

Distributing the Heat Equation

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1 Cellular automata

Question 1.

Lemma 1. N^2 applications of function δ are necessary to compute X^t from X^{t-1} .

Proof. Each cell $X_{i,j}^t$ needs one application of δ to be computed from $X_{i,j}^{t-1}$. There are N^2 cells, so N^2 applications of δ are needed. \square

Property 2. tN^2 applications of function δ are necessary to compute X^t on $\llbracket 0, N-1 \rrbracket^2$.

Proof. X^t is obtained after t applications of δ^+ on X^0 . Each application needs N^2 calls to δ according to lemma 1. The whole computation needs tN^2 applications of δ . \square

Question 2. Let p be the number of processors.

For the sake of simplicity, we will suppose that p is a perfect square (i.e. $\sqrt{p} \in \mathcal{N}$), and that \sqrt{p} divides N . Take $n = \frac{N}{\sqrt{p}}$.

We divide the grid into square zones of size n . Each of this zones is given to one processor, which stores the data in its own memory and performs the computation of δ for all its cells. See figure 1 for an example.

The computation of δ for the cells at the edges of the zones requires communication to retrieve the current states of their neighbours in other zones.

The general case can be treated in a similar fashion.

Figure 1: Graphical representation of the topology for $N = 6$ and $p = 9$.

