中国科学技术大学
University of Science and Technology of China

# TECHNICAL REPORT:

## The Latest Research Results on CVRP, 2L-CVRP, and 3L-CVRP


Yangguang Wang

Research Technologist & Research Assistant Intelligent Computing and Application Lab (ICA-LAB)
ygw@mail.ustc.edu.cn

Supervisor:

Professor of School of Management
Dean of School of Software Engineering
hpchen@.ustc.edu.cn


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**Abstract**

Distributors are faced with loading constraints in their route planning, e.g.,multi-dimensional packing constraints, unloading sequence constraints, stability constraints and axle weight limits. Ignoring these constraints impairs planning and induces last-minute changes resulting in additional costs. Developing vehicle routing models incorporating loading constraints is critical to more efficient route planning. The research on 2L-CVRP and 3L-CVRP has received extensive attention from academia and industry in recent years due to its great application value. A vivid example is that China's largest logistics enterprises, like JD Logistics and SF Logistics, are trying to promote the integration of automatic planning of vehicle routes and unmanned loading of cargo, thus building more intelligent and integrated supply chain systems.

My contribution is twofold. First, I succeeded in designing sate-of-the-art metaheuristics called AMA-ENS to solve the well-known 2L-CVRP and 3L-CVRP. New best solutions are found on corresponding well-studied benchmark data sets. Second, I further considered time-dependent travel time on the original models, which are more complicated and practical. These two new models are called 2L-TDCVRP and 3L-TDCVRP. The metaheuristic methods designed for the original 2L-CVRP and 3L-CVRP can efficiently solve the new 2L-TDCVRP and 3L-TDCVRP problem.

Without considering loading constraints, AMA-ENS can be used to solve the capacitated vehicle routing problem (CVRP). Computational experiments indicates that AMA-ENS can compete with the state-of-the-art SISR algorithm (Christiaens et al. 2020. *Transportation Science*) and FILO algorithm (Accorsi at al.2021. *Transportation Science*) for the well-known CVRP problem.

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# 1 Results of AMA-ENS for the 2L-CVRP Problem

We executed AMA-ENS ten times for each instance by setting the random seed from 1 to 10. We compare our AMA-ENS with some of the most efficient approaches for 2L-CVRP. All these approaches were also executed 10 times for each instance. All four versions of 2L-CVRP are solved in our study. In the following tables, the cost listed is the best cost achieved over 10 runs. We list the details of only the best-known solution (BKS) among all previous approaches and the three methods with excellent performance: PRMP, VNS, SA for the 2UOL and 2SOL versions, and ACO, MS-BR, and SA for the 2URL and 2ORL versions (only these three papers studied the rotation allowed versions).

- ACO ( Fuellerer et al., 2009 )
- SA1 ( Leung et al., 2010 ), EGTS + LBFH ( Leung et al., 2011 )
- EGTS + LBFH ( Leung et al., 2011 )
- GRASPxELS ( Duhamel et al., 2011 )
- PRMP ( Zachariadis et al., 2013 )
- MS-BR ( Dominguez et al., 2014; Dominguez et al., 2016)
- VNS ( Wei et al., 2015)
- SA (Wei at al. 2018)
- **AMA-ENS**: the adaptive memetic algorithm with extended Neighbourhood search (Wang at al. 2021)

**The charts and tables in this section are extracted from the following papers:**

[1] **Wang Y.**, Liu C., Zhou S., Chen H. *, "An Adaptive Memetic Algorithm with Extended Neighbourhood Search for the Vehicle Routing Problem with Backhauls and Two-dimensional Loading Constraints" submitted to Annals of Operations Research.

[2] **Wang Y.**, Zhou S., Chen Z., Chen H.*, "A Metaheuristic Algorithm for the Time-dependent Capacitated Vehicle Routing Problem with Two-dimensional Loading Constraints" submitted to European Journal of Operational Research.

## 1.1 2L-CVRP under orientated loading

Table 1: Comparative results on the instances of Class 1

| Inst. | BKS[1] | VNS | | PRMP | | ACO | | BR-LNS | | SA | | AMA-ENS | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Cost | Time (s) | Cost | Time (s) | Cost | Time (s) | Cost | Time (s) | Cost | Time (s) | Cost | Time (s) | Gap (%)[2] |
| 1 | 278.73 | 278.73 | 0.00 | 278.73 | 0.00 | 278.73 | 0.10 | 278.73 | 0.01 | 278.73 | 0.90 | 278.73 | 0.00 | 0.00 |
| 2 | 334.96 | 334.96 | 0.00 | 334.96 | 0.00 | 334.96 | 0.10 | 334.96 | 0.00 | 334.96 | 0.30 | 334.96 | 0.00 | 0.00 |
| 3 | 358.40 | 358.40 | 0.10 | 358.40 | 0.00 | 358.40 | 0.20 | 358.40 | 0.00 | 358.40 | 1.00 | 358.40 | 0.00 | 0.00 |
| 4 | 430.88 | 430.89 | 0.00 | 430.88 | 0.00 | 430.88 | 0.30 | 430.88 | 0.00 | 430.89 | 0.90 | 430.89 | 0.00 | 0.00 |
| 5 | 375.28 | 375.28 | 0.00 | 375.28 | 0.00 | 375.28 | 0.30 | 375.28 | 0.00 | 375.28 | 0.00 | 375.28 | 0.00 | 0.00 |
| 6 | 495.85 | 495.85 | 0.10 | 495.85 | 0.00 | 495.85 | 0.30 | 495.85 | 0.06 | 495.85 | 2.50 | 495.85 | 0.01 | 0.00 |
| 7 | 568.56 | 568.56 | 0.00 | 568.56 | 0.00 | 568.56 | 0.20 | 568.56 | 0.00 | 568.56 | 0.00 | 568.56 | 0.00 | 0.00 |
| 8 | 568.56 | 568.56 | 0.00 | 568.56 | 0.00 | 568.56 | 0.20 | 568.56 | 0.00 | 568.56 | 0.00 | 568.56 | 0.00 | 0.00 |
| 9 | 607.65 | 607.65 | 0.10 | 607.65 | 0.00 | 607.65 | 0.60 | 607.65 | 0.00 | 607.65 | 1.10 | 607.65 | 0.00 | 0.00 |
| 10 | 535.74 | 535.80 | 0.10 | 535.80 | 0.10 | 535.80 | 2.30 | 535.80 | 5.18 | 535.80 | 5.80 | 535.80 | 0.01 | 0.01 |
| 11 | 505.01 | 505.01 | 0.00 | 505.01 | 0.00 | 505.01 | 0.80 | 505.01 | 0.18 | 505.01 | 0.60 | 505.01 | 0.00 | 0.00 |
| 12 | 610.00 | 610.00 | 0.90 | 610.00 | 0.20 | 610.00 | 1.60 | 610.00 | 0.46 | 610.00 | 5.40 | 610.00 | 0.32 | 0.00 |
| 13 | 2006.34 | 2006.34 | 0.10 | 2006.34 | 0.30 | 2006.34 | 1.30 | 2006.34 | 0.08 | 2006.34 | 0.00 | 2006.34 | 0.02 | 0.00 |
| 14 | 837.67 | 837.67 | 0.10 | 837.67 | 0.10 | 837.67 | 4.10 | 837.67 | 0.20 | 837.67 | 0.00 | 837.67 | 0.01 | 0.00 |
| 15 | 837.67 | 837.67 | 0.10 | 837.67 | 0.40 | 837.67 | 2.80 | 837.67 | 0.18 | 837.67 | 0.00 | 837.67 | 0.01 | 0.00 |
| 16 | 698.61 | 698.61 | 1.10 | 698.61 | 0.30 | 698.61 | 2.00 | 698.61 | 0.16 | 698.61 | 4.00 | 698.61 | 0.01 | 0.00 |
| 17 | 861.79 | 861.79 | 4.00 | 861.79 | 1.60 | 861.79 | 3.30 | 861.79 | 185.65 | 861.79 | 22.20 | 861.79 | 1.97 | 0.00 |
| 18 | 723.54 | 723.54 | 1.40 | 723.54 | 3.60 | 723.54 | 9.50 | 723.54 | 1.10 | 723.54 | 6.70 | 723.54 | 0.02 | 0.00 |
| 19 | 524.61 | 524.61 | 2.00 | 524.61 | 2.10 | 524.61 | 7.90 | 524.61 | 7.29 | 524.61 | 9.00 | 524.61 | 0.02 | 0.00 |
| 20 | 241.97 | 241.97 | 3.50 | 241.97 | 7.20 | 241.97 | 55.70 | 241.97 | 2.85 | 241.97 | 14.60 | 241.97 | 0.19 | 0.00 |
| 21 | 687.60 | 687.60 | 74.90 | 687.60 | 3.80 | 690.20 | 26.70 | 687.60 | 164.12 | 687.60 | 343.80 | 687.60 | 1.78 | 0.00 |
| 22 | 740.66 | 740.66 | 21.20 | 740.66 | 2.80 | 742.91 | 56.90 | 740.66 | 12.63 | 740.66 | 101.10 | 740.66 | 5.98 | 0.00 |
| 23 | 835.26 | 835.26 | 159.70 | 835.26 | 48.70 | 845.34 | 55.90 | 835.26 | 30.74 | 835.26 | 838.00 | 835.26 | 3.35 | 0.00 |
| 24 | 1024.69 | 1024.69 | 175.90 | 1024.69 | 38.10 | 1030.25 | 49.80 | 1024.69 | 490.50 | 1024.69 | 1250.20 | 1024.69 | 7.21 | 0.00 |
| 25 | 826.14 | 826.14 | 332.20 | 826.14 | 8.60 | 830.82 | 167.50 | 826.14 | 44.48 | 826.14 | 418.00 | 826.14 | 2.90 | 0.00 |
| 26 | 819.56 | 819.56 | 1.70 | 819.56 | 11.20 | 819.56 | 173.30 | 819.56 | 0.77 | 819.56 | 1.60 | 819.56 | 0.11 | 0.00 |
| 27 | 1082.65 | 1082.65 | 445.50 | 1082.65 | 172.30 | 1100.22 | 191.00 | 1082.65 | 9.50 | 1082.65 | 1306.00 | 1082.65 | 25.12 | 0.00 |
| 28 | 1040.70 | 1042.12 | 1021.50 | 1042.12 | 71.20 | 1062.23 | 252.20 | 1042.12 | 136.28 | 1042.12 | 24.60 | 1042.12 | 1.12 | 0.14 |
| 29 | 1162.96 | 1162.96 | 172.90 | 1162.96 | 121.90 | 1168.13 | 765.00 | 1162.96 | 147.85 | 1162.96 | 35.90 | 1162.96 | 10.25 | 0.00 |
| 30 | 1028.42 | 1028.42 | 1570.00 | 1028.42 | 267.50 | 1041.05 | 313.90 | 1028.42 | 371.68 | 1029.79 | 1435.80 | 1028.42 | 37.15 | 0.00 |
| 31 | 1299.21 | 1302.48 | 1813.80 | 1299.56 | 353.80 | 1341.89 | 517.80 | 1299.21 | 312.86 | 1301.03 | 1884.00 | **1293.68** | 97.61 | **-0.45** |
| 32 | 1296.18 | 1300.22 | 1976.10 | 1296.91 | 312.00 | 1334.26 | 519.70 | 1296.18 | 372.05 | 1300.30 | 2006.90 | **1291.45** | 116.56 | **-0.42** |
| 33 | 1296.13 | 1298.02 | 2204.10 | 1299.55 | 434.10 | 1331.69 | 479.20 | 1297.50 | 161.80 | 1296.13 | 1884.20 | **1291.45** | 197.53 | **-0.36** |
| 34 | 708.39 | 708.39 | 2125.20 | 709.82 | 328.20 | 712.32 | 621.40 | 709.08 | 554.20 | 708.66 | 1658.30 | **707.57** | 136.36 | **-0.12** |
| 35 | 862.79 | 865.39 | 2050.40 | 866.06 | 396.30 | 868.12 | 1468.20 | 864.63 | 382.43 | 862.79 | 1611.00 | **857.19** | 254.25 | **-0.65** |
| 36 | 583.98 | 586.49 | 2420.20 | 585.46 | 228.90 | 616.69 | 1589.80 | 590.16 | 560.74 | 583.98 | 1276.30 | **579.71** | 1106.82 | **-0.73** |
| Avg. | 769.37 | 769.80 | 460.53 | 769.70 | 78.20 | 776.04 | 203.94 | 769.69 | 109.89 | 769.62 | 448.63 | 768.69 | 55.74 | -0.07 |

Bold type represents the new best solution values found.

[1] The best known solution in the literature.

[2] The percentage improvement (%) on the current BKS level. Lower numbers equate to better performance.

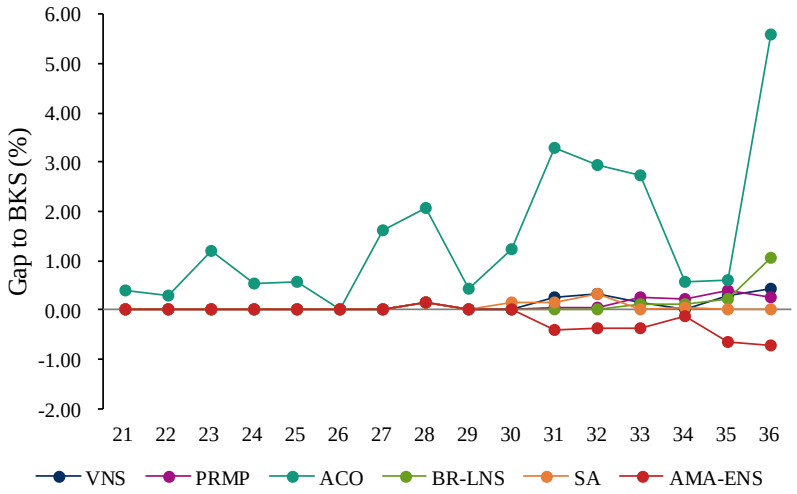


Figure 1: Gap to the BKS[1] on the instances of class 1

The X-axis represents the the current BKS level in the literature. The Lower, the better

[a]Gap to BKS (%): Percentage improvement between the solutions and BKS, Gap=100*(solution-BKS)/BKS.

Table 2: Results for the 2UOL version of 2L-CVRP

| Inst. | Class 2 | | | Class 3 | | | Class 4 | | | Class 5 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | BKS[1] | AMA-ENS | Gap (%)[2] | BKS[1] | AMA-ENS | Gap (%)[2] | BKS[1] | AMA-ENS | Gap (%)[2] | BKS[1] | AMA-ENS | Gap (%)[2] |
| 1 | 278.73 | 278.73 | 0.00 | 284.52 | **279.49** | **-1.77** | 282.95 | 282.95 | 0.00 | 278.73 | 278.73 | 0.00 |
| 2 | 334.96 | 334.96 | 0.00 | 352.16 | **349.92** | **-0.64** | 334.96 | 334.96 | 0.00 | 334.96 | 334.96 | 0.00 |
| 3 | 387.70 | **379.77** | **-2.05** | 394.72 | **385.32** | **-2.38** | 362.41 | **358.40** | **-1.11** | 358.40 | 358.40 | 0.00 |
| 4 | 430.88 | 430.89 | 0.00 | 430.88 | 430.89 | 0.00 | 447.37 | 447.37 | 0.00 | 430.88 | 430.89 | 0.00 |
| 5 | 375.28 | 375.28 | 0.00 | 381.69 | **375.28** | **-1.68** | 383.87 | 383.88 | 0.00 | 375.28 | 375.28 | 0.00 |
| 6 | 495.85 | 495.85 | 0.00 | 497.17 | 498.16 | 0.20 | 498.32 | 498.32 | 0.00 | 495.75 | 495.85 | 0.02 |
| 7 | 725.46 | **708.61** | **-2.32** | 678.75 | **660.53** | **-2.68** | 700.72 | **686.26** | **-2.06** | 657.77 | 657.77 | 0.00 |
| 8 | 674.55 | **664.30** | **-1.52** | 738.43 | **724.16** | **-1.93** | 692.47 | **688.32** | **-0.60** | 609.90 | 609.90 | 0.00 |
| 9 | 607.65 | 607.65 | 0.00 | 607.65 | 607.65 | 0.00 | 621.23 | **607.65** | **-2.19** | 607.65 | 607.65 | 0.00 |
| 10 | 689.68 | **665.76** | **-3.47** | 615.68 | **611.54** | **-0.67** | 710.87 | **703.64** | **-1.02** | 678.66 | **678.62** | **-0.01** |
| 11 | 684.21 | **642.78** | **-6.06** | 706.73 | **698.30** | **-1.19** | 784.88 | **765.04** | **-2.53** | 624.82 | 624.82 | 0.00 |
| 12 | 610.57 | **610.00** | **-0.09** | 610.00 | 610.00 | 0.00 | 614.23 | **610.23** | **-0.65** | 610.00 | 610.00 | 0.00 |
| 13 | 2585.72 | **2512.14** | **-2.85** | 2436.56 | **2370.66** | **-2.70** | 2548.06 | **2544.09** | **-0.16** | 2334.78 | **2334.59** | **-0.01** |
| 14 | 1038.09 | **1028.80** | **-0.89** | 996.25 | **989.08** | **-0.72** | 981.00 | **954.06** | **-2.75** | 871.22 | 871.22 | 0.00 |
| 15 | 1013.29 | **1002.91** | **-1.02** | 1149.99 | **1096.97** | **-4.61** | 1181.30 | **1164.77** | **-1.40** | 1159.94 | 1159.94 | 0.00 |
| 16 | 698.61 | 698.61 | 0.00 | 698.61 | 698.61 | 0.00 | 703.35 | 703.35 | 0.00 | 698.61 | 698.61 | 0.00 |
| 17 | 863.66 | **861.79** | **-0.22** | 861.79 | 861.79 | 0.00 | 861.79 | 861.79 | 0.00 | 861.79 | 861.79 | 0.00 |
| 18 | 1004.99 | **983.06** | **-2.18** | 1069.45 | **1013.72** | **-5.21** | 1116.45 | **1095.30** | **-1.89** | 917.94 | 917.94 | 0.00 |
| 19 | 754.53 | **715.31** | **-5.20** | 771.66 | **747.39** | **-3.15** | 775.87 | **759.63** | **-2.09** | 644.59 | 644.59 | 0.00 |
| 20 | 524.91 | **488.68** | **-6.90** | 521.31 | **513.53** | **-1.49** | 537.56 | **533.58** | **-0.74** | 470.33 | **468.69** | **-0.35** |
| 21 | 992.83 | **965.43** | **-2.76** | 1116.58 | **1087.87** | **-2.57** | 970.37 | **959.84** | **-1.09** | 873.25 | **870.82** | **-0.28** |
| 22 | 1035.66 | **979.29** | **-5.44** | 1052.98 | **1025.12** | **-2.65** | 1045.91 | **1044.08** | **-0.17** | 930.83 | **929.08** | **-0.19** |
| 23 | 1035.18 | **977.37** | **-5.58** | 1074.30 | **1047.37** | **-2.51** | 1071.30 | **1045.71** | **-2.39** | 930.09 | **922.34** | **-0.83** |
| 24 | 1178.07 | **1135.03** | **-3.65** | 1080.88 | **1073.75** | **-0.66** | 1100.76 | **1093.55** | **-0.66** | 1028.04 | 1042.37 | 1.39 |
| 25 | 1407.86 | **1334.66** | **-5.20** | 1365.37 | **1326.00** | **-2.88** | 1398.02 | **1365.25** | **-2.34** | 1150.04 | 1150.69 | 0.06 |
| 26 | 1272.87 | **1234.91** | **-2.98** | 1342.19 | **1315.86** | **-1.96** | 1390.99 | **1375.88** | **-1.09** | 1213.04 | 1216.90 | 0.32 |
| 27 | 1313.12 | **1268.01** | **-3.44** | 1369.44 | **1341.79** | **-2.02** | 1314.05 | **1294.35** | **-1.50** | 1240.32 | **1236.52** | **-0.31** |
| 28 | 2551.41 | **2515.35** | **-1.41** | 2592.73 | **2557.44** | **-1.36** | 2585.92 | **2526.69** | **-2.29** | 2294.40 | 2307.46 | 0.57 |
| 29 | 2196.00 | **2134.77** | **-2.79** | 2087.15 | **2058.57** | **-1.37** | 2240.18 | **2203.36** | **-1.64** | 2127.60 | **2126.97** | **-0.03** |
| 30 | 1803.26 | **1726.50** | **-4.26** | 1821.83 | **1779.06** | **-2.35** | 1820.46 | **1802.24** | **-1.00** | 1521.91 | **1517.09** | **-0.32** |
| 31 | 2254.47 | **2188.87** | **-2.91** | 2268.64 | **2216.04** | **-2.32** | 2366.80 | **2335.26** | **-1.33** | 1987.08 | 1989.38 | 0.12 |
| 32 | 2241.02 | **2170.92** | **-3.13** | 2227.66 | **2195.11** | **-1.46** | 2252.39 | **2228.37** | **-1.07** | 1949.34 | **1944.97** | **-0.22** |
| 33 | 2249.68 | **2177.36** | **-3.21** | 2348.25 | **2304.74** | **-1.85** | 2373.63 | **2344.96** | **-1.21** | 1975.14 | **1969.26** | **-0.30** |
| 34 | 1170.77 | **1136.53** | **-2.92** | 1196.20 | **1169.77** | **-2.21** | 1193.18 | **1183.02** | **-0.85** | 1014.76 | 1015.48 | 0.07 |
| 35 | 1364.35 | **1323.39** | **-3.00** | 1436.52 | **1406.93** | **-2.06** | 1486.29 | **1470.64** | **-1.05** | 1236.42 | **1229.22** | **-0.58** |
| 36 | 1681.82 | **1650.36** | **-1.87** | 1757.43 | **1738.61** | **-1.07** | 1638.66 | **1621.28** | **-1.06** | 1470.26 | 1478.07 | 0.53 |
| Avg. | 1125.77 | 1094.57 | -2.48 | 1137.28 | 1115.75 | -1.72 | 1149.68 | 1135.50 | -1.11 | 1026.79 | 1026.86 | -0.01 |

