Model of Consumption/Saving and Current Account with Endogenous Output

Introduction

So have looked at a 2-period model of consumption/saving and CA with exogenous output (2-period a model of consumption and saving with a household choosing consumption/saving to max utility, no capital accumulation (output falls from the sky))

Now we'll add investment (i.e. make output endogenous) in the consumption/saving model .

We'll add a lump-sum tax while we're developing the model.

In this note we will again confine ourselves to the 2-period setting but we'll explore both the SOE model as well as a two-country global equilibrium.

Production & Investment

So far when we had micro foundations in the model (household maximizing utility) we have been assuming that output is exogenous (i.e. given, not determined inside the model). We'll relax this assumption by assuming that output is produced using capital, labor and technology according to

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Out[ ]=

$$Y_t = A_t F(K_t, L_t)$$

$$F_K > 0,\ F_L > 0$$
$$F_{KK} < 0,\ F_{LL} < 0$$

Also, we will soften make the following assumption

$$Y_t = A_t\, F\,(K_t,\, L_t) = A_t\, K_t^{\alpha}\, L_t^{1-\alpha},\ 0 \leqslant \alpha \leqslant 1.$$

Initial capital K_1 will be GIVEN (exogenous) just like initial assets B_1 were (and will be here as well, keep in mind (see NOTE above), this is what we used to call A_0).

Of course, K_1 cannot be assumed to equal 0 as we often do with B_1. (Why? because there would be zero output in year 1 and things wouldn't get off the ground)

K_2 will be determined by investment decisions.

We won't have much to say about labor for now so let's assume: $L_t = 1.$ So the production function an be written more compactly as

Out[ ]=

$$Y_t = A_t F(K_t, L_t) = A_t F(K_t, 1) = A_t F(K_t)$$

Nature of Capital

We will make the so-called putty-putty assumption: namely we will assume that capital and output can be costlessly turned into one another. That is, you can take a unit of Y and -- at no cost --make it a unit of K, and then you can -- again at no cost -- turn it back into output to be consumed.

Q: What does this imply about the price of capital?

A: It must be = 1!

What are the alternatives?

Closed Economy: PPF

First, we'll consider a closed economy. We'll also add a government which taxes (T) and spends (G). Let the taxes be lump sum.
Recall that the endowment cons/saving economy that was closed, was "forced" to consume its endowment:

$Y_1 = C_1$
$Y_2 = C_2$

And net borrowing/lending had to be zero B =0.
Here, we'll still have B =0 (still no one to borrow from/lend to in a closed economy). But we'll also see that things are somewhat different here.

Let's start by thinking about the budget constraints.

Each period, the household has its income Y minus taxes T it has to pay, to spend. What does it spend it on? Consumption (as before) and investment (this is new).

$Y_1 - T_1 = C_1 + I_1$
$Y_2 - T_2 = C_2 + I_2$

or
$A_1 F(K_1, L_1) - T_1 = C_1 + I_1$
$A_2 F(K_2, L_2) - T_2 = C_2 + I_2$

In addition capital evolves according to (recall that K_1 is given.)

$$K_2 = (1-\delta) K_1 + I_1$$
$$K_3 = (1-\delta) K_2 + I_2$$

where δ is the depreciation rate. If -- for now -- we assume that $\delta = 0$ (for simplicity), and recall that capital can be costlessly converted to output, we have to conclude that $K_3 = 0$ (same argument as for assets in the endowment cons/sav model) and so

$$K_2 = K_1 + I_1$$
$$I_2 = -K_2$$

Together this gives us:

$$\begin{aligned} A_1 F(K_1) - T_1 &= C_1 + I_1 \\ A_2 F(K_2) - T_2 &= C_2 + I_2 \\ &= C_2 - K_2 \end{aligned}$$

Rewrite the second constraint as

Out[ ]=
$$C_2 = A_2 F(K_2) - T_2 + K_2$$

This says that consumption in the second period is equal to the disposable income (Y_2 - T_2) plus all the capital that was accumulated before this period K_2 (recall, there is no depreciation; with depreciation the second term would be whatever is left of the capital after depreciation has taken its toll: $(1 - \delta)\ K_2$).
Continuing with the transformation of period 2 constraint, we can substitute $K_2 = K_1 + I_1$ to get

$$\begin{aligned} A_2\ F\ (K_2) - T_2 &= C_2 - K_2 \\ &= C_2 - (K_1 + I_1) \end{aligned}$$

At this point, note that from the period-1 constraint equation we have $I_1 = A_1\ F\ (K_1) - T_1 - C_1$, which we can substitute for I_1 in the above equation to get:

$$A_2\ F\ (K_2) - T_2 = C_2 - K_1 - (A_1\ F\ (K_1) - T_1) + C_1$$
$$A_2\ F\ (K_1 + I_1) - T_2 = C_2 - K_1 - (A_1\ F\ (K_1) - T_1) + C_1$$
$$A_2\ F\ (K_1 + A_1\ F\ (K_1) - T_1 - C_1) - T_2 = C_2 - K_1 - (A_1\ F\ (K_1) - T_1) + C_1$$

or re-arranging terms

Out[ ]=
$$C_2 = A_2 F\left(K_1 + A_1 F(K_1) - T_1 - C_1\right) - T_2 + K_1 + \left(A_1 F(K_1) - T_1 - C_1\right)$$

This gives us all the feasible C_2 as a function of C_1; this is the so-called Production Possibilities Frontier (PPF).

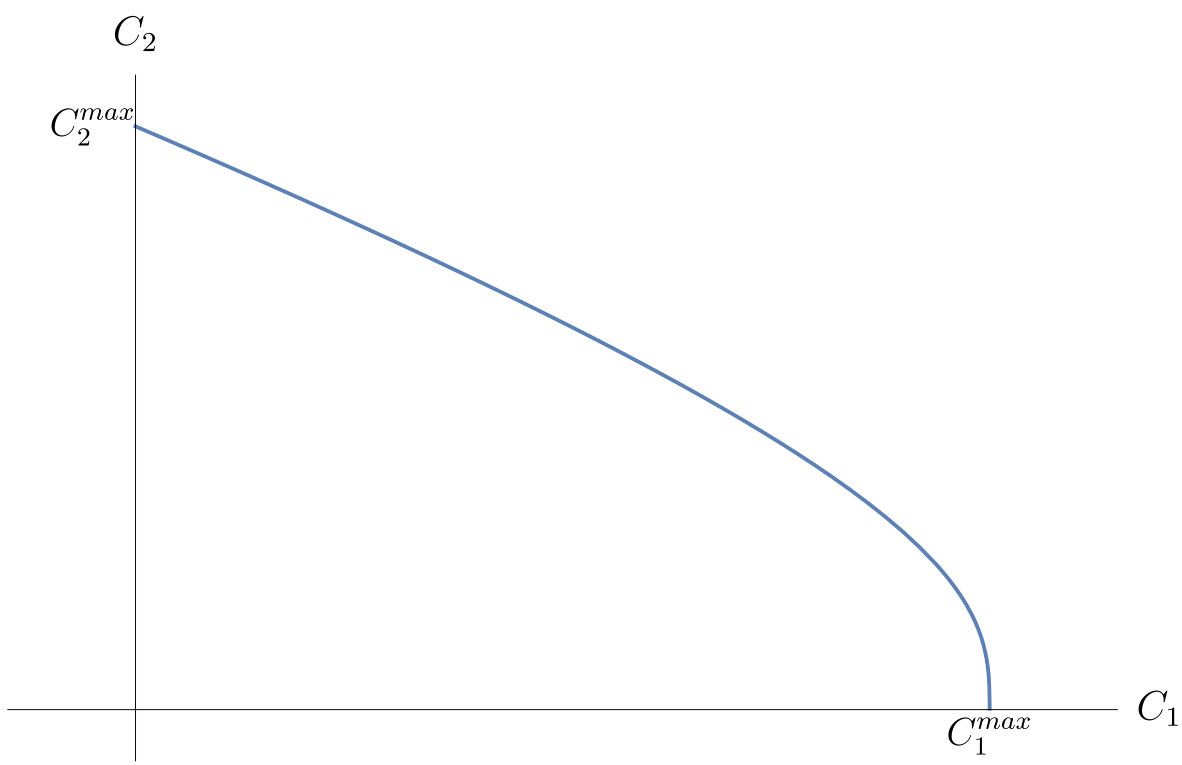
Closed Economy: PPF

Here is what the PPF looks like

Out[ ]=

$$C_2 = A_2 F\Big(K_1 + A_1 F(K_1) - T_1 - C_1\Big) - T_2 + K_1 + \Big(A_1 F(K_1) - T_1 - C_1\Big)$$

Out[ ]=



What are the intercepts of the PPF?

