Modeling COVID-19 Spatio-temporal Dynamics

So far we have modeled individual countries, in this project we will put these models together into a model for the whole world. We will use a Network model, where countries are tied together by a graph which models international travel.

We first extend our model to the whole world by introducing variables $S_i, E_i, I_i, R_i, C_i, D_i$ where $i = 1, 2, ..., N$ and N is the number of countries we have data for. First, modify your code to run all countries simultaneously with the same model as before:

$$\begin{aligned}
\dot{S}_i &= -\beta_i S_i I_i/(S_i + E_i + I_i + R_i) \\
\dot{E}_i &= \beta_i S_i I_i/(S_i + E_i + I_i + R_i) - \sigma E_i \\
\dot{I}_i &= \sigma E_i - \gamma I_i - \gamma\tau I_i - \gamma\mu I_i \\
\dot{R}_i &= \gamma I_i \\
\dot{C}_i &= \gamma\tau_i I_i \\
\dot{D}_i &= \gamma\mu_i I_i
\end{aligned}$$

We will assume that σ, γ are the same for every country, but the reporting rate τ_i and mortality rate μ_i may be different for each country and also the infection rate β_i may be different for each country.

Problem 1: Download `loadfulldata.m`, `loadLocations.m`, `locations.csv`, `fulldata.csv` and run the new `loadfulldata.m` (this may take a few minutes to run). Use the command `plot(AllCases');` to plot all the cases for every country on one plot. Use the command `plot(AllDeaths');` to plot all the cases for every country on one plot.

You will now have the variables:

- `N` which is the number of countries with data available.
- `T` which is the number of dates.
- `P` which is an $N \times 1$ vector containing the populations for each country.
- `Contintent` which is an $N \times 1$ vector containing either $\{1, 2, ..., 7\}$ for each country.
- `AllCases` which is an $N \times T$ matrix containing the number of cases for each country over time.
- `AllDeaths` which is an $N \times T$ matrix containing the number of deaths for each country over time.

Problem 2: Create a new version of your original `SEIRCD.m` code and call it `SEIRCDworld.m` and modify your `ydot` function by adding a for-loop that runs computes the SEIRCD differential equation for each country. You will also need to modify your initial condition `ic` to have length $6 * N$ since there are 6 initial conditions for each country, the first initial condition for each country should be the population found in P. For now, just use the same parameter values for each country.

Next we need to connect the countries with travel, since different countries have different relationships, we will use a matrix to represent the connections between countries. We will define an $N \times N$ matrix `M` where M_{ij} will represent the level of travel between country i and country j. A simple model for travel is the *gravity model*:

$$M_{ij} = \theta_1 \frac{P_i P_j}{1 + \theta_2 C_{ij}} \tag{1}$$

where P_i and P_j are the populations of the two countries and $C_{ij} = 0$ if the two countries are on the same continent and $C_{ij} = 1$ if the two countries are on different continents.

Problem 3: First build the C matrix by initializing `C = zeros(N,N);` and writing a double for-loop and if `Continent(i) ~= Continent(j)` then setting `C(i,j) = 1;` Then set `theta1 = 1/sum(P)^2;` and `theta2 = 5;` and write another double for-loop to build the M matrix.

We are now ready to add travel to our global model, we exchange people in each category by adding on a summation to each term:

$$\dot{S}_i = -\beta_i S_i I_i/(S_i + E_i + I_i + R_i) + \sum_{j=1}^{N} M_{ij} S_j - M_{ji} S_i$$

$$\dot{E}_i = \beta_i S_i I_i/(S_i + E_i + I_i + R_i) - \sigma E_i + \sum_{j=1}^{N} M_{ij} E_j - M_{ji} E_i$$

$$\dot{I}_i = \sigma E_i - \gamma I_i - \gamma\tau I_i - \gamma\mu I_i + \sum_{j=1}^{N} M_{ij} I_j - M_{ji} I_i$$

$$\dot{R}_i = \gamma I_i + \sum_{j=1}^{N} M_{ij} R_j - M_{ji} R_i$$

$$\dot{C}_i = \gamma\tau_i I_i$$

$$\dot{D}_i = \gamma\mu_i I_i$$

Problem 4: Modify the `ydot` function in your `SEIRCDworld.m` code to implement the model above.

Problem 5: Design an error function and use fminsearch to fit some parameters to the data set.