

Econ8410 Course paper

Yan Hao

Dec. 5, 2021

1 Introduction

I will reproduce most part of the Sheffrin and Woo's results in this course paper. It includes six countries, United Kingdom, Thailand, Sweden, United States, Canada, and Denmark (KTSUCD). Balance of Payment data are from International Monetary Fund. Data using to compute Net output are from Penn World Table and cover the period 1977-2019 (42 years).

2 Theoretical Model

Similar to what we discussed in class the model assumes that 1) there is a small open economy with a given world interest rate; 2) Consumption depends solely on a country's wealth; 3) The wealth is measured by the sum of NO and the stock of external assets (B); 4) The first order difference of NO is stationary, which implies the level of current account is stationary; 5) Agents cannot borrow money without a limit (Transversality condition)

$$\lim_{i \rightarrow \infty} \left(\frac{1}{1+r} \right)^i b_{t+i} = 0,$$

where $\beta = \frac{1}{1+r}$, and b_{t+i} is the stock of external assets at the beginning of period $t+i$.

The accumulation of external assets follows the rule below:

$$b_{t+i} = (1+r)b_t + [y_t - c_t - i_t - g_t]. \quad (1)$$

By iterating equation (1),

$$\begin{aligned} b_t &= \frac{1}{1+r}b_{t+1} - \frac{1}{1+r}(y_t - c_t - i_t - g_t) \\ &= \beta b_{t+1} - \beta x_t, \text{ where } x_t = y_t - c_t - i_t - g_t \text{ is the net export} \\ &= \beta(\beta b_{t+2} - \beta x_{t+1}) - \beta x_t \\ &= \beta^2 b_{t+2} - \beta^2 x_{t+1} - \beta x_t \\ &\dots \end{aligned}$$

$$b_t = - \sum_{i=1}^{\infty} \beta^i x_{t+i-1}. \quad (2)$$

It says the external asset in period t is the present discounted value of the stream of future trade deficits.

The consumption for individual agent is the difference between income and the sum of investment, government spending, and net export,

$$c_{t+i-1} = y_{t+i-1} - i_{t+i-1} - g_{t+i-1} - x_{t+i-1}. \quad (3)$$

The sum of present discounted value can be written as

$$\sum_{i=1}^{\infty} \beta^i c_{t+i-1} = \sum_{i=1}^{\infty} \beta^i [y_{t+i-1} - i_{t+i-1} - g_{t+i-1}] - \sum_{i=1}^{\infty} \beta^i x_{t+i-1}. \quad (4)$$

Plug in equation (2), we obtain agent's intertemporal budget constraint,

$$\sum_{i=1}^{\infty} \beta^i c_{t+i-1} = \left(\sum_{i=1}^{\infty} \beta^i (NO)_{t+i-1} \right) + b_t, \quad (5)$$

where the Net Output (NO) is defined as GDP less the sum of investment plus government spending, $NO = Y - (I + G)$.

Now we can write down the optimization problem:

$$\begin{aligned} U &= \mathbb{E}_0 \left(\sum_{i=0}^{\infty} \beta^i u(c_i) \right) \\ s.t. \quad & \sum_{i=1}^{\infty} \beta^i c_{t+i-1} = \left(\sum_{i=1}^{\infty} \beta^i (NO)_{t+i-1} \right) + b_t. \end{aligned}$$

With quadratic utility function, we can find the closed-form solution for consumption,

$$c_t = \frac{r}{1+r} \left\{ (1+r)b_t + \sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}_t(NO_{t+i}) \right\}. \quad (6)$$

Current account is the difference between savings and investment, and it can be written as a function of net output,

$$CA_t = S_t - I_t = y_t + rb_t - i_t - g_t - c_t = NO_t + rb_t - c_t. \quad (7)$$

Plug in equation (6), we can rewrite CA as a function of the change of NO ,

$$CA_t = NO_t + rb_t - \left\{ \frac{r}{1+r} \left\{ (1+r)b_t + \sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}_t(NO_{t+i}) \right\} \right\} \quad (8)$$

$$= - \left(\frac{r}{1+r} \right) \sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}_t(NO_{t+i} - NO_t) \quad (9)$$

$$= - \sum_{i=1}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}_t(\Delta NO_{t+i}). \quad (10)$$

Clearly, ΔNO is the first difference of NO . If it is stationary, then the current account will also be stationary in its level. We will examine this latter.