Horizontal: C_1 is maximized but C_2 is 0.

Out[ ]=
$$C_1^{max} = A_1 F(K_1) - T_1 + K_1$$

Vertical : C_2 is maximized but C_1 is 0.

Out[ ]=
$$C_2^{max} = A_2 F\left(A_1 F(K_1) - T_1 + K_1\right) - T_2 + K_1 + A_1 F(K_1) - T_1$$

What is the slope of the PPF?

Out[ ]=
$$\frac{dC_2}{dC_1} = -A_2 F'(K_2) - 1 = -(1 + A_2 F'(K_2))$$

When we are at the horizontal intercept, C_1 is maximized: all period-1 output and capital are eaten in period 1 But this means no investment and K_2 =0! This makes the PPF vertical, since $A_2 F'(0)$ is infinite. As we move left, period-1 investment increases and the slope of the PPF flattens.

Closed Economy: Solution

The problem is therefore reduced to

$$\max_{\{C_1, C_2\}} u(C_1) + \beta u(C_2)$$

$$\text{subject to}$$

$$C_2 = A_2 F(K_1 + A_1 F(K_1) - T_1 - C_1) - T_2 + K_1 + (A_1 F(K_1) - T_1) - C_1$$

By substituting the budget constraint into the objective function, we get

Out[ ]=

$$\max_{C_1, C_2} u(C_1, C_2) = u(C_1) + \beta u\left(A_2 F\left(A_1 F(K_1) - T_1 + K_1 - C_1\right) - T_2 + K_1 + A_1 F(K_1) - T_1 - C_1\right)$$

and taking the first-order condition with respect to C_1 we get:

Out[ ]=

$$u'(C_1) = \beta\left(1 + A_2 F'(K_2)\right) u'(C_2)$$

(If we didn't assume depreciation away, the condition would be: $u'(C_1) = \beta u'(C_2)(1 + A_2 F'(K_2) - \delta)$, which is what we will derive later for the infinite horizon model. Can you derive it? Go back to teh PPF and derive it without imposing $\delta = 0$, then re-do the household maximization problem.)

This is another version of the Euler equation. In the perhaps more familiar form $u'(C_1) / \beta u'(C_2) = 1 + A_2 F'(K_2)$ it says that the MRS = MRT, or the indifference curve is tangent to the PPF.

For the purposes of the graphs, we'll assume taxes are zero (to make room for labels)

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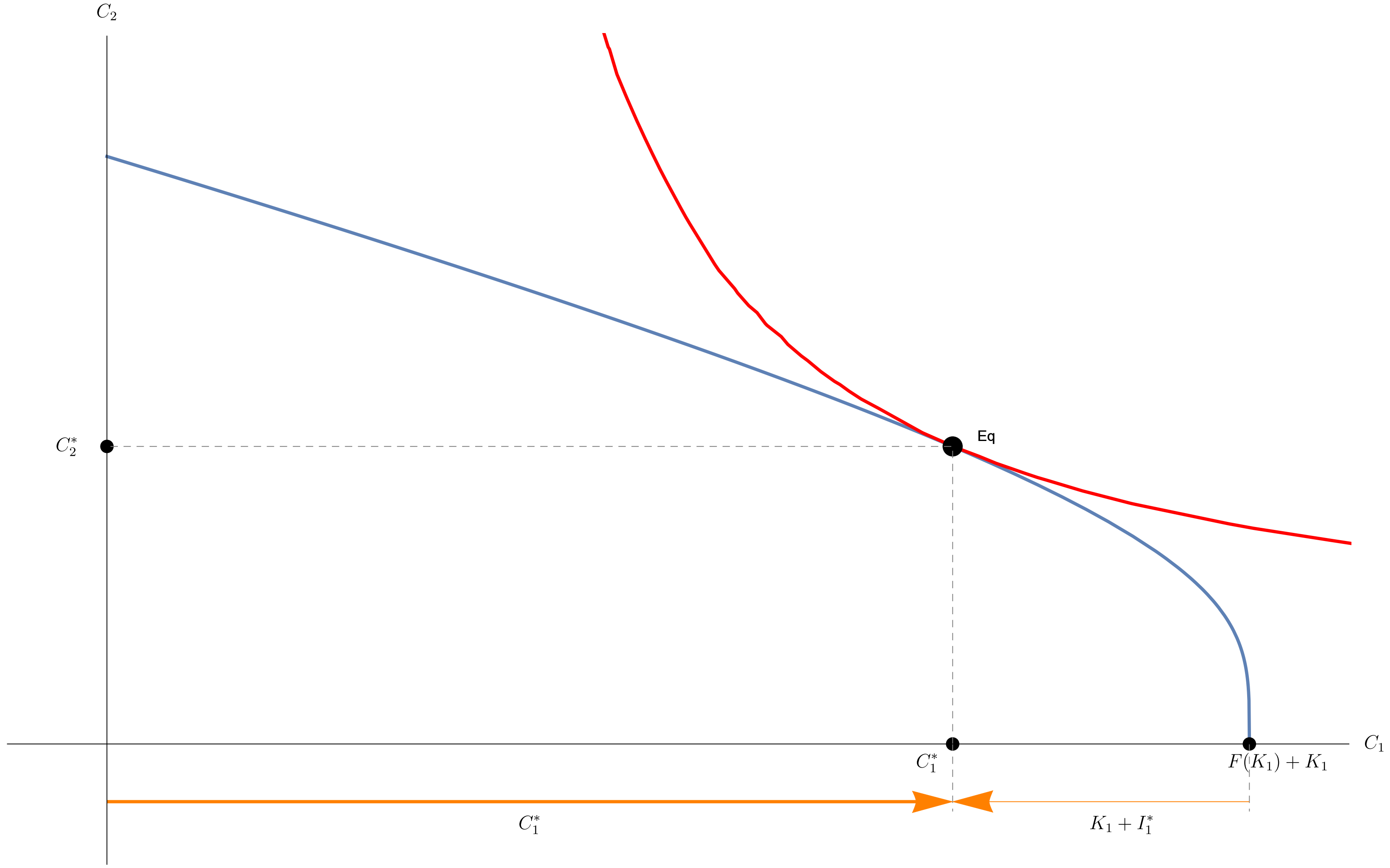

Closed Economy: Closed-form Solution

Getting a closed form solution would require a bit of work, since one would have to solve the following system of equations (this version does not impose δ = 0)

Out[ ]=

$$C_2 = A_2 F\left(A_1 F(K_1) - T_1 + (1-\delta) K_1 - C_1\right) - T_2 + (1-\delta)\big((1-\delta) K_1 + A_1 F(K_1) - T_1 - C_1\big)$$

$$u'(C_1) = \beta\left(1 + A_2 F'(K_2) - \delta\right) u'(C_2)$$

which could be very nonlinear.

There is one case, however, when the solution is pretty straightforward: log utility, Cobb-Douglas production function, and δ =1 (yes, that's 100% depreciation) . Do the math and, hopefully, you will get

and taking the first-order condition with respect to C_1 we get:

Out[ ]=

$$C_1 = \frac{1}{1+\alpha\beta} A_1 K_1^{\alpha}$$

Closed Economy: Productivity Growth

An interesting question is the following: Suppose future productivity (A_2) increases (what about current productivity?), what effect does this have on consumption and investment?
Turns out we can have two cases. Because C and I are tied in year 1 via $Y_1 - T_1 = C_1 + I_1$, if one goes up, the other must fall.
This is really too bad since you'd like both to occur. Why? think about it: higher A_2 means that marginal product of capital will be high and this is a good time to invest, i.e. you'd like to increase investment I_1 but at the same time higher A_2 means future output will be higher (even more so if you do respond and boost investment!) so consumption smoothing tells you, to increase C_1. Unfortunately, in closed economy cannot do both!

