Linear Direct Current Electromagnetic Motor with Liquid Eutectic Gallium-Indium Alloy Coil

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Abstract

Abstract placeholder

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Contents

1 Table of Notation

Symbol	Description	Units
P_{in}	Electrical power input	W
P_{out}	Total power output	W
P_{mech}	Mechanical power output	W
P_{heat}	Heat power output	W
f	Frequency	Hz
$\mathcal{F}$	Magnetomotive force	A
$\mathcal{R}$	Magnetic reluctance	H^{-1}
$\mathcal{R}_{mag}$	Magnetic reluctance of the magnet	H^{-1}
Φ	Magnetic flux	Wb
Φ_0	Short circuit magnetic flux, i.e. flux in circuit when poles of magnet are connected with minimal reluctance	Wb
L_{Wtotal}	Total length of wire in motor	m
L_{mag}	Axial length of cylindrical magnet	m
B	Magnetic field density	T
B_r	Residual flux density of magnet	T
I	Electric current	A
d_{travel}	Required travel distance of the motor bobbin	m
c_{pWire}	Specific heat capacity of wire	$JKg^{-1}K^{-1}$
μ_0	Vaccum permeability, $4\pi \times 10^{-7} Hm^{-1}$ [1]	Hm^{-1}
μ_s	Relative magnetic permeability of mild steel, 760 [2]	NA
μ_m	Relative magnetic permeability of neodymium magnet, 1.05 [1]	NA

2 Introduction

Background

Soft robots important

Methods of locomotion all new, each with different advantages and drawbacks

Lack of traditional locomotion options

Liquid metal wired electromagnetic motors presents a possible solution

Transferral of existing robotics locomotion corpus to soft robots

Also presents a possible advantage re: cooling by circulating the wires

Question Statement

Is it feasible to build an electromagnetic motor using liquid metal for wiring that can also be cooled via circulating metal in the wiring?

Aims

Design, build and characterise an electromagnetic motor with liquid metal coils

3 Design, Manufacturing and Assembly

3.1 Mathematical Modelling

3.1.1 Motor Force

$$\mathcal{F} = \mathcal{R}\Phi \tag{1}$$

Magnetomotive force of a magnet can be calculated using total magnetic reluctance in a magnetic circuit and magnetic flux in that circuit, seen in equation 1. This is analogous to electrical circuits following Ohm's law: magnetomotive force is analogous to voltage, reluctance is analogous to resistance and flux is analogous to current. Magnetomotive force in cylindrical magnets can be calculated using a short circuit scenario, i.e. by assuming poles of magnet are connected with minimal reluctance. Under the assumption that magnetic field density is uniform, the short circuit flux is equal to the remanence field density of the magnet multiplied by the cross-sectional area of the magnet normal to the direction of the magnetic field. In the case of an axially magnetised cylindrical magnet, this area is equal to the radial cross-section area of the cylinder.

$$\Phi_0 = B_r A_{mag} \tag{2}$$

Substituting equation 2 into equation 1 gives a way to simply calculate $\mathcal{F}$ using axial length of magnet and remanent field density:

$$\begin{aligned} \mathcal{F} &= \mathcal{R}_{mag}\Phi_0 \\ &= \frac{L_{mag}}{A_{mag}} B_r A_{mag} \\ &= L_{mag} B_r \end{aligned} \tag{3}$$

The magnetic circuit of a voice coil motor travels through four components in series: the magnet, the core, the "air" gap where the coils are and the shell, seen in figure 3.1.1. This