Bold type represents the new best solution values found.

[1] The best known solution in the literature.

[2] The percentage improvement (%) on the current BKS level. Lower numbers equate to better performance.

Table 3: Comparison for the 2UOLversion of 2L-CVRP (averaged over Classes 2–5)

| Inst. | BKS[1] | PRMP | | | VNS | | | SA | | | AMA-ENS | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Cost | Time (s) | Gap (%)[2] | Cost | Time (s) | Gap (%)[2] | Cost | Time (s) | Gap (%)[2] | Cost | Time (s) | Gap (%)[2] |
| 1 | 281.23 | 281.23 | 0.40 | 0.00 | 281.23 | 1.20 | 0.00 | 281.23 | 5.70 | 0.00 | **279.97** | 1.09 | **-0.45** |
| 2 | 339.26 | 339.26 | 0.30 | 0.00 | 339.26 | 0.10 | 0.00 | 339.26 | 0.40 | 0.00 | **338.70** | 0.36 | **-0.17** |
| 3 | 375.81 | 376.32 | 0.40 | 0.14 | 376.32 | 0.90 | 0.14 | 375.81 | 0.70 | 0.00 | **370.47** | 2.17 | **-1.42** |
| 4 | 435.00 | 435.01 | 0.30 | 0.00 | 435.01 | 0.30 | 0.00 | 435.01 | 1.00 | 0.00 | 435.01 | 1.54 | 0.00 |
| 5 | 379.03 | 379.03 | 1.10 | 0.00 | 379.03 | 1.50 | 0.00 | 379.03 | 0.60 | 0.00 | **377.43** | 0.97 | **-0.42** |
| 6 | 496.77 | 497.04 | 0.30 | 0.05 | 497.04 | 1.00 | 0.05 | 497.04 | 1.80 | 0.05 | 497.04 | 1.01 | 0.05 |
| 7 | 690.67 | 690.67 | 1.60 | 0.00 | 690.67 | 2.50 | 0.00 | 690.67 | 1.90 | 0.00 | **678.29** | 1.47 | **-1.79** |
| 8 | 678.84 | 678.84 | 2.60 | 0.00 | 678.84 | 3.80 | 0.00 | 679.44 | 5.30 | 0.09 | **671.67** | 1.47 | **-1.06** |
| 9 | 611.05 | 612.01 | 1.60 | 0.16 | 612.01 | 1.30 | 0.16 | 612.01 | 2.40 | 0.16 | **607.65** | 2.64 | **-0.56** |
| 10 | 674.88 | 676.75 | 26.90 | 0.28 | 674.92 | 25.10 | 0.01 | 674.88 | 16.70 | 0.00 | **664.89** | 9.31 | **-1.48** |
| 11 | 701.07 | 703.22 | 27.20 | 0.31 | 702.47 | 59.90 | 0.20 | 701.07 | 23.70 | 0.00 | **682.73** | 3.83 | **-2.62** |
| 12 | 611.20 | 611.26 | 1.40 | 0.01 | 611.20 | 3.30 | 0.00 | 611.20 | 6.90 | 0.00 | **610.06** | 15.64 | **-0.19** |
| 13 | 2480.73 | 2491.18 | 52.70 | 0.42 | 2484.16 | 25.20 | 0.14 | 2480.73 | 22.00 | 0.00 | **2440.37** | 10.68 | **-1.63** |
| 14 | 973.23 | 975.88 | 164.30 | 0.27 | 975.06 | 295.50 | 0.19 | 973.23 | 53.40 | 0.00 | **960.79** | 99.31 | **-1.28** |
| 15 | 1128.18 | 1132.91 | 20.10 | 0.42 | 1128.60 | 246.30 | 0.04 | 1128.18 | 445.30 | 0.00 | **1106.15** | 100.01 | **-1.95** |
| 16 | 699.79 | 699.79 | 4.10 | 0.00 | 699.79 | 1.60 | 0.00 | 699.79 | 4.00 | 0.00 | 699.79 | 19.51 | 0.00 |
| 17 | 862.26 | 864.05 | 2.40 | 0.21 | 864.05 | 4.00 | 0.21 | 864.05 | 18.20 | 0.21 | **861.79** | 30.65 | **-0.05** |
| 18 | 1027.45 | 1031.95 | 33.00 | 0.44 | 1027.98 | 79.80 | 0.05 | 1027.45 | 160.50 | 0.00 | **1002.50** | 81.09 | **-2.43** |
| 19 | 737.40 | 741.78 | 24.30 | 0.59 | 737.73 | 250.50 | 0.04 | 737.40 | 206.60 | 0.00 | **716.73** | 91.38 | **-2.80** |
| 20 | 513.53 | 515.44 | 552.20 | 0.37 | 515.92 | 794.20 | 0.47 | 513.53 | 855.90 | 0.00 | **501.12** | 267.26 | **-2.42** |
| 21 | 988.30 | 992.78 | 241.50 | 0.45 | 991.63 | 751.10 | 0.34 | 988.30 | 1658.40 | 0.00 | **970.99** | 347.64 | **-1.75** |
| 22 | 1017.33 | 1023.01 | 166.60 | 0.56 | 1019.03 | 885.20 | 0.17 | 1017.56 | 1740.10 | 0.02 | **994.39** | 402.99 | **-2.25** |
| 23 | 1029.32 | 1032.36 | 336.80 | 0.30 | 1030.40 | 853.10 | 0.10 | 1029.32 | 1353.20 | 0.00 | **998.20** | 369.98 | **-3.02** |
| 24 | 1099.57 | 1104.64 | 319.60 | 0.46 | 1102.53 | 572.10 | 0.27 | 1100.64 | 923.10 | 0.10 | **1086.18** | 328.10 | **-1.22** |
| 25 | 1330.32 | 1341.26 | 921.70 | 0.82 | 1333.76 | 998.70 | 0.26 | 1330.32 | 1833.30 | 0.00 | **1294.15** | 490.37 | **-2.72** |
| 26 | 1306.59 | 1311.79 | 403.50 | 0.40 | 1306.60 | 1050.60 | 0.00 | 1306.59 | 1466.80 | 0.00 | **1285.89** | 503.69 | **-1.58** |
| 27 | 1309.92 | 1318.04 | 438.20 | 0.62 | 1311.27 | 874.50 | 0.10 | 1309.92 | 1696.00 | 0.00 | **1285.17** | 864.46 | **-1.89** |
| 28 | 2506.12 | 2530.46 | 3701.90 | 0.97 | 2519.35 | 2259.20 | 0.53 | 2506.12 | 2222.30 | 0.00 | **2476.74** | 2688.55 | **-1.17** |
| 29 | 2163.06 | 2173.02 | 1835.70 | 0.46 | 2166.14 | 2232.80 | 0.14 | 2163.06 | 2169.90 | 0.00 | **2130.92** | 2812.31 | **-1.49** |
| 30 | 1741.87 | 1760.59 | 2151.80 | 1.07 | 1746.82 | 2495.40 | 0.28 | 1741.87 | 1337.10 | 0.00 | **1706.22** | 2342.04 | **-2.05** |
| 31 | 2220.22 | 2244.13 | 2927.40 | 1.08 | 2227.79 | 2952.90 | 0.34 | 2220.22 | 2080.90 | 0.00 | **2182.39** | 2665.39 | **-1.70** |
| 32 | 2167.60 | 2196.85 | 3713.80 | 1.35 | 2177.66 | 2648.70 | 0.46 | 2167.60 | 1954.70 | 0.00 | **2134.84** | 2603.74 | **-1.51** |
| 33 | 2236.73 | 2261.68 | 1964.80 | 1.12 | 2239.91 | 2942.70 | 0.14 | 2236.73 | 1949.90 | 0.00 | **2199.08** | 2132.24 | **-1.68** |
| 34 | 1144.14 | 1157.22 | 3551.70 | 1.14 | 1147.67 | 2459.60 | 0.31 | 1144.14 | 2472.60 | 0.00 | **1126.20** | 3091.90 | **-1.57** |
| 35 | 1380.90 | 1401.17 | 2756.50 | 1.47 | 1388.55 | 2620.60 | 0.55 | 1380.90 | 2447.20 | 0.00 | **1357.55** | 3119.45 | **-1.69** |
| 36 | 1637.04 | 1669.44 | 4245.60 | 1.98 | 1656.00 | 3012.00 | 1.16 | 1637.04 | 2953.50 | 0.00 | **1622.08** | 2967.55 | **-0.91** |
| Avg. | 1110.46 | 1118.11 | 849.84 | 0.50 | 1113.23 | 872.42 | 0.19 | 1110.59 | 891.44 | 0.02 | 1093.17 | 790.88 | -1.41 |

Bold type represents the new best solution values found.

[1] The best known solution in the literature.

[2] The percentage improvement (%) on the current BKS level. Lower numbers equate to better performance.

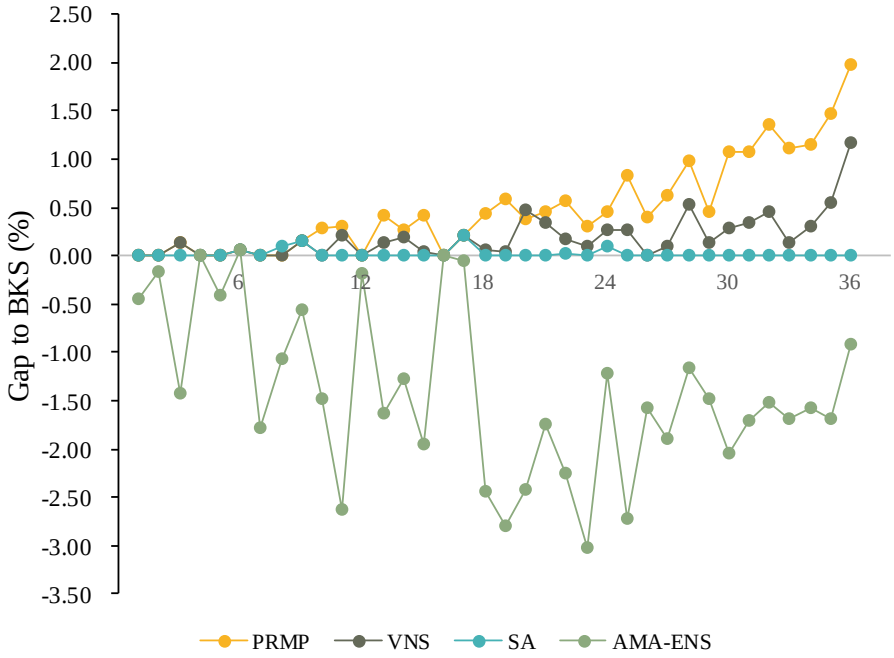


Figure 2: Gap to the BKS[1] for the $2|UO|L$ version of 2L-CVRP (averaged over Class 2–5)

The X-axis represents the the current BKS level in the literature. The Lower, the better

[a]Gap to BKS (%): Percentage improvement between the solutions and BKS, Gap=100*(solution-BKS)/BKS.