Either C increases and I falls or I increases but C falls. Which one will be a better choice depends on preferences.

If the household cares a lot about smoothing of consumption, C will rise and I will fall, as illustrated in the figure below. Notice that the distance from the horizontal intercept of the PPF (given by $F(K_1) + K_1$) and C_1 is equal to $K_1 + I_1$but since $K_1$is constant (it's an exogenously given initial condition), the change in that distance is the change in investment $I_{1.}$ In the graph below, we can use this to see that C increases while I shrinks.

CASE 1: Consumption increases but investment falls.

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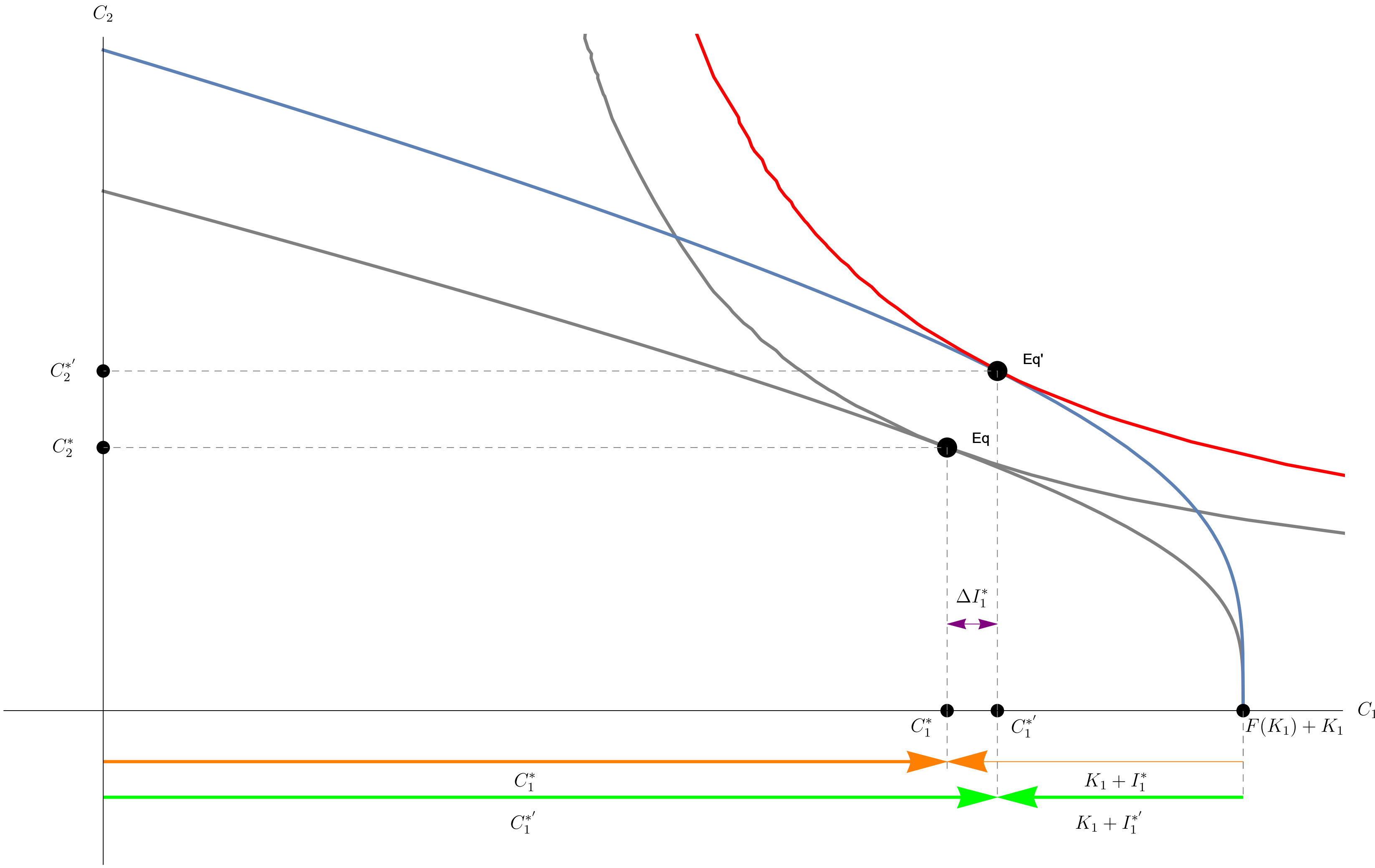
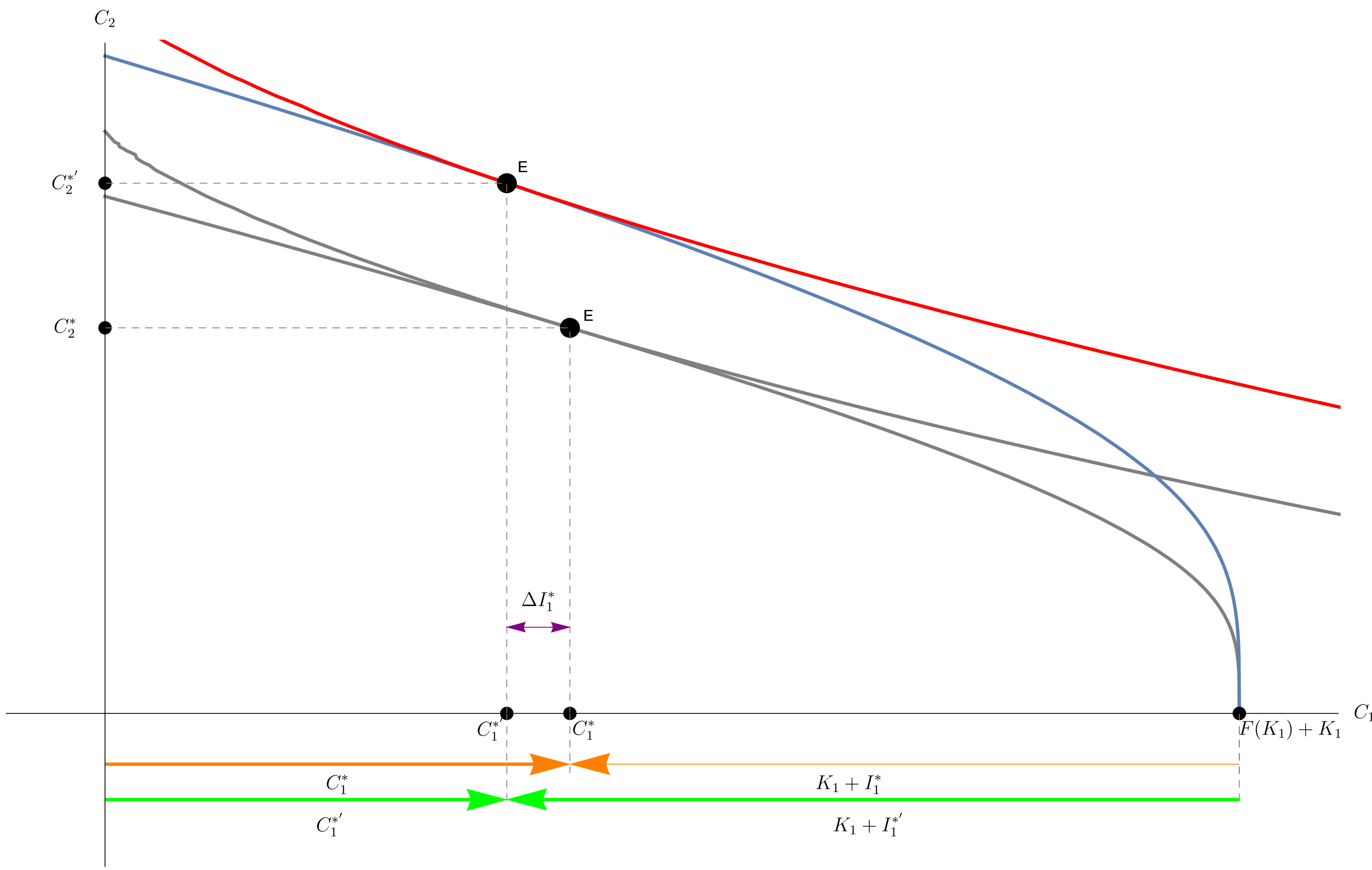

C_2
C_1
$C_2^{*\prime}$
C_2^*
Eq'
Eq
ΔI_1^*
C_1^*
$C_1^{*\prime}$
$F(K_1)+K_1$
C_1^*
$K_1+I_1^*$
$C_1^{*\prime}$
$K_1+I_1^{*\prime}$

If the household does not care much about smoothing of consumption, C will fall and I will rise, as illustrated in the figure below. Notice that the distance from the horizontal intercept of the PPF (given by $F(K_1) + K_1$) and C_1 is equal to $K_1 + I_1$ but since K_1 is constant (it's an exogenously given initial condition), the change in that distance is the change in investment I_1. In the graph below, we can use this to see that C falls while I increases.

