컴퓨터공학부 201814121 이연희

Baysian Inference

1 linear Classification

- 입력 데이터

$\omega_1 : (1,2)^T, (3,1)^T, (5,2)^T, (3,3)^T$

$\omega_2 : (6,6)^T, (8,5)^T, (10,6)^T, (8,7)^T$

- 확률분포 모델링

$$\mu_1 = \frac{1}{4}\left(\begin{pmatrix}1\\2\end{pmatrix}+\begin{pmatrix}3\\1\end{pmatrix}+\begin{pmatrix}5\\2\end{pmatrix}+\begin{pmatrix}3\\3\end{pmatrix}\right)=\begin{pmatrix}3\\2\end{pmatrix}, \quad \Sigma_1 = \begin{pmatrix}\frac{8}{3} & 0\\ 0 & \frac{2}{3}\end{pmatrix}$$

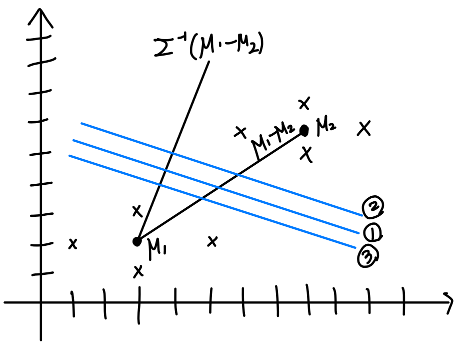
$$\mu_2 = \frac{1}{4}\left(\begin{pmatrix}6\\6\end{pmatrix}+\begin{pmatrix}8\\5\end{pmatrix}+\begin{pmatrix}10\\6\end{pmatrix}+\begin{pmatrix}8\\7\end{pmatrix}\right)=\begin{pmatrix}8\\6\end{pmatrix}, \quad \Sigma_2 = \begin{pmatrix}\frac{8}{3} & 0\\ 0 & \frac{2}{3}\end{pmatrix}$$

- 식별함수

$$g_{12}(x) = \left(\begin{pmatrix}\frac{2}{3} & 0\\ 0 & \frac{8}{3}\end{pmatrix}\begin{pmatrix}3-8\\ 2-6\end{pmatrix}\right)^T x + \left(\ln P(\omega_1) - \ln P(\omega_2) - \frac{1}{2}(3 \;\; 2)\begin{pmatrix}\frac{3}{8} & 0\\ 0 & \frac{2}{3}\end{pmatrix}\begin{pmatrix}3\\2\end{pmatrix} + \frac{1}{2}(8 \;\; 6)\begin{pmatrix}\frac{3}{8} & 0\\ 0 & \frac{2}{3}\end{pmatrix}\begin{pmatrix}8\\6\end{pmatrix}\right)$$

$$= \left(-\frac{15}{8} \;\; -6\right)\begin{pmatrix}x_1\\x_2\end{pmatrix} + \left(\ln P(\omega_1) - \ln P(\omega_2) - \frac{1}{2}\left(\frac{9}{8} \;\; \frac{4}{3}\right)\begin{pmatrix}3\\2\end{pmatrix} + \frac{1}{2}(3 \;\; 4)\begin{pmatrix}8\\6\end{pmatrix}\right)$$

$$= -\frac{15}{8}x_1 - 6x_2 + \left(\ln P(\omega_1) - \ln P(\omega_2) + 34.3125\right)$$



① $P(\omega_1)=0.5$, $P(\omega_2)=0.5$

$: 5x_1 + 16x_2 - 91.5 = 0$

② $P(\omega_1)=0.8$, $P(\omega_2)=0.2$

$: 5x_1 + 16x_2 - 95.197 = 0$

③ $P(\omega_1)=0.2$, $P(\omega_2)=0.8$

$: 5x_1 + 16x_2 - 87.803 = 0$

2. non-linear classification

- 입력데이터

$\omega_1 : (1,2)^T, (3,1)^T, (5,2)^T, (3,3)^T$

$\omega_2 : (7,6)^T, (8,4)^T, (9,6)^T, (8,8)^T$

- 확률분포 모델링

$$\mu_1 = \frac{1}{4}\left(\begin{pmatrix}1\\2\end{pmatrix}+\begin{pmatrix}3\\1\end{pmatrix}+\begin{pmatrix}5\\2\end{pmatrix}+\begin{pmatrix}3\\3\end{pmatrix}\right)=\begin{pmatrix}3\\2\end{pmatrix}, \quad \Sigma_1 = \begin{pmatrix}\frac{8}{3} & 0\\ 0 & \frac{2}{3}\end{pmatrix}$$