Table 4: Results for the 2URLversion of 2L-CVRP

| Inst. | Class2 | | | Class 3 | | | Class 4 | | | Class 5 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | BKS[1] | AMA-ENS | Gap (%)[2] | BKS[1] | AMA-ENS | Gap (%)[2] | BKS[1] | AMA-ENS | Gap (%)[2] | BKS[1] | AMA-ENS | Gap (%)[2] |
| 1 | 278.73 | 278.73 | 0.00 | 282.95 | **279.49** | **-1.22** | 282.95 | 282.95 | 0.00 | 278.73 | 278.73 | 0.00 |
| 2 | 334.96 | 334.96 | 0.00 | 352.16 | **349.92** | **-0.64** | 334.96 | 334.96 | 0.00 | 334.96 | 334.96 | 0.00 |
| 3 | 380.35 | **371.72** | **-2.27** | 385.32 | 385.32 | 0.00 | 358.40 | 358.40 | 0.00 | 358.40 | 358.40 | 0.00 |
| 4 | 430.88 | 430.89 | 0.00 | 430.88 | 430.89 | 0.00 | 447.37 | 447.37 | 0.00 | 430.88 | 430.89 | 0.00 |
| 5 | 375.28 | 375.28 | 0.00 | 379.94 | **375.28** | **-1.23** | 383.87 | 383.88 | 0.00 | 375.28 | 375.28 | 0.00 |
| 6 | 495.85 | 495.85 | 0.00 | 498.16 | 498.16 | 0.00 | 498.32 | 498.32 | 0.00 | 495.85 | 495.85 | 0.00 |
| 7 | 715.02 | **699.52** | **-2.17** | 664.96 | **659.66** | **-0.80** | 686.26 | 686.26 | 0.00 | 657.77 | 657.77 | 0.00 |
| 8 | 665.17 | **664.30** | **-0.13** | 738.43 | **724.16** | **-1.93** | 688.32 | 688.32 | 0.00 | 609.90 | 609.90 | 0.00 |
| 9 | 607.65 | 607.65 | 0.00 | 607.65 | 607.65 | 0.00 | 625.10 | **607.65** | **-2.79** | 607.65 | 607.65 | 0.00 |
| 10 | 667.42 | **648.94** | **-2.77** | 591.61 | **584.80** | **-1.15** | 703.64 | 703.64 | 0.00 | 678.62 | 678.62 | 0.00 |
| 11 | 664.48 | **637.12** | **-4.12** | 699.35 | **685.80** | **-1.94** | 771.93 | **760.53** | **-1.48** | 624.82 | 624.82 | 0.00 |
| 12 | 610.00 | 610.00 | 0.00 | 610.00 | 610.00 | 0.00 | 610.23 | 610.23 | 0.00 | 610.00 | 610.00 | 0.00 |
| 13 | 2502.65 | **2463.55** | **-1.56** | 2377.39 | **2345.10** | **-1.36** | 2533.79 | **2500.85** | **-1.30** | 2334.59 | 2334.59 | 0.00 |
| 14 | 1029.34 | **1025.87** | **-0.34** | 988.79 | 988.80 | 0.00 | 955.09 | **954.06** | **-0.11** | 871.22 | 871.22 | 0.00 |
| 15 | 1001.51 | **1000.68** | **-0.08** | 1116.07 | **1096.97** | **-1.71** | 1164.63 | **1164.39** | **-0.02** | 1159.94 | 1159.94 | 0.00 |
| 16 | 698.61 | 698.61 | 0.00 | 698.61 | 698.61 | 0.00 | 703.35 | 703.35 | 0.00 | 698.61 | 698.61 | 0.00 |
| 17 | 861.79 | 861.79 | 0.00 | 861.79 | 861.79 | 0.00 | 861.79 | 861.79 | 0.00 | 861.79 | 861.79 | 0.00 |
| 18 | 987.10 | **971.48** | **-1.58** | 986.30 | **985.97** | **-0.03** | 1100.52 | **1095.12** | **-0.49** | 917.94 | 917.94 | 0.00 |
| 19 | 723.93 | **701.53** | **-3.09** | 749.43 | **742.27** | **-0.96** | 747.03 | **739.92** | **-0.95** | 644.59 | 644.59 | 0.00 |
| 20 | 488.69 | **483.60** | **-1.04** | 511.46 | **510.06** | **-0.27** | 533.77 | **528.33** | **-1.02** | 466.79 | 466.79 | 0.00 |
| 21 | 964.49 | **944.12** | **-2.11** | 1086.72 | **1071.20** | **-1.43** | 959.82 | **952.83** | **-0.73** | 870.82 | 870.82 | 0.00 |
| 22 | 976.70 | **957.04** | **-2.01** | 1024.11 | **1010.08** | **-1.37** | 1041.80 | **1033.58** | **-0.79** | 928.02 | 928.02 | 0.00 |
| 23 | 984.00 | **961.68** | **-2.27** | 1041.60 | **1031.44** | **-0.98** | 1047.32 | **1032.80** | **-1.39** | 922.34 | 922.34 | 0.00 |
| 24 | 1140.13 | **1112.35** | **-2.44** | 1066.15 | **1062.81** | **-0.31** | 1086.09 | **1081.90** | **-0.39** | 1042.37 | 1042.37 | 0.00 |
| 25 | 1345.89 | **1301.14** | **-3.32** | 1333.64 | **1314.75** | **-1.42** | 1366.28 | **1357.98** | **-0.61** | 1149.66 | 1149.66 | 0.00 |
| 26 | 1257.00 | **1220.09** | **-2.94** | 1311.11 | **1295.86** | **-1.16** | 1362.22 | **1359.75** | **-0.18** | 1209.34 | 1209.34 | 0.00 |
| 27 | 1271.10 | **1242.87** | **-2.22** | 1329.33 | **1315.87** | **-1.01** | 1284.94 | **1278.65** | **-0.49** | 1231.52 | **1222.66** | **-0.72** |
| 28 | 2491.86 | **2439.92** | **-2.08** | 2541.02 | **2522.66** | **-0.72** | 2510.29 | **2499.13** | **-0.44** | 2276.71 | 2291.78 | 0.66 |
| 29 | 2129.10 | **2081.81** | **-2.22** | 2040.83 | **2028.67** | **-0.60** | 2199.79 | **2193.98** | **-0.26** | 2115.53 | 2116.51 | 0.05 |
| 30 | 1740.87 | **1689.74** | **-2.94** | 1767.72 | **1753.16** | **-0.82** | 1784.14 | **1772.14** | **-0.67** | 1512.71 | 1513.42 | 0.05 |
| 31 | 2162.88 | **2120.38** | **-1.96** | 2196.26 | **2170.36** | **-1.18** | 2314.76 | **2302.72** | **-0.52** | 1968.89 | 1974.17 | 0.27 |
| 32 | 2165.96 | **2112.12** | **-2.49** | 2166.18 | **2149.86** | **-0.75** | 2206.72 | **2181.48** | **-1.14** | 1938.96 | **1938.42** | **-0.03** |
| 33 | 2157.23 | **2096.28** | **-2.83** | 2276.31 | **2235.41** | **-1.80** | 2318.77 | **2308.14** | **-0.46** | 1946.51 | 1951.79 | 0.27 |
| 34 | 1121.67 | **1091.89** | **-2.65** | 1165.57 | **1148.94** | **-1.43** | 1163.96 | **1160.03** | **-0.34** | 1006.38 | 1012.15 | 0.57 |
| 35 | 1310.33 | **1282.76** | **-2.10** | 1393.90 | **1374.65** | **-1.38** | 1452.59 | **1448.03** | **-0.31** | 1224.21 | **1218.27** | **-0.49** |
| 36 | 1625.42 | **1595.50** | **-1.84** | 1708.05 | **1678.23** | **-1.75** | 1605.00 | **1600.33** | **-0.29** | 1457.05 | 1459.39 | 0.16 |
| Avg. | 1093.45 | 1072.55 | -1.60 | 1110.55 | 1099.57 | -0.87 | 1130.44 | 1124.27 | -0.48 | 1022.76 | 1023.32 | 0.02 |

Bold type represents the new best solution values found.

[1] The best known solution in the literature.

[2] The percentage improvement (%) on the current BKS level. Lower numbers equate to better performance.

Table 5: Comparison for the 2URLversion of 2L-CVRP (averaged over Classes 2–5)

| Inst. | BKS[1] | ACO | | | MS-BR | | | SA | | | AMA-ENS | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Cost | Time (s) | Gap (%)[2] | Cost | Time (s) | Gap (%)[2] | Cost | Time (s) | Gap (%)[2] | Cost | Time (s) | Gap (%)[2] |
| 1 | 280.84 | 281.16 | – | 0.11 | 280.84 | 3.30 | 0.00 | 281.13 | 0.70 | 0.10 | **279.97** | 0.48 | **-0.31** |
| 2 | 339.26 | 341.02 | – | 0.52 | 339.26 | 0.80 | 0.00 | 339.26 | 0.40 | 0.00 | **338.70** | 0.39 | **-0.17** |
| 3 | 370.62 | 372.93 | – | 0.62 | 370.62 | 34.80 | 0.00 | 371.62 | 2.10 | 0.27 | **368.46** | 0.91 | **-0.58** |
| 4 | 435.00 | 435.01 | – | 0.00 | 435.00 | 2.00 | 0.00 | 435.00 | 1.10 | 0.00 | 435.01 | 0.40 | 0.00 |
| 5 | 378.59 | 378.59 | – | 0.00 | 378.59 | 30.00 | 0.00 | 378.59 | 0.70 | 0.00 | **377.43** | 0.63 | **-0.31** |
| 6 | 497.04 | 497.04 | – | 0.00 | 497.05 | 3.80 | 0.00 | 497.04 | 1.70 | 0.00 | 497.04 | 0.25 | 0.00 |
| 7 | 681.00 | 688.50 | – | 1.10 | 681.00 | 20.30 | 0.00 | 681.00 | 1.20 | 0.00 | **675.80** | 0.64 | **-0.76** |
| 8 | 675.45 | 678.75 | – | 0.49 | 675.46 | 17.00 | 0.00 | 675.45 | 5.10 | 0.00 | **671.67** | 0.50 | **-0.56** |
| 9 | 612.01 | 612.02 | – | 0.00 | 612.01 | 32.30 | 0.00 | 612.01 | 2.80 | 0.00 | **607.65** | 1.44 | **-0.71** |
| 10 | 660.43 | 671.00 | – | 1.60 | 667.65 | 182.00 | 1.09 | 660.43 | 5.40 | 0.00 | **654.00** | 2.03 | **-0.97** |
| 11 | 690.56 | 698.25 | – | 1.11 | 690.56 | 146.50 | 0.00 | 690.56 | 5.20 | 0.00 | **677.07** | 4.72 | **-1.95** |
| 12 | 610.06 | 611.12 | – | 0.17 | 611.06 | 15.30 | 0.16 | 610.06 | 6.80 | 0.00 | 610.06 | 8.22 | 0.00 |
| 13 | 2437.15 | 2468.19 | – | 1.27 | 2437.15 | 114.50 | 0.00 | 2437.58 | 4.90 | 0.02 | **2411.02** | 5.16 | **-1.07** |
| 14 | 961.11 | 974.80 | – | 1.42 | 968.55 | 176.50 | 0.77 | 961.11 | 10.90 | 0.00 | **959.99** | 12.15 | **-0.12** |
| 15 | 1110.54 | 1132.49 | – | 1.98 | 1112.00 | 183.50 | 0.13 | 1110.54 | 46.40 | 0.00 | **1105.50** | 24.33 | **-0.45** |
| 16 | 699.79 | 699.79 | – | 0.00 | 699.80 | 26.50 | 0.00 | 699.79 | 5.10 | 0.00 | 699.79 | 8.49 | 0.00 |
| 17 | 861.79 | 862.36 | – | 0.07 | 861.79 | 12.50 | 0.00 | 861.79 | 15.40 | 0.00 | 861.79 | 14.34 | 0.00 |
| 18 | 997.97 | 1012.19 | – | 1.42 | 999.22 | 182.50 | 0.13 | 997.97 | 23.70 | 0.00 | **992.63** | 41.29 | **-0.54** |
| 19 | 716.24 | 726.96 | – | 1.50 | 722.17 | 268.80 | 0.83 | 716.24 | 168.40 | 0.00 | **707.08** | 39.72 | **-1.28** |
| 20 | 500.18 | 508.69 | – | 1.70 | 501.90 | 210.50 | 0.34 | 500.18 | 143.90 | 0.00 | **497.19** | 79.67 | **-0.60** |
| 21 | 971.45 | 989.24 | – | 1.83 | 977.03 | 348.80 | 0.57 | 971.45 | 1346.40 | 0.00 | **959.74** | 123.35 | **-1.21** |
| 22 | 994.77 | 1008.52 | – | 1.38 | 1001.75 | 297.30 | 0.70 | 994.77 | 1402.00 | 0.00 | **982.18** | 147.64 | **-1.27** |
| 23 | 998.81 | 1024.25 | – | 2.55 | 1011.19 | 420.50 | 1.24 | 998.81 | 903.80 | 0.00 | **987.07** | 197.71 | **-1.18** |
| 24 | 1083.69 | 1098.60 | – | 1.38 | 1092.90 | 213.50 | 0.85 | 1083.69 | 986.40 | 0.00 | **1074.86** | 160.60 | **-0.81** |
| 25 | 1298.87 | 1323.84 | – | 1.92 | 1320.27 | 362.50 | 1.65 | 1298.87 | 1389.20 | 0.00 | **1280.88** | 238.94 | **-1.39** |
| 26 | 1284.92 | 1314.34 | – | 2.29 | 1302.52 | 332.00 | 1.37 | 1284.92 | 1145.90 | 0.00 | **1271.26** | 267.51 | **-1.06** |
| 27 | 1279.22 | 1309.76 | – | 2.39 | 1304.14 | 362.00 | 1.95 | 1279.22 | 1529.90 | 0.00 | **1265.01** | 322.06 | **-1.11** |
| 28 | 2454.97 | 2526.81 | – | 2.93 | 2518.51 | 401.00 | 2.59 | 2454.97 | 1796.60 | 0.00 | **2438.37** | 2581.86 | **-0.68** |
| 29 | 2121.31 | 2175.33 | – | 2.55 | 2161.43 | 417.80 | 1.89 | 2121.31 | 1520.70 | 0.00 | **2105.24** | 1955.18 | **-0.76** |
| 30 | 1701.36 | 1742.15 | – | 2.40 | 1742.01 | 337.80 | 2.39 | 1701.36 | 1717.40 | 0.00 | **1682.12** | 893.57 | **-1.13** |
| 31 | 2160.70 | 2227.74 | – | 3.10 | 2204.44 | 472.80 | 2.02 | 2160.70 | 2244.60 | 0.00 | **2141.91** | 1820.12 | **-0.87** |
| 32 | 2119.46 | 2180.18 | – | 2.86 | 2167.61 | 394.00 | 2.27 | 2119.46 | 2036.50 | 0.00 | **2095.47** | 1441.45 | **-1.13** |
| 33 | 2174.71 | 2239.04 | – | 2.96 | 2222.42 | 459.50 | 2.19 | 2174.71 | 1925.10 | 0.00 | **2147.91** | 1732.20 | **-1.23** |
| 34 | 1114.40 | 1149.87 | – | 3.18 | 1142.25 | 458.00 | 2.50 | 1114.40 | 2399.10 | 0.00 | **1103.25** | 1992.90 | **-1.00** |
| 35 | 1345.26 | 1387.45 | – | 3.14 | 1392.05 | 471.80 | 3.48 | 1345.26 | 2376.00 | 0.00 | **1330.93** | 1979.94 | **-1.07** |
| 36 | 1598.88 | 1670.67 | – | 4.49 | 1653.05 | 375.30 | 3.39 | 1598.88 | 2646.50 | 0.00 | **1583.36** | 2754.05 | **-0.97** |
| Avg. | 1089.40 | 1111.63 | – | 1.57 | 1104.31 | 216.33 | 0.96 | 1089.45 | 772.72 | 0.01 | 1079.93 | 523.75 | -0.76 |

Bold type represents the new best solution values found.

[1] The best known solution in the literature.

[2] The percentage improvement (%) on the current BKS level. Lower numbers equate to better performance.

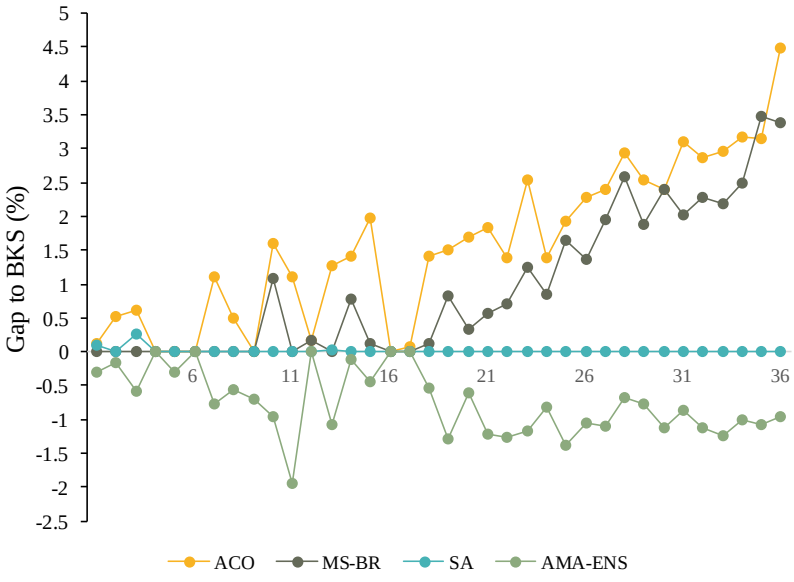


Figure 3: Gap to the BKS[1] for the 2URL version of 2L-CVRP (averaged over Class 2-5)

The X-axis represents the the current BKS level in the literature. The Lower, the better

[a]Gap to BKS (%): Percentage improvement between the solutions and BKS, Gap=100*(solution-BKS)/BKS.

## 1.2 2L-CVRP under rotated loading

Table 6: Results for 2L-CVRP (2SOL)