CASE 2: Consumption falls but investment increase.

Out[ ]=


C_2
E
$C_2^{*\prime}$
E
C_2^*
ΔI_1^*
C_1^*
C_1^*
$F(K_1)+K_1$
C_1
C_1^*
$K_1+I_1^*$
$C_1^{*\prime}$
$K_1+I_1^{*\prime}$

Open Economy: Budget Constraints with Investment & Government

First, let’s assume that two assets are available: riskless bonds (B) and physical capital (K). Riskless bonds pay a fixed real interest rate r; this is again our small open economy assumption.
We’ll also add a government which taxes (T) and spends (G). Capital is owned domestically. Let the taxes be lump sum.

Let’s start by thinking about the budget constraints.

Recall that K_1 and B_1 are given.

$B_2 = (1+r) B_1 + Y_1 - T_1 - C_1 - I_1$

$B_3 = (1+r) B_3 + Y_2 - T_2 - C_2 - I_2$

Now let’s combine these assuming $B_1 = 0$ and recalling that B_3 must be 0. We get:

$$Y_1 - T_1 + \frac{Y_2 - T_2}{1+r} = C_1 + I_1 + \frac{C_2 + I_2}{1+r}$$

In addition capital evolves according to

$K_2 = (1-\delta) K_1 + I_1$

$K_3 = (1-\delta) K_2 + I_2$

where δ is the depreciation rate. If we assume that it is 0 (for simplicity), and recalling that capital can be costlessly converted to output we have to conclude that $K_3 = 0$ and so

$K_2 = K_1 + I_1$

$I_2 = -K_2$

Together this gives (assuming, as before $B_1 = 0$).

$$(1+r)(Y_1 - T_1 - (C_1 + I_1)) + (Y_2 - T_2) = C_2 + I_2$$
$$(1+r)(Y_1 - T_1 - (C_1 + I_1)) + (Y_2 - T_2) = C_2 - K_2$$
$$C_2 = (1+r)(Y_1 - T_1 - (C_1 + I_1)) + (Y_2 - T_2) + K_1 + I_1$$

$$C_2 = (1 + r)\ (Y_1 - T_1 - (C_1 + I_1)) + A_2\, F\ (K_1 + I_1) - T_2 + K_1 + I_1$$

The general statement of the problem is:

$$\max_{\{C_1,C_2,I_1\}} u(C_1) + \beta u(C_2)$$
$$\text{subject to}$$
$$B_{t+1} = (1+r) B_t + Y_t - C_t - I_t \text{ for } t = 1, 2$$
$$K_{t+1} = (1-\delta) K_t + I_t \text{ for } t = 1, 2$$

But with the above transformations we can express it as:

$$\max_{\{C_1,C_2,I_1\}} u(C_1) + \beta u(C_2)$$
$$\text{subject to}$$
$$C_2 = (1+r)(Y_1 - T_1 - (C_1 + I_1)) + A_2 F(K_1 + I_1) - T_2 + K_1 + I_1$$

Which can be solved by substituting the constraint into the objective function to get

$$\max_{\{C_1,I_1\}} u(C_1) + \beta u[(1+r)(Y_1 - T_1 - (C_1 + I_1)) + A_2 F(K_1 + I_1) - T_2 + K_1 + I_1]$$

The first order conditions for this problem are:

$$u'(C_1) = \beta(1+r) u'(C_2)$$
$$\beta u'(C_2)(A_2 F'(K_2) - r) = 0$$

Or simply:

$$u'(C_1) = \beta(1+r) u'(C_2)$$
$$A_2 F'(K_2) = r$$

Investment and consumption decisions are separated. The FONC's say that to get to the optimum you need
1. Pick I_1 according to $A_2 F'(K_2) = r$. Your consumption choice is irrelevant here.
2. Pick consumption optimally just like in the economy without investment; That is max $u(C_1) + \beta u(C_2)$ subject to $Y_1 - T_1 + \frac{Y_2 - T_2}{1+r} = C_1 + I_1 + \frac{C_2 + I_2}{1+r}$, where Y_2, I_1 and I_2 follow from your choice in (1) so are given. Thus, it's just like exogenous income path we studied before!

Open Economy: Solution

How does the open economy equilibrium differ from that of the closed economy? There are two cases to consider: Either the world interest rate r (the slope of the life-time budget constraint) is higher or lower than the autarky interest rate at home (i.e. the marginal product of capital, which is the slope of the PPF at the closed economy equilibrium).

Case I. The world interest rate is higher. In this case, when the economy is closed, the equilibrium is the one marked “Closed”. When the economy opens up, its MPK (at “Closed”) is lower than r. This means investing in domestic capital is a worse option compared to international lending. The optimal choice is therefore to cut back on domestic investment and lend to foreign countries. The household has preferences such that it actually prefers to curtail C_1 in order to take advantage of the high world interest rate (thus it does not care much about smoothing; I used CES utility with $\sigma = 1.4$).

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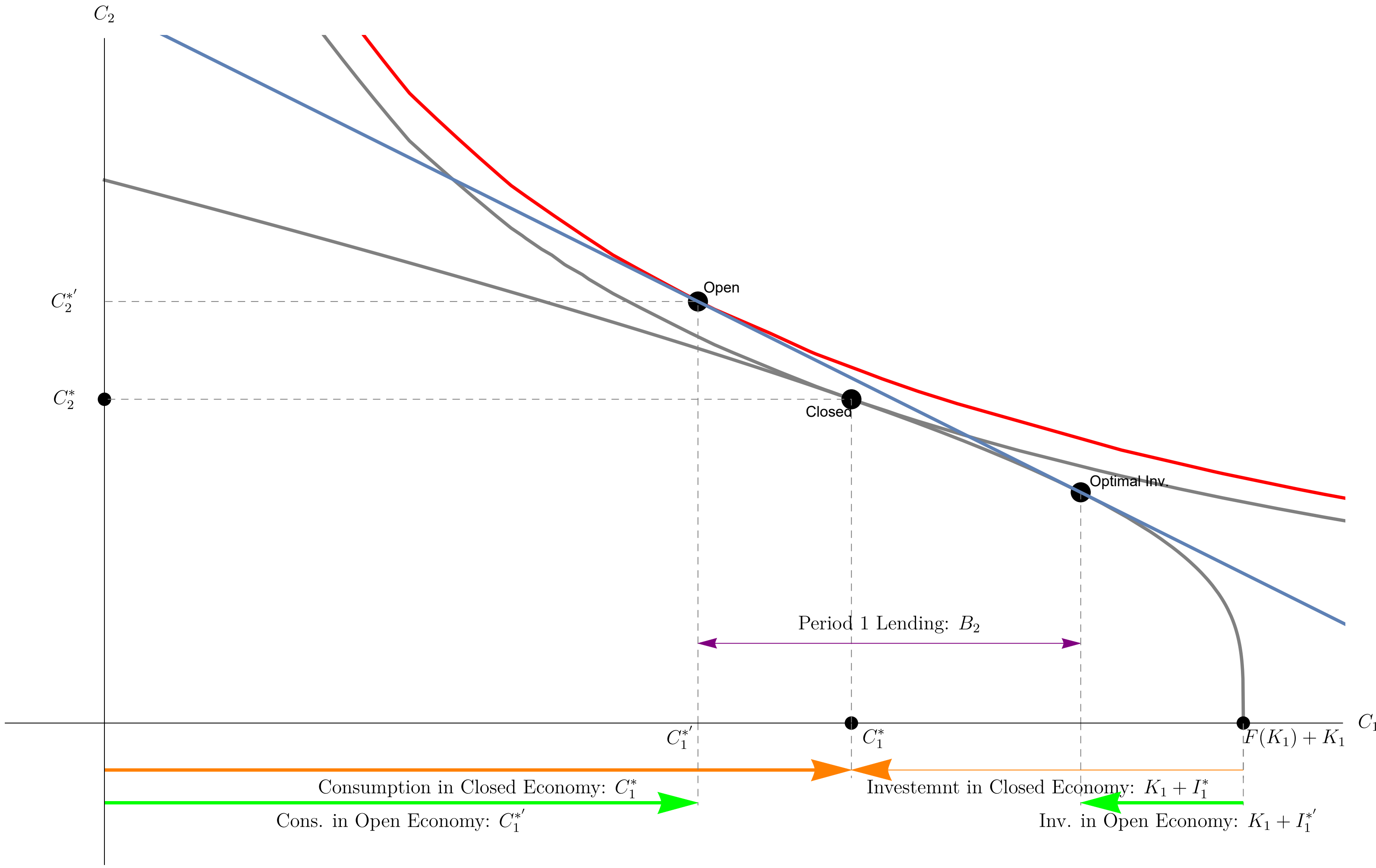
C_2
$C_2^{*'}$
C_2^*
Open
Closed
Optimal Inv.
Period 1 Lending: B_2
$C_1^{*'}$
C_1^*
$F(K_1)+K_1$
C_1
Consumption in Closed Economy: C_1^*
Investemnt in Closed Economy: $K_1+I_1^*$
Cons. in Open Economy: $C_1^{*'}$
Inv. in Open Economy: $K_1+I_1^{*'}$

