A4q3

Part a

$$Y_{ijk} = \mu + \alpha_i + \lambda_j + \tau_{ij} + R_{ijk}$$

where mu is the over all mean
alpha is the factor A effect
lambda is the factor b effect
tau is the interaction effect R is the random Error
where i is 1 to ta, ta = 2(# of A); j is 1 to tb, tb = 2(# of B); k is 1 to r, r = 4(# of blcok)
constraints:

$$\sum_{i=1}^{ta} \alpha_i = 0, \sum_{j=1}^{tb} \lambda_j = 0, \sum_i \tau_{ij} = 0, for \quad \forall i; \sum_j \tau_{ij} = 0, for \quad \forall j$$

part b:) $\hat{\tau}_{ij} = \overline{y_{ij\cdot}} - \overline{y_{i\cdot\cdot}} - \overline{y_{\cdot j\cdot}} + \overline{y_{\cdot\cdot\cdot}}$

$\hat{\tau}_{11} = \overline{y_{11\cdot}} - \overline{y_{1\cdot\cdot}} - \overline{y_{\cdot 1\cdot}} + \overline{y_{\cdot\cdot\cdot}} = 145 - 145.125 - 137.25 + 137.9375 = 0.5625$

Since $\sum_i \hat{\tau}_{ij} = 0$, $\sum_j \hat{\tau}_{ij} = 0$

$\hat{\tau}_{12} + \hat{\tau}_{11} = 0 \quad \hat{\tau}_{21} + \hat{\tau}_{11} = 0 \quad \hat{\tau}_{22} + \hat{\tau}_{21} = 0$

$\hat{\tau}_{12} = \cancel{0.5625}\ -0.5625$

$\hat{\tau}_{21} = -0.5625$

$\hat{\tau}_{22} = 0.5625$

(c) 1. Hypothesis: $H_0: \theta = 0$ where $\theta = \tau_1 - \tau_2$ for factor A has same level of effect of Factor B.

2. Test Statistic: $\hat{\theta} = (\mu_{11} - \mu_{12}) - (\mu_{21} - \mu_{22}) = -3.25$ > from code below

$\hat{\sigma}^2 = 23.84$

Test Statistic $= \dfrac{|-3.25 - 0|}{\sqrt{23.84}\sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}} = 0.6656$

p-value $=$ ~~0.517270~~ 0.5223 → from code below

Thus there is NO evidence against H_0, thus the effect of A and B is same.

Code for part c: (next page)

```
> q2block =matrix(NA, nrow=4, ncol=4)
> q2block[1,] =c(143,141,150,146)
> q2block[2,] =c(152,149,137,143)
> q2block[3,] =c(134,136,132,127)
> q2block[4,] =c(129,127,132,129)
> rownames(q2block) =c('1','2','3','4')
> mu_11 = mean(q2block[1,])
> mu_12 = mean(q2block[2,])
> mu_21 = mean(q2block[3,])
> mu_22 = mean(q2block[4,])
> tube = c(q2block[1,],q2block[2,],q2block[3,],q2block[4,])
> treatment = rep(c(1:4),each = 4)
> treatment = factor(treatment)
> cond = rep(c(1:4),4)
> cond = factor(cond)
>
> m1 = aov(tube~treatment+cond)
> summary(m1)
            Df Sum Sq Mean Sq F value  Pr(>F)
treatment    3  844.7  281.56  11.810 0.00179 **
cond         3   21.7    7.23   0.303 0.82247
Residuals    9  214.6   23.84
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
> sigma_sq = 23.84
> theta_hat = (mu_11 - mu_12) - (mu_21 - mu_22)
> theta_hat
[1] -3.25
> T_obs = abs(theta_hat) / sqrt(sigma_sq)
> T_obs
[1] 0.6656259
> 2*pt(T_obs,df=9,lower.tail = FALSE)
[1] 0.5223419
>
```

Part d

(i) since the p-value 0.00179 < 0.05 (from part(c) code), there is STRONG evidence against H0.

Whitch means there is no defference among treatments.

(ii) since the p-value 0.5223 > 0.5, there is NO evidence against H0.

YES, interaction is significantly defferent from 0(code show below)

Code for d(ii)

```
>
> f_a = rep(c('low','high'),each = 8)
> f_b = rep(rep(c('low','high'),each = 4),2)
> m2 = aov(tube~ f_a*f_b + cond)
> summary(m2)
            Df Sum Sq Mean Sq F value   Pr(>F)
f_a          1  826.6   826.6  34.671 0.000232 ***
f_b          1    7.6     7.6   0.317 0.587041
cond         3   21.7     7.2   0.303 0.822469
f_a:f_b      1   10.6    10.6   0.443 0.522344
Residuals    9  214.6    23.8
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```