Local Average Treatment Effect

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The Problem

- A device (Z) which can randomly assign people into treatments (X) is a specific case where we have the perfect scenario instrumental variable estimator.
- However, sometimes we cannot assign the subjects directly into treatment groups. Very often, what we have are some devices which can induce subjects to enter into different treatment groups.
- In this case, we can still use this device as an instrument, but we need to be more careful with the interpretation since the treatment effect is not necessarily the treatment effect of everyone.

Type of Compliance

We define the type of people in the sample by what the will do (X) given which group (Z) they are in.

Z = 0: no pizza for obs

		$Z = 0$	
		$X = 0$ not go	$X = 1$ go to TA session
Z = 1: give pizza $Z = 1$	$X = 0$	Never Taker(N)	Defier(D)
	$X = 1$	Complier(C)	Always Taker(A)

Notice that we only know the data (X and Z), but not the types.
Given the data, here are the possible types.

	$X = 0$	$X = 1$
$Z = 0$	Complier(C)/Never Taker(N)	Defier(D)/Always Taker(A)
$Z = 1$	Defier(D)/Never Taker(N)	Complier(C)/Always Taker(A)

Proportion of Compliance Types

- Let π_A, π_N, π_C, and π_D are the proportion of always taker, never taker, complier, and defiers respectively.
- With the no defier assumption, we know $\pi_D = 0$.
- For π_A, π_N, and π_C:

$$\pi_A = E[X|Z=0] \tag{1}$$

$$\pi_A + \pi_C = E[X|Z=1] \tag{2}$$

$$\pi_C = E[X|Z=1] - E[X|Z=0] \tag{3}$$

$$\pi_N = 1 - E[X|Z=1] \tag{4}$$

Expected Y

x = go to tutorial or not

- $Y(X)$ depends on X and the expected value of $Y(X)$ depends on Z. z = pizza or not
- With the proportions we can calculate the expected value of Y: y = exam score

probability to be a complier

$$E[Y(0)|Z=0] = \frac{\pi_C}{\pi_C+\pi_N}E[Y(0)|C] + \frac{\pi_N}{\pi_C+\pi_N}E[Y(0)|N] \quad (5)$$

i.e.
pai(A) = 0.2
pai(C) = 0.5
pai(N) = 0.3

$$E[Y(0)|Z=1] = E[Y(0)|N] \quad (6)$$

$$E[Y(1)|Z=0] = E[Y(1)|A] \quad (7)$$

$$E[Y(1)|Z=1] = \frac{\pi_C}{\pi_C+\pi_A}E[Y(1)|C] + \frac{\pi_A}{\pi_C+\pi_A}E[Y(1)|A] \quad (8)$$

Results

- $E[X|Z=0]$, $E[X|Z=1]$, $E[Y(0)|Z=0]$, $E[Y(0)|Z=1]$, $E[Y(1)|Z=0]$, $E[Y(1)|Z=1]$ can be estimate by the sample analog.
- For example, we can estimate $E[X|Z=0]$ by $\frac{1}{N_{Z=0}}\sum_{Z=0} x_i$.
- And we can estimate $E[Y(1)|Z=0]$ by $\frac{1}{N_{Z=0,X=1}}\sum_{Z=0,X=1} y_i$
- From equation (1) to (4), we can estimate the π_A, π_N, and π_C.
- From equation (5) to (8), we can estimate the $E[Y(0)|N]$, $E[Y(1)|A]$, $E[Y(0)|C]$, $E[Y(1)|C]$

in this method we always assume that there's no difier.
use given C result to convince your advisor, with two reason:
1. there're lots of complier in your data set.
the more you have ,
the more effect it is.
2. no difier

Results

Complier = give pizza --> go tutorial

subtraction because both are given C

- Therefore, we get an estimate of $E[Y(1) - Y(0)|C]$, which is as close as we can get to estimate $E[Y(1) - Y(0)]$.

  they are close but not equal as the upper equation estimates the expectation of compliers.
- This is the Local Average Treatment Effect, which is the treatment effect for the compliers.

  loacal average treatment effect = complier
- If we look at the difference between $E[Y|Z = 1]$ and $E[Y|Z = 0]$,

$$E[Y|Z = 1] - E[Y|Z = 0] = E[Y(1) - Y(0)|C]\pi_C$$

$$E[Y(1) - Y(0)|C] = \frac{E[Y|Z = 1] - E[Y|Z = 0]}{E[X|Z = 1] - E[X|Z = 0]}$$

substitute pai(C) into the upper EQ

- Which is the IV estimator when Z is a dummy variable. This is also known as the Wald estimator.

given a data set, you don't know if variables are IV or not. You run regression.
How to judge if it is a local average treatment effect or not(only work for complier or not):
depends on your data set: In your experiment, if you can induce your participants, then you should probably get a local....
Then you should argue why the local one also works.