Case II. The world interest rate is lower. In this case, when the economy is closed, the equilibrium is the one marked "Closed". When the economy opens up, its MPK (at "Closed") is higher than *r*. This means investing in domestic capital is a better option compared to international lending. The optimal choice is therefore to ramp up domestic investment by borrowing from foreign countries. The household has preferences are such that it actually prefers to additionally increase C_1 by borrowing at the low world interest rate.

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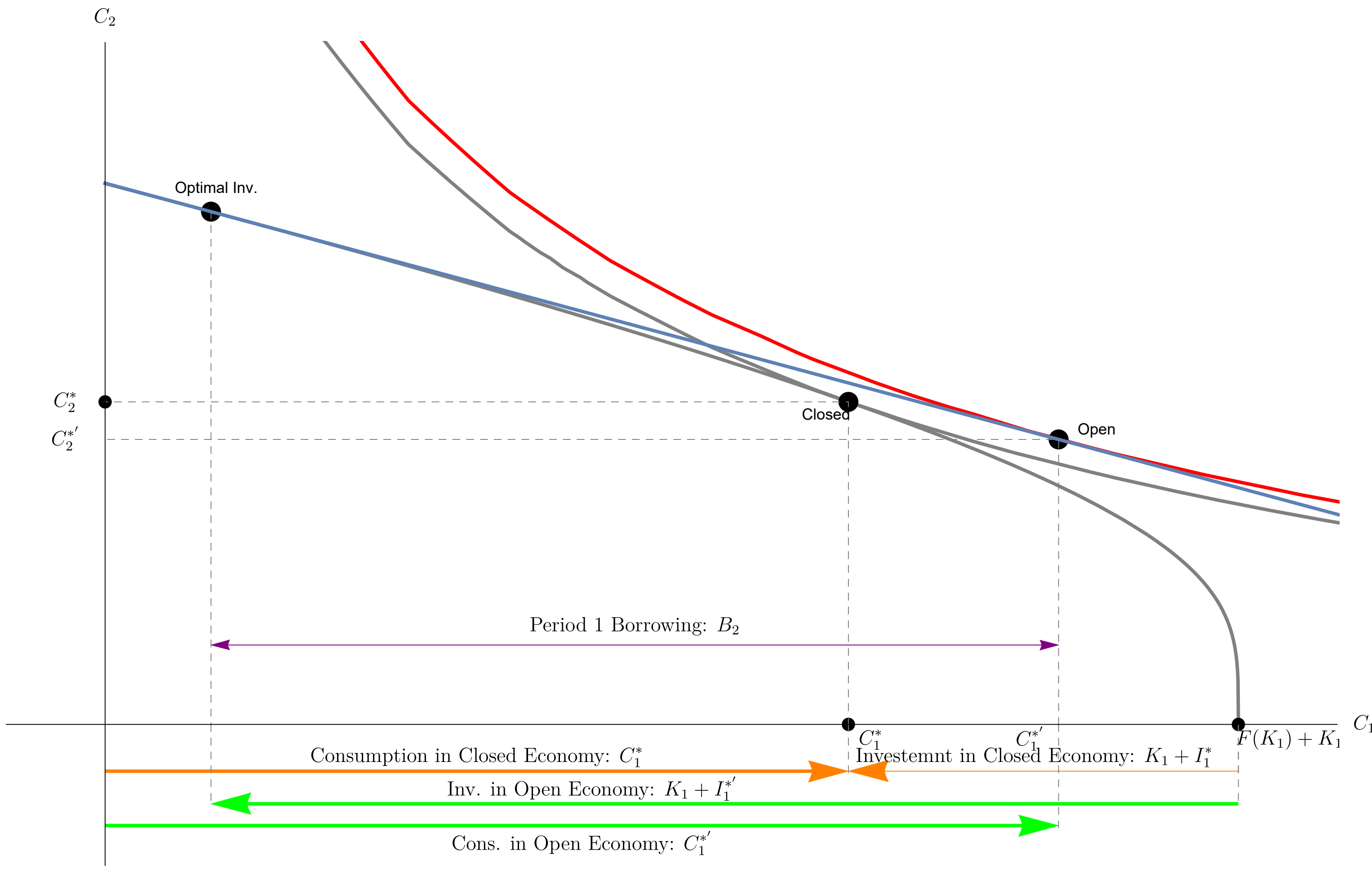

Open Economy: Productivity Growth

Let’s revisit the question of productivity growth: Suppose future productivity increases, what effect does this have on consumption and investment?

Recall that in the closed economy we had two cases. Because C and I were tied in year 1 via $Y_1 - T_1 = C_1 + I_1$, if one went up, the other had to fall.

Here things are different! Investment and consumption are separated! In response to an increase in A_2 , we have both consumption and investment go up!

Of course, that means current borrowing increases.