| Inst. | Class 2 | | | Class 3 | | | Class 4 | | | Class 5 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | BKS[1] | AMA-ENS | Gap (%)[2] | BKS[1] | AMA-ENS | Gap (%)[2] | BKS[1] | AMA-ENS | Gap (%)[2] | BKS[1] | AMA-ENS | Gap (%)[2] |
| 1 | 290.84 | 290.84 | 0.00 | 284.52 | **280.80** | **-1.31** | 280.80 | **280.80** | **0.00** | 278.73 | 278.73 | 0.00 |
| 2 | 347.73 | 347.73 | 0.00 | 352.16 | **346.18** | **-1.70** | 346.18 | **338.40** | **-2.25** | 334.96 | 334.96 | 0.00 |
| 3 | 403.93 | **397.46** | **-1.60** | 394.72 | **389.95** | **-1.21** | 389.95 | **365.99** | **-6.14** | 358.40 | 358.40 | 0.00 |
| 4 | 440.94 | **436.32** | **-1.05** | 440.68 | **434.91** | **-1.31** | 447.37 | **442.95** | **-0.99** | 430.88 | 430.88 | 0.00 |
| 5 | 388.72 | **384.61** | **-1.06** | 381.69 | **376.64** | **-1.32** | 383.87 | **378.53** | **-1.39** | 375.28 | 375.28 | 0.00 |
| 6 | 499.08 | **490.71** | **-1.68** | 504.68 | **499.67** | **-0.99** | 498.32 | **491.27** | **-1.41** | 495.85 | 495.85 | 0.00 |
| 7 | 734.65 | **722.53** | **-1.65** | 702.59 | **691.59** | **-1.57** | 703.49 | **697.39** | **-0.87** | 658.64 | 658.64 | 0.00 |
| 8 | 725.91 | 725.91 | 0.00 | 741.12 | **728.70** | **-1.68** | 697.92 | **692.50** | **-0.78** | 621.85 | 621.85 | 0.00 |
| 9 | 611.49 | 611.49 | 0.00 | 613.90 | **604.77** | **-1.49** | 625.10 | **621.01** | **-0.65** | 607.65 | 607.65 | 0.00 |
| 10 | 700.20 | 700.20 | 0.00 | 628.94 | **618.20** | **-1.71** | 715.82 | **710.06** | **-0.80** | 690.96 | 690.96 | 0.00 |
| 11 | 721.54 | **713.24** | **-1.15** | 717.37 | **708.59** | **-1.22** | 815.68 | **807.27** | **-1.03** | 624.82 | 624.82 | 0.00 |
| 12 | 619.63 | **611.86** | **-1.25** | 610.00 | **601.42** | **-1.41** | 618.23 | **610.68** | **-1.22** | 610.00 | **608.88** | **-0.18** |
| 13 | 2669.39 | **2630.50** | **-1.46** | 2486.44 | 2486.44 | 0.00 | 2609.36 | **2587.07** | **-0.85** | 2416.04 | **2416.01** | **0.00** |
| 14 | 1092.51 | **1075.41** | **-1.57** | 1039.06 | 1039.06 | 0.00 | 982.25 | **970.08** | **-1.24** | 922.75 | **921.35** | **-0.15** |
| 15 | 1041.75 | **1032.38** | **-0.90** | 1181.68 | **1167.75** | **-1.18** | 1246.49 | **1235.93** | **-0.85** | 1229.95 | **1225.94** | **-0.33** |
| 16 | 698.61 | **689.32** | **-1.33** | 698.61 | **686.32** | **-1.76** | 708.20 | **700.83** | **-1.04** | 698.61 | 698.61 | 0.00 |
| 17 | 870.86 | **861.18** | **-1.11** | 861.79 | **853.50** | **-0.96** | 861.79 | **851.28** | **-1.22** | 861.79 | 861.79 | 0.00 |
| 18 | 1053.09 | **1043.18** | **-0.94** | 1102.17 | **1094.21** | **-0.72** | 1134.11 | **1120.10** | **-1.24** | 926.34 | 926.34 | 0.00 |
| 19 | 792.07 | **787.28** | **-0.60** | 801.13 | **790.09** | **-1.38** | 801.21 | **790.20** | **-1.37** | 652.15 | 652.15 | 0.00 |
| 20 | 545.68 | **536.18** | **-1.74** | 541.58 | **534.75** | **-1.26** | 551.72 | **547.37** | **-0.79** | 478.15 | 478.15 | 0.00 |
| 21 | 1060.72 | **1040.78** | **-1.88** | 1149.90 | **1141.63** | **-0.72** | 1000.25 | **988.56** | **-1.17** | 886.00 | 886.00 | 0.00 |
| 22 | 1081.44 | **1070.39** | **-1.02** | 1094.66 | **1083.73** | **-1.00** | 1089.27 | **1075.78** | **-1.24** | 948.60 | **944.99** | **-0.38** |
| 23 | 1093.27 | **1073.83** | **-1.78** | 1117.54 | **1100.87** | **-1.49** | 1093.01 | **1077.94** | **-1.38** | 948.68 | **946.57** | **-0.22** |
| 24 | 1222.43 | **1211.27** | **-0.91** | 1118.44 | **1106.84** | **-1.04** | 1141.97 | **1129.43** | **-1.10** | 1046.08 | 1046.08 | 0.00 |
| 25 | 1453.98 | **1439.77** | **-0.98** | 1433.92 | **1423.40** | **-0.73** | 1435.18 | **1423.98** | **-0.78** | 1183.63 | **1179.86** | **-0.32** |
| 26 | 1323.23 | **1309.63** | **-1.03** | 1392.43 | **1372.76** | **-1.41** | 1447.03 | **1437.92** | **-0.63** | 1252.65 | 1252.65 | 0.00 |
| 27 | 1367.85 | **1355.55** | **-0.90** | 1423.74 | **1396.75** | **-1.90** | 1353.06 | **1343.52** | **-0.71** | 1259.17 | 1259.17 | 0.00 |
| 28 | 2632.55 | **2615.47** | **-0.65** | 2737.42 | **2705.31** | **-1.17** | 2690.69 | **2669.67** | **-0.78** | 2399.25 | 2399.25 | 0.00 |
| 29 | 2285.84 | **2255.95** | **-1.31** | 2150.35 | **2134.91** | **-0.72** | 2299.32 | **2276.92** | **-0.97** | 2179.12 | 2179.12 | 0.00 |
| 30 | 1875.38 | **1841.07** | **-1.83** | 1912.09 | **1899.18** | **-0.68** | 1904.42 | **1879.27** | **-1.32** | 1565.96 | **1562.08** | **-0.25** |
| 31 | 2341.08 | **2308.28** | **-1.40** | 2354.21 | **2332.46** | **-0.92** | 2459.59 | **2425.00** | **-1.41** | 2053.57 | **2045.41** | **-0.40** |
| 32 | 2365.99 | **2348.70** | **-0.73** | 2320.35 | **2285.14** | **-1.52** | 2343.29 | **2310.29** | **-1.41** | 2016.58 | 2016.58 | 0.00 |
| 33 | 2349.98 | **2335.20** | **-0.63** | 2447.20 | **2407.80** | **-1.61** | 2446.05 | **2423.17** | **-0.94** | 2044.88 | **2039.12** | **-0.28** |
| 34 | 1217.24 | **1200.51** | **-1.37** | 1249.07 | **1235.21** | **-1.11** | 1241.13 | **1232.61** | **-0.69** | 1062.18 | **1060.09** | **-0.20** |
| 35 | 1434.99 | **1410.19** | **-1.73** | 1511.66 | **1497.24** | **-0.95** | 1550.24 | **1529.38** | **-1.35** | 1278.90 | **1275.19** | **-0.29** |
| 36 | 1755.33 | **1727.37** | **-1.59** | 1833.97 | **1813.94** | **-1.09** | 1713.71 | **1694.85** | **-1.10** | 1541.07 | **1533.39** | **-0.50** |
| Avg. | 1169.72 | 1156.45 | -1.08 | 1175.88 | 1163.08 | -1.17 | 1184.06 | 1171.06 | -1.20 | 1053.89 | 1052.58 | -0.10 |

Bold type represents the new best solution values found.

[1] The best known solution in the literature.

[2] The percentage improvement (%) on the current BKS level. Lower numbers equate to better performance.

Table 7: Comparison for the 2SOL version of 2L-CVRP (averaged over Classes 2-5)

| Inst. | BKS[1] | PRMP | | | VNS | | | SA | | | AMA-ENS | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Cost | Time (s) | Gap (%)[2] | Cost | Time (s) | Gap (%)[2] | Cost | Time (s) | Gap (%)[2] | Cost | Time (s) | Gap (%)[2] |
| 1 | 287.08 | 287.08 | 1.40 | 0.00 | 287.08 | 4.50 | 0.00 | 287.08 | 5.30 | 0.00 | **282.79** | 5.03 | **-1.49** |
| 2 | 344.21 | 344.21 | 1.00 | 0.00 | 344.21 | 0.50 | 0.00 | 347.21 | 0.60 | 0.87 | **341.82** | 0.52 | **-0.69** |
| 3 | 381.40 | 381.40 | 1.30 | 0.00 | 381.40 | 1.40 | 0.00 | 389.40 | 0.90 | 2.10 | **377.95** | 0.81 | **-0.90** |
| 4 | 439.97 | 439.97 | 1.60 | 0.00 | 439.97 | 0.90 | 0.00 | 439.97 | 1.30 | 0.00 | **436.27** | 1.16 | **-0.84** |
| 5 | 382.39 | 382.39 | 2.60 | 0.00 | 382.39 | 4.20 | 0.00 | 382.39 | 1.80 | 0.00 | **378.77** | 1.68 | **-0.95** |
| 6 | 499.48 | 499.48 | 5.60 | 0.00 | 499.48 | 1.70 | 0.00 | 499.48 | 2.50 | 0.00 | **494.38** | 2.04 | **-1.02** |
| 7 | 699.84 | 702.27 | 5.30 | 0.35 | 701.63 | 12.20 | 0.26 | 699.84 | 6.80 | 0.00 | **692.54** | 6.15 | **-1.04** |
| 8 | 696.70 | 699.54 | 7.00 | 0.41 | 696.70 | 21.30 | 0.00 | 696.70 | 45.80 | 0.00 | **692.24** | 37.29 | **-0.64** |
| 9 | 614.53 | 615.93 | 6.20 | 0.23 | 614.53 | 3.70 | 0.00 | 614.53 | 4.00 | 0.00 | **611.23** | 3.40 | **-0.54** |
| 10 | 686.09 | 688.48 | 55.00 | 0.35 | 686.09 | 115.60 | 0.00 | 686.52 | 294.80 | 0.06 | **679.86** | 240.73 | **-0.91** |
| 11 | 722.22 | 725.83 | 75.30 | 0.50 | 722.84 | 54.30 | 0.09 | 722.22 | 173.90 | 0.00 | **713.48** | 163.21 | **-1.21** |
| 12 | 614.47 | 614.52 | 7.10 | 0.01 | 614.52 | 7.50 | 0.01 | 614.47 | 130.90 | 0.00 | **608.21** | 115.32 | **-1.02** |
| 13 | 2545.31 | 2554.93 | 119.60 | 0.38 | 2546.77 | 53.40 | 0.06 | 2545.31 | 274.50 | 0.00 | **2530.01** | 228.06 | **-0.60** |
| 14 | 1009.14 | 1027.38 | 637.00 | 1.81 | 1026.21 | 416.70 | 1.69 | 1009.14 | 632.80 | 0.00 | **1001.48** | 521.66 | **-0.76** |
| 15 | 1175.08 | 1189.97 | 68.10 | 1.27 | 1175.08 | 298.50 | 0.00 | 1175.19 | 344.20 | 0.01 | **1165.50** | 306.14 | **-0.82** |
| 16 | 701.00 | 701.00 | 14.20 | 0.00 | 701.00 | 3.40 | 0.00 | 701.00 | 6.10 | 0.00 | **693.77** | 5.57 | **-1.03** |
| 17 | 864.05 | 864.05 | 40.90 | 0.00 | 864.05 | 4.40 | 0.00 | 864.05 | 19.40 | 0.00 | **856.94** | 18.25 | **-0.82** |
| 18 | 1054.29 | 1058.00 | 95.20 | 0.35 | 1054.29 | 396.20 | 0.00 | 1056.07 | 480.10 | 0.17 | **1045.96** | 397.52 | **-0.79** |
| 19 | 761.64 | 766.05 | 188.30 | 0.58 | 761.83 | 297.40 | 0.02 | 761.64 | 435.40 | 0.00 | **754.93** | 394.74 | **-0.88** |
| 20 | 529.91 | 534.87 | 1660.90 | 0.94 | 530.26 | 921.70 | 0.07 | 529.91 | 1541.70 | 0.00 | **524.11** | 1337.34 | **-1.09** |
| 21 | 1024.22 | 1041.77 | 420.20 | 1.71 | 1027.74 | 1003.00 | 0.34 | 1024.22 | 1850.20 | 0.00 | **1014.24** | 1725.56 | **-0.97** |
| 22 | 1053.49 | 1066.56 | 524.20 | 1.24 | 1053.49 | 1107.50 | 0.00 | 1055.62 | 1403.60 | 0.20 | **1043.72** | 1234.77 | **-0.93** |
| 23 | 1063.52 | 1076.19 | 519.50 | 1.19 | 1063.52 | 953.10 | 0.00 | 1064.15 | 1499.30 | 0.06 | **1049.80** | 1231.06 | **-1.29** |
| 24 | 1132.24 | 1139.12 | 1064.30 | 0.61 | 1132.88 | 841.40 | 0.06 | 1132.24 | 1339.60 | 0.00 | **1123.41** | 1116.31 | **-0.78** |
| 25 | 1378.55 | 1401.00 | 2319.50 | 1.63 | 1378.55 | 1306.10 | 0.00 | 1379.16 | 1716.90 | 0.04 | **1366.75** | 1551.94 | **-0.86** |
| 26 | 1354.05 | 1370.78 | 1491.20 | 1.24 | 1355.92 | 1240.30 | 0.14 | 1354.05 | 1550.00 | 0.00 | **1343.24** | 1398.38 | **-0.80** |
| 27 | 1352.11 | 1372.49 | 4163.80 | 1.51 | 1354.92 | 1242.50 | 0.21 | 1352.11 | 1795.20 | 0.00 | **1338.75** | 1606.14 | **-0.99** |
| 28 | 2615.65 | 2669.07 | 8640.10 | 2.04 | 2646.59 | 2423.00 | 1.18 | 2615.65 | 2206.80 | 0.00 | **2597.43** | 1999.53 | **-0.70** |
| 29 | 2228.66 | 2263.76 | 5484.30 | 1.57 | 2241.65 | 2672.80 | 0.58 | 2228.66 | 2281.90 | 0.00 | **2211.73** | 2164.88 | **-0.76** |
| 30 | 1814.92 | 1853.02 | 4676.90 | 2.10 | 1819.25 | 2502.10 | 0.24 | 1814.92 | 1583.80 | 0.00 | **1795.40** | 1439.26 | **-1.08** |
| 31 | 2302.11 | 2358.26 | 5845.40 | 2.44 | 2317.82 | 2760.60 | 0.68 | 2302.11 | 2406.80 | 0.00 | **2277.79** | 2224.63 | **-1.06** |
| 32 | 2261.55 | 2322.71 | 9433.20 | 2.70 | 2274.88 | 2664.00 | 0.59 | 2261.55 | 2704.20 | 0.00 | **2240.18** | 2455.73 | **-0.94** |
| 33 | 2322.03 | 2394.32 | 5662.50 | 3.11 | 2342.87 | 2614.50 | 0.90 | 2322.03 | 2864.50 | 0.00 | **2301.32** | 2448.45 | **-0.89** |
| 34 | 1192.59 | 1225.54 | 13141.80 | 2.76 | 1196.33 | 2825.70 | 0.31 | 1192.59 | 2568.80 | 0.00 | **1182.11** | 2157.23 | **-0.88** |
| 35 | 1443.95 | 1494.32 | 8989.60 | 3.49 | 1454.42 | 3052.90 | 0.73 | 1443.95 | 2809.40 | 0.00 | **1428.00** | 2354.17 | **-1.10** |
| 36 | 1711.02 | 1762.17 | 10059.60 | 2.99 | 1736.03 | 3282.60 | 1.46 | 1711.02 | 3282.20 | 0.00 | **1692.39** | 3017.78 | **-1.09** |
| Avg. | 1146.10 | 1163.57 | 2373.05 | 1.10 | 1150.76 | 975.32 | 0.27 | 1146.56 | 1062.94 | 0.10 | 1135.79 | 942.01 | -0.92 |

Bold type represents the new best solution values found.

[1] The best known solution in the literature.

[2] The percentage improvement (%) on the current BKS level. Lower numbers equate to better performance.

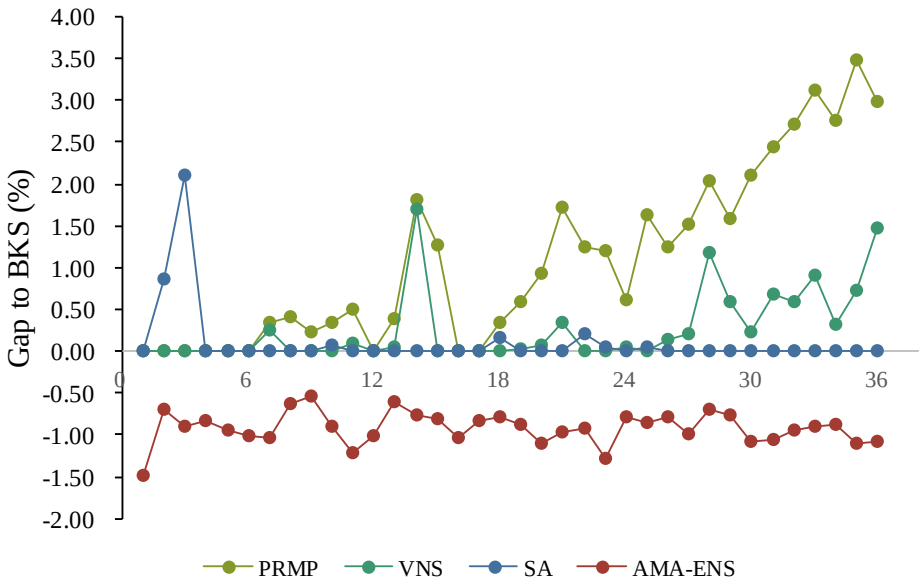


Figure 4: Gap to the BKS[1] for the 2SOL version of 2L-CVRP (averaged over Class 2–5)

The X-axis represents the the current BKS level in the literature. The Lower, the better

[a]Gap to BKS (%): Percentage improvement between the solutions and BKS, Gap=100*(solution- BKS)/BKS.