3 Regression Model

To test this theoretical result (equation (10)), we can run a second-order vector autoregression (VAR) to capture the properties of the series. The VAR can be written as the following,

$$\begin{bmatrix} \Delta NO_t \\ \Delta NO_{t-1} \\ CA_t \\ CA_{t-1} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & b_1 & b_2 \\ 1 & 0 & 0 & 0 \\ c_1 & c_2 & d_1 & d_2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta NO_{t-1} \\ \Delta NO_{t-2} \\ CA_{t-1} \\ CA_{t-2} \end{bmatrix} + \begin{bmatrix} u_t \\ 0 \\ v_t \\ 0 \end{bmatrix}, \quad (11)$$

or it can be written in a compact form, $Z_t = AZ_{t-1} + w_t$, where $\mathbb{E}(Z_{t+i}) = A^i Z_t$ (This can be derived by iterating the equation of Z_t , then take the expectation of it). Considering the restrictions on current account, equation (10), we can write the following,

$$\begin{aligned} CA_t &= - \sum_{i=1}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}_t(\Delta NO_{t+i}) \\ CA_t' &= - \sum_i \left(\frac{1}{1+r} \right)^i \mathbf{h}' \mathbf{A}^i \mathbf{Z}_t \\ &= - \mathbf{h}' \sum_i \left(\frac{\mathbf{A}}{1+r} \right)^i \mathbf{Z}_t \\ &= \mathbf{K} \mathbf{Z}_t, \end{aligned}$$

where $\mathbf{h}' = [1, 0, 0, 0]$,

$$\mathbf{K} = -\mathbf{h}' \frac{\left(\frac{\mathbf{A}}{1+r} \right)}{\left[1 - \left(\frac{\mathbf{A}}{1+r} \right) \right]}.$$

We know \mathbf{h}' is a 1×4 row vector, the ratio of \mathbf{A} is still a 4×4 matrix. Therefore, \mathbf{K} is a

1×4 row matrix, \mathbf{Z}_t is a 4×1 matrix,

$$\mathbf{K} = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \end{bmatrix}$$

$$\mathbf{Z}_t = \begin{bmatrix} \Delta NO_t \\ \Delta NO_{t-1} \\ CA_t \\ CA_{t-1} \end{bmatrix}$$

Hence, we would like to see γ_1, γ_2 , and γ_4 are zeros. And we can use this to predict the current account series, then evaluate the performance of the model by comparing the prediction of CA series and the actual ones.

4 Estimations, and results

As we discussed in section 2, we want to first check if the first difference of NO and the level of CA are stationary. This can be done by running regressions of the form:

$$\Delta NO_t = a + b_1 \Delta NO_{t-1} + b_2 \Delta NO_{t-2} + c NO_{t-1} + d \cdot time + \eta_t, \quad (12)$$

and test if the coefficient c is negative and significantly different than zero using the Dickey-Fuller statistics, τ_τ . For the net output series, we want to fail to reject the null; for the current account series, we would like to reject the null. Since we have 42 observations, criterion for sample size of 50 could be the appropriate one. The critical values for 10% and 5% significant level are -3.18 and -3.5.

For KTSUCD countries, t statistics for the net output series are -1.833, -1.532, -0.868, -2.986, -0.997, -1.914. Except for the United States, these are below the critical value to reject the null. For the current account series, statistics for KTSUCD countries are -3.805, -2.924, -2.361, -2.215, -1.951, -1.573.

Table 1 to Table 6 present the result of VAR and the estimated K matrix for KTSUCD countries. Figure 1 to Figure 6 are the comparison between the prediction based on vector \mathbf{K} and the actual current account series (Dashed-line: prediction, Solid line: actual series). Except for Sweden and Denmark, other four countries are run in a current account deficits during this period. Denmark's current account series is exploding. From the figure, even though the \mathbf{K} vector for the US does not satisfy the requirements of theoretical model, the prediction of it performs well.

Table 1: United Kingdom

	ΔNO_t	CA_t	4%	14%
ΔNO_{t-1}	0.215 (0.162)	-0.010 (0.071)	-0.682	-0.695
ΔNO_{t-2}	0.267 (0.159)	-0.102 (0.070)	0.367	0.353
CA_{t-1}	-0.069 (0.381)	1.072 (0.167)	-3.819	-3.672
CA_{t-2}	0.002 (0.393)	-0.124 (0.173)	4.590	4.413
R^2	0.188	0.925		

Table 2: Thailand

	ΔNO_t	CA_t	4%	14%
ΔNO_{t-1}	0.779 (0.167)	-0.078 (0.063)	-0.077	-0.041
ΔNO_{t-2}	-0.069 (0.191)	-0.130 (0.072)	-0.935	-0.971
CA_{t-1}	0.311 (0.425)	0.553 (0.160)	2.946	3.061
CA_{t-2}	-0.660 (0.390)	0.055 (0.147)	-2.362	-2.455
R^2	0.676	0.831		