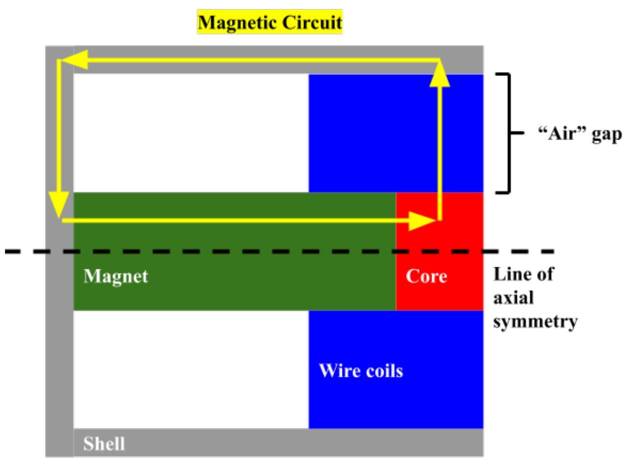

Figure 1: Simplified axisymmetric diagram of typical moving-wire voice coil motor

means the total reluctance in the circuit can be represented by equation 4, similar to how resistance is calculated in an electrical circuit.

$$\mathcal{R}_{total} = \mathcal{R}_{gap} + \mathcal{R}_{mag} + \mathcal{R}_{core} + \mathcal{R}_{shell} \tag{4}$$

Magnetic reluctance can be calculated as the proportion of magnetic path length l_m and the product of material permeability μ and cross-sectional area normal to magnetic flux A, seen in equation 5 [3].

$$\mathcal{R} = \frac{l_m}{\mu A} \tag{5}$$

Assuming the shell and core are made out of magnetically conductive material such as mild steel, the permeability of shell and core will be hundreds of times higher than the permeability of "air" gap and magnet [1]. Therefore, the reluctance of shell and core can be approximated to zero for the purposes of this calculation. Substituting simplified equation 4 into equation 1 produces:

$$\mathcal{F} = (\mathcal{R}_{gap} + \mathcal{R}_{mag})\Phi \tag{6}$$

Reluctance between two coaxial cylinders, in this case between the magnet and the shell, can be calculated as in equation 7, assuming all magnetic flux flows through the core [4]. Relative permeability of the gap is approximated to 1 as none of air, silicone or eGaIn are ferromagnetic [1].

$$\mathcal{R}_{gap} = \frac{\mu_0 \ln \frac{r_{out}}{r_{in}}}{2\pi L_{core}} \tag{7}$$

Reluctance through the magnet can be calculated as seen in equation 8, similar to how resistance would be calculated for a cylinder.

$$\mathcal{R}_{mag} = \frac{\mu_m L_{mag}}{2\pi r_{mag}^2} \tag{8}$$

Substituting equations 7, 8 and 3 into equation 6 gives equation 9 which yields flux in the motor magnetic circuit.

$$\begin{aligned} L_m B_r &= \frac{\ln \frac{r_{out}}{r_{in}}}{2\pi L_c}\Phi \\ \frac{2\pi L_m L_c B_r}{\ln \frac{r_{out}}{r_{in}}} &= \Phi \end{aligned} \tag{9}$$

To calculate the magnetic field density in the "air" gap, which can then be used to calculate force acting on the moving coils, the area of action that the flux is distributed across is also required as seen in equation 10.

$$\Phi = B_{gap} A_{active} \tag{10}$$

Substituting equation 10 into equation 9 gives equation 11, which can be used to calculate the magnetic field density B_{gap} across the "air" gap.

$$B_{gap} = \frac{2\pi L_{mag} L_{core} B_r}{\ln \frac{r_{out}}{r_{in}} A_{active}} \tag{11}$$

Using Lorentz's force law substituted with equation 11, equation 12 gives a minimum current I can be obtained given a known required force F and a known length of wire in the magnetic field L_{active}.

$$\begin{aligned} F &= B_{gap} I L_{active} \\ I &= \frac{F}{B_{gap} L_{active}} \\ I &= \frac{F A_{active} \ln \frac{r_{out}}{r_{in}}}{2\pi L_{mag} L_{core} B_r L_{active}} \end{aligned} \tag{12}$$

The active area in the magnetic field is different for each layer of wire in the motor. Layers of wire further away from the magnet, meaning a larger area and therefore lower magnetic field density. The total active area that magnetic flux is distributed across can be calculated as shown in equation 13, where n is the total number of wire layers.

$$A_{active} = \sum_{1}^{n} A_{active}^{layer} = 2\pi L_c (r_{core} - r_{wireout} + 2r_{wireout} \frac{(n+1)n}{2}) \tag{13}$$