Table 8: Results for the 2SRL version of 2L-CVRP

| Inst. | Class 2 | | | Class 3 | | | Class 4 | | | Class 5 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | BKS[1] | AMA-ENS | Gap (%)[2] | BKS[1] | AMA-ENS | Gap (%)[2] | BKS[1] | AMA-ENS | Gap (%)[2] | BKS[1] | AMA-ENS | Gap (%)[2] |
| 1 | 278.73 | 278.73 | 0.00 | 284.23 | 284.23 | 0.00 | 282.95 | **278.49** | **-1.58** | 278.73 | 278.73 | 0.00 |
| 2 | 334.96 | 334.96 | 0.00 | 352.16 | 352.16 | 0.00 | 334.96 | **330.92** | **-1.21** | 334.96 | 334.96 | 0.00 |
| 3 | 384.93 | 384.93 | 0.00 | 385.32 | 385.32 | 0.00 | 364.45 | 364.45 | 0.00 | 358.40 | 358.40 | 0.00 |
| 4 | 430.89 | 430.89 | 0.00 | 430.89 | **423.39** | **-1.74** | 447.37 | 447.37 | 0.00 | 430.89 | 430.89 | 0.00 |
| 5 | 375.28 | **372.97** | **-0.62** | 379.94 | **372.96** | **-1.84** | 383.88 | 383.88 | 0.00 | 375.28 | 375.28 | 0.00 |
| 6 | 498.16 | **494.87** | **-0.66** | 498.16 | **491.77** | **-1.28** | 498.32 | 498.32 | 0.00 | 495.85 | 495.85 | 0.00 |
| 7 | 716.82 | **711.55** | **-0.74** | 668.39 | **657.27** | **-1.66** | 686.26 | 686.26 | 0.00 | 657.77 | 657.77 | 0.00 |
| 8 | 674.20 | **670.46** | **-0.55** | 738.43 | **733.86** | **-0.62** | 692.47 | **682.89** | **-1.38** | 609.90 | 609.90 | 0.00 |
| 9 | 607.65 | **602.65** | **-0.82** | 607.65 | 607.65 | 0.00 | 625.10 | **617.87** | **-1.16** | 607.65 | 607.65 | 0.00 |
| 10 | 684.70 | **672.10** | **-1.84** | 615.68 | 615.68 | 0.00 | 710.87 | **701.59** | **-1.31** | 680.26 | 680.26 | 0.00 |
| 11 | 694.60 | **687.12** | **-1.08** | 704.77 | 704.77 | 0.00 | 776.69 | **768.03** | **-1.12** | 624.82 | 624.82 | 0.00 |
| 12 | 610.00 | **599.59** | **-1.71** | 610.00 | **605.92** | **-0.67** | 614.24 | **608.38** | **-0.95** | 610.00 | 610.00 | 0.00 |
| 13 | 2526.07 | **2497.53** | **-1.13** | 2436.41 | **2423.89** | **-0.51** | 2561.65 | **2547.29** | **-0.56** | 2334.78 | **2314.73** | **-0.86** |
| 14 | 1032.96 | **1015.88** | **-1.65** | 1006.69 | **991.19** | **-1.54** | 981.90 | **976.41** | **-0.56** | 921.45 | **909.27** | **-1.32** |
| 15 | 1005.26 | **988.49** | **-1.67** | 1146.66 | **1128.40** | **-1.59** | 1172.43 | **1164.03** | **-0.72** | 1160.96 | **1142.64** | **-1.58** |
| 16 | 698.61 | **693.95** | **-0.67** | 698.61 | **688.15** | **-1.50** | 703.35 | **699.69** | **-0.52** | 698.61 | **685.64** | **-1.86** |
| 17 | 861.79 | **855.89** | **-0.68** | 861.79 | **850.62** | **-1.30** | 861.79 | **856.94** | **-0.56** | 861.79 | **846.64** | **-1.76** |
| 18 | 988.37 | **982.36** | **-0.61** | 1031.49 | **1023.47** | **-0.78** | 1118.18 | **1108.76** | **-0.84** | 921.29 | **921.29** | **0.00** |
| 19 | 732.64 | **726.17** | **-0.88** | 757.59 | **749.43** | **-1.08** | 776.59 | **771.34** | **-0.68** | 651.97 | **651.97** | **0.00** |
| 20 | 495.01 | **488.72** | **-1.27** | 519.43 | **516.21** | **-0.62** | 541.17 | **533.06** | **-1.50** | 472.09 | **468.52** | **-0.76** |
| 21 | 986.35 | **976.52** | **-1.00** | 1104.72 | **1098.40** | **-0.57** | 977.00 | **965.11** | **-1.22** | 881.38 | **869.11** | **-1.39** |
| 22 | 1001.03 | **993.70** | **-0.73** | 1044.34 | **1037.14** | **-0.69** | 1061.43 | **1054.24** | **-0.68** | 935.74 | **924.60** | **-1.19** |
| 23 | 1001.74 | **987.13** | **-1.46** | 1064.72 | **1055.25** | **-0.89** | 1076.34 | **1070.13** | **-0.58** | 942.10 | **925.20** | **-1.79** |
| 24 | 1173.04 | **1161.78** | **-0.96** | 1076.30 | **1056.79** | **-1.81** | 1102.37 | **1092.83** | **-0.87** | 1042.43 | **1033.30** | **-0.88** |
| 25 | 1371.51 | **1345.79** | **-1.88** | 1364.61 | **1350.86** | **-1.01** | 1395.23 | **1384.08** | **-0.80** | 1173.55 | **1163.71** | **-0.84** |
| 26 | 1278.62 | **1257.48** | **-1.65** | 1341.01 | **1321.63** | **-1.45** | 1395.93 | **1386.11** | **-0.70** | 1226.01 | **1202.74** | **-1.90** |
| 27 | 1292.78 | **1281.32** | **-0.89** | 1367.12 | **1341.89** | **-1.85** | 1324.92 | **1314.50** | **-0.79** | 1250.91 | **1240.92** | **-0.80** |
| 28 | 2524.37 | **2511.47** | **-0.51** | 2588.78 | **2564.20** | **-0.95** | 2647.03 | **2631.62** | **-0.58** | 2368.67 | **2327.12** | **-1.75** |
| 29 | 2170.03 | **2154.51** | **-0.72** | 2089.73 | **2060.72** | **-1.39** | 2247.12 | **2232.24** | **-0.66** | 2152.16 | **2130.54** | **-1.00** |
| 30 | 1774.70 | **1764.33** | **-0.58** | 1821.19 | **1804.82** | **-0.90** | 1838.87 | **1816.89** | **-1.20** | 1541.85 | **1517.39** | **-1.59** |
| 31 | 2231.33 | **2207.55** | **-1.07** | 2260.59 | **2244.64** | **-0.71** | 2380.00 | **2346.15** | **-1.42** | 2025.75 | **2006.04** | **-0.97** |
| 32 | 2226.32 | **2189.07** | **-1.67** | 2234.74 | **2216.97** | **-0.80** | 2277.96 | **2246.83** | **-1.37** | 1986.14 | **1954.86** | **-1.57** |
| 33 | 2209.40 | **2192.23** | **-0.78** | 2350.43 | **2306.24** | **-1.88** | 2381.54 | **2360.18** | **-0.90** | 2010.55 | **1984.73** | **-1.28** |
| 34 | 1156.30 | **1139.55** | **-1.45** | 1195.80 | **1186.40** | **-0.79** | 1201.39 | **1180.68** | **-1.72** | 1041.41 | **1022.70** | **-1.80** |
| 35 | 1352.64 | **1328.02** | **-1.82** | 1430.28 | **1415.07** | **-1.06** | 1504.19 | **1486.84** | **-1.15** | 1261.97 | **1253.09** | **-0.70** |
| 36 | 1673.51 | **1644.27** | **-1.75** | 1759.82 | **1744.59** | **-0.87** | 1656.29 | **1641.88** | **-0.87** | 1511.25 | **1490.66** | **-1.36** |
| Avg. | 1112.65 | 1100.68 | -0.99 | 1134.12 | 1122.55 | -0.95 | 1155.62 | 1145.45 | -0.84 | 1039.98 | 1029.22 | -0.80 |

Bold type represents the new best solution values found.

[1] The best known solution in the literature.

[2] The percentage improvement (%) on the current BKS level. Lower numbers equate to better performance.

# 2 Results of AMA-ENS for the 3L-CVRP Problem

In the three-dimensional loading CVRP (3L-CVRP), the three dimensions of the vehicle are taken into account and the customer's demand also consists of three-dimensional items. Since the height dimension is considered, additional loading constraints concerning fragility and vertical stability of the cargo may be specified. This problem is frequently encountered in distribution logistics when items may be stacked on top of each other in a container. Examples of applications of the 3L-CVRP are found in the distribution of furniture, household appliances, soft drinks and staple goods (Ruan et al. 2013).

There are two sets of the 3L-CVRP benchmark data. The first set includes 27 instances and was introduced by Gendreau et al. (2006).The second set was proposed by Tarantilis et al. (2009); it contains 12 instances and has more variation in customer size and item dimensions. The first set was derived from classic CVRP data by introducing 3-D items for each customer. More precisely, the length L, width W, and height H of vehicles are set to 60, 25, and 30, respectively. The customer distribution and demand are the same as in the CVRP instance. However, for each customer, the item number is a random number within the [1, 3] interval, and the item dimensions are randomly generated in the interval between 20% and 60% of the corresponding vehicle edges. Each item is given a 20% probability of being fragile. The second set contains 50-125 customers located at (rcos ($\vartheta$), rsin ($\vartheta$)), and the depot location is (0,0); r and $\vartheta$ are randomly selected from [10, 100] and [0, $2\pi$] respectively. The vehicle dimensions are set as L = 60, W = 30, and H = 30, respectively. There are three classes of items: Class 1 contains small items with dimensions equal to 20%-40% of the corresponding vehicle dimensions; Class 2 contains large items whose dimensions are within the interval [40%, 60%] of vehicle dimensions; and Class 3 contains diverse items by setting the factor interval as [10%, 70%]. Each item has a 20% probability of being fragile. The number of items required by each customer is randomly taken from the intervals [2, 4], [1, 2], and [1, 3] for classes 1, 2, and 3, respectively. The demand of each customer is randomly distributed in the interval [5, 35].

**The charts and tables in this section are extracted from the following paper:**

**Wang Y.**, Zhou S., Chen H., Andrew Lim*, "Three-dimensional Capacitated Vehicle Routing Problem with Time-Dependent Travel Times", (Working paper), to be submitted to Transportation Research Part B: Methodological.

Table 9: Comparison between AMA-ENS and existing approaches in 3L-CVRP set1 instances. All constraints are imposed.

| Id | Name | BKS | DMTS | | VRLH1 | | HA | | TS-ILA | | ELS | | AMA-ENS | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Id | Name | BKS[1] | Avg | time (s) | Avg | time (s) | Avg | time (s) | Avg | time (s) | Avg | time (s) | Avg | time (s) | Gap (%)[2] |
| 1 | E016-03m | 302.02 | 302.23 | 85.10 | 302.02 | 72.30 | 303.21 | 98.85 | 302.02 | 53.50 | 302.02 | 3.20 | 302.02 | 3.15 | 0.00 |
| 2 | E016-05m | 334.96 | 334.96 | 3.50 | 334.96 | 0.90 | 334.96 | 4.55 | 334.96 | 6.30 | 334.96 | 0.18 | 334.96 | 0.17 | 0.00 |
| 3 | E021-04m | 381.37 | 409.44 | 450.10 | 401.44 | 182.00 | 398.05 | 93.86 | 381.37 | 116.20 | 385.53 | 365.61 | 381.37 | 355.83 | 0.00 |
| 4 | E021-06m | 437.19 | 439.98 | 51.20 | 437.54 | 16.10 | 440.68 | 46.75 | 437.19 | 14.00 | 437.19 | 20.26 | 437.19 | 19.95 | 0.00 |
| 5 | E022-04g | 436.79 | 447.36 | 287.80 | 451.03 | 182.60 | 452.56 | 63.98 | 436.79 | 149.30 | 443.17 | 151.91 | 436.79 | 149.93 | 0.00 |
| 6 | E022-06m | 498.32 | 499.99 | 130.60 | 498.38 | 23.60 | 498.56 | 196.90 | 498.32 | 31.60 | 501.06 | 3.44 | 498.32 | 3.35 | 0.00 |
| 7 | E023-03g | 768.94 | 773.31 | 421.00 | 772.49 | 133.10 | 790.23 | 317.02 | 768.94 | 84.90 | 771.07 | 19.54 | 768.94 | 19.06 | 0.00 |
| 8 | E023-05s | 805.77 | 807.59 | 548.20 | 821.35 | 139.10 | 820.67 | 98.90 | 805.77 | 120.40 | 813.13 | 325.20 | 805.77 | 317.97 | 0.00 |
| 9 | E026-08m | 630.13 | 630.13 | 95.50 | 645.81 | 24.30 | 635.50 | 353.07 | 631.68 | 13.70 | 630.13 | 6.26 | 630.13 | 6.11 | 0.00 |
| 10 | E030-03g | 824.69 | 839.75 | 601.50 | 827.29 | 175.10 | 836.21 | 410.90 | 828.99 | 258.60 | 824.69 | 473.48 | 824.69 | 463.50 | 0.00 |
| 11 | E030-04s | 776.19 | 790.47 | 434.40 | 815.62 | 136.40 | 825.75 | 197.76 | 780.61 | 278.90 | 776.19 | 220.54 | **764.83** | 214.96 | **-1.46** |
| 12 | E031-09h | 610.23 | 615.05 | 224.80 | 630.46 | 14.00 | 626.59 | 89.47 | 614.60 | 145.80 | 610.23 | 21.60 | **602.09** | 21.09 | **-1.33** |
| 13 | E033-03n | 2636.85 | 2732.85 | 654.60 | 2694.81 | 268.40 | 2739.80 | 319.78 | 2636.85 | 369.40 | 2656.72 | 494.99 | **2600.00** | 484.81 | **-1.40** |
| 14 | E033-04g | 1369.22 | 1460.34 | 2659.30 | 1413.59 | 311.60 | 1469.38 | 268.39 | 1398.77 | 588.10 | 1369.22 | 1079.37 | **1338.71** | 1055.49 | **-2.23** |
| 15 | E033-05s | 1338.35 | 1386.75 | 984.60 | 1355.50 | 311.50 | 1369.69 | 356.55 | 1352.76 | 615.90 | 1338.35 | 1295.01 | **1312.97** | 1265.43 | **-1.90** |
| 16 | E036-11h | 698.61 | 698.69 | 50.20 | 705.05 | 3.40 | 703.15 | 431.74 | 698.92 | 4.00 | 698.61 | 6.01 | **683.21** | 5.90 | **-2.20** |
| 17 | E041-14h | 866.40 | 869.96 | 177.20 | 917.96 | 2.50 | 872.05 | 374.84 | 866.40 | 9.50 | 866.40 | 17.30 | **856.14** | 16.86 | **-1.18** |
| 18 | E045-04f | 1223.64 | 1252.67 | 2258.60 | 1228.98 | 309.50 | 1250.86 | 325.74 | 1228.47 | 1634.10 | 1223.64 | 1104.38 | **1212.02** | 1083.64 | **-0.95** |
| 19 | E051-05e | 744.33 | 777.96 | 1407.20 | 753.87 | 416.50 | 780.37 | 1374.84 | 763.09 | 718.40 | 744.33 | 1494.95 | **729.43** | 1472.96 | **-2.00** |
| 20 | E072-04f | 570.82 | 600.82 | 7466.00 | 596.42 | 427.00 | 605.59 | 1336.97 | 590.99 | 2941.80 | 570.82 | 2441.63 | **565.65** | 2373.08 | **-0.91** |
| 21 | E076-07s | 1063.20 | 1140.11 | 2848.60 | 1107.00 | 443.40 | 1119.45 | 1247.86 | 1096.53 | 2301.40 | 1063.20 | 2510.78 | **1052.77** | 2441.71 | **-0.98** |
| 22 | E076-08s | 1140.54 | 1199.14 | 1890.00 | 1171.49 | 423.50 | 1167.28 | 1294.57 | 1155.81 | 1241.80 | 1140.54 | 2689.20 | **1118.18** | 2640.55 | **-1.96** |
| 23 | E076-10e | 1094.33 | 1176.07 | 2829.50 | 1135.46 | 425.80 | 1171.77 | 1105.74 | 1130.08 | 1924.90 | 1094.33 | 2887.94 | **1087.41** | 2826.04 | **-0.63** |
| 24 | E076-14s | 1101.67 | 1161.87 | 2391.60 | 1128.82 | 411.10 | 1136.27 | 2001.05 | 1122.80 | 2526.80 | 1101.67 | 2282.75 | **1088.06** | 2224.28 | **-1.24** |
| 25 | E101-08e | 1359.25 | 1442.62 | 3580.30 | 1428.80 | 453.00 | 1426.34 | 1458.80 | 1417.09 | 4536.20 | 1359.25 | 2846.14 | **1347.16** | 2769.55 | **-0.89** |
| 26 | E101-10c | 1545.67 | 1614.56 | 2968.70 | 1625.31 | 430.60 | 1585.46 | 3354.72 | 1605.11 | 3017.70 | 1545.67 | 2958.29 | **1527.89** | 2901.70 | **-1.15** |
| 27 | E101-14s | 1479.73 | 1571.38 | 2837.80 | 1550.85 | 435.00 | 1562.18 | 3140.18 | 1538.10 | 6025.70 | 1479.73 | 3065.46 | **1453.82** | 3018.86 | **-1.75** |
| Avg. | - | 927.38 | 962.08 | 1419.92 | 953.79 | 228.60 | 960.10 | 754.21 | 941.59 | 1101.07 | 928.96 | 1066.13 | 917.06 | 1042.81 | -0.90 |

Bold type represents the new best solution values found.

[1] The best known solution in the literature.

[2] The percentage improvement (%) on the current BKS level. Lower numbers equate to better performance.

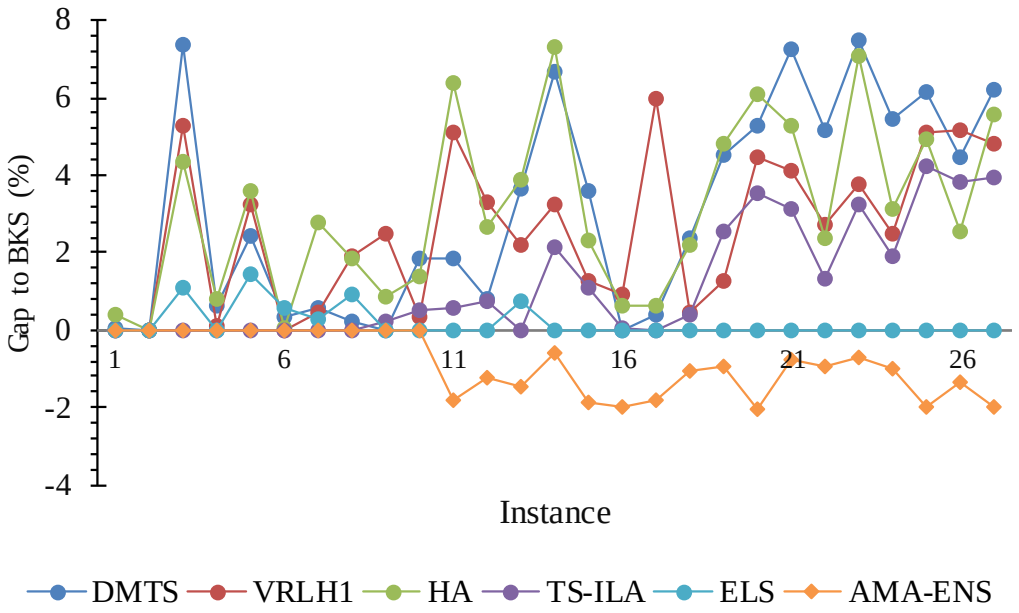


Figure 5: Gap to the BKS[1] for 3L-CVRP

The X-axis represents the the current BKS level in the literature. The Lower, the better

[a]Gap to BKS (%): Percentage improvement between the solutions and BKS, Gap=100*(solution-BKS)/BKS.

# 3 Results of AMA-ENS for the CVRP problem

CVRP is the cornerstone of VRP related research and has been widely used in the field of logistics. Since the CVRP benchmark data sets published earlier has many disadvantages, like becaming too easy for current algorithms, being too homogeneous, not covering the wide range of characteristics found in real applications, etc., we ran our algorithm on the newly published CVRP benchmark data set (Uchoa at al. 2017. EJOR), whose customer size ranges from 100 to 1000.

For comparison purposes, we consider recent state-of-the-art algorithms:

- Google OR: Google Operations Research (OR) Solver
  https://developers.google.com/optimization/
- HILS: Hybrid iterated local search
  (Subramanian at el. 2013)
- LKH-3: Lin-Kernighan-Helsgaun heuristic
  (Helsgaun at el. 2017)
- KGLS: Knowledge guided local search
  (Arnold et al. 2018)
- SISR: Slack induction by string removals
  (Christiaens at el. 2020. *Transportations Science*)
- FILO: Fast iterative localized optimization algorithm
  (Accorsi at al. 2021. *Transportations Science*)
- **AMA-ENS**: Adaptive memetic algorithm with extended neighborhood search
  (Wang at al. 2021.)

