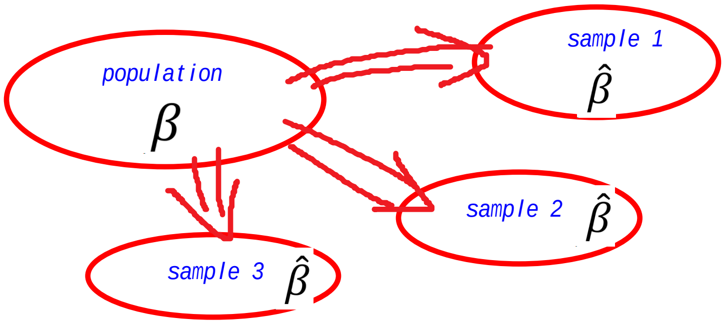
Bootstrapping



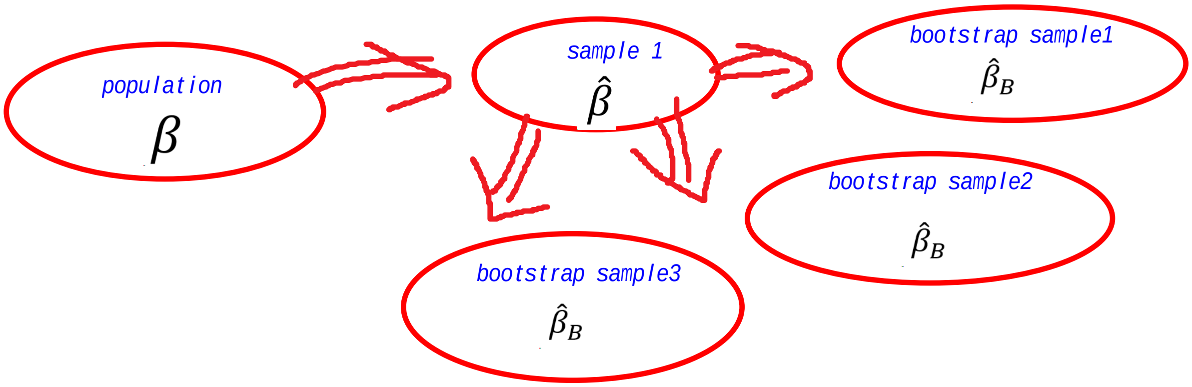
every time you draw a sample from population, you'll get a beta hat. And if you draw many times you'll get s.d of beta hat. But in fact, in real world you cannot draw many times.

then you find that you have learn the equation of variance (standard error of estimator) of beta hat

$\sigma^2(X'X)^{-1}$

However, this is for OLS. what if you have heteroskadasticity? what if you have a very complicated model? In real research, it is hard for us to calculate standard error.

Then we need bootstrapping. we consider sample 1 is representative for population. then we can draw samples from our samples



We can draw as many as we want from sample 1. then, we'll have many bootstrap samples, and many $\hat{\beta}_B$ *then we can calculate the standard error of* $\hat{\beta}_B$

when we draw bootstrap from sample 1, we should draw with replace. we assume sample 1 is the population,
if ten numbers in sample 1: 1,3,2,5,4,2,4,5,5,7, there are 3 fives in sample 1, it means, 30% of the population
are 5. Draw with replacement will sustain this probability distribution.
So, if we do not draw with replacement, the distribution will be distorted.!!! important!!

we draw many times from sample 1. every time we draw 10 numbers from sample 1, we calculate the mean of it. After many draws, we have many bootstrap samples, and their means. then we can compute standard deviation(standard error) of the means.!!!

SAME THING WHEN WE COMPUTE ESTIMATORS BETA.
every time we get a estimator beta, then, when we use these betas to compute std error of beta

Notice:
evert time, we get a beta, and we use standard deviation formula to compute standard error, rather use

OLS general equation: $\sigma^2(X'X)^{-1}$

std error of sample mean = std dev of the sample / square root(sample size)
SEM

$$SEM = \frac{SD}{\sqrt{n}}.$$

if you have this formula, you do not need to bootstrap any more.

Consistency of Sample Mean

$$
\begin{aligned}
Var[\bar{x}] &= Var\left[\frac{1}{n}\sum_{i=1}^{n} x_i\right] \\
&= \sum_{i=1}^{n} Var\left[\frac{x_i}{n}\right] \\
&= \sum_{i=1}^{n} \frac{\sigma^2}{n^2} \\
&= \frac{\sigma^2}{n}
\end{aligned}
$$

Bootstrapping cannot solve endogenous problem. you should use IV