Table 3: Sweden

	ΔNO_t	CA_t	4%	14%
ΔNO_{t-1}	0.507 (0.174)	-0.059 (0.119)	1.616	1.551
ΔNO_{t-2}	0.276 (0.176)	0.113 (0.120)	-1.238	-1.207
CA_{t-1}	0.097 (0.265)	0.963 (0.181)	-3.685	-3.592
CA_{t-2}	-0.093 (0.279)	-0.149 (0.191)	3.601	3.510
R^2	0.541	0.669		

Table 4: United States

	ΔNO_t	CA_t	4%	14%
ΔNO_{t-1}	0.466 (0.158)	-0.124 (0.112)	-0.224	-0.240
ΔNO_{t-2}	0.284 (0.156)	0.110 (0.110)	-0.094	-0.092
CA_{t-1}	-0.453 (0.229)	1.206 (0.162)	0.199	0.195
CA_{t-2}	0.370 (0.230)	-0.224 (0.163)	0.352	0.345
R^2	0.731	0.964		

Table 5: Canada

	ΔNO_t	CA_t	4%	14%
ΔNO_{t-1}	-0.086 (0.227)	-0.063 (0.080)	-0.194	-0.188
ΔNO_{t-2}	-0.140 (0.159)	-0.076 (0.056)	0.012	0.012
CA_{t-1}	-0.341 (0.621)	1.222 (0.220)	0.403	0.406
CA_{t-2}	0.055 (0.656)	-0.313 (0.232)	-0.300	-0.302
R^2	0.1	0.863		

Table 6: Canada

	ΔNO_t	CA_t	4%	14%
ΔNO_{t-1}	0.505 (0.202)	0.134 (0.163)	-0.461	-0.458
ΔNO_{t-2}	-0.020 (0.189)	-0.061 (0.152)	0.048	0.048
CA_{t-1}	-0.340 (0.226)	0.786 (0.181)	1.089	1.096
CA_{t-2}	0.403 (0.238)	0.270 (0.191)	-0.750	-0.755
R^2	0.198	0.946		

