

**1**

(a) The linear velocity of point C is:

$$[0, 0, 0]^T$$

(b) The linear velocity of point A is:

$$[2v, 0, 0]^T$$

(c) Assume that the center of this cylinder is  $D$ , The velocity of the body fixed point coincides with  $C$  is:

$${}^O v_C = {}^O v_D + {}^O \omega \times \overrightarrow{DC} = \begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{v}{r} \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(d) The velocity of the body fixed point coincides with A is:

$${}^O v_A = {}^O v_D + {}^O \omega \times \overrightarrow{DA} = \begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{v}{r} \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ r \end{bmatrix} = \begin{bmatrix} 2v \\ 0 \\ 0 \end{bmatrix}$$

(e) Choose O as the reference point:

$$\omega = [0, \frac{v}{r}, 0]^T$$

$${}^O v_O = {}^O v_D + {}^O \omega \times \overrightarrow{DO} = \begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{v}{r} \\ 0 \end{bmatrix} \times \begin{bmatrix} -C_x(t) \\ 0 \\ -r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{C_x(t)v}{r} \end{bmatrix}$$

$$\mathcal{V} = [0, \frac{v}{r}, 0, 0, 0, \frac{C_x(t)v}{r}]^T$$

(f)

$${}^c X_O = \begin{bmatrix} {}^c R_O & O \\ [p] {}^c R_O & {}^c R_O \end{bmatrix}$$

$${}^c \mathcal{V} = {}^c X_O {}^O \mathcal{V} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -C_x(t) & 0 & 1 & 0 \\ 0 & C_x(t) & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{v}{r} \\ 0 \\ 0 \\ 0 \\ \frac{C_x(t)v}{r} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{v}{r} \\ 0 \\ 0 \\ 0 \\ \frac{2C_x(t)v}{r} \end{bmatrix}$$

### 3.21

(a) From the  ${}^aT_b$ :

$${}^a p_c = \begin{bmatrix} 0 \\ 800 \\ 0 \end{bmatrix}, {}^a p_b = \begin{bmatrix} -100 \\ 300 \\ 500 \end{bmatrix}$$

$${}^a R_b = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The vector  $r$  is:

$${}^a r = \overrightarrow{OC} - \overrightarrow{OB} = {}^a p_c - {}^a p_b = \begin{bmatrix} 100 \\ 500 \\ -500 \end{bmatrix}$$

$${}^b r = {}^b R_a {}^a r = \begin{bmatrix} 500 \\ -100 \\ -500 \end{bmatrix}$$

(b)  ${}^a R_c$  can get from the rotation of frame  $\{c\}$ :

$${}^a R_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$${}^b R_c = {}^b R_a {}^a R_c = {}^a R_b^T {}^a R_c = \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

Then we can get  ${}^b T_c$ :

$${}^b T_c = \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 500 \\ -1 & 0 & 0 & -100 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & -500 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### 3.28

The body's angular velocity:

$$\omega_b = {}^b R_s^T \omega_s = \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}$$

## 5.5

(a)

$${}^s P = \begin{bmatrix} L + d \sin \theta \\ L - d \cos \theta \\ 0 \end{bmatrix}$$

(b)

$$v_p = \begin{bmatrix} d \dot{\theta} \cos \theta \\ d \dot{\theta} \sin \theta \\ 0 \end{bmatrix}$$

(c)

$${}^s T_b = \begin{bmatrix} 1 & 0 & 0 & L + d \sin \theta \\ 0 & \cos \theta & -\sin \theta & L - d \cos \theta \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(d)

$$\dot{T}_{sb} = \begin{bmatrix} -\dot{\theta} \sin \theta & -\dot{\theta} \cos \theta & 0 & d \dot{\theta} \cos \theta \\ \dot{\theta} \cos \theta & -\dot{\theta} \sin \theta & 0 & d \dot{\theta} \sin \theta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad T_{sb}^{-1} = \begin{bmatrix} \cos \theta & \sin \theta & 0 & -L(\cos \theta + \sin \theta) \\ -\sin \theta & \cos \theta & 0 & -L(\cos \theta - \sin \theta) + d \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$[\mathcal{V}_b] = T_{sb}^{-1} \dot{T}_{sb} = \begin{bmatrix} 0 & -\dot{\theta} & 0 & d \dot{\theta} \\ \dot{\theta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \mathcal{V}_b = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \\ d \dot{\theta} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \omega_b \\ v_b \end{bmatrix}.$$

(e)

$$[\mathcal{V}_s] = \dot{T}_{sb} T_{sb}^{-1} = \begin{bmatrix} 0 & -\dot{\theta} & 0 & L\dot{\theta} \\ \dot{\theta} & 0 & 0 & -L\dot{\theta} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \mathcal{V}_s = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \\ L\dot{\theta} \\ -L\dot{\theta} \\ 0 \end{bmatrix} = \begin{bmatrix} \omega_s \\ v_s \end{bmatrix}.$$

(f)

$$[\mathcal{V}_s] = T_{sb}[\mathcal{V}_b]T_{sb}^{-1}.$$

(g) Because:

$$\dot{p}_P = \dot{p}_{sb}, \quad R_{sb}^{-1} \dot{p}_P = v_b.$$

(h) From:

$$\dot{R}_{sb} R_{sb}^{-1} = \omega_s, \quad -\omega_s p_P + \dot{p}_P = v_s.$$

## 5.6

(a) Given

$$\theta_1 = t, \theta_2 = t, \dot{\theta}_1 = \dot{\theta}_2 = 1$$

$$\mathcal{V}_b = J_b(\theta)\dot{\theta}$$

$$J_b(\theta) = [\mathcal{V}_{b1}(\theta) \quad \mathcal{V}_{b2}(\theta)]$$

where:

$$\mathcal{V}_{b1}(\theta) = \begin{bmatrix} \sin \theta_2 \\ \cos \theta_2 \\ 0 \\ -20 \cos \theta_2 \\ 20 \sin \theta_2 \\ -10 \cos \theta_2 \end{bmatrix}, \quad \mathcal{V}_{b2}(\theta) = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 10 \\ 0 \end{bmatrix}$$

Thus,

$$\mathcal{V}_b = J_b(\theta)\dot{\theta} = \begin{bmatrix} \sin t & 0 \\ \cos t & 0 \\ 0 & 1 \\ -20 \cos t & 0 \\ 20 \sin t & 10 \\ -10 \cos t & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin t \\ \cos t \\ 1 \\ -20 \cos t \\ 20 \sin t + 10 \\ -10 \cos t \end{bmatrix} = \begin{bmatrix} \omega_b \\ v_b \end{bmatrix}$$

Therefore,

$$\omega_b = \begin{bmatrix} \sin t \\ \cos t \\ 1 \end{bmatrix}, \quad v_b = \begin{bmatrix} -20 \cos t \\ 20 \sin t + 10 \\ -10 \cos t \end{bmatrix}$$

**(b)** The linear velocity of the rider in the fixed frame {s} coordinates is  $\dot{p}$ :

$$\dot{p} = R_{sb}v_b$$

Expanding this:

$$\begin{aligned} \dot{p}(t) &= \begin{bmatrix} \cos t & 0 & \sin t \\ 0 & 1 & 0 \\ -\sin t & 0 & \cos t \end{bmatrix} \begin{bmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -20 \cos t \\ 20 \sin t + 10 \\ -10 \cos t \end{bmatrix} \\ &= \begin{bmatrix} -20 \cos t - 20 \cos t \sin t \\ 10 \cos t \\ 20 \sin t + 10 \sin^2 t - 10 \cos^2 t \end{bmatrix} \end{aligned}$$