The length of wire in the magnetic field can be calculated in a similar fashion, shown in equation 14.

$$L_{active} = L_{core}\pi(layers + 1)layers + \frac{L_{core} r_{mag} \pi}{r_{wireOut}} \tag{14}$$

Combining equations 12, 13 and 14 yields a final current calculation in equation 15.

$$I = \frac{F \cdot 2\pi L_c (r_{core} - r_{wireout} + 2r_{wireout} \frac{(n+1)n}{2}) \ln \frac{r_{out}}{r_{in}}}{2\pi L_{mag} L_{core} B_r L_{core}\pi(layers + 1)layers + \frac{L_{core} r_{mag} \pi}{r_{wireOut}}} \tag{15}$$

3.1.2 Heat and Temperature

Using heat energy can calculate temperature in wires Safety

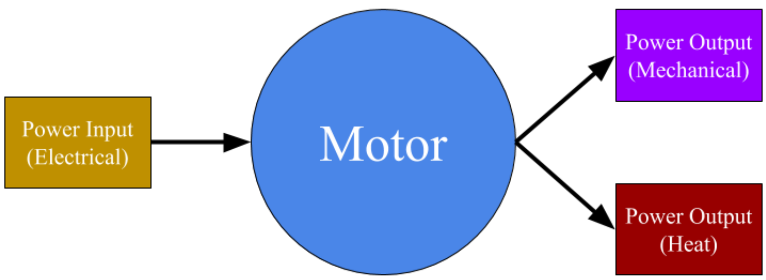

Figure 2: Conservation of energy in the motor system

$$P_{in} = IV = I^2 R \tag{16}$$

$$R = \frac{L_{Wtotal}}{A\sigma} \tag{17}$$

$$P_{in} = P_{out} = P_{mech} + P_{heat} \tag{18}$$

$$\begin{aligned} P_{mech} &= \vec{F}\vec{d} \\ &= BIL \cdot d_{travel} \cdot f \end{aligned} \tag{19}$$

Combining equations 16, 17, 18 and 19 gives a formulation for temperature change of wires, assuming heat energy is conserved in wires:

$$\begin{aligned} P_{heat} &= P_{in} - P_{mech} \\ \rho Q c_{pWire} \Delta T &= \frac{I^2 L_{Wtotal}}{A\sigma} - BIL \cdot d \cdot f \\ \Delta T &= \frac{\frac{I^2 L_{Wtotal}}{A\sigma} - BIL \cdot d \cdot f}{\rho Q c_{pWire}} \end{aligned} \tag{20}$$

3.2 Optimisation Algorithm

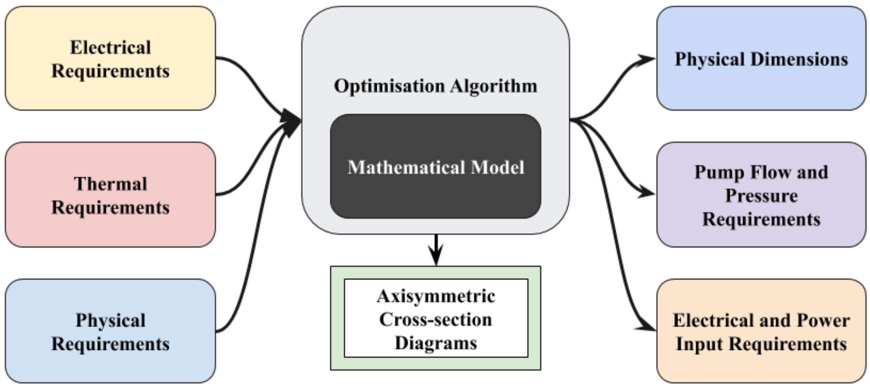

Figure 3: Diagrammatic representation of optimisation algorithm inputs and outputs

Grid-search optimisation for even numbered layers.

Optimised for an optimisation variable $\mathcal{P}$, which is the product of total motor mass and electrical input power.

Runs requirement checking function on each result and discards invalid solutions.

