linear_regression (1)

August 19, 2022

[1]:
```
import pandas as pd
%matplotlib inline
import matplotlib.pyplot as plt
import scipy.stats as stat
import time
```

[2]:
```
data = pd.read_csv("regr.txt", sep=" ", header=None, names=['l', 't'])
print(data.head())
print(data.tail())
```

```
      l        t
0  0.10  0.69004
1  0.11  0.69497
2  0.12  0.74252
3  0.13  0.75360
4  0.14  0.83568
       l       t
85  0.95  1.9841
86  0.96  2.0066
87  0.97  2.0493
88  0.98  2.0503
89  0.99  2.0214
```

[3]:
```
l = data['l'].values
t = data['t'].values
tsq = t * t
```

[4]:
```
data
```

[4]:

	l	t
0	0.10	0.69004
1	0.11	0.69497
2	0.12	0.74252
3	0.13	0.75360
4	0.14	0.83568
..	…	…
85	0.95	1.98410

```
86   0.96   2.00660
87   0.97   2.04930
88   0.98   2.05030
89   0.99   2.02140

[90 rows x 2 columns]
```

We quickly summarise the essenial parts of the Gradient Descent method:

y=mx+c

E = 1/n ni=1(yi−y)2

 E m = 2/n ni=1−xi(yi−(mxi+c))

 E c = 2/n ni=1−(yi−(mxi+c))

[5]:
```python
def train(x, y, m, c, eta):
    const = - 2.0/len(y)
    ycalc = m * x + c
    delta_m = const * sum(x * (y - ycalc))
    delta_c = const * sum(y - ycalc)
    m = m - delta_m * eta
    c = c - delta_c * eta
    error = sum((y - ycalc)**2)/len(y)
    return m, c, error

def train_on_all(x, y, m, c, eta, iterations=1000):
    for steps in range(iterations):
        m, c, err = train(x, y, m, c, eta)
    return m, c, err
```

TRAIN Let us visualize the training: = 0.01 Training for 1000 iterations, plotting after every 100 iterations:

[6]:
```python
# Init m, c
m, c = 0, 0
```

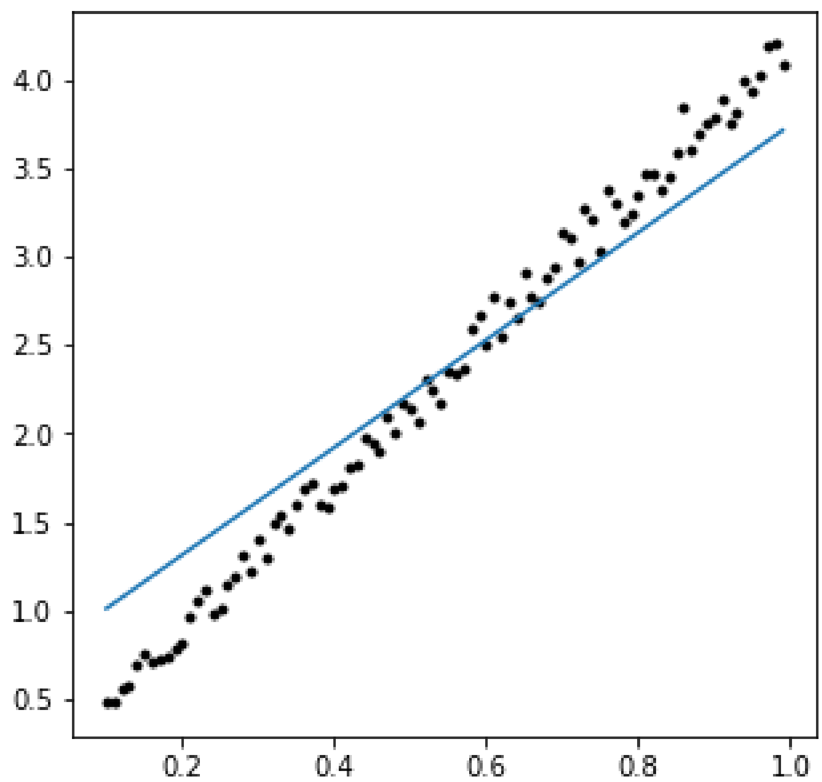
[7]:
```python
# Learning rate
lr = 0.01
```

