

Distributing the Heat Equation

Tom Cornebize

Yassine Hamoudi

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1 Cellular automata

Question 1.

Lemma 1. N^2 applications of function δ are necessary to compute X^t from X^{t-1} .

Proof. Each cell $X_{i,j}^t$ needs one application of δ to be computed from $X_{i,j}^{t-1}$. There are N^2 cells, so N^2 applications of δ are needed. \square

Property 2. tN^2 applications of function δ are necessary to compute X^t on $\llbracket 0, N-1 \rrbracket^2$.

Proof. X^t is obtained after t applications of δ^+ on X^0 . Each application needs N^2 calls to δ according to lemma 1. The whole computation needs tN^2 applications of δ . \square

Question 2. Let p^2 be the number of processors.

For the sake of simplicity, we will suppose that p divides N . Take $n = \frac{N}{p}$.

We divide the grid into square zones of size n . Each of this zones is given to one processor, which stores the data in its own memory and performs the computation of δ for all its cells. See figure 1 for an example.

At each step of computation, each processor updates its sub-matrix cells using a temporary sub-matrix that replaces the old one once the computation step is finished. Indeed, if we update the cells “in place”, we overwrite values that are still necessary to compute other cells.

The computation of δ for the cells at the edges of the zones requires communication to retrieve the current states of their neighbours in other zones.

Figure 1: Graphical representation of the topology for $N = 6$ and $p^2 = 9$.