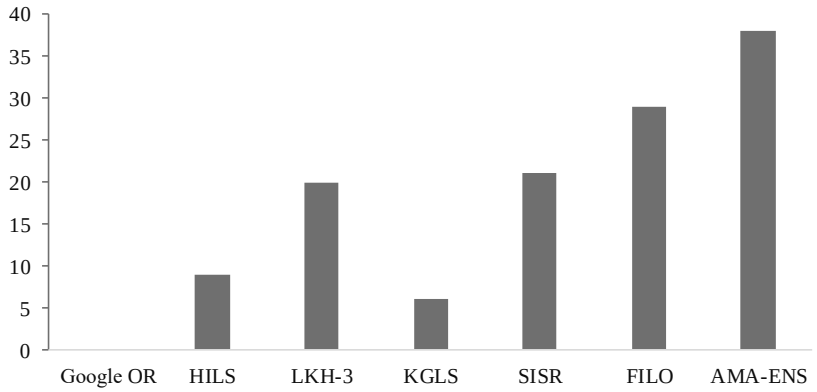


Figure 6: Number of Instances matching the BKS

AMA-ENS succeeds in matching 38 instances with its corresponding BKS value, and the number for SISR and FILO are 21 and 29, respectively. AMA-ENS performs better than SISR and FILO.

Table 10: Comparison of average solution quality for CVRP

| Inst. | BKS | Google OR | | LKH-3 | | HILS | | KGLS | | SISR | | FILO | | AMA-ENS | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Avg Cost | Gap | Avg Cost | Gap | Avg Cost | Gap | Avg Cost | Gap | Avg Cost | Gap | Avg Cost | Gap | Avg Cost | Gap |
| X-n101-k25 | 27591 | 27977.2 | 1.40 | 27639.2 | 0.17 | 27591.0 | 0.00 | 27631.9 | 0.15 | 27593.3 | 0.01 | 27591.0 | 0.00 | 27591.0 | 0.00 |
| X-n106-k14 | 26362 | 26757.5 | 1.50 | 26406.8 | 0.17 | 26391.1 | 0.11 | 26413.2 | 0.19 | 26380.9 | 0.07 | 26373.3 | 0.04 | 26380.9 | 0.07 |
| X-n110-k13 | 14971 | 15099.8 | 0.86 | 14993.9 | 0.15 | 14971.0 | 0.00 | 14971.0 | 0.00 | 14972.1 | 0.01 | 14971.0 | 0.00 | 14971.0 | 0.00 |
| X-n115-k10 | 12747 | 12808.3 | 0.48 | 12747.0 | 0.00 | 12747.0 | 0.00 | 12747.1 | 0.00 | 12747.0 | 0.00 | 12747.0 | 0.00 | 12747.0 | 0.00 |
| X-n120-k6 | 13332 | 13501.9 | 1.27 | 13332.8 | 0.01 | 13333.7 | 0.01 | 13332.0 | 0.00 | 13332.0 | 0.00 | 13332.0 | 0.00 | 13332.0 | 0.00 |
| X-n125-k30 | 55539 | 56853.4 | 2.37 | 55907.4 | 0.66 | 55846.5 | 0.55 | 55740.8 | 0.36 | 55559.8 | 0.04 | 55693.7 | 0.28 | 55539.0 | 0.00 |
| X-n129-k18 | 28940 | 29722.3 | 2.70 | 29083.3 | 0.50 | 28972.1 | 0.11 | 28971.6 | 0.11 | 28948.9 | 0.03 | 28948.4 | 0.03 | 28948.9 | 0.03 |
| X-n134-k13 | 10916 | 11171.0 | 2.34 | 10970.6 | 0.50 | 10947.5 | 0.29 | 10940.5 | 0.22 | 10937.7 | 0.20 | 10927.9 | 0.11 | 10937.7 | 0.20 |
| X-n139-k10 | 13590 | 13741.2 | 1.11 | 13654.9 | 0.48 | 13591.2 | 0.01 | 13590.0 | 0.00 | 13590.4 | 0.00 | 13590.0 | 0.00 | 13590.0 | 0.00 |
| X-n143-k7 | 15700 | 16135.6 | 2.77 | 15767.8 | 0.43 | 15735.7 | 0.23 | 15730.6 | 0.19 | 15727.8 | 0.18 | 15723.8 | 0.15 | 15700.0 | 0.00 |
| X-n148-k46 | 43448 | 44598.5 | 2.65 | 43518.9 | 0.16 | 43448.0 | 0.00 | 43588.3 | 0.32 | 43464.1 | 0.04 | 43480.5 | 0.07 | 43448.0 | 0.00 |
| X-n153-k22 | 21220 | 21789.3 | 2.68 | 21240.8 | 0.10 | 21452.3 | 1.09 | 21386.0 | 0.78 | 21228.6 | 0.04 | 21232.9 | 0.06 | 21225.0 | 0.02 |
| X-n157-k13 | 16876 | 17137.7 | 1.55 | 16879.1 | 0.02 | 16876.0 | 0.00 | 16877.5 | 0.01 | 16878.2 | 0.01 | 16876.0 | 0.00 | 16876.0 | 0.00 |
| X-n162-k11 | 14138 | 14262.2 | 0.88 | 14173.7 | 0.25 | 14152.4 | 0.10 | 14147.0 | 0.06 | 14159.0 | 0.15 | 14157.5 | 0.14 | 14138.0 | 0.00 |
| X-n167-k10 | 20557 | 21176.4 | 3.01 | 20706.1 | 0.73 | 20603.7 | 0.23 | 20586.9 | 0.15 | 20558.6 | 0.01 | 20557.0 | 0.00 | 20557.0 | 0.00 |
| X-n172-k51 | 45607 | 46874.9 | 2.78 | 45788.1 | 0.40 | 45665.3 | 0.13 | 45802.8 | 0.43 | 45622.6 | 0.03 | 45607.0 | 0.00 | 45607.0 | 0.00 |
| X-n176-k26 | 47812 | 49260.2 | 3.03 | 48104.1 | 0.61 | 48218.5 | 0.85 | 47991.6 | 0.38 | 47823.7 | 0.02 | 47985.0 | 0.36 | 47823.7 | 0.02 |
| X-n181-k23 | 25569 | 25935.6 | 1.43 | 25627.0 | 0.23 | 25572.1 | 0.01 | 25602.3 | 0.13 | 25575.1 | 0.02 | 25569.2 | 0.00 | 25575.1 | 0.02 |
| X-n186-k15 | 24145 | 24908.0 | 3.16 | 24277.7 | 0.55 | 24170.7 | 0.11 | 24178.3 | 0.14 | 24166.2 | 0.09 | 24154.6 | 0.04 | 24166.2 | 0.09 |
| X-n190-k8 | 16980 | 17421.9 | 2.60 | 17074.7 | 0.56 | 17108.0 | 0.75 | 17033.5 | 0.32 | 16982.8 | 0.02 | 16984.3 | 0.03 | 16983.3 | 0.02 |
| X-n195-k51 | 44225 | 46151.1 | 4.36 | 44478.8 | 0.57 | 44305.0 | 0.18 | 44427.2 | 0.46 | 44292.0 | 0.15 | 44265.7 | 0.09 | 44225.0 | 0.00 |
| X-n200-k36 | 58578 | 60447.9 | 3.19 | 58913.6 | 0.57 | 58784.0 | 0.35 | 58828.0 | 0.43 | 58635.6 | 0.10 | 58806.9 | 0.39 | 58578.0 | 0.00 |
| X-n204-k19 | 19565 | 20348.4 | 4.00 | 19731.3 | 0.85 | 19617.6 | 0.27 | 19621.0 | 0.29 | 19653.2 | 0.45 | 19568.4 | 0.02 | 19565.0 | 0.00 |
| X-n209-k16 | 30656 | 31775.5 | 3.65 | 30925.0 | 0.88 | 30739.0 | 0.27 | 30709.7 | 0.18 | 30661.7 | 0.02 | 30684.4 | 0.09 | 30656.0 | 0.00 |
| X-n214-k11 | 10856 | 11374.0 | 4.77 | 11103.4 | 2.28 | 11077.2 | 2.04 | 10944.3 | 0.81 | 10894.4 | 0.35 | 10884.3 | 0.26 | 10860.5 | 0.04 |
| X-n219-k73 | 117595 | 118038.0 | 0.38 | 117669.3 | 0.06 | 117595.0 | 0.00 | 117689.1 | 0.08 | 117623.7 | 0.02 | 117595.1 | 0.00 | 117596.1 | 0.00 |
| X-n223-k34 | 40437 | 42046.6 | 3.98 | 40750.9 | 0.78 | 40549.8 | 0.28 | 40714.4 | 0.69 | 40535.5 | 0.24 | 40502.8 | 0.16 | 40437.0 | 0.00 |
| X-n228-k23 | 25742 | 26613.4 | 3.39 | 25879.8 | 0.54 | 25803.7 | 0.24 | 25836.8 | 0.37 | 25814.3 | 0.28 | 25781.7 | 0.15 | 25742.8 | 0.00 |
| X-n233-k16 | 19230 | 19883.9 | 3.40 | 19345.8 | 0.60 | 19296.0 | 0.34 | 19328.6 | 0.51 | 19285.7 | 0.29 | 19293.9 | 0.33 | 19230.0 | 0.00 |
| X-n237-k14 | 27042 | 27927.5 | 3.27 | 27164.0 | 0.45 | 27068.8 | 0.10 | 27095.9 | 0.20 | 27081.1 | 0.14 | 27050.8 | 0.03 | 27081.1 | 0.14 |
| X-n242-k48 | 82751 | 85518.0 | 3.34 | 83353.0 | 0.73 | 82867.9 | 0.14 | 83209.2 | 0.55 | 82885.6 | 0.16 | 82876.1 | 0.15 | 82885.6 | 0.16 |
| X-n247-k50 | 37274 | 38282.8 | 2.71 | 37412.2 | 0.37 | 37502.3 | 0.61 | 37388.4 | 0.31 | 37379.6 | 0.28 | 37453.6 | 0.48 | 37379.6 | 0.28 |
| X-n251-k28 | 38684 | 40087.6 | 3.63 | 38982.0 | 0.77 | 38859.4 | 0.45 | 38893.3 | 0.54 | 38765.2 | 0.21 | 38783.5 | 0.26 | 38765.2 | 0.21 |
| X-n256-k16 | 18839 | 19294.5 | 2.42 | 19086.6 | 1.31 | 18880.8 | 0.22 | 18891.6 | 0.28 | 18887.3 | 0.26 | 18880.0 | 0.22 | 18887.3 | 0.26 |
| X-n261-k13 | 26558 | 27920.6 | 5.13 | 27115.6 | 2.10 | 26808.2 | 0.94 | 26717.5 | 0.60 | 26595.8 | 0.14 | 26682.4 | 0.47 | 26558.2 | 0.00 |
| X-n266-k58 | 75478 | 77660.8 | 2.89 | 76117.7 | 0.85 | 75611.4 | 0.18 | 75954.6 | 0.63 | 75609.2 | 0.17 | 75767.0 | 0.38 | 75564.7 | 0.11 |
| X-n270-k35 | 35291 | 36700.5 | 3.99 | 35523.3 | 0.66 | 35352.9 | 0.18 | 35462.1 | 0.48 | 35364.4 | 0.21 | 35348.3 | 0.16 | 35303.0 | 0.03 |
| X-n275-k28 | 21245 | 22087.3 | 3.96 | 21340.5 | 0.45 | 21262.4 | 0.08 | 21299.4 | 0.26 | 21250.5 | 0.03 | 21251.1 | 0.03 | 21245.0 | 0.00 |
| X-n280-k17 | 33503 | 35055.6 | 4.63 | 33933.6 | 1.29 | 33803.4 | 0.90 | 33670.1 | 0.50 | 33648.6 | 0.43 | 33652.6 | 0.45 | 33543.2 | 0.12 |
| X-n284-k15 | 20215 | 21137.9 | 4.57 | 20521.2 | 1.51 | 20415.9 | 0.99 | 20360.0 | 0.72 | 20287.6 | 0.36 | 20273.5 | 0.29 | 20245.5 | 0.15 |
| X-n289-k60 | 95151 | 98560.9 | 3.58 | 96055.6 | 0.95 | 95515.0 | 0.38 | 95882.8 | 0.77 | 95345.8 | 0.20 | 95556.3 | 0.43 | 95300.9 | 0.16 |
| X-n294-k50 | 47161 | 49301.8 | 4.54 | 47538.6 | 0.80 | 47262.0 | 0.21 | 47454.1 | 0.62 | 47251.9 | 0.19 | 47273.3 | 0.24 | 47184.1 | 0.05 |
| X-n298-k31 | 34231 | 36970.5 | 8.00 | 34571.7 | 1.00 | 34383.7 | 0.45 | 34377.4 | 0.43 | 34267.8 | 0.11 | 34283.3 | 0.15 | 34267.8 | 0.11 |
| X-n303-k21 | 21736 | 22573.7 | 3.85 | 22008.0 | 1.25 | 21900.7 | 0.76 | 21903.4 | 0.77 | 21772.9 | 0.17 | 21809.1 | 0.34 | 21772.9 | 0.17 |
| X-n308-k13 | 25859 | 27141.4 | 4.96 | 26194.9 | 1.30 | 26058.6 | 0.77 | 26076.4 | 0.84 | 26281.0 | 1.63 | 25937.7 | 0.30 | 26281.0 | 1.63 |
| X-n313-k71 | 94043 | 97497.4 | 3.67 | 94974.7 | 0.99 | 94290.3 | 0.26 | 94763.8 | 0.77 | 94155.7 | 0.12 | 94351.6 | 0.33 | 94112.2 | 0.07 |
| X-n317-k53 | 78355 | 79211.0 | 1.09 | 78553.5 | 0.25 | 78355.0 | 0.00 | 78413.5 | 0.07 | 78386.1 | 0.04 | 78358.6 | 0.00 | 78355.4 | 0.00 |
| X-n322-k28 | 29834 | 31488.5 | 5.55 | 30253.4 | 1.41 | 29996.5 | 0.54 | 30038.0 | 0.68 | 29892.5 | 0.20 | 29934.9 | 0.34 | 29848.7 | 0.05 |
| X-n327-k20 | 27532 | 28777.6 | 4.52 | 27905.1 | 1.36 | 27815.8 | 1.03 | 27646.8 | 0.42 | 27644.7 | 0.41 | 27610.7 | 0.29 | 27540.8 | 0.03 |
