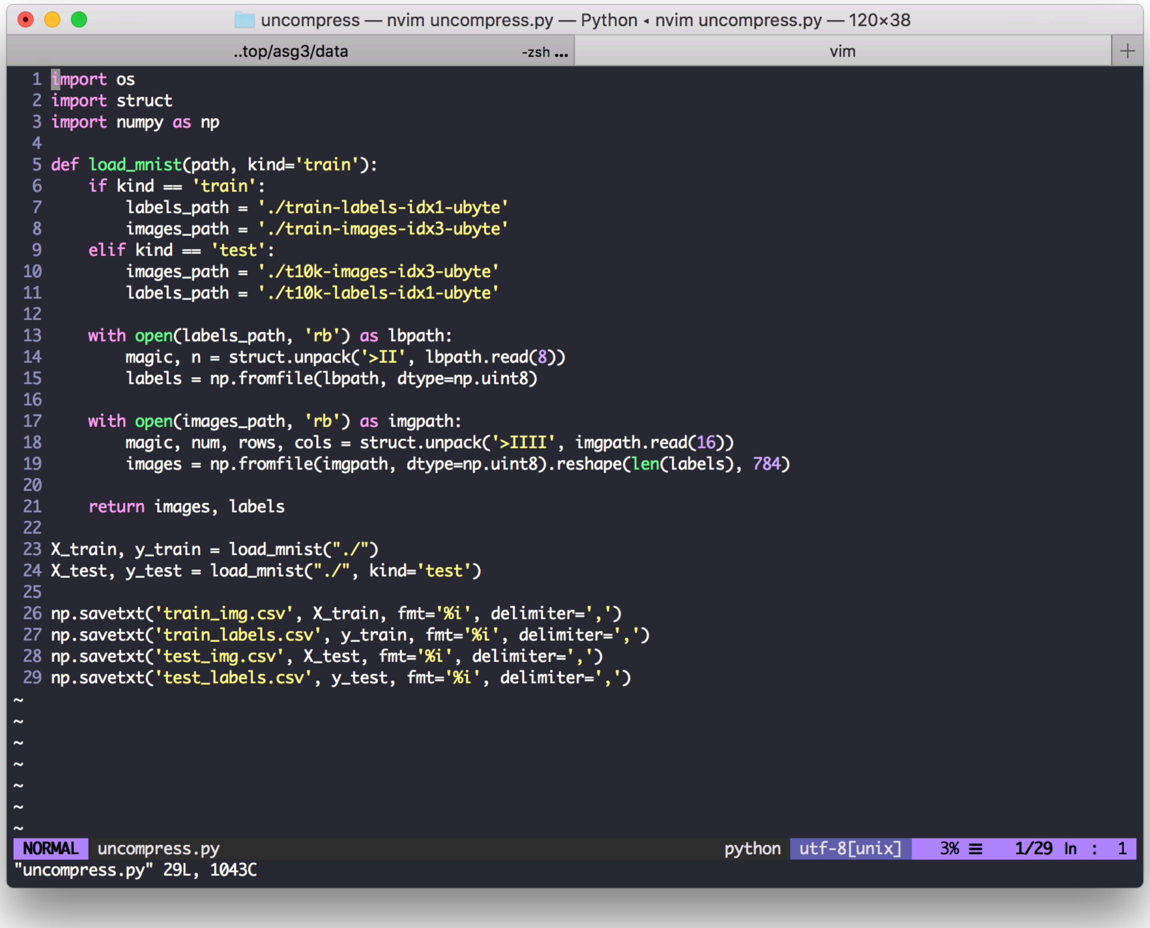
I declare that the assignment submitted on Elearning system is original except for source material explicitly acknowledged, and that the same or related material has not been previously submitted for another course. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the website http://www.cuhk.edu.hk/policy/academichonesty/.

Signed (Student Cao Yuhang ) Date: 04/07/2018
Name Cao Yuhang SID 1155092180

# Uncompress:

Convert original data to “csv” format. Each row is a 784 dimension data, each column is a pixel feature.