[8]:
```python
# Training for 1000 iterations, plotting after every 100 iterations:
fig = plt.figure(figsize=(5, 5))
ax = fig.add_subplot(111)
plt.ion()
fig.show()
fig.canvas.draw()

for num in range(10):
    m, c, error = train_on_all(l, tsq, m, c, lr, iterations=100)
```

```
    print("m = {0:.6} c = {1:.6} Error = {2:.6}".format(m, c, error))
    y = m * l + c
    ax.clear()
    ax.plot(l, tsq, '.k')
    ax.plot(l, y)
    fig.canvas.draw()
    time.sleep(1)
```

```
m = 1.28677 c = 1.54235 Error = 0.567336
m = 1.62739 c = 1.50603 Error = 0.438432
m = 1.87741 c = 1.37352 Error = 0.358112
m = 2.099 c = 1.24699 Error = 0.292845
m = 2.29864 c = 1.13234 Error = 0.239713
m = 2.47876 c = 1.02887 Error = 0.196459
m = 2.64127 c = 0.9355 Error = 0.161247
m = 2.78789 c = 0.851259 Error = 0.132582
m = 2.92019 c = 0.775251 Error = 0.109246
m = 3.03955 c = 0.706672 Error = 0.0902493
```



Clearly this is not enough.

Let us train for a 1000 more iterations, with a greater learning rate:

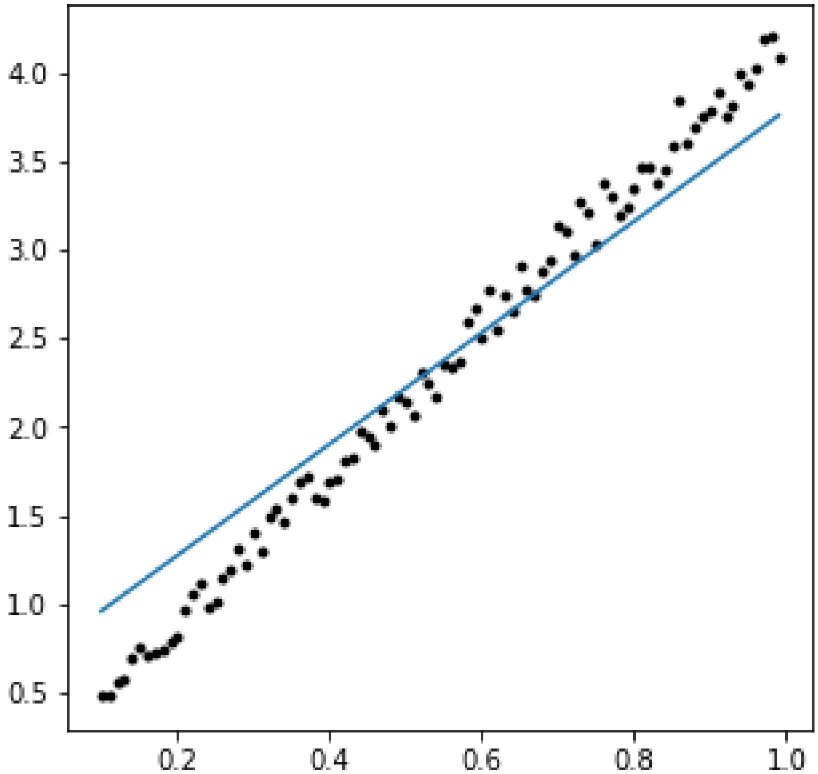
$= 0.01$

[9]:
```python
# Learning rate
lr = 0.01
```

[10]:
```python
# Training for 1000 iterations, plotting after every 100 iterations:
fig = plt.figure(figsize=(5, 5))
ax = fig.add_subplot(111)
plt.ion()
fig.show()
fig.canvas.draw()

for num in range(10):
    m, c, error = train_on_all(l, tsq, m, c, lr, iterations=100)
    print("m = {0:.6} c = {1:.6} Error = {2:.6}".format(m, c, error))
    y = m * l + c
    ax.clear()
    ax.plot(l, tsq, '.k')
    ax.plot(l, y)
    fig.canvas.draw()
    time.sleep(1)
```

```
m = 1.6271 c = 1.50581 Error = 0.438483
m = 1.87729 c = 1.37356 Error = 0.358145
m = 2.0989 c = 1.24705 Error = 0.292872
m = 2.29856 c = 1.13239 Error = 0.239735
m = 2.47868 c = 1.02891 Error = 0.196477
m = 2.6412 c = 0.935541 Error = 0.161262
m = 2.78783 c = 0.851296 Error = 0.132594
m = 2.92013 c = 0.775284 Error = 0.109256
m = 3.0395 c = 0.706702 Error = 0.0902572
m = 3.14721 c = 0.644823 Error = 0.0747905
```

