

A4q3

Part a

The randomization performed by randomly assigning treatments to the bolt. Which means we have random chemical X and chemical y within each bolt.

Part b

$$Y_{ijk} = \mu + \alpha_i + \lambda_j + \tau_{ij} + \beta_k + R_{ijk}$$

where μ is the over all mean

α is the chemical X effect

λ is the chemical Y effect

τ is the interaction effect

β is the Bolt effect

R is the random Error

where i is 1 to t_a , $t_a = 2$ (# of A); j is 1 to t_b , $t_b = 2$ (# of B); k is 1 to r , $r = 4$ (# of block)

constraints:

$$\sum_{i=1}^{t_a} \alpha_i = 0, \sum_{j=1}^{t_b} \lambda_j = 0, \sum_{k=1}^r \beta_k = 0, \sum_i \tau_{ij} = 0, \text{ for } \forall i; \sum_j \tau_{ij} = 0, \text{ for } \forall j$$

Part c

C) 1. Hypothesis $H_0: \theta = 0$

$$\text{Since } \theta = \tau_1 - \tau_2$$

Test Statistic

2. ~~T-obs~~: find all $\bar{y}_{11}, \bar{y}_{12}, \bar{y}_{21}, \bar{y}_{22}$ by R code

$$\hat{\theta} = -0.6 \text{ by R code}$$

$\hat{\sigma}^2 = 1.82$ by R code from ml, since $\hat{\sigma}^2 = \text{mean square of Residual mode.}$

$$T\text{-obs} = \frac{-0.6}{\sqrt{1.82 \left(\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} \right)}} \approx -0.444704972$$

$$p\text{-value} = 0.628$$

There is no evidence against H_0 . means the X and Y have same effect.

The code is showed in the next page.

```

> q3block = matrix(NA, nrow=4, ncol=5)
> q3block[1,] = c(73,68,74,71,67)
> q3block[2,] = c(73,67,75,72,70)
> q3block[3,] = c(75,68,78,73,68)
> q3block[4,] = c(73,71,75,75,69)
> rownames(q3block) = c('1','2','3','4')
> mu_11 = mean(q3block[1,])
> mu_12 = mean(q3block[2,])
> mu_21 = mean(q3block[3,])
> mu_22 = mean(q3block[4,])
> baths = c(q3block[1,],q3block[2,],q3block[3,],q3block[4,])
> treatment = rep(c(1:4),each = 5)
> treatment = factor(treatment)
> bolt = rep(c(1:5),4)
> bolt = factor(bolt)
> m1 = aov(baths~treatment+bolt)
> summary(m1)

```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
treatment	3	12.95	4.32	2.376	0.121
bolt	4	157.00	39.25	21.606	2.06e-05 ***
Residuals	12	21.80	1.82		

```

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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> sigma_sq = 1.82
> theta_hat = (mu_11 - mu_12) - (mu_21 - mu_22)
> T_obs = abs(theta_hat) / (sqrt(sigma_sq) * sqrt(4/5))
> T_obs
[1] 0.4972452
> 2*pt(T_obs,df=12,lower.tail = FALSE)
[1] 0.6280016

```

Part d

- (i) since the p-value $0.121 > 0.05$ (from part(c) code), there is no evidence against H_0
- (ii) since the p-value $0.6277 > 0.5$, there is NO evidence against H_0 .

Code of part d:

```

> h_x = rep(c('low','high'),each = 10)
> h_y = rep(rep(c('low','high'),each = 5),2)
> m2 = aov(baths~h_x*h_y + bolt)
> summary(m2)

```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
h_x	1	11.25	11.25	6.193	0.0285 *
h_y	1	1.25	1.25	0.688	0.4230
bolt	4	157.00	39.25	21.606	2.06e-05 ***
h_x:h_y	1	0.45	0.45	0.248	0.6277
Residuals	12	21.80	1.82		

```

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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```