Robust Standard Errors

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Recall the Variance of OLS estimator

$$
\begin{aligned}
E[(\hat{\beta}-\beta)(\hat{\beta}-\beta)'|X] &= E[((X'X)^{-1}X'\epsilon)((X'X)^{-1}X'\epsilon)'|X] \\
&= E[(X'X)^{-1}X'\epsilon\epsilon'X(X'X)^{-1}] \\
&= (X'X)^{-1}X'E[\epsilon\epsilon'|X]X(X'X)^{-1} \\
&= (X'X)^{-1}X'\sigma^2 IX(X'X)^{-1} \\
&= \sigma^2 I(X'X)^{-1}X'X(X'X)^{-1} \\
&= \sigma^2(X'X)^{-1}
\end{aligned}
$$

Assume Heteroskadasticity

If we assume heteroskadasticity, we have:

$$\cancel{E[\epsilon\epsilon|X]} = \sigma^2\Omega \tag{1}$$

$$E[\epsilon\epsilon'|X]$$

Therefore,

$$\begin{aligned}
E[(\hat{\beta}-\beta)(\hat{\beta}-\beta)'|X] &= E[((X'X)^{-1}X'\epsilon)((X'X)^{-1}X'\epsilon)'|X] \\
&= E[(X'X)^{-1}X'\epsilon\epsilon'X(X'X)^{-1}] \\
&= (X'X)^{-1}X'E[\epsilon\epsilon'|X]X(X'X)^{-1} \\
&= (X'X)^{-1}X'\sigma^2\Omega X(X'X)^{-1}
\end{aligned}$$

Robust Standard Errors

We can directly construct an estimator for

$$(X'X)^{-1}X'E[\epsilon\epsilon'|X]X(X'X)^{-1} \tag{2}$$

And using the sample analog of $E[\epsilon\epsilon'|X]$

$$\hat{Var}(\hat{\beta}) = (X'X)^{-1}X' \begin{bmatrix} \hat{\epsilon}_1^2 & 0 & \dots & 0 \\ 0 & \hat{\epsilon}_2^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \hat{\epsilon}_n^2 \end{bmatrix} X(X'X)^{-1} \tag{3}$$

Clustered Standard Errors

- The same logic can be applied to clustered standard error.
- Sometimes observations are related to each other but the relationship only happens within groups
- In this case we do not have homoskadasticity, and we need cannot use $\sigma^2(X'X)^{-1}$

Clustered Standard Errors

Similar to previous case, we directly construct an estimator for

$$(X'X)^{-1}X'E[\epsilon\epsilon'|X]X(X'X)^{-1} \tag{4}$$

Applying the assumption of no intergroup correlation,

$$\begin{bmatrix} \hat{\epsilon}_{11}^2 & \hat{\epsilon}_{11}\hat{\epsilon}_{12} & \dots & \dots & 0 & \dots & \dots & 0 \\ \hat{\epsilon}_{12}\hat{\epsilon}_{11} & \hat{\epsilon}_{12}^2 & \dots & \dots & 0 & \dots & \dots & 0 \\ \vdots & \vdots & \ddots & \dots & 0 & \dots & \dots & 0 \\ \vdots & \vdots & \ddots & \hat{\epsilon}_{1n_1}^2 & 0 & \dots & \dots & 0 \\ 0 & 0 & \dots & 0 & \hat{\epsilon}_{21}^2 & \hat{\epsilon}_{21}\hat{\epsilon}_{22} & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \hat{\epsilon}_{22}\hat{\epsilon}_{21} & \hat{\epsilon}_{22}^2 & \dots & 0 \\ 0 & 0 & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \dots & \dots & \dots & \hat{\epsilon}_{Gn_G}^2 \end{bmatrix} \tag{5}$$