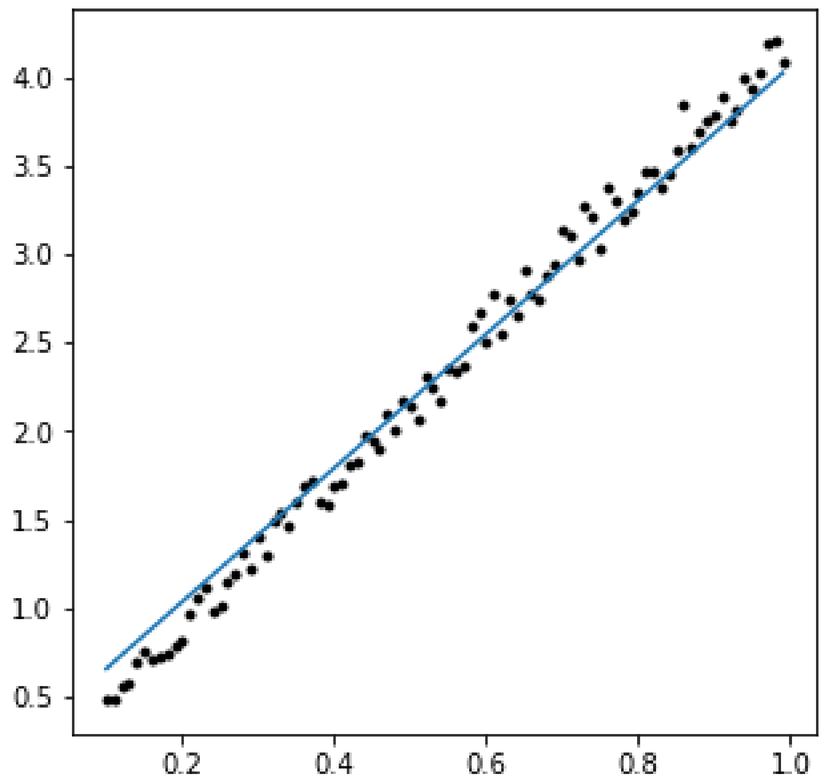


[11]:
```
# Training for 1000 iterations, plotting after every 100 iterations:
fig = plt.figure(figsize=(5, 5))
ax = fig.add_subplot(111)
plt.ion()
fig.show()
fig.canvas.draw()

for num in range(10):
    m, c, error = train_on_all(l, tsq, m, c, lr, iterations=100)
    print("m = {0:.6} c = {1:.6} Error = {2:.6}".format(m, c, error))
    y = m * l + c
    ax.clear()
    ax.plot(l, tsq, '.k')
    ax.plot(l, y)
    fig.canvas.draw()
    time.sleep(1)
```

```
m = 3.24438 c = 0.588992 Error = 0.0621994
m = 3.33206 c = 0.538617 Error = 0.0519493
m = 3.41117 c = 0.493166 Error = 0.0436049
m = 3.48255 c = 0.452157 Error = 0.0368119
m = 3.54695 c = 0.415156 Error = 0.0312818
```

```
m = 3.60506 c = 0.381772 Error = 0.0267799
m = 3.65749 c = 0.35165 Error = 0.023115
m = 3.70479 c = 0.324473 Error = 0.0201315
m = 3.74747 c = 0.299951 Error = 0.0177027
m = 3.78598 c = 0.277826 Error = 0.0157254
```



This seems correct.

Comparison with standard library Let us compare the values we found with the values found by the standard scipy library function ”stats”:

[12]:
```python
print("From our Gradient Descent   m = {0:.06} c = {1:.06}".format(m, c))

msp, csp, _, _, _ = stat.linregress(l,tsq)
print("From scipy.stats.linregress m = {0:.06} c = {1:.06}".format(msp, csp))
```

```
From our Gradient Descent   m = 3.78598 c = 0.277826
From scipy.stats.linregress m = 4.14148 c = 0.0735804
```

