iterative Control Behavior

- Consider

```
(define fact
  (lambda (n)
    (if (zero? n) 1 (* n (fact (- n 1))))))
```

- The trace

```
   (fact 4)
= (* 4 (fact 3))
= (* 4 (* 3 (fact 2)))
= (* 4 (* 3 (* 2 (fact 1))))
= (* 4 (* 3 (* 2 (* 1 (fact 0)))))
= (* 4 (* 3 (* 2 (* 1 1))))
= (* 4 (* 3 (* 2 1)))
= (* 4 (* 3 2))
= (* 4 6)
= 24
```

# Recursive vs. Iterative Control Behavior

- Consider

```
(define fact-iter
  (lambda (n)
    (fact-iter-acc n 1)))

(define fact-iter-acc
  (lambda (n a)
    (if (zero? n) a (fact-iter-acc (- n 1) (* n a)))))
```

- The trace

```
  (fact-iter 4)
= (fact-iter-acc 4 1)
= (fact-iter-acc 3 4)
= (fact-iter-acc 2 12)
= (fact-iter-acc 1 24)
= (fact-iter-acc 0 24)
= 24
```

# What is the key difference between the two versions?

- What do we do after each call?
- How does the control context grow?
- Continuation:
  - Captures the control context
  - Describes what needs to be done next!

# A CPS Interpreter

- The environment grows as we evaluate expressions
- Now we need to keep around a list of things to do after the evaluation of each expression.
- Introduce apply-cont
  - Example:

*FinalAnswer* = *ExpVal*
**apply-cont** : *Cont* × *ExpVal* → *FinalAnswer*

```
(apply-cont (end-cont) val)
= (begin
    (eopl:printf "End of computation.~%")
    val)
```

# Evaluation

- Value-of-program

**value-of-program** : *Program* → *FinalAnswer*

```
(define value-of-program
  (lambda (pgm)
    (cases program pgm
      (a-program (exp1)
        (value-of/k exp1 (init-env) (end-cont))))))
```

- Value-of/k

**value-of/k** : *Exp* × *Env* × *Cont* → *FinalAnswer*

```
(define value-of/k
  (lambda (exp env cont)
    (cases expression exp
      (const-exp (num) (apply-cont cont (num-val num)))
      (var-exp (var) (apply-cont cont (apply-env env var)))
      (proc-exp (var body)
        (apply-cont cont
          (proc-val (procedure var body env))))
      ...)))
```

# Evaluation

- Letrec

```
(letrec-exp (p-name b-var p-body letrec-body)
  (value-of/k letrec-body
    (extend-env-rec p-name b-var p-body env)
    cont))
```

- Zero?

```
(zero?-exp (exp1)
  (value-of/k exp1 env
    (zero1-cont cont)))
```

```
(apply-cont (zero1-cont cont) val)
= (apply-cont cont
     (bool-val
       (zero? (expval->num val))))
```

# Evaluation

- Let
  - Before

```
(let-exp (var exp1 body)
  (let ((val1 (value-of exp1 env)))
    (value-of body
      (extend-env var val1 env))))
```

  - After

```
(let-exp (var exp1 body)
  (value-of/k exp1 env
    (let-exp-cont var body env cont)))
```

```
(apply-cont (let-exp-cont var body env cont) val)
= (value-of/k body (extend-env var val env) cont)
```

# Evaluation

- If

```
(if-exp (exp1 exp2 exp3)
  (value-of/k exp1 env
    (if-test-cont exp2 exp3 env cont)))
```

```
(apply-cont (if-test-cont exp2 exp3 env cont) val)
= (if (expval->bool val)
     (value-of/k exp2 env cont)
     (value-of/k exp3 env cont))
```

# Example

(value-of/k <<letrec p(x) = x in if b then 3 else 4>>
$\rho_0$ $cont_0$)

= *letting* $\rho_1$ *be* (extend-env-rec ... $\rho_0$)

(value-of/k <<if b then 3 else 4>> $\rho_1$ $cont_0$)

