Lab 3: Exercises

ML701: Machine Learning
MBZUAI

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Problem 1. Assume there exists a hyperplane parametrized by w that perfectly separates data, i.e., $y_i \langle w, x_i \rangle > 0,\ \forall i \in [n]$. Show that then there exists a hyperplane parametrized by w' that solves the perception problem, i.e., $y_i \langle w', x_i \rangle \geq 1,\ \forall i \in [n]$.

Solution. Let $\alpha = \min_{i\in[n]} y_i \langle w, x_i \rangle > 0$. Then, let $w' = \alpha w$. Therefore, $\forall i \in [n]$

$$y_i \langle w', x_i \rangle = \frac{1}{\alpha} y_i \langle w, x_i \rangle \geq \frac{1}{\alpha}\alpha = 1.$$

Problem 2. Show that when we can optimize hinge loss for the perceptron, i.e.,

$$\min_{w\in\mathbb{R}^d} \frac{1}{n}\sum_{i=1}^{n} \max\{1 - y_i \langle w, x_i \rangle, 0\},$$

to zero, then the data are perfectly separable.

Solution. By definition $\max\{1-y_i \langle w, x_i \rangle, 0\} = 0$ if and only if $1-y_i \langle w, x_i \rangle \leq 0$ that implies $y_i \langle w, x_i \rangle \geq 1 > 0$. Therefore, for all the data with $y_i = 1$, we have $\langle w, x_i \rangle > 0$. While for data with negative label -1, we have $\langle w, x_i \rangle < 0$. Therefore, the data are perfectly separable.