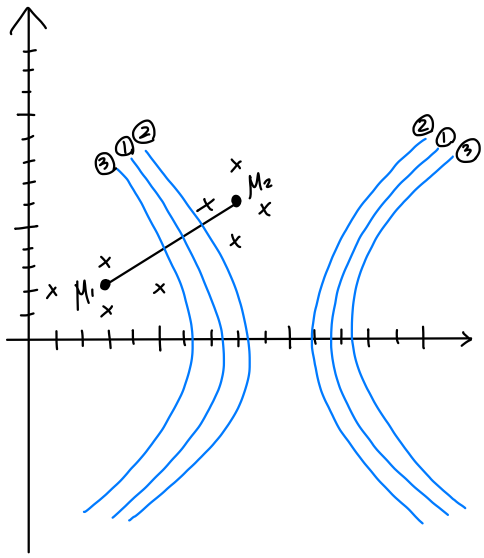
$$\mu_2 = \frac{1}{4}\left(\begin{pmatrix}7\\6\end{pmatrix}+\begin{pmatrix}8\\4\end{pmatrix}+\begin{pmatrix}9\\6\end{pmatrix}+\begin{pmatrix}8\\8\end{pmatrix}\right)=\begin{pmatrix}8\\6\end{pmatrix}, \quad \Sigma_2 = \begin{pmatrix}\frac{2}{3} & 0\\ 0 & \frac{8}{3}\end{pmatrix}$$

- 식별함수

$$g_{12}(x) = -\frac{1}{2}x^T\begin{pmatrix}\frac{8}{3} & 0\\ 0 & \frac{2}{3}\end{pmatrix}^{-1}x + \frac{1}{2}x^T\begin{pmatrix}\frac{2}{3} & 0\\ 0 & \frac{8}{3}\end{pmatrix}^{-1}x + (3 \;\; 2)\begin{pmatrix}\frac{8}{3} & 0\\ 0 & \frac{2}{3}\end{pmatrix}^{-1}x - (8 \;\; 6)\begin{pmatrix}\frac{2}{3} & 0\\ 0 & \frac{8}{3}\end{pmatrix}^{-1}x$$

$$+\left(-\frac{1}{2}(3 \;\; 2)\begin{pmatrix}\frac{8}{3} & 0\\ 0 & \frac{2}{3}\end{pmatrix}^{-1}\begin{pmatrix}3\\2\end{pmatrix} - \frac{1}{2}\ln\left|\begin{pmatrix}\frac{8}{3} & 0\\ 0 & \frac{2}{3}\end{pmatrix}\right| + \ln P(\omega_1)\right) - \left(-\frac{1}{2}(8 \;\; 6)\begin{pmatrix}\frac{2}{3} & 0\\ 0 & \frac{8}{3}\end{pmatrix}^{-1}\begin{pmatrix}8\\6\end{pmatrix} - \frac{1}{2}\ln\left|\begin{pmatrix}\frac{2}{3} & 0\\ 0 & \frac{8}{3}\end{pmatrix}\right| + \ln P(\omega_2)\right)$$

$$= \frac{9}{16}x_1^2 - \frac{9}{16}x_2^2 - \frac{87}{8}x_1 + \frac{3}{4}x_2 + \frac{801}{16} + \ln P(\omega_1) - \ln P(\omega_2)$$



① $P(W_1) = 0.5$, $P(W_2) = 0.5$

. $3X_1^2 - 3X_2^2 - 58X_1 + 4X_2 - 267 = 0$

② $P(W_1) = 0.8$, $P(W_2) = 0.2$

: $3X_1^2 - 3X_2^2 - 58X_1 + 4X_2 + 274.3936 = 0$

③ $P(W_1) = 0.2$, $P(W_2) = 0.8$

: $3X_1^2 - 3X_2^2 - 58X_1 + 4X_2 + 259.6064 = 0$