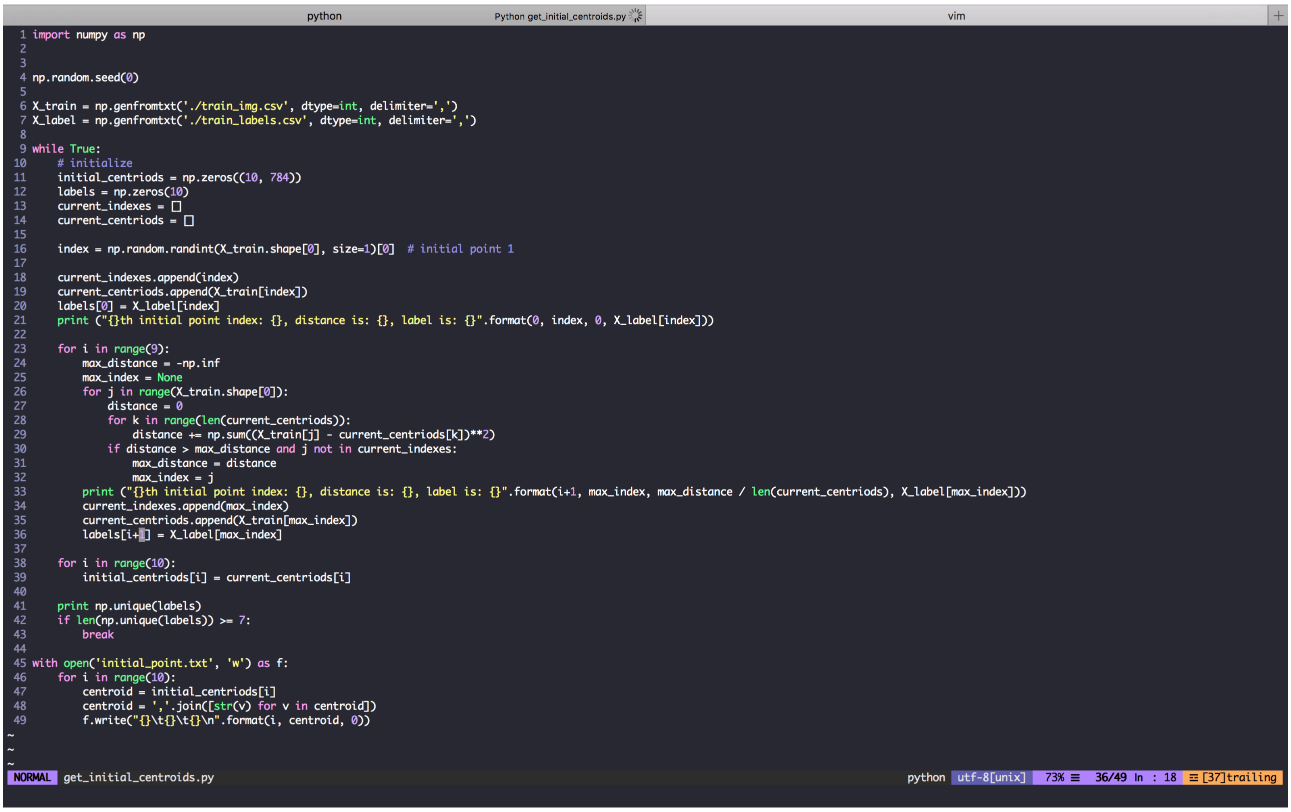
```python
import os
import struct
import numpy as np

def load_mnist(path, kind='train'):
    if kind == 'train':
        labels_path = './train-labels-idx1-ubyte'
        images_path = './train-images-idx3-ubyte'
    elif kind == 'test':
        images_path = './t10k-images-idx3-ubyte'
        labels_path = './t10k-labels-idx1-ubyte'

    with open(labels_path, 'rb') as lbpath:
        magic, n = struct.unpack('>II', lbpath.read(8))
        labels = np.fromfile(lbpath, dtype=np.uint8)

    with open(images_path, 'rb') as imgpath:
        magic, num, rows, cols = struct.unpack('>IIII', imgpath.read(16))
        images = np.fromfile(imgpath, dtype=np.uint8).reshape(len(labels), 784)

    return images, labels

X_train, y_train = load_mnist("./")
X_test, y_test = load_mnist("./", kind='test')

np.savetxt('train_img.csv', X_train, fmt='%i', delimiter=',')
np.savetxt('train_labels.csv', y_train, fmt='%i', delimiter=',')
np.savetxt('test_img.csv', X_test, fmt='%i', delimiter=',')
np.savetxt('test_labels.csv', y_test, fmt='%i', delimiter=',')
```

```
"uncompress.py" 29L, 1043C
```

# Get Initial Centroids:

1. Choose first centroid randomly.
2. Select next initial centroids in such a way that the Euclidean distance of that point is maximum from other selected initial centroids.
3. Repeat step 2 until we get k initial centroids.
4. The program will ensure initial points contain at least 7 different labels.

The result is stored in “cluster_id \t centroid \t cluster_number” format, cluster_number represents how many images in this cluster, at beginning, this is just a dump number, just for the consistent in later processing.



```python
import numpy as np


np.random.seed(0)

X_train = np.genfromtxt('./train_img.csv', dtype=int, delimiter=',')
X_label = np.genfromtxt('./train_labels.csv', dtype=int, delimiter=',')

while True:
    # initialize
    initial_centriods = np.zeros((10, 784))
    labels = np.zeros(10)
    current_indexes = []
    current_centriods = []

    index = np.random.randint(X_train.shape[0], size=1)[0]  # initial point 1

    current_indexes.append(index)
    current_centriods.append(X_train[index])
    labels[0] = X_label[index]
    print ("{}th initial point index: {}, distance is: {}, label is: {}".format(0, index, 0, X_label[index]))

    for i in range(9):
        max_distance = -np.inf
        max_index = None
        for j in range(X_train.shape[0]):
            distance = 0
            for k in range(len(current_centriods)):
                distance += np.sum((X_train[j] - current_centriods[k])**2)
            if distance > max_distance and j not in current_indexes:
                max_distance = distance
                max_index = j
        print ("{}th initial point index: {}, distance is: {}, label is: {}".format(i+1, max_index, max_distance / len(current_centriods), X_label[max_index]))
        current_indexes.append(max_index)
        current_centriods.append(X_train[max_index])
        labels[i+1] = X_label[max_index]

    for i in range(10):
        initial_centriods[i] = current_centriods[i]

    print np.unique(labels)
    if len(np.unique(labels)) >= 7:
        break

with open('initial_point.txt', 'w') as f:
    for i in range(10):
        centroid = initial_centriods[i]
        centroid = ','.join([str(v) for v in centroid])
        f.write("{}\t{}\t{}\n".format(i, centroid, 0))
~
~
~
```

# Q1 (a):

Mapper:
input: split of data
output: “cluster_id \t partial sum of this cluster \t image number of this cluster”