Variable	Requirement	Justification
Output force	Must be over 9 N	Minimum to drive liquid metal pump
Acceptable temperature change in wires	Must be under 30 K	Higher temperatures may be a health and safety hazard
Travel distance of motor	Must be over 9 mm	Minimum to drive liquid metal pump
Bobbin length	Must be shorter than combined length of core and magnet	Not physically valid otherwise
Shell internal diameter	Must be under 80 mm	Not practical for manufacturing, assembly and handling if bigger
Circuit current	Must be under 10 A	No power supply available to drive more than 10 A
Total volume of wire	Must be under 15 ml	Limit to amount of eGaIn available and difficult to assemble with larger volume
Flow rate of liquid metal in wires	Must be positive	Validity check
Electrical power input	Must be positive	Validity check

Optimisation was conducted assuming the use of 2 mm internal diameter, 3 mm external diameter silicone tubing for wiring. This was the tubing with largest internal volume to total volume ratio that was commercially available.

The optimisation software produced physical dimensions, pump flow and pressure requirements, electrical input requirements and an axisymmetric cross-section diagram to show scale of the optimised motor for each number of layers that the optimisation was run for.

Variable	4-layer solution	6-layer solution	Units
Magnet length	35	42.5	mm
Magnet radius	20	20	mm
Core length	12	8	mm
Bobbin length	27.5	23.75	mm
Shell internal radius	33.5	39.5	mm
Total wire length	6.68	9.55	m
Total wire resistance	0.15	0.22	Ω
Minimum required current	7.75	6.27	A
Minimum required voltage	1.19	1.38	V
Minimum power input	9.22	8.63	W
Flow rate required for circulation cooling	0.135	0.125	$ml \cdot s^{-1}$
Pressure required to drive circulation cooling	5.51	7.28	kPa
Total mass of motor	1.21	1.50	kg
Optimisation Variable $\mathcal{P}$	11.19	12.98	$kg \cdot W$

Choice between 4 layer design and 6 layer design

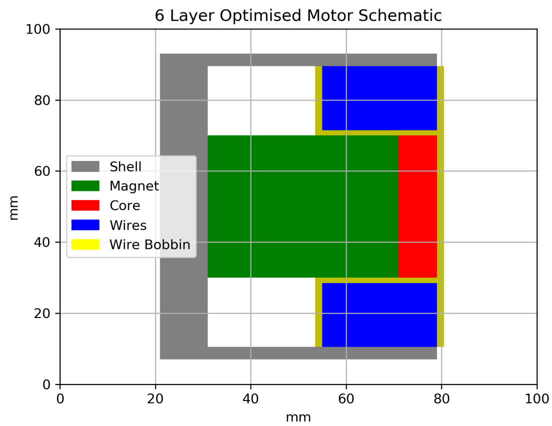

Figure 4: Graphical representation of cross-section of final motor design

3.3 Design Validation

Abacus Maxwell 2D axisymmetric electromagnetic simulation

3.4 Physical Design and Manufacturing

4 axial, 40 mm diameter, 10 mm tall N45 grade neodymium disc magnets were used in series, to substitute for the required 40 mm diameter, 42.5 mm tall N42 grade neodymium magnet. The disc magnets also come with countersunk unthreaded M6 holes for fastening.

3.4.1 Shell

3.4.2 Core

3.4.3 Wire Bobbin

3.5 Assembly

3.5.1 Shell and Magnet Assembly

3.5.2 Wire Assembly

3.5.3 Health, Safety and Containment

4 Experiment Designs and Results

4.1 Experiment 1: Force Characterisation

4.1.1 Method

4.1.2 Results

4.2 Experiment 2: Circulation Thermoregulation

4.2.1 Method

4.2.2 Results

5 Discussion

Application to the original physical situation

Comparison with related problems and other solutions

Critical assessment of significance

Difficulty of the problem and how well it has been tackled

[5]

6 Conclusion

7 Appendices

References to previous works should be made in a consistent way. Specific references should be itemised in the Reference list, with any other more general material listed in a Bibliography. Only those books and papers actually consulted should be included. There are several variations on layout of reference lists; obtain advice from your supervisor and the library staff.

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