| X-n331-k15 | 31102 | 32648.2 | 4.97 | 31336.1 | 0.75 | 31227.4 | 0.40 | 31200.1 | 0.32 | 31124.5 | 0.07 | 31103.1 | 0.00 | 31103.0 | 0.00 |
| X-n336-k84 | 139111 | 143294.8 | 3.01 | 140226.2 | 0.80 | 139560.0 | 0.32 | 140831.3 | 1.24 | 139429.8 | 0.23 | 139585.7 | 0.34 | 139273.5 | 0.12 |
| X-n344-k43 | 42050 | 44036.4 | 4.72 | 42625.4 | 1.37 | 42307.5 | 0.61 | 42350.5 | 0.71 | 42122.7 | 0.17 | 42174.2 | 0.30 | 42075.6 | 0.06 |
| X-n351-k40 | 25896 | 27433.6 | 5.94 | 26266.6 | 1.43 | 26134.7 | 0.92 | 26190.7 | 1.14 | 25976.5 | 0.31 | 25994.5 | 0.38 | 25943.6 | 0.18 |
| X-n359-k29 | 51505 | 53858.4 | 4.57 | 52128.4 | 1.21 | 52089.2 | 1.13 | 51901.3 | 0.77 | 51549.8 | 0.09 | 51598.3 | 0.18 | 51549.8 | 0.09 |
| X-n367-k17 | 22814 | 23874.0 | 4.65 | 23080.4 | 1.17 | 22985.5 | 0.75 | 22944.7 | 0.57 | 22836.1 | 0.10 | 22818.6 | 0.02 | 22836.1 | 0.10 |
| X-n376-k94 | 147713 | 148775.7 | 0.72 | 147950.1 | 0.16 | 147713.4 | 0.00 | 147854.1 | 0.10 | 147763.5 | 0.03 | 147717.0 | 0.00 | 147763.5 | 0.03 |
| X-n384-k52 | 65940 | 69022.0 | 4.67 | 66625.8 | 1.04 | 66407.8 | 0.71 | 66443.0 | 0.76 | 66113.6 | 0.26 | 66107.7 | 0.25 | 66113.6 | 0.26 |
| X-n393-k38 | 38260 | 40785.6 | 6.60 | 38694.9 | 1.14 | 38515.7 | 0.67 | 38466.4 | 0.54 | 38384.5 | 0.33 | 38299.3 | 0.10 | 38384.5 | 0.33 |
| X-n401-k29 | 66163 | 68249.2 | 3.15 | 66813.6 | 0.98 | 66729.5 | 0.86 | 66501.9 | 0.51 | 66239.5 | 0.12 | 66259.8 | 0.15 | 66239.5 | 0.12 |
| X-n411-k19 | 19712 | 20810.6 | 5.57 | 20057.0 | 1.75 | 19970.8 | 1.31 | 19924.8 | 1.08 | 19776.7 | 0.33 | 19776.9 | 0.33 | 19720.3 | 0.04 |
| X-n420-k130 | 107798 | 111594.0 | 3.52 | 108574.8 | 0.72 | 107838.0 | 0.04 | 108295.3 | 0.46 | 107853.4 | 0.05 | 107923.5 | 0.12 | 107839.8 | 0.04 |
| X-n429-k61 | 65449 | 68858.4 | 5.21 | 66198.4 | 1.15 | 65786.8 | 0.52 | 65857.5 | 0.62 | 65539.3 | 0.14 | 65565.8 | 0.18 | 65502.7 | 0.08 |
| X-n439-k37 | 36391 | 37655.3 | 3.47 | 36590.1 | 0.55 | 36448.5 | 0.16 | 36483.8 | 0.26 | 36457.7 | 0.18 | 36397.3 | 0.02 | 36395.5 | 0.01 |
| X-n449-k29 | 55233 | 58427.1 | 5.78 | 56515.9 | 2.32 | 56272.8 | 1.88 | 55770.7 | 0.97 | 55388.8 | 0.28 | 55420.9 | 0.34 | 55368.5 | 0.25 |
| X-n459-k26 | 24139 | 25834.9 | 7.03 | 24570.6 | 1.79 | 24479.3 | 1.41 | 24251.0 | 0.46 | 24228.3 | 0.37 | 24195.5 | 0.23 | 24163.8 | 0.10 |
| X-n469-k138 | 221824 | 230963.3 | 4.12 | 223845.1 | 0.91 | 222189.0 | 0.16 | 223468.0 | 0.74 | 222253.9 | 0.19 | 222988.5 | 0.52 | 222170.1 | 0.16 |
| X-n480-k70 | 89449 | 92923.0 | 3.88 | 90186.5 | 0.82 | 89857.0 | 0.46 | 89986.3 | 0.60 | 89515.1 | 0.07 | 89628.2 | 0.20 | 89524.4 | 0.08 |
| X-n491-k59 | 66487 | 70817.2 | 6.51 | 67522.2 | 1.56 | 67238.7 | 1.13 | 67145.6 | 0.99 | 66606.9 | 0.18 | 66677.8 | 0.29 | 66641.5 | 0.23 |
| X-n502-k39 | 69226 | 70166.5 | 1.36 | 69377.3 | 0.22 | 69380.4 | 0.22 | 69333.9 | 0.16 | 69271.4 | 0.07 | 69247.7 | 0.03 | 69239.5 | 0.02 |
| X-n513-k21 | 24201 | 25845.9 | 6.80 | 24506.7 | 1.26 | 24406.9 | 0.85 | 24360.7 | 0.66 | 24293.9 | 0.38 | 24242.1 | 0.17 | 24201.0 | 0.00 |
| X-n524-k153 | 154593 | 156897.0 | 1.49 | 154840.6 | 0.16 | 155176.6 | 0.38 | 155699.6 | 0.72 | 154894.6 | 0.20 | 154892.3 | 0.19 | 154747.6 | 0.10 |
| X-n536-k96 | 94868 | 99575.6 | 4.96 | 96764.7 | 2.00 | 95713.0 | 0.89 | 95864.7 | 1.05 | 95145.9 | 0.29 | 95560.2 | 0.73 | 95091.9 | 0.24 |
| X-n548-k50 | 86700 | 89382.6 | 3.09 | 87133.1 | 0.50 | 86976.2 | 0.32 | 86938.6 | 0.28 | 86789.1 | 0.10 | 86742.8 | 0.05 | 86778.4 | 0.09 |
| X-n561-k42 | 42717 | 45758.6 | 7.12 | 43210.9 | 1.16 | 43095.7 | 0.89 | 43031.7 | 0.74 | 42875.0 | 0.37 | 42829.7 | 0.26 | 42742.7 | 0.06 |
| X-n573-k30 | 50673 | 52436.5 | 3.48 | 51179.8 | 1.00 | 51203.9 | 1.05 | 50957.2 | 0.56 | 50842.6 | 0.33 | 50821.3 | 0.29 | 50813.0 | 0.28 |
| X-n586-k159 | 190316 | 198347.2 | 4.22 | 191756.9 | 0.76 | 190835.6 | 0.27 | 191411.4 | 0.58 | 190640.1 | 0.17 | 190952.2 | 0.33 | 190588.1 | 0.14 |
| X-n599-k92 | 108451 | 113380.7 | 4.55 | 110086.5 | 1.51 | 109460.7 | 0.93 | 109356.1 | 0.83 | 108684.8 | 0.22 | 108754.2 | 0.28 | 108656.0 | 0.19 |
| X-n613-k62 | 59535 | 64073.6 | 7.62 | 60616.7 | 1.82 | 60457.8 | 1.55 | 60201.2 | 1.12 | 59705.6 | 0.29 | 59699.4 | 0.28 | 59696.3 | 0.27 |
| X-n627-k43 | 62164 | 64897.9 | 4.40 | 63084.1 | 1.48 | 63052.4 | 1.43 | 62568.1 | 0.65 | 62291.8 | 0.21 | 62251.9 | 0.14 | 62371.6 | 0.33 |
| X-n641-k35 | 63694 | 66862.3 | 4.97 | 64825.9 | 1.78 | 64709.8 | 1.59 | 64094.3 | 0.63 | 63851.8 | 0.25 | 63835.4 | 0.22 | 63874.2 | 0.28 |
| X-n655-k131 | 106780 | 107815.9 | 0.97 | 107044.4 | 0.25 | 106785.7 | 0.01 | 106956.8 | 0.17 | 106841.6 | 0.06 | 106805.6 | 0.02 | 106808.8 | 0.03 |
| X-n670-k130 | 146332 | 151874.1 | 3.79 | 147704.1 | 0.94 | 148272.8 | 1.33 | 147654.2 | 0.90 | 146961.5 | 0.43 | 147490.8 | 0.79 | 146777.7 | 0.30 |
| X-n685-k75 | 68205 | 74085.5 | 8.62 | 69443.7 | 1.82 | 68988.4 | 1.15 | 68854.7 | 0.95 | 68379.6 | 0.26 | 68440.0 | 0.34 | 68343.1 | 0.20 |
| X-n701-k44 | 81923 | 87060.3 | 6.27 | 83261.3 | 1.63 | 83159.4 | 1.51 | 82513.5 | 0.72 | 82053.9 | 0.16 | 82083.5 | 0.20 | 82237.3 | 0.38 |
| X-n716-k35 | 43387 | 46012.9 | 6.05 | 44441.3 | 2.43 | 44264.0 | 2.02 | 43730.4 | 0.79 | 43492.0 | 0.24 | 43492.6 | 0.24 | 43505.8 | 0.27 |
| X-n733-k159 | 136190 | 143829.1 | 5.61 | 137413.2 | 0.90 | 137014.7 | 0.61 | 137299.3 | 0.81 | 136445.2 | 0.19 | 136428.1 | 0.17 | 136426.9 | 0.17 |
| X-n749-k98 | 77314 | 82813.4 | 7.11 | 78910.4 | 2.06 | 78323.3 | 1.31 | 78211.9 | 1.16 | 77534.9 | 0.29 | 77551.0 | 0.31 | 77655.4 | 0.44 |
| X-n766-k71 | 114454 | 123106.2 | 7.56 | 116096.4 | 1.43 | 115858.3 | 1.23 | 115186.0 | 0.64 | 114836.0 | 0.33 | 114840.8 | 0.34 | 114764.5 | 0.27 |
| X-n783-k48 | 72394 | 77518.9 | 7.08 | 73933.2 | 2.13 | 73765.3 | 1.89 | 73043.8 | 0.90 | 72637.3 | 0.34 | 72573.8 | 0.25 | 72790.7 | 0.55 |
| X-n801-k40 | 73305 | 76428.2 | 4.26 | 74001.2 | 0.95 | 74141.6 | 1.14 | 73590.5 | 0.39 | 73412.0 | 0.15 | 73396.5 | 0.12 | 73500.4 | 0.27 |
| X-n819-k171 | 158121 | 165074.0 | 4.40 | 160305.2 | 1.38 | 159363.2 | 0.79 | 159572.5 | 0.92 | 158424.5 | 0.19 | 158918.8 | 0.50 | 158511.6 | 0.25 |
| X-n837-k142 | 193737 | 201836.8 | 4.18 | 195548.5 | 0.94 | 195053.8 | 0.68 | 195135.0 | 0.72 | 193946.6 | 0.11 | 194232.7 | 0.26 | 194231.3 | 0.26 |
| X-n856-k95 | 88965 | 91613.9 | 2.98 | 89530.6 | 0.64 | 89266.2 | 0.34 | 89333.5 | 0.41 | 89111.1 | 0.16 | 89040.1 | 0.08 | 89037.5 | 0.08 |
| X-n876-k59 | 99299 | 103576.1 | 4.31 | 100700.2 | 1.41 | 100487.3 | 1.20 | 100115.7 | 0.82 | 99484.5 | 0.19 | 99528.2 | 0.23 | 99682.7 | 0.39 |
| X-n895-k37 | 53860 | 58191.7 | 8.04 | 56627.0 | 5.14 | 55023.1 | 2.16 | 54306.0 | 0.83 | 54072.3 | 0.39 | 54033.3 | 0.32 | 54070.6 | 0.39 |
| X-n916-k207 | 329179 | 342127.2 | 3.93 | 331668.2 | 0.76 | 331158.4 | 0.60 | 331111.0 | 0.59 | 329584.3 | 0.12 | 330164.7 | 0.30 | 329852.0 | 0.20 |
| X-n936-k151 | 132725 | 140479.3 | 5.84 | 134477.5 | 1.32 | 135052.1 | 1.75 | 133831.4 | 0.83 | 133497.1 | 0.58 | 133259.4 | 0.40 | 133369.9 | 0.49 |
| X-n957-k87 | 85465 | 88603.0 | 3.67 | 86089.1 | 0.73 | 85979.8 | 0.60 | 85746.6 | 0.33 | 85559.8 | 0.11 | 85526.2 | 0.07 | 85550.1 | 0.10 |
| X-n979-k58 | 118987 | 123885.2 | 4.12 | 121339.6 | 1.98 | 120569.7 | 1.33 | 119600.1 | 0.52 | 119108.2 | 0.10 | 119202.8 | 0.18 | 119247.5 | 0.22 |
| X-n1001-k43 | 72359 | 78084.7 | 7.91 | 74151.1 | 2.48 | 74158.5 | 2.49 | 72998.9 | 0.88 | 72533.1 | 0.24 | 72518.9 | 0.22 | 72748.8 | 0.54 |
| Avg | 63108.61 | 65685.1 | 4.01 | 63751.7 | 1.00 | 63540.8 | 0.66 | 63493.4 | 0.53 | 63227.6 | 0.19 | 63266.1 | 0.20 | 63216.7 | 0.14 |