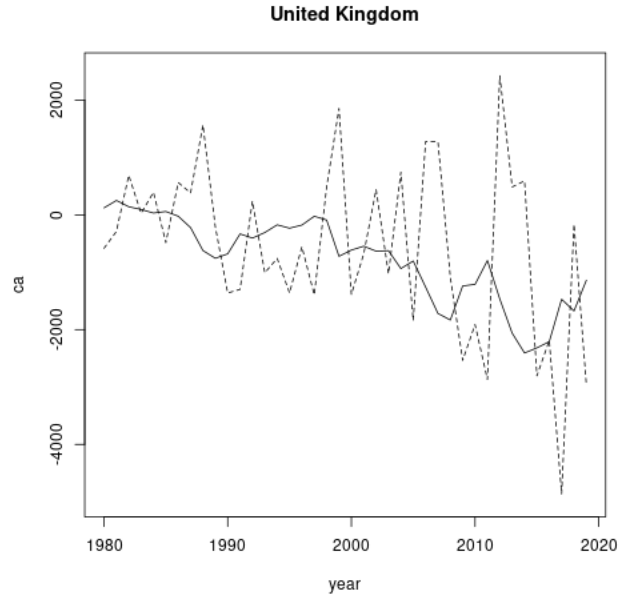


Figure 1

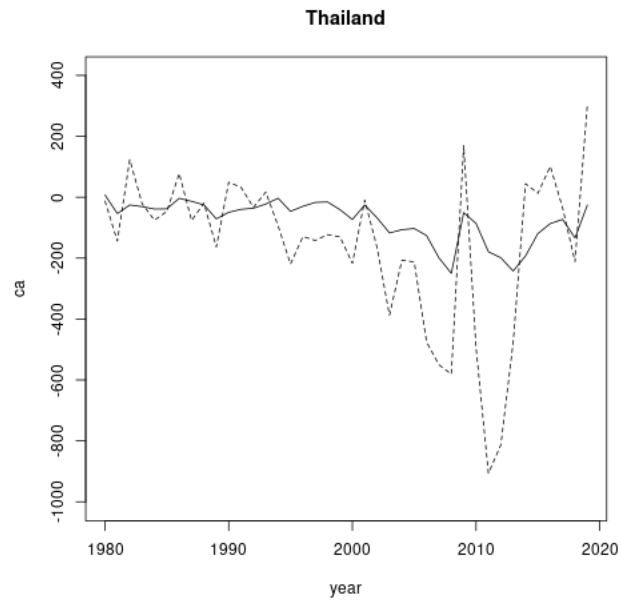


Figure 2

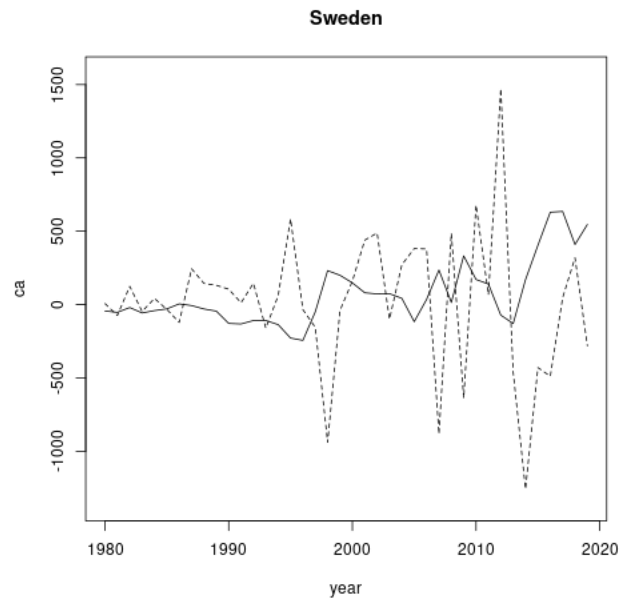


Figure 3

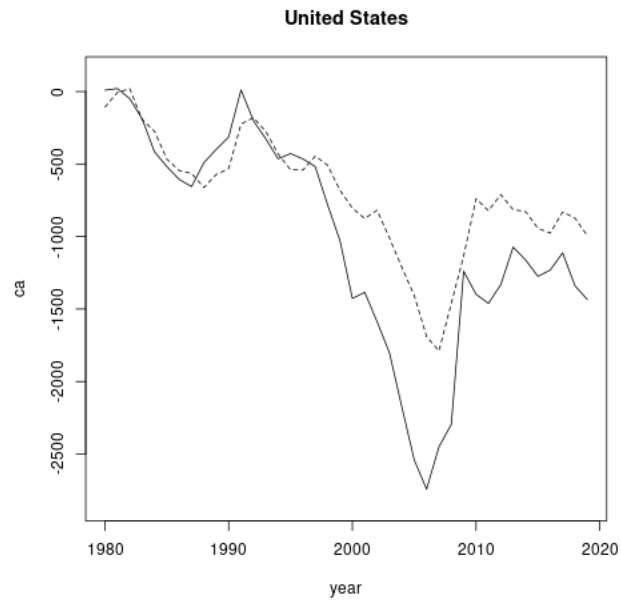


Figure 4

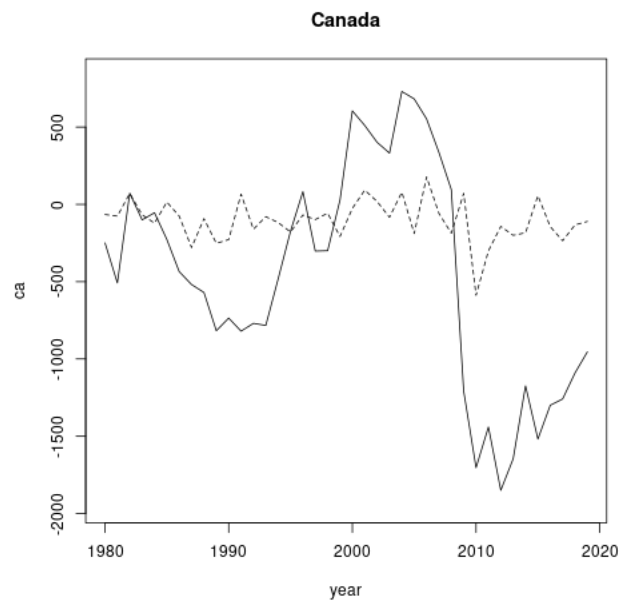


Figure 5

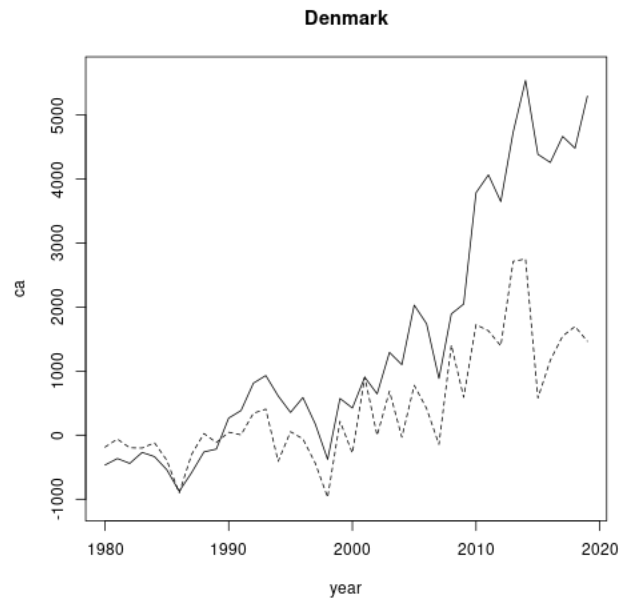


Figure 6