Chungsang Tom Lam¹

¹Department of Economics Clemson University

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OLS estimation

• Consider the regression

$$y = D\gamma + X\beta + \epsilon \tag{1}$$

- where D is a dummy variable and X are the control variables.
- γ is the variable we are interested in
- An example would be an estimation of gender gap, where y is In(wages), D is the gender dummy and X contains other factors which explains y.

• We can run the regression separately

$$y_{D=1} = X_{D=1}\beta_{D=1} + \epsilon_{D=1}$$

 $y_{D=0} = X_{D=0}\beta_{D=0} + \epsilon_{D=0}$

Or simply

$$y_1 = X_1 \beta_1 + \epsilon_1$$
$$y_0 = X_0 \beta_0 + \epsilon_0$$

• By running OLS regression separately, we can get $\hat{\beta}_1$ and $\hat{\beta}_0$ as the estimators of β_1 and β_0

Then we can examine the difference in the mean of y

$$mean(y_1) - mean(y_0)$$

- This is the average difference in y for individuals in group 1 and group 0.
- We can decompose this difference:

$$\begin{split} & \textit{mean}(y_1) - \textit{mean}(y_0) \\ &= \textit{mean}(X_1 \hat{\beta}_1 + \epsilon_1) - \textit{mean}(X_0 \hat{\beta}_0 + \epsilon_0) \\ &= \hat{\beta}_1 \textit{mean}(X_1) - \hat{\beta}_0 \textit{mean}(X_0) \\ &= \hat{\beta}_1 \textit{mean}(X_1) - \hat{\beta}_1 \textit{mean}(X_0) + \hat{\beta}_1 \textit{mean}(X_0) - \textit{mean}(X_0) \hat{\beta}_0 \\ &= \hat{\beta}_1 (\textit{mean}(X_1) - \textit{mean}(X_0)) + \textit{mean}(X_0) (\hat{\beta}_1 - \hat{\beta}_0) \end{split}$$

• We can write the decomposition in a variety of ways:

$$\begin{split} \hat{\beta}_{1} \textit{mean}(X_{1}) - \hat{\beta}_{0} \textit{mean}(X_{0}) \\ = & \frac{1}{2} \hat{\beta}_{1} \textit{mean}(X_{1}) + \frac{1}{2} \hat{\beta}_{1} \textit{mean}(X_{1}) - (\frac{1}{2} \hat{\beta}_{0} \textit{mean}(X_{0}) + \frac{1}{2} \hat{\beta}_{0} \textit{mean}(X_{0})) \\ = & (\frac{1}{2} \hat{\beta}_{1} + \frac{1}{2} \hat{\beta}_{0}) (\textit{mean}(X_{1}) - \textit{mean}(X_{0})) + (\hat{\beta}_{1} - \hat{\beta}_{0}) (\frac{1}{2} \textit{mean}(X_{1}) + \frac{1}{2} \hat{\beta}_{0}) (\frac{1}{2} \textit{mean}(X_{1})) + (\hat{\beta}_{1} - \hat{\beta}_{0}) (\frac{1}{2} \textit{mean}(X_{1}) + \frac{1}{2} \hat{\beta}_{0}) (\frac{1}{2} \textit{mean}(X_{1}) + \frac$$