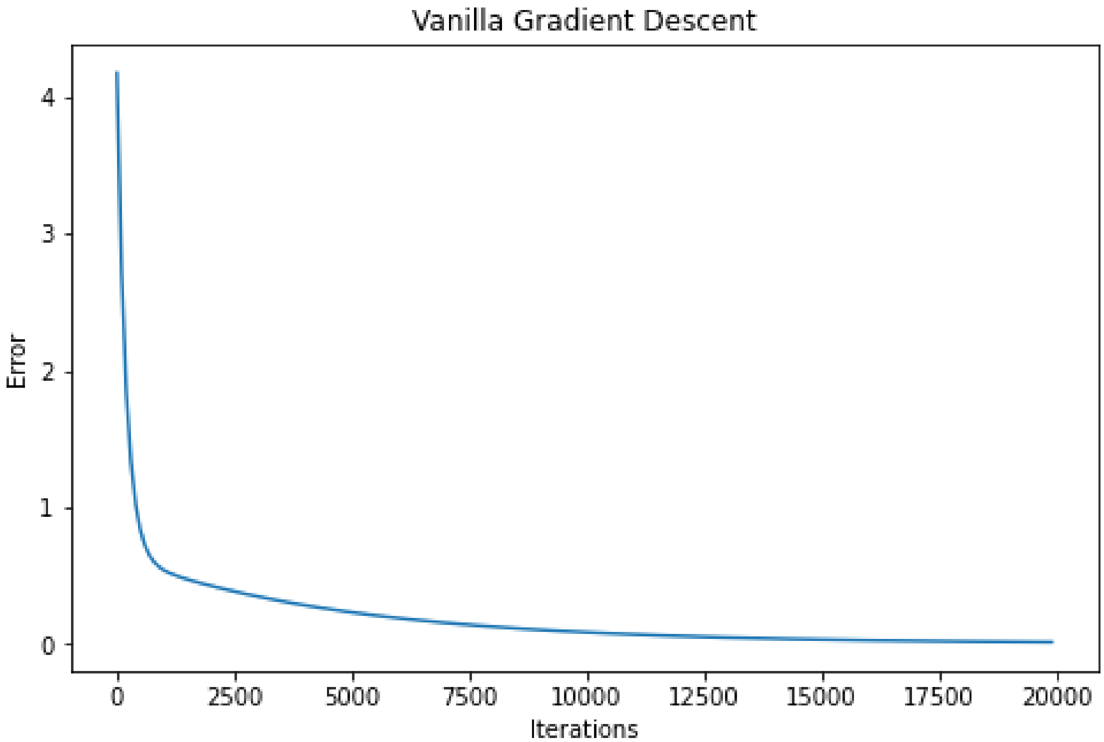
Close enough!

Plotting error vs iterations So far we have seen how the Gradient Descent works by looking at the fit of the regression line. Let us change perspectives and plot the error at various stages. This just shows that the process is converging and gives us a feel for the rate at which it is converging.

E=1n ni=1(yi−y)2

=1n ni=1(yi−mxi−c)2

[13]:
```
ms, cs,errs = [], [], []
m, c = 0, 0
eta = 0.001
for times in range(200):
    m, c, error = train_on_all(l, tsq, m, c, eta, iterations=100) # We will␣
 ↪plot the value of for every 100 iterations
    ms.append(m)
    cs.append(c)
    errs.append(error)
epochs = range(0, 20000,100)
plt.figure(figsize=(8,5))
plt.plot(epochs, errs)
plt.xlabel("Iterations")
plt.ylabel("Error")
plt.title("Vanilla Gradient Descent")
plt.show()
```



We see that the error at saturation is around 0.01.

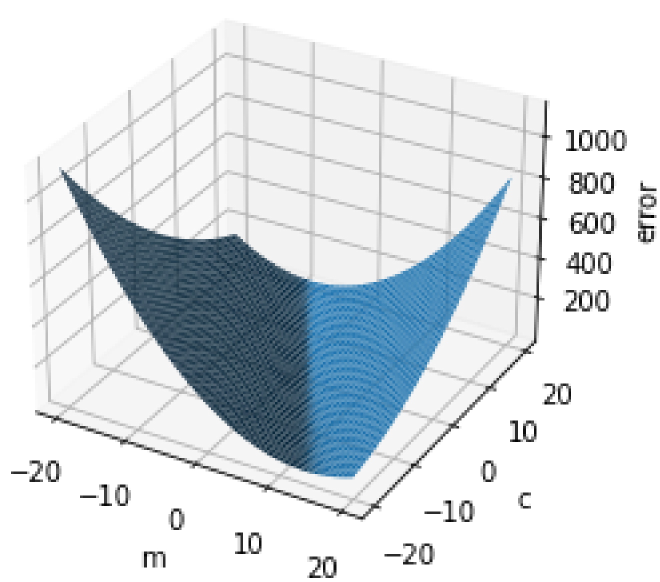
Error vs m, c

Let us visualize the error as a function of m and c

[14]:
```
def error(x,y,m,c):
    ycalc = m * x + c
    error = sum((y - ycalc)**2) / len(y)
    return error
```

[15]:
```
from mpl_toolkits.mplot3d import Axes3D
import numpy as np
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ms1 = np.arange(-20, 20, 0.1)
cs1 = np.arange(-20, 20, 0.1)
X, Y = np.meshgrid(ms1, cs1)
err = []
for i in range(len(ms1)):
    for j in range(len(cs1)):
        err.append(error(l,tsq,ms1[i],cs1[j]))
err = np.array(err)
Z = np.reshape(err,(-1,len(ms1)))
print(X.shape, Y.shape, Z.shape)
ax.plot_surface(X, Y, Z)
ax.set_xlabel('m')
ax.set_ylabel('c')
ax.set_zlabel('error')
plt.show()
```

```
(400, 400) (400, 400) (400, 400)
```



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