= *next, evaluate the test expression*

(value-of/k <<b>> $\rho_1$ (test-cont <<3>> <<4>> $\rho_1$ $cont_0$))

= *send the value of* b *to the continuation*

(apply-cont (test-cont <<3>> <<4>> $\rho_1$ $cont_0$)
(bool-val #t))

= *evaluate the then-expression*

(value-of/k <<3>> $\rho_1$ $cont_0$)

= *send the value of the expression to the continuation*

(apply-cont $cont_0$ (num-val 3))

= *invoke the final continuation with the final answer*

(begin (eopl:printf ...) (num-val 3))

# Evaluation

- diff

```
(diff-exp (exp1 exp2)
  (value-of/k exp1 env
    (diff1-cont exp2 env cont)))
```

```
(apply-cont (diff1-cont exp2 env cont) val1)
= (value-of/k exp2 env
    (diff2-cont val1 cont))
```

```
(apply-cont (diff2-cont val1 cont) val2)
= (let ((num1 (expval->num val1))
        (num2 (expval->num val2)))
    (apply-cont cont
      (num-val (- num1 num2))))
```

# Example

```
(value-of/k
  <<-(-(44,11),3)>>
  ρ0
   #(struct:end-cont))
```
= *start working on first operand*
```
(value-of/k
  <<-(44,11)>>
  ρ0
  #(struct:diff1-cont <<3>> ρ0
     #(struct:end-cont)))
```
= *start working on first operand*
```
(value-of/k
  <<44>>
  ρ0
  #(struct:diff1-cont <<11>> ρ0
     #(struct:diff1-cont <<3>> ρ0
        #(struct:end-cont))))
```
= *send value of <<44>> to continuation*
```
(apply-cont
  #(struct:diff1-cont <<11>> ρ0
     #(struct:diff1-cont <<3>> ρ0
        #(struct:end-cont)))
  (num-val 44))
```
= *now start working on second operand*
```
(value-of/k
  <<11>>
  ρ0
  #(struct:diff2-cont (num-val 44)
     #(struct:diff1-cont <<3>> ρ0
        #(struct:end-cont))))
```
= *send value to continuation*
```
(apply-cont
  #(struct:diff2-cont (num-val 44)
     #(struct:diff1-cont <<3>> ρ0
        #(struct:end-cont)))
  (num-val 11))
```
= $44 - 11$ *is* 33*, send that to the continuation*
```
(apply-cont
  #(struct:diff1-cont <<3>> ρ0
     #(struct:end-cont))
  (num-val 33))
```
= *start working on second operand <<3>>*
```
(value-of/k
  <<3>>
  ρ0
  #(struct:diff2-cont (num-val 33)
     #(struct:end-cont)))
```
= *send value to continuation*
```
(apply-cont
  #(struct:diff2-cont (num-val 33)
     #(struct:end-cont))
  (num-val 3))
```
= $33 - 3$ *is* 30*, send that to the continuation*
```
(apply-cont
  #(struct:end-cont)
  (num-val 30))
```

# Evaluation

- Procedure application
  - Before

```
(call-exp (rator rand)
  (let ((proc1 (expval->proc (value-of rator env)))
        (val (value-of rand env)))
    (apply-procedure proc1 val)))
```

  - After

```
(call-exp (rator rand)
  (value-of/k rator env
    (rator-cont rand env cont)))
```

```
(apply-cont (rator-cont rand env cont) val1)
= (value-of/k rand env
    (rand-cont val1 cont))
```

```
(apply-cont (rand-cont val1 cont) val2)
= (let ((proc1 (expval->proc val1)))
    (apply-procedure/k proc1 val2 cont))
```

**apply-procedure/k** : *Proc* × *ExpVal* × *Cont* → *FinalAnswer*

```
(define apply-procedure/k
  (lambda (proc1 val cont)
    (cases proc proc1
      (procedure (var body saved-env)
        (value-of/k body
          (extend-env var val saved-env)
          cont)))))
```