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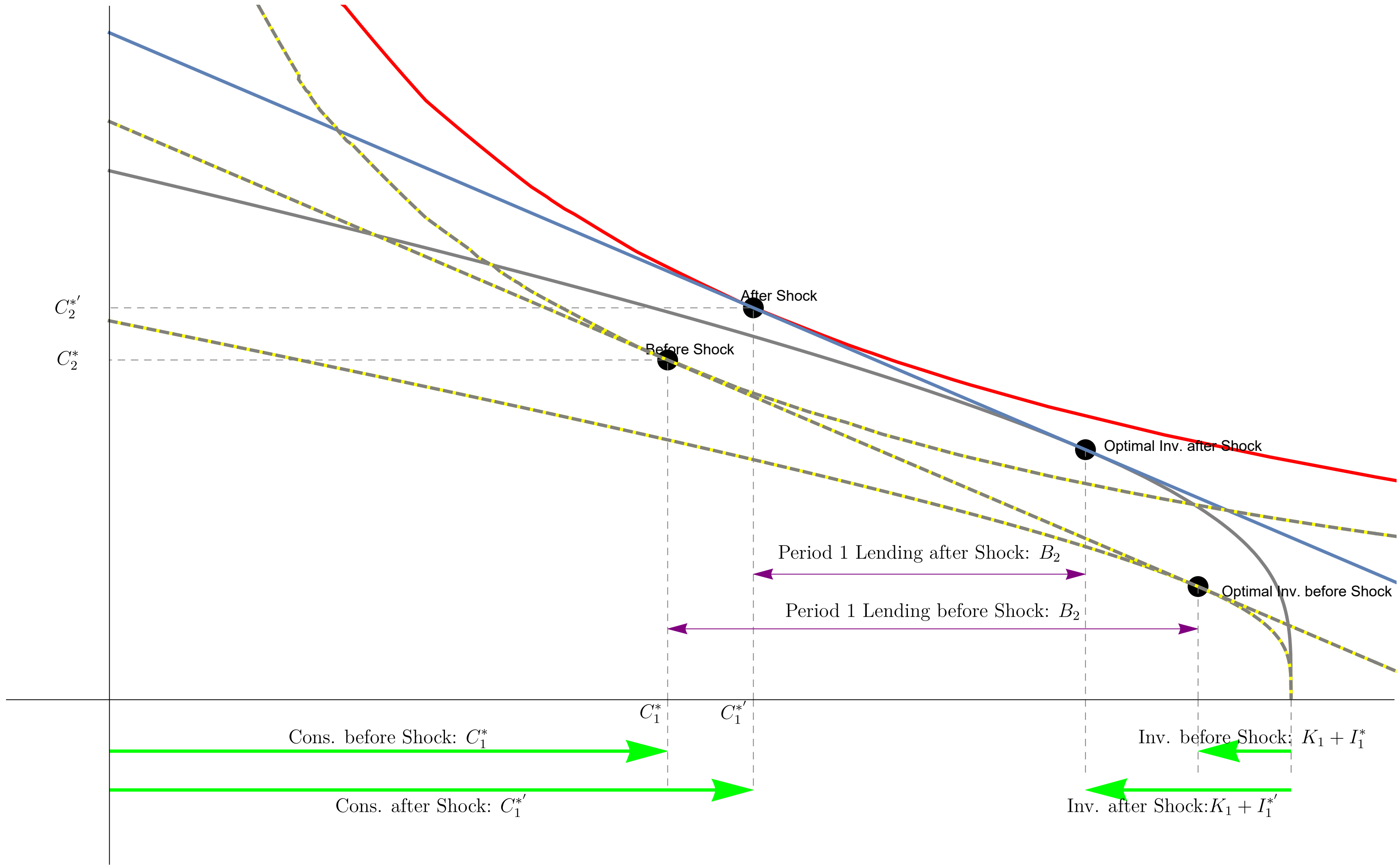
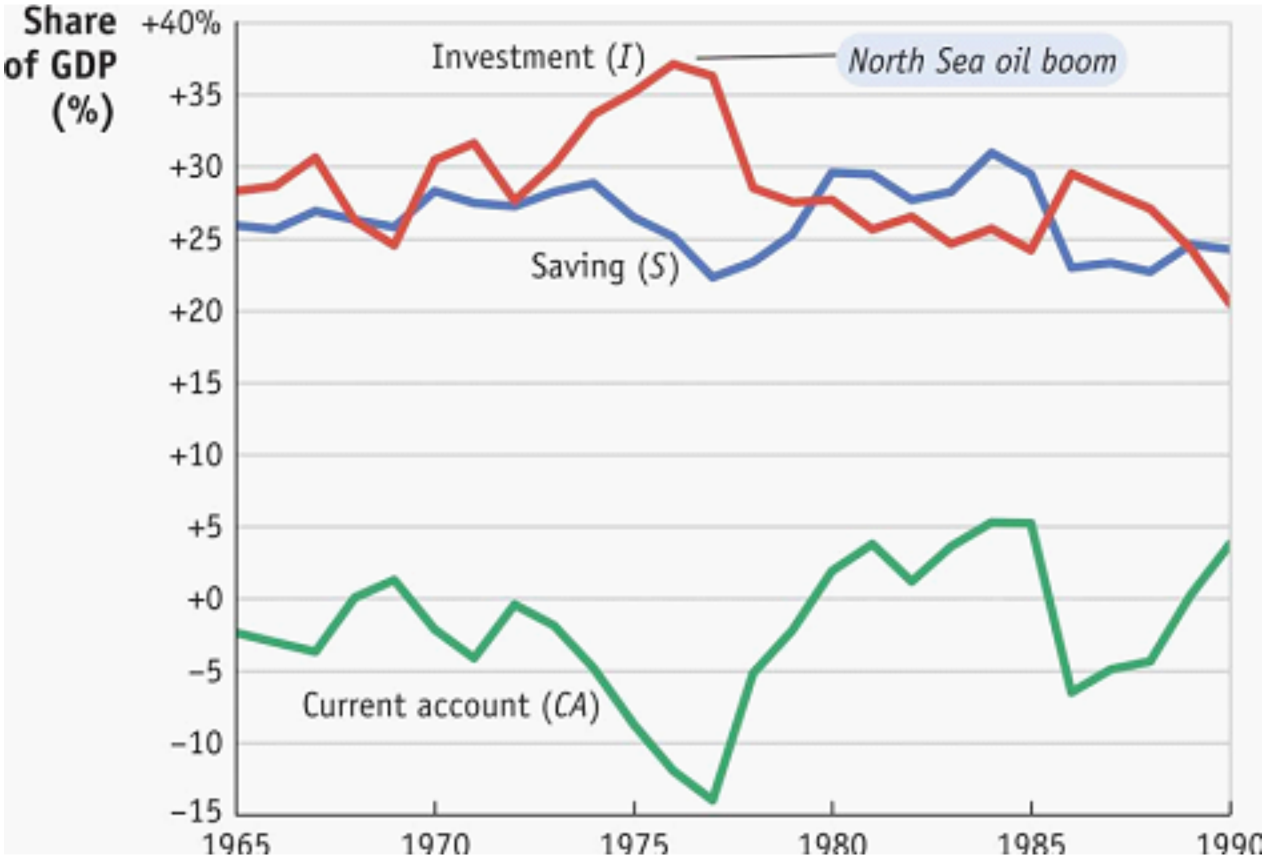


After Shock
Before Shock
Optimal Inv. after Shock
Optimal Inv. before Shock
$C_2^{*\prime}$
C_2^*
Period 1 Lending after Shock: B_2
Period 1 Lending before Shock: B_2
C_1^*
$C_1^{*\prime}$
Cons. before Shock: C_1^*
Cons. after Shock: $C_1^{*\prime}$
Inv. before Shock: $K_1 + I_1^*$
Inv. after Shock: $K_1 + I_1^{*\prime}$

An example: Norway

Norway discovers oil under the North Sea and this follows:



Following a large increase in oil prices in the early 1970s, Norway invested heavily to exploit oil fields in the North Sea. Norway took advantage of openness to finance a temporary increase in investment by running a very large current account deficit, thus increasing its indebtedness to the rest of the world. At its peak, the current account deficit was more than 10% of GDP.

Increase in Current Productivity Level

What would happen if current productivity (A_1) went up instead?