Table 11: Comparison of results of best solution quality for CVRP

| Inst | BKS | Google OR | | HILS | | LKH-3 | | KGLS | | SISR | | FILO | | AMA-ENS | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Cost | Gap | Cost | Gap | Cost | Gap | Cost | Gap | Cost | Gap | Cost | Gap | Cost | Gap |
| X-n101-k25 | 27591 | 27865 | 0.99 | 27591 | 0.00 | 27591 | 0.00 | 27595 | 0.01 | 27591 | 0.00 | 27591 | 0.00 | 27591 | 0.00 |
| X-n106-k14 | 26362 | 26747 | 1.46 | 26381 | 0.07 | 26381 | 0.07 | 26375 | 0.05 | 26368 | 0.02 | 26362 | 0.00 | 26364 | 0.01 |
| X-n110-k13 | 14971 | 14986 | 0.10 | 14971 | 0.00 | 14971 | 0.00 | 14971 | 0.00 | 14971 | 0.00 | 14971 | 0.00 | 14971 | 0.00 |
| X-n115-k10 | 12747 | 12768 | 0.16 | 12747 | 0.00 | 12747 | 0.00 | 12747 | 0.00 | 12747 | 0.00 | 12747 | 0.00 | 12747 | 0.00 |
| X-n120-k6 | 13332 | 13458 | 0.95 | 13332 | 0.00 | 13332 | 0.00 | 13332 | 0.00 | 13332 | 0.00 | 13332 | 0.00 | 13332 | 0.00 |
| X-n125-k30 | 55539 | 56601 | 1.91 | 55701 | 0.29 | 55713 | 0.31 | 55670 | 0.24 | 55539 | 0.00 | 55539 | 0.00 | 55539 | 0.00 |
| X-n129-k18 | 28940 | 29668 | 2.52 | 28948 | 0.03 | 28954 | 0.05 | 28954 | 0.05 | 28940 | 0.00 | 28940 | 0.00 | 28940 | 0.00 |
| X-n134-k13 | 10916 | 11096 | 1.65 | 10937 | 0.19 | 10929 | 0.12 | 10930 | 0.13 | 10918 | 0.02 | 10916 | 0.00 | 10916 | 0.00 |
| X-n139-k10 | 13590 | 13693 | 0.76 | 13612 | 0.16 | 13590 | 0.00 | 13590 | 0.00 | 13590 | 0.00 | 13590 | 0.00 | 13590 | 0.00 |
| X-n143-k7 | 15700 | 16019 | 2.03 | 15718 | 0.11 | 15723 | 0.15 | 15726 | 0.17 | 15700 | 0.00 | 15700 | 0.00 | 15700 | 0.00 |
| X-n148-k46 | 43448 | 44334 | 2.04 | 43448 | 0.00 | 43448 | 0.00 | 43507 | 0.14 | 43448 | 0.00 | 43448 | 0.00 | 43448 | 0.00 |
| X-n153-k22 | 21220 | 21605 | 1.81 | 21225 | 0.02 | 21225 | 0.02 | 21375 | 0.73 | 21225 | 0.02 | 21225 | 0.02 | 21225 | 0.02 |
| X-n157-k13 | 16876 | 17086 | 1.24 | 16876 | 0.00 | 16876 | 0.00 | 16876 | 0.00 | 16876 | 0.00 | 16876 | 0.00 | 16876 | 0.00 |
| X-n162-k11 | 14138 | 14238 | 0.71 | 14171 | 0.23 | 14138 | 0.00 | 14147 | 0.06 | 14138 | 0.00 | 14147 | 0.06 | 14138 | 0.00 |
| X-n167-k10 | 20557 | 21158 | 2.92 | 20583 | 0.13 | 20557 | 0.00 | 20557 | 0.00 | 20557 | 0.00 | 20557 | 0.00 | 20557 | 0.00 |
| X-n172-k51 | 45607 | 46695 | 2.39 | 45607 | 0.00 | 45607 | 0.00 | 45763 | 0.34 | 45607 | 0.00 | 45607 | 0.00 | 45607 | 0.00 |
| X-n176-k26 | 47812 | 48986 | 2.46 | 47897 | 0.18 | 48140 | 0.69 | 47958 | 0.31 | 47812 | 0.00 | 47812 | 0.00 | 47812 | 0.00 |
| X-n181-k23 | 25569 | 25787 | 0.85 | 25598 | 0.11 | 25569 | 0.00 | 25594 | 0.10 | 25569 | 0.00 | 25569 | 0.00 | 25569 | 0.00 |
| X-n186-k15 | 24145 | 24908 | 3.16 | 24149 | 0.02 | 24147 | 0.01 | 24156 | 0.05 | 24151 | 0.02 | 24147 | 0.01 | 24151 | 0.02 |
| X-n190-k8 | 16980 | 17380 | 2.36 | 16995 | 0.09 | 17029 | 0.29 | 17001 | 0.12 | 16980 | 0.00 | 16980 | 0.00 | 16980 | 0.00 |
| X-n195-k51 | 44225 | 45757 | 3.46 | 44388 | 0.37 | 44225 | 0.00 | 44396 | 0.39 | 44241 | 0.04 | 44225 | 0.00 | 44241 | 0.04 |
| X-n200-k36 | 58578 | 60338 | 3.00 | 58773 | 0.33 | 58617 | 0.07 | 58756 | 0.30 | 58587 | 0.02 | 58620 | 0.07 | 58578 | 0.00 |
| X-n204-k19 | 19565 | 20212 | 3.31 | 19610 | 0.23 | 19565 | 0.00 | 19581 | 0.08 | 19565 | 0.00 | 19565 | 0.00 | 19565 | 0.00 |
| X-n209-k16 | 30656 | 31740 | 3.54 | 30700 | 0.14 | 30702 | 0.15 | 30685 | 0.09 | 30656 | 0.00 | 30659 | 0.01 | 30656 | 0.00 |
| X-n214-k11 | 10856 | 11228 | 3.43 | 11033 | 1.63 | 10917 | 0.56 | 10913 | 0.53 | 10874 | 0.17 | 10870 | 0.13 | 10856 | 0.00 |
| X-n219-k73 | 117595 | 117924 | 0.28 | 117595 | 0.00 | 117595 | 0.00 | 117651 | 0.05 | 117596 | 0.00 | 117595 | 0.00 | 117595 | 0.00 |
| X-n223-k34 | 40437 | 41794 | 3.36 | 40604 | 0.41 | 40490 | 0.13 | 40686 | 0.62 | 40504 | 0.17 | 40445 | 0.02 | 40437 | 0.00 |
| X-n228-k23 | 25742 | 26396 | 2.54 | 25806 | 0.25 | 25745 | 0.01 | 25808 | 0.26 | 25782 | 0.16 | 25743 | 0.00 | 25742 | 0.00 |
| X-n233-k16 | 19230 | 19682 | 2.35 | 19232 | 0.01 | 19276 | 0.24 | 19268 | 0.20 | 19232 | 0.01 | 19230 | 0.00 | 19232 | 0.01 |
| X-n237-k14 | 27042 | 27809 | 2.84 | 27042 | 0.00 | 27042 | 0.00 | 27044 | 0.01 | 27043 | 0.00 | 27042 | 0.00 | 27043 | 0.00 |
| X-n242-k48 | 82751 | 85518 | 3.34 | 83052 | 0.36 | 82809 | 0.07 | 83136 | 0.47 | 82805 | 0.07 | 82775 | 0.03 | 82805 | 0.07 |
| X-n247-k50 | 37274 | 37853 | 1.55 | 37292 | 0.05 | 37300 | 0.07 | 37317 | 0.12 | 37274 | 0.00 | 37274 | 0.00 | 37274 | 0.00 |
| X-n251-k28 | 38684 | 40007 | 3.42 | 38918 | 0.60 | 38798 | 0.29 | 38847 | 0.42 | 38687 | 0.01 | 38723 | 0.10 | 38687 | 0.01 |
| X-n256-k16 | 18839 | 19067 | 1.21 | 18986 | 0.78 | 18880 | 0.22 | 18888 | 0.26 | 18880 | 0.22 | 18880 | 0.22 | 18839 | 0.00 |
| X-n261-k13 | 26558 | 27760 | 4.53 | 26844 | 1.08 | 26692 | 0.50 | 26671 | 0.43 | 26558 | 0.00 | 26612 | 0.20 | 26558 | 0.00 |
| X-n266-k58 | 75478 | 77275 | 2.38 | 75855 | 0.50 | 75478 | 0.00 | 75793 | 0.42 | 75549 | 0.09 | 75664 | 0.25 | 75478 | 0.00 |
| X-n270-k35 | 35291 | 36401 | 3.15 | 35432 | 0.40 | 35324 | 0.09 | 35447 | 0.44 | 35325 | 0.10 | 35309 | 0.05 | 35303 | 0.03 |
| X-n275-k28 | 21245 | 21918 | 3.17 | 21257 | 0.06 | 21245 | 0.00 | 21265 | 0.09 | 21245 | 0.00 | 21245 | 0.00 | 21245 | 0.00 |
| X-n280-k17 | 33503 | 34859 | 4.05 | 33690 | 0.56 | 33725 | 0.66 | 33598 | 0.28 | 33545 | 0.13 | 33608 | 0.31 | 33506 | 0.01 |
| X-n284-k15 | 20215 | 20872 | 3.25 | 20373 | 0.78 | 20325 | 0.54 | 20323 | 0.53 | 20261 | 0.23 | 20257 | 0.21 | 20231 | 0.08 |
| X-n289-k60 | 95151 | 97868 | 2.86 | 95754 | 0.63 | 95401 | 0.26 | 95770 | 0.65 | 95245 | 0.10 | 95429 | 0.29 | 95242 | 0.10 |
| X-n294-k50 | 47161 | 49010 | 3.92 | 47430 | 0.57 | 47240 | 0.17 | 47413 | 0.53 | 47199 | 0.08 | 47240 | 0.17 | 47167 | 0.01 |
| X-n298-k31 | 34231 | 36296 | 6.03 | 34391 | 0.47 | 34318 | 0.25 | 34359 | 0.37 | 34234 | 0.01 | 34234 | 0.01 | 34231 | 0.00 |
| X-n303-k21 | 21736 | 22376 | 2.94 | 21878 | 0.65 | 21806 | 0.32 | 21845 | 0.50 | 21753 | 0.08 | 21792 | 0.26 | 21739 | 0.01 |
| X-n308-k13 | 25859 | 26934 | 4.16 | 25992 | 0.51 | 25989 | 0.50 | 25999 | 0.54 | 26224 | 1.41 | 25862 | 0.01 | 25862 | 0.01 |
| X-n313-k71 | 94043 | 96958 | 3.10 | 94778 | 0.78 | 94216 | 0.18 | 94652 | 0.65 | 94098 | 0.06 | 94246 | 0.22 | 94045 | 0.00 |
| X-n317-k53 | 78355 | 78863 | 0.65 | 78408 | 0.07 | 78355 | 0.00 | 78391 | 0.05 | 78361 | 0.01 | 78355 | 0.00 | 78355 | 0.00 |
| X-n322-k28 | 29834 | 30932 | 3.68 | 30078 | 0.82 | 29923 | 0.30 | 30010 | 0.59 | 29861 | 0.09 | 29878 | 0.15 | 29834 | 0.00 |
| X-n327-k20 | 27532 | 28592 | 3.85 | 27786 | 0.92 | 27767 | 0.85 | 27613 | 0.29 | 27611 | 0.29 | 27565 | 0.12 | 27532 | 0.00 |
| X-n331-k15 | 31102 | 32493 | 4.47 | 31153 | 0.16 | 31136 | 0.11 | 31111 | 0.03 | 31122 | 0.06 | 31103 | 0.00 | 31102 | 0.00 |
| X-n336-k84 | 139111 | 142905 | 2.73 | 139655 | 0.39 | 139351 | 0.17 | 140716 | 1.15 | 139272 | 0.12 | 139324 | 0.15 | 139205 | 0.07 |
| X-n344-k43 | 42050 | 43560 | 3.59 | 42450 | 0.95 | 42190 | 0.33 | 42229 | 0.43 | 42081 | 0.07 | 42089 | 0.09 | 42061 | 0.03 |
| X-n351-k40 | 25896 | 27093 | 4.62 | 26142 | 0.95 | 26050 | 0.59 | 26150 | 0.98 | 25965 | 0.27 | 25960 | 0.25 | 25924 | 0.11 |
| X-n359-k29 | 51505 | 53541 | 3.95 | 51852 | 0.67 | 51820 | 0.61 | 51662 | 0.30 | 51514 | 0.02 | 51514 | 0.02 | 51566 | 0.12 |
| X-n367-k17 | 22814 | 23597 | 3.43 | 22959 | 0.64 | 22956 | 0.62 | 22867 | 0.23 | 22821 | 0.03 | 22814 | 0.00 | 22814 | 0.00 |
| X-n376-k94 | 147713 | 148630 | 0.62 | 147876 | 0.11 | 147713 | 0.00 | 147801 | 0.06 | 147736 | 0.02 | 147713 | 0.00 | 147713 | 0.00 |
| X-n384-k52 | 65940 | 68550 | 3.96 | 66489 | 0.83 | 66351 | 0.62 | 66363 | 0.64 | 66046 | 0.16 | 66036 | 0.15 | 65997 | 0.09 |
| X-n393-k38 | 38260 | 40303 | 5.34 | 38607 | 0.91 | 38360 | 0.26 | 38433 | 0.45 | 38338 | 0.20 | 38290 | 0.08 | 38260 | 0.00 |
| X-n401-k29 | 66163 | 67913 | 2.64 | 66584 | 0.64 | 66597 | 0.66 | 66466 | 0.46 | 66222 | 0.09 | 66227 | 0.10 | 66209 | 0.07 |
| X-n411-k19 | 19712 | 20571 | 4.36 | 19860 | 0.75 | 19834 | 0.62 | 19782 | 0.36 | 19757 | 0.23 | 19758 | 0.23 | 19716 | 0.02 |
| X-n420-k130 | 107798 | 110857 | 2.84 | 108292 | 0.46 | 107801 | 0.00 | 108175 | 0.35 | 107809 | 0.01 | 107826 | 0.03 | 107810 | 0.01 |
| X-n429-k61 | 65449 | 68113 | 4.07 | 65939 | 0.75 | 65689 | 0.37 | 65795 | 0.53 | 65494 | 0.07 | 65509 | 0.09 | 65494 | 0.07 |
| X-n439-k37 | 36391 | 37171 | 2.14 | 36491 | 0.27 | 36402 | 0.03 | 36445 | 0.15 | 36402 | 0.03 | 36395 | 0.01 | 36402 | 0.03 |
| X-n449-k29 | 55233 | 58066 | 5.13 | 56212 | 1.77 | 56154 | 1.67 | 55675 | 0.80 | 55296 | 0.11 | 55358 | 0.23 | 55296 | 0.11 |
| X-n459-k26 | 24139 | 25435 | 5.37 | 24479 | 1.41 | 24363 | 0.93 | 24234 | 0.39 | 24187 | 0.20 | 24157 | 0.07 | 24187 | 0.20 |
| X-n469-k138 | 221824 | 230460 | 3.89 | 223289 | 0.66 | 221939 | 0.05 | 223086 | 0.57 | 222090 | 0.12 | 222543 | 0.32 | 222090 | 0.12 |
| X-n480-k70 | 89449 | 92457 | 3.36 | 90034 | 0.65 | 89629 | 0.20 | 89926 | 0.53 | 89458 | 0.01 | 89540 | 0.10 | 89458 | 0.01 |
| X-n491-k59 | 66487 | 69944 | 5.20 | 67280 | 1.19 | 67098 | 0.92 | 67034 | 0.82 | 66502 | 0.02 | 66605 | 0.18 | 66502 | 0.02 |
| X-n502-k39 | 69226 | 70032 | 1.16 | 69275 | 0.07 | 69340 | 0.16 | 69307 | 0.12 | 69238 | 0.02 | 69227 | 0.00 | 69238 | 0.02 |
| X-n513-k21 | 24201 | 25295 | 4.52 | 24384 | 0.76 | 24316 | 0.48 | 24293 | 0.38 | 24237 | 0.15 | 24201 | 0.00 | 24201 | 0.00 |
| X-n524-k153 | 154593 | 156322 | 1.12 | 154657 | 0.04 | 154656 | 0.04 | 155422 | 0.54 | 154758 | 0.11 | 154610 | 0.01 | 154646 | 0.03 |
| X-n536-k96 | 94868 | 98815 | 4.16 | 96214 | 1.42 | 95663 | 0.84 | 95781 | 0.96 | 95071 | 0.21 | 95485 | 0.65 | 95040 | 0.18 |
| X-n548-k50 | 86700 | 89066 | 2.73 | 87059 | 0.41 | 86813 | 0.13 | 86901 | 0.23 | 86710 | 0.01 | 86707 | 0.01 | 86710 | 0.01 |
| X-n561-k42 | 42717 | 45330 | 6.12 | 43070 | 0.83 | 42918 | 0.47 | 42989 | 0.64 | 42799 | 0.19 | 42756 | 0.09 | 42726 | 0.02 |
| X-n573-k30 | 50673 | 52080 | 2.78 | 51013 | 0.67 | 51102 | 0.85 | 50849 | 0.35 | 50777 | 0.21 | 50757 | 0.17 | 50757 | 0.17 |
| X-n586-k159 | 190316 | 197853 | 3.96 | 191412 | 0.58 | 190695 | 0.20 | 191260 | 0.50 | 190454 | 0.07 | 190865 | 0.29 | 190470 | 0.08 |
| X-n599-k92 | 108451 | 112831 | 4.04 | 109646 | 1.10 | 109119 | 0.62 | 109125 | 0.62 | 108598 | 0.14 | 108654 | 0.19 | 108605 | 0.14 |
| X-n613-k62 | 59535 | 63561 | 6.76 | 60217 | 1.15 | 60333 | 1.34 | 60111 | 0.97 | 59609 | 0.12 | 59584 | 0.08 | 59636 | 0.17 |
| X-n627-k43 | 62164 | 64590 | 3.90 | 62755 | 0.95 | 62928 | 1.23 | 62486 | 0.52 | 62221 | 0.09 | 62228 | 0.10 | 62238 | 0.12 |
| X-n641-k35 | 63694 | 66652 | 4.64 | 64638 | 1.48 | 64591 | 1.41 | 63952 | 0.41 | 63802 | 0.17 | 63769 | 0.12 | 63782 | 0.14 |
| X-n655-k131 | 106780 | 107710 | 0.87 | 106970 | 0.18 | 106780 | 0.00 | 106936 | 0.15 | 106808 | 0.03 | 106780 | 0.00 | 106785 | 0.00 |
| X-n670-k130 | 146332 | 151071 | 3.24 | 147139 | 0.55 | 147974 | 1.12 | 147477 | 0.78 | 146676 | 0.24 | 147247 | 0.63 | 146640 | 0.21 |
| X-n685-k75 | 68205 | 73090 | 7.16 | 69234 | 1.51 | 68843 | 0.94 | 68628 | 0.62 | 68271 | 0.10 | 68355 | 0.22 | 68288 | 0.12 |
| X-n701-k44 | 81923 | 86604 | 5.71 | 82863 | 1.15 | 82982 | 1.29 | 82447 | 0.64 | 82007 | 0.10 | 82006 | 0.10 | 82075 | 0.19 |
| X-n716-k35 | 43387 | 45704 | 5.34 | 44074 | 1.58 | 44058 | 1.55 | 43627 | 0.55 | 43449 | 0.14 | 43461 | 0.17 | 43459 | 0.17 |
| X-n733-k159 | 136190 | 142650 | 4.74 | 137172 | 0.72 | 136848 | 0.48 | 137185 | 0.73 | 136344 | 0.11 | 136317 | 0.09 | 136323 | 0.10 |
| X-n749-k98 | 77314 | 82083 | 6.17 | 78612 | 1.68 | 78213 | 1.16 | 78109 | 1.03 | 77399 | 0.11 | 77467 | 0.20 | 77563 | 0.32 |
| X-n766-k71 | 114454 | 121645 | 6.28 | 115732 | 1.12 | 115396 | 0.82 | 115011 | 0.49 | 114751 | 0.26 | 114703 | 0.22 | 114679 | 0.20 |
| X-n783-k48 | 72394 | 76764 | 6.04 | 73718 | 1.83 | 73512 | 1.54 | 72974 | 0.80 | 72544 | 0.21 | 72486 | 0.13 | 72665 | 0.37 |
| X-n801-k40 | 73305 | 76262 | 4.03 | 73849 | 0.74 | 73970 | 0.91 | 73500 | 0.27 | 73362 | 0.08 | 73322 | 0.02 | 73396 | 0.12 |
| X-n819-k171 | 158121 | 164377 | 3.96 | 159697 | 1.00 | 159261 | 0.72 | 159396 | 0.81 | 158344 | 0.14 | 158661 | 0.34 | 158391 | 0.17 |
| X-n837-k142 | 193737 | 201518 | 4.02 | 195308 | 0.81 | 194857 | 0.58 | 194988 | 0.65 | 193868 | 0.07 | 194142 | 0.21 | 194103 | 0.19 |
| X-n856-k95 | 88965 | 91109 | 2.41 | 89327 | 0.41 | 89082 | 0.13 | 89218 | 0.28 | 89042 | 0.09 | 88996 | 0.03 | 88986 | 0.02 |
| X-n876-k59 | 99299 | 103017 | 3.74 | 100539 | 1.25 | 100418 | 1.13 | 100048 | 0.75 | 99405 | 0.11 | 99421 | 0.12 | 99526 | 0.23 |
| X-n895-k37 | 53860 | 57607 | 6.96 | 56334 | 4.59 | 54856 | 1.85 | 54240 | 0.71 | 53982 | 0.23 | 53966 | 0.20 | 54029 | 0.31 |
| X-n916-k207 | 329179 | 340947 | 3.57 | 331018 | 0.56 | 330920 | 0.53 | 331006 | 0.56 | 329418 | 0.07 | 329882 | 0.21 | 329572 | 0.12 |
| X-n936-k151 | 132725 | 139456 | 5.07 | 133944 | 0.92 | 134732 | 1.51 | 133713 | 0.74 | 133190 | 0.35 | 132999 | 0.21 | 133129 | 0.30 |
| X-n957-k87 | 85465 | 88222 | 3.23 | 85893 | 0.50 | 85864 | 0.47 | 85656 | 0.22 | 85493 | 0.03 | 85493 | 0.03 | 85506 | 0.05 |
| X-n979-k58 | 118987 | 123379 | 3.69 | 120791 | 1.52 | 120142 | 0.97 | 119559 | 0.48 | 119065 | 0.07 | 119145 | 0.13 | 119120 | 0.11 |
| X-n1001-k43 | 72359 | 77117 | 6.58 | 73994 | 2.26 | 73749 | 1.92 | 72882 | 0.72 | 72414 | 0.08 | 72443 | 0.12 | 72528 | 0.23 |
| Max Gap | - | - | 7.162232 | - | 4.59339 | - | 1.920977 | - | 1.153755 | - | 1.411501 | - | 0.650377 | - | 0.37434 |
| Min Gap | - | - | 0.100194 | - | 0 | - | 0 | - | 0 | - | 0 | - | 0 | - | 0 |
| Average Gap | - | - | 3.322353 | - | 0.641092 | - | 0.443209 | - | 0.40076 | - | 0.095248 | - | 0.101099 | - | 0.061363 |
| NB BKS found | - | 0 | - | 9 | - | 20 | - | 6 | - | 21 | - | 29 | - | 38 | - |

The gray cell indicates that the current solution matches the known best solution (BKS).

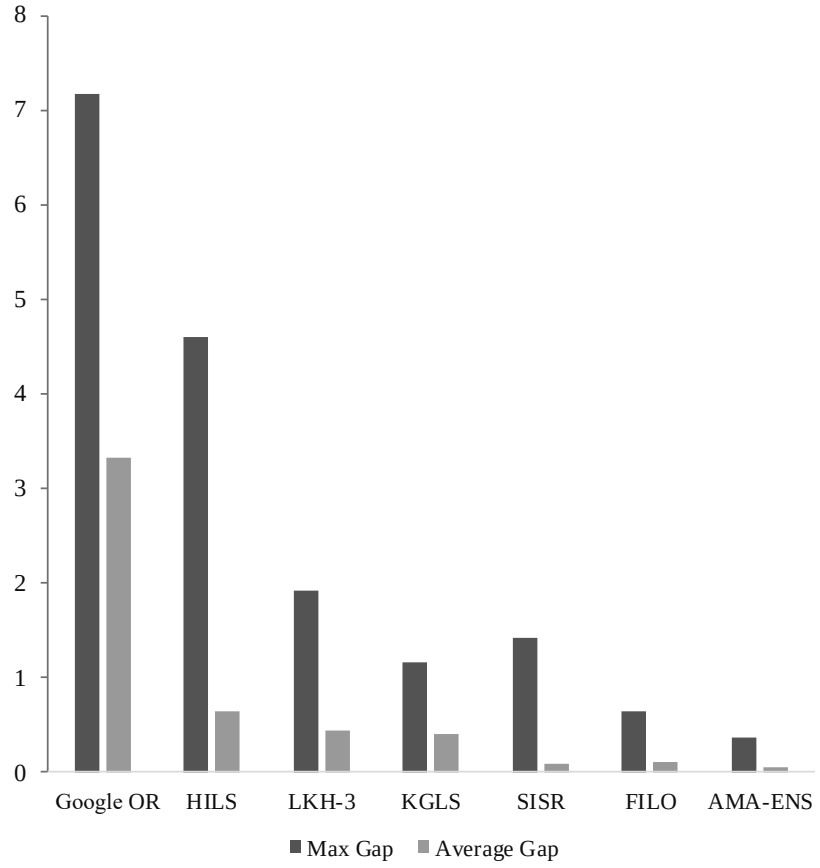


Figure 7: Average Gap[1] and Max Gap[2] for CVRP

[a]Average Gap(%): average gap between the solution and BKS for all the instances. The smaller, the better.

[b]Max Gap(%): the maximal gap between the solution and BKS among all the instances. The smaller, the better.

The average gap and max gap of AMA-ENS are the smallest among the compared algorithms, indicating that AMA-ENS performs the best.