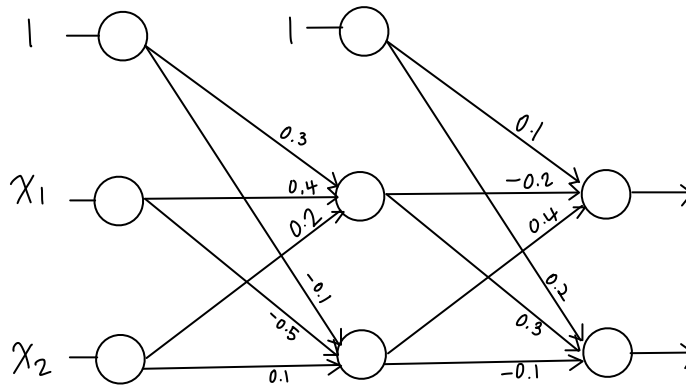


#MLP 학습예제

- 아래 그림은 $d=2, p=2, m=2$ 인 이커넥터를 가진 MLP이다. 가중치는 그림에서처럼 초기화 되어 있다고 하자. 활성화 함수는 양극 시그모이드를 사용하고 학습률은 0.2라고 하자. 아래 샘플을 가지고 시뮬레이션 하여라.

$$X = (0.7, 0.2)^T, t = (-1, 1)^T$$



· 전방전산 (forward computation)

- 1) 은닉층의 j 번째 노드, $1 \leq j \leq p$:

$$\left. \begin{aligned} Z\text{-sum}_j &= \sum_{i=1}^n x_i u_{ij} + u_{0j} \\ Z_j &= \tau(Z\text{-sum}_j) \end{aligned} \right\}$$

- 2) 출력층의 k 번째 노드, $1 \leq k \leq m$:

$$\left. \begin{aligned} O\text{-sum}_k &= \sum_{j=1}^p Z_j v_{jk} + v_{0k} \\ O_k &= \tau(O\text{-sum}_k) \end{aligned} \right\}$$

· 아진시그모이드 함수

$$1) \tau_1(x) = \frac{1}{1 + e^{-\alpha x}}$$

$$\tau_1'(x) = \alpha \tau_1(x)(1 - \tau_1(x))$$

$$2) \tau_2(x) = \frac{2}{1 + e^{-\alpha x}} - 1$$

$$\tau_2'(x) = \frac{\alpha}{2} (1 + \tau_2(x))(1 - \tau_2(x))$$

· 손실

$$E = \frac{1}{2} \sum_{k=1}^m (t_k - O_k)^2$$

$$\delta_k = (t_k - O_k) \tau_2'(O\text{-sum}_k), 1 \leq k \leq m$$

$$\Delta v_{jk} = -\rho \frac{\partial E}{\partial v_{jk}} = \rho \delta_k Z_j, 0 \leq j \leq p, 1 \leq k \leq m$$

$$\eta_j = \tau_1'(Z\text{-sum}_j) \sum_{k=1}^m \delta_k v_{jk}, 1 \leq j \leq p$$

$$\Delta u_{ij} = -\rho \frac{\partial E}{\partial u_{ij}} = \rho \eta_j x_i, 0 \leq i \leq d, 1 \leq j \leq p$$

• 품이과정

전방계산을 해보자

$$Z_Sum1 = 1 \times 0.7 + 0.7 \times 0.4 + 0.2 \times 0.2 = 0.62$$

$$Z_Sum2 = 1 \times (-0.1) + 0.7 \times (-0.5) + 0.2 \times (0.1) = -0.43$$

$$Z_1 = T_2(0.62) = 2 / (1 + e^{-0.62}) - 1 = 0.30044$$

$$Z_2 = T_2(-0.43) = 2 / (1 + e^{0.43}) - 1 = -0.21175$$

$$O_Sum1 = 1 \times 0.1 + 0.30044 \times (-0.2) + (-0.21175) \times 0.4 = -0.04479$$

$$O_Sum2 = 1 \times 0.2 + 0.30044 \times 0.3 + (-0.21175) \times (-0.1) = 0.31131$$

$$O_1 = T_2(-0.04479) = 2(1 + e^{0.04479}) - 1 = -0.02239$$

$$O_2 = T_2(0.31131) = 2(1 + e^{-0.31131}) - 1 = 0.15441$$

이 다층 퍼셉트론은 입력 $X = (0.7, 0.2)^T$ 에 대해 $O = (-0.02239, 0.15441)^T$ 를 출력했다.

기대하는 값 $t = (-1, 1)^T$ 과의 오류는 아래와 같이 구할 수 있다.

$$E = 0.5 \times ((-1.0 - (-0.02239))^2 + (1.0 - 0.15441)^2) = 0.83537$$

오류역전파 단계를 계산해보자.

$$\delta_1 = (-1.0 + 0.02239) T_2'(-0.04479) = -0.97761 \times 0.5 \times (1 + T_2(-0.04479))(1 - T_2(-0.04479)) = -0.48856$$

$$\delta_2 = (1.0 - 0.15441) T_2'(0.31131) = 0.84559 \times 0.5 \times (1 + T_2(0.31131))(1 - T_2(0.31131)) = 0.41271$$

$$\Delta V_{01} = 0.2 \times (-0.48856) \times 1.0 = -0.09771$$

$$\Delta V_{02} = 0.2 \times 0.41271 \times 1.0 = 0.08254$$

$$\Delta V_{11} = 0.2 \times (-0.48856) \times 0.30044 = -0.02936$$

$$\Delta V_{12} = 0.2 \times 0.41271 \times 0.30044 = 0.02480$$

$$\Delta V_{21} = 0.2 \times (-0.48856) \times (-0.21175) = 0.02069$$

$$\Delta V_{22} = 0.2 \times 0.41271 \times (-0.21175) = -0.01748$$

$$\eta_1 = T_2'(0.62) \times ((-0.48856) \times (-0.2) + 0.41271 \times 0.3) = 0.10076$$

$$\eta_2 = T_2'(-0.43) \times ((-0.48856) \times (0.4) + 0.41271 \times (-0.1)) = -0.11304$$

$$\Delta U_{01} = 0.2 \times 0.10076 \times 1.0 = 0.02015$$

$$\Delta U_{02} = 0.2 \times (-0.11304) \times 1.0 = -0.02261$$

$$\Delta U_{11} = 0.2 \times 0.10076 \times 0.7 = 0.01411$$

$$\Delta U_{12} = 0.2 \times (-0.11304) \times 0.7 = -0.01583$$

$$\Delta U_{21} = 0.2 \times 0.10076 \times 0.2 = 0.00403$$

$$\Delta U_{22} = 0.2 \times (-0.11304) \times 0.2 = -0.00452$$

가중치 갱신 단계

$$V_{01} = 0.1 - 0.09771 = 0.00229$$

$$V_{02} = 0.2 + 0.08254 = 0.28254$$

$$V_{11} = -0.2 - 0.02936 = -0.22936$$

$$V_{12} = 0.3 + 0.02480 = 0.32480$$

$$V_{21} = 0.4 + 0.02069 = 0.42069$$

$$V_{22} = -0.1 - 0.01748 = -0.11748$$

$$U_{01} = 0.3 + 0.02015 = 0.32015$$

$$U_{02} = -0.1 - 0.02261 = -0.12261$$

$$U_{11} = 0.4 + 0.01411 = 0.41411$$

$$U_{12} = -0.5 - 0.01583 = -0.51583$$

$$U_{21} = 0.2 + 0.00403 = 0.20403$$

$$U_{22} = 0.1 - 0.00452 = 0.09548$$

효과를 확인해보자

$$Z_sum1 = 1.0 \times 0.32015 + 0.7 \times 0.41411 + 0.2 \times 0.20403 = 0.65083$$

$$Z_sum2 = 1.0 \times (-0.12261) + 0.7 \times (-0.51583) + 0.2 \times 0.09548 = -0.46460$$

$$Z1 = 0.31440$$

$$Z2 = -0.22821$$

$$O_sum1 = 1.0 \times 0.00229 + 0.31440 \times (-0.22936) + (-0.22821) \times 0.42069 = -0.16582$$

$$O_sum2 = 1.0 \times 0.28254 + 0.31440 \times (0.32480) + (-0.22821) \times (-0.11748) = 0.41447$$

$$O1 = -0.08272$$

$$O2 = 0.20288$$

$O = (-0.08272, 0.20288)^T$ 를 얻었다. $t = (-1, 1)^T$ 에 가까워졌으며, 오류도 $E = 0.77840$ 으로 이전보다 줄어 들었다.