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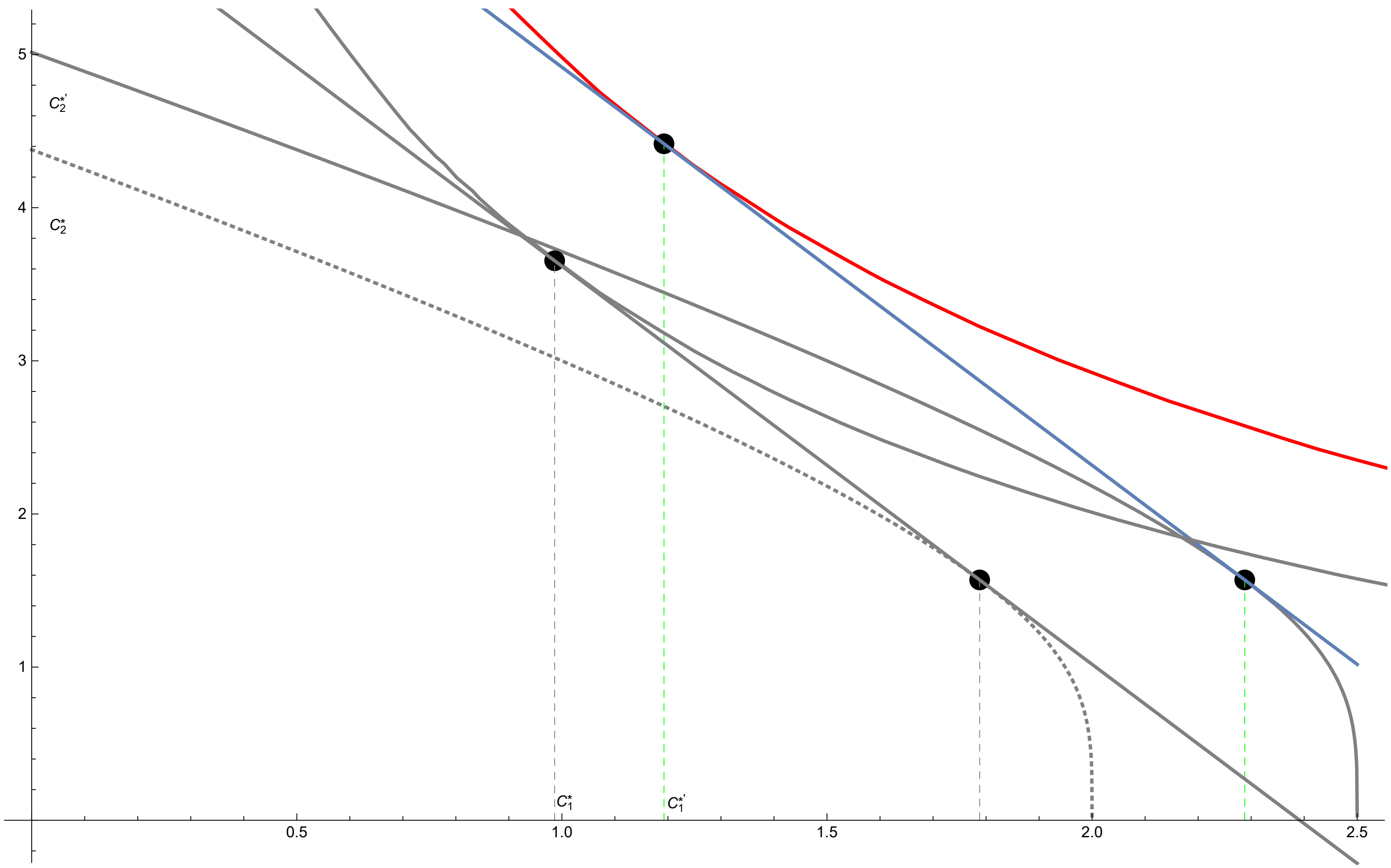

World Interest Rate Equilibrium with Investment

In the model with investment, we can again ask what will determine the equilibrium world interest rate.
Here the equilibrium condition will be

$$CA_1(r) + CA_1^*(r) = 0$$
$$S_1(r) - I_1(r) + S_1^*(r) - I_1^*(r) = 0$$

Q: what will dS/dr look like now?
A: the same as before!

Start with the Euler equation $u'(C_1) = (1+r)\beta u'(C_2)$ and substitute for C_2 using the budget constraint (ignore taxes) to get

$$u'(C_1) = (1+r)\beta u[(1+r)(Y_1 - (C_1 + I_1)) + A_2 F(K_1 + I_1) + K_1 + I_1]$$

And totally differentiate wrt r and C_1.

$$u''(C_1)\, dC_1 =$$
$$\beta u'(C_2)\, dr - (1+r)^2 \beta u''(C_2)\, dC_1 + (1+r)\beta u''(C_2)(Y_1 - T_1 - (C_1 + I_1))\, dr + (1+r)\beta u''(C_2)\left(A_2 F'(K_2)\frac{\partial I_1}{\partial r} - r\frac{\partial I_1}{\partial r}\right)$$

The last part comes from the fact that now change sin r affect investment, which in turn affects the budget constraint. However, note that this term is zero at the opium!

$$\left(A_2 F'(K_2)\frac{\partial I_1}{\partial r} - r\frac{\partial I_1}{\partial r}\right) = (A_2 F'(K_2) - r)\frac{\partial I_1}{\partial r} = 0$$

So the equation becomes

$$(u''(C_1) + 1 + r)^2 \beta u''(C_2))\, dC_1 = (\beta u'(C_2) + (1+r)\beta u''(C_2)(Y_1 - (C_1 + I_1)))\, dr$$

Collecting terms and re-arranging

$$\frac{dC_1}{dr} = \frac{\beta u'(C_2) + (1+r)\beta u''(C_2)(Y_1 - (C_1 + I_1))}{u''(C_1) + (1+r)^2 \beta u''(C_2)}$$
$$= \frac{\beta u'(C_2)/u'(C_1) + (1+r)\beta u''(C_2)/u'(C_1)(Y_1 - (C_1 + I_1))}{u''(C_1)/u'(C_1) + (1+r)^2 \beta u''(C_2)/u'(C_1)}$$
$$= \frac{1/(1+r) + (1+r)\frac{C_2 u''(C_2)\beta u'(C_2)}{C_2 u'(C_1) u'(C_2)}(Y_1 - (C_1 + I_1))}{\frac{1}{C_1}\frac{C_1 u''(C_1)}{u'(C_1)} + -(1+r)^2 \frac{C_2 u''(C_2)\beta u'(C_2)}{C_2 u'(C_1) u'(C_2)}}$$

Defining, as before, σ to be the intertemporal elasticity of substitution (or inverse of the coefficient of relative risk aversion)

$$\sigma(C) = -\frac{u'(C)}{C\,u''(C)}$$

$$\frac{dC_1}{dr} = \frac{1/(1+r) - \frac{1}{\sigma(C_2)\,C_2}(Y_1 - (C_1 + I_1))}{\frac{1}{C_1}\frac{1}{\sigma(C_1)} - (1+r)\frac{1}{\sigma(C_2)\,C_2}}$$

Assuming σ is constant and multiplying by $C_2\,\sigma$

$$\frac{dC_1}{dr} = \frac{(Y_1 - C_1 - I_1) - \sigma C_2/(1+r)}{1 + r + \frac{C_2}{C_1}}$$

World Interest Rate Equilibrium with Investment

World Interest Rate Equilibrium : world saving = world investment

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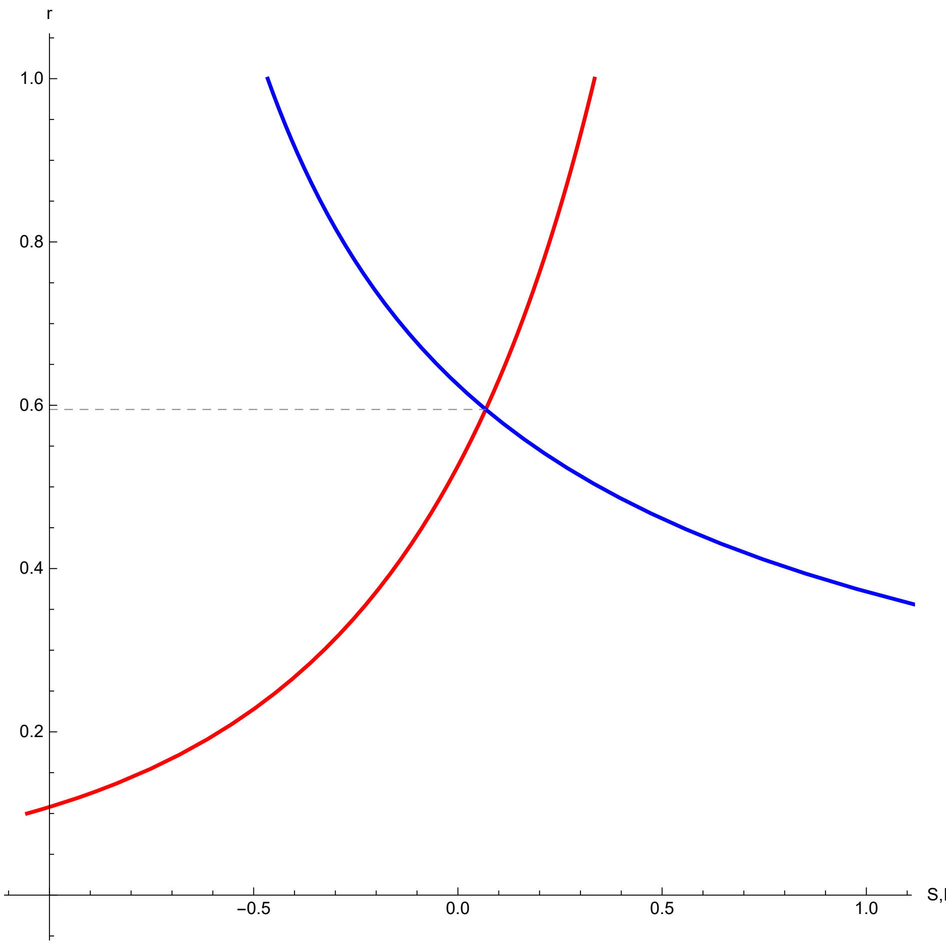


