1

(a) The linear velocity of point C is:

$$[0,0,0]^T$$

(b) The linear velocity of point A is:

$$[2v,0,0]^T$$

(c) Assume that the center of this cylinder is D,The velocity of the body fixed point coincides with C is:

$${}^{O}v_C = {}^{O}v_D + {}^{O}\omega \times \overrightarrow{DC} = \begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{v}{r} \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(d) The velocity of the body fixed point coincides with A is:

$${}^{O}v_A = {}^{O}v_D + {}^{O}\omega \times \overrightarrow{DA} = \begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{v}{r} \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ r \end{bmatrix} = \begin{bmatrix} 2v \\ 0 \\ 0 \end{bmatrix}$$

(e) Choose O as the reference point:

$$\omega = [0, \frac{v}{r}, 0]^T$$

$${}^{O}v_O = {}^{O}v_D + {}^{O}\omega \times \overrightarrow{DO} = \begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{v}{r} \\ 0 \end{bmatrix} \times \begin{bmatrix} -C_x(t) \\ 0 \\ -r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{C_x(t)v}{r} \end{bmatrix}$$

$$\mathcal{V} = [0, \frac{v}{r}, 0, 0, 0, \frac{C_x(t)v}{r}]^T$$

(f)

$${}^{C}X_O = \begin{bmatrix} {}^{C}R_O & O \\ [p]^{C}R_O & {}^{C}R_O \end{bmatrix}$$

$${}^{C}\mathcal{V} = {}^{C}X_O^{O}\mathcal{V} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -C_x(t) & 0 & 1 & 0 \\ 0 & C_x(t) & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{v}{r} \\ 0 \\ 0 \\ 0 \\ \frac{C_x(t)v}{r} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{v}{r} \\ 0 \\ 0 \\ 0 \\ \frac{2C_x(t)v}{r} \end{bmatrix}$$

3.21

(a) From the ${}^aT_b$:

$$ {}^a p_c = \begin{bmatrix} 0 \\ 800 \\ 0 \end{bmatrix}, {}^a p_b = \begin{bmatrix} -100 \\ 300 \\ 500 \end{bmatrix} $$

$$ {}^a R_b = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

The vector r is:

$$ {}^a r = \overrightarrow{OC} - \overrightarrow{OB} = {}^a p_c - {}^a p_b = \begin{bmatrix} 100 \\ 500 \\ -500 \end{bmatrix} $$

$$ {}^b r = {}^b R_a {}^a r = \begin{bmatrix} 500 \\ -100 \\ -500 \end{bmatrix} $$

(b) ${}^a R_c$ can get from the rotation of frame $\{c\}$:

$$ {}^a R_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} $$

$$ {}^b R_c = {}^b R_a {}^a R_c = {}^a R_b^T {}^a R_c = \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} $$

Then we can get ${}^b T_c$:

$$ {}^b T_c = \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 500 \\ -1 & 0 & 0 & -100 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & -500 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

3.28

The body's angular velocity:

$$\omega_b = {}^b R_s^T \omega_s = \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}$$

5.5

(a)

$${}^s P = \begin{bmatrix} L + dsin\theta \\ L - dcos\theta \\ 0 \end{bmatrix}$$

(b)

$$v_p = \begin{bmatrix} d\dot{\theta}cos\theta \\ d\dot{\theta}sin\theta \\ 0 \end{bmatrix}$$

(c)

$${}^s T_b = \begin{bmatrix} 1 & 0 & 0 & L + dsin\theta \\ 0 & cos\theta & -sin\theta & L - dcos\theta \\ 0 & sin\theta & cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(d)

$$\dot{T}_{sb} = \begin{bmatrix} -\dot{\theta}\sin\theta & -\dot{\theta}\cos\theta & 0 & d\dot{\theta}\cos\theta \\ \dot{\theta}\cos\theta & -\dot{\theta}\sin\theta & 0 & d\dot{\theta}\sin\theta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad T_{sb}^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & -L(\cos\theta + \sin\theta) \\ -\sin\theta & \cos\theta & 0 & -L(\cos\theta - \sin\theta) + d \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$[\mathcal{V}_b] = T_{sb}^{-1}\dot{T}_{sb} = \begin{bmatrix} 0 & -\dot{\theta} & 0 & d\dot{\theta} \\ \dot{\theta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \mathcal{V}_b = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \\ d\dot{\theta} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \omega_b \\ v_b \end{bmatrix}.$$

(e)

$$
[\mathcal{V}_s] = \dot{T}_{sb}T_{sb}^{-1} = \begin{bmatrix} 0 & -\dot{\theta} & 0 & L\dot{\theta} \\ \dot{\theta} & 0 & 0 & -L\dot{\theta} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \mathcal{V}_s = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \\ L\dot{\theta} \\ -L\dot{\theta} \\ 0 \end{bmatrix} = \begin{bmatrix} \omega_s \\ v_s \end{bmatrix}.
$$

(f)

$$
[\mathcal{V}_s] = T_{sb}[\mathcal{V}_b]T_{sb}^{-1}.
$$

(g) Because:

$$
\dot{p}_P = \dot{p}_{sb}, \quad R_{sb}^{-1}\dot{p}_P = v_b.
$$

(h) From:

$$
\dot{R}_{sb}R_{sb}^{-1} = \omega_s, \quad -\omega_s p_P + \dot{p}_P = v_s.
$$

5.6

(a) Given

$$
\theta_1 = t,\ \theta_2 = t,\ \dot{\theta}_1 = \dot{\theta}_2 = 1
$$

$$
\mathcal{V}_b = J_b(\theta)\dot{\theta}
$$

$$
J_b(\theta) = \begin{bmatrix} \mathcal{V}_{b1}(\theta) & \mathcal{V}_{b2}(\theta) \end{bmatrix}
$$

where:

$$
\mathcal{V}_{b1}(\theta) = \begin{bmatrix} \sin\theta_2 \\ \cos\theta_2 \\ 0 \\ -20\cos\theta_2 \\ 20\sin\theta_2 \\ -10\cos\theta_2 \end{bmatrix}, \quad \mathcal{V}_{b2}(\theta) = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 10 \\ 0 \end{bmatrix}
$$

Thus,