Distributing the Heat Equation

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1 Question 1

Lemma 1. *N^2 applications of function δ are necessary to compute X^t from X^{t-1}.*

Proof. Each cell $X^t_{i,j}$ needs one application of δ to be computed from $X^{t-1}_{i,j}$. There are N^2 cells, so N^2 applications of δ are needed. $\square$

Property 2. *tN^2 applications of function δ are necessary to compute X^t on $[\![0, N-1]\!]^2$.*

Proof. X^t is obtained after t applications of $\delta^\dagger$ on X^0. Each application needs N^2 calls to δ according to lemma 1. The whole computation needs tN^2 applications of δ. $\square$

2 Question 2

Let p be the number of processors.

For the sake of simplicity, we will suppose that p is a perfect square (i.e. $\sqrt{p} \in \mathcal{N}$), and that $\sqrt{p}$ divides N. Take $n = \frac{N}{\sqrt{p}}$.

We divide the grid into square zones of size n. Each of this zones is given to one processor, which stores the data in its own memory and performs the computation of δ for all its cells. See figure 1 for an example.

The computation of δ for the cells at the edges of the zones requires communication to retrieve the current states of their neighbours in other zones.

The general case can be treated in a similar fashion.

Figure 1: Graphical representation of the topology for $N = 6$ and $p = 9$.