Barro and Sala-i-Martin (1990)

So here is an interesting question related to R. Barro and X. Sala-i-Martin paper.
During the 1980's the world real interest (r) was unusually high. BSM want to explain this.

Figure 3 WORLD ACTUAL AND EXPECTED REAL INTEREST RATES

In order to do that they will run regressions of the form:

$$r_t^e = X_t \beta + \epsilon_t$$

Where r_t^e is the expected real interest rate.

The first question they must confront is how to measure the world real interest rate. They choose a weighted average of 10 largest OECD economies' short term interest rates. Is short term rate what we mean by *r*? Not really but they claim that rates of different maturities move together.

The second problem is that that they only have data on the nominal interest rates (i_t), where (using "e" to denote "expected") and what they really want is the expected real interest rate. The Fisher identity tells us that

$$r_t^e = i_t - \pi_t^e$$

where π_t^e is the expected inflation rate.

They could just use realized ("ex-post") real rates given by:

$$r_t = i_t - \pi_t$$

where π_t is the actual (realized) inflation.

Combining the last two equations:

$$r_t^e = r_t - (\pi_t^e - \pi_t)$$

where the last term $(\pi_t^e - \pi_t)$ is the "inflation forecast error".

So if the use ex-post rates in place of expected they end up with

$$r_t^e = X_t \beta + \epsilon_t$$

$$r_t - (\pi_t^e - \pi_t) = X_t \beta + \epsilon_t$$

$$r_t = X_t \beta + \epsilon_t + (\pi_t^e - \pi_t) = X_t \beta + \mu_t$$

The problem with this regression is that even if $\in_t$ is uncorrelated with X_t, μ_t may be. The key question is whether:

$$E_t[X_t\ (\pi_t^e - \pi_t)] = 0$$

If this is true, then using ex-post rates is OK.
Turns out this condition is true under Rational Expectations. But they are not willing to just use the ex-post real interest rates. assume Instead they use a different way of getting r_t^e. They run same autoregressive (ARMA(1,1)) regressions to come up with a good forecast of inflation π_t^e and use $r_t^e = i_t - \pi_t^e$, to compute expected rates. This is also consistent with rational expectations if past inflation is the only variable you need to know to forecast future inflation...
[ADD: notes on distributed lag models?]

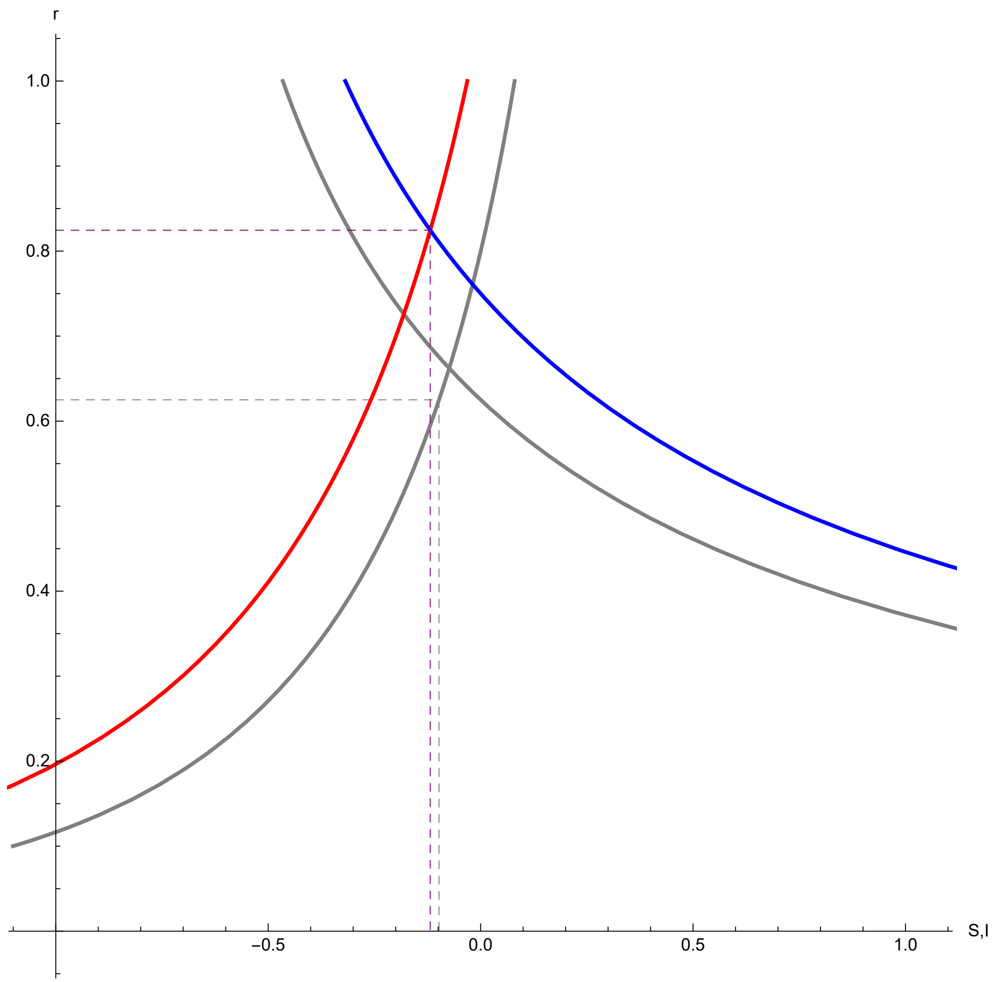
The next problem is that the interest rate is determined by supply and demand. When we run the regression of price P on quantity Q, are you estimating the demand or the supply curve?’’

You could do an IV or estimate a system of equations (under some conditions the two are the same). That is what they do, more or less...

We used the structural model to determine a reduced form for the "world" expected real interest rate and ratio of investment to GDP. The main predictions are that more favorable stock returns raise the real interest rate and investment, higher oil prices increase the real interest rate but decrease investment, higher monetary growth lowers the real interest rate and stimulates investment, and greater fiscal expansion raises the real interest rate and reduces investment.

So they concluded that part of the story is higher anticipated productivity growth.
But what does the theory say about that?
Suppose future productivity A_2 is expected to increase. What will happen to r and investment?
Well, we know that the investment demand curve will shift up (look back at the FONC if you need convincing). That by itself would push r and I up. But at the same time the saving schedule will shift to the left as well (consumption smoothing). This will work to push r further up and it will also work to push equilibrium investment down.
The result is that r unambiguously goes up but I may go up or down. Here is a case for log utility where I actually falls.

Out[296]=



Now Barro and Sala-i-Martin thought r went up because of high anticipated A2 (measured by stock market returns). They also pointed to higher levels of investment as evidence that demand for I went up. So, when would that happen?
The answer is: it will occur as long as the shift in the saving supply is smaller relative to that in I demand. But that takes a large value of intertemporal elasticity of substitution. Here is an example with $\sigma = 3$.
But most studies find $\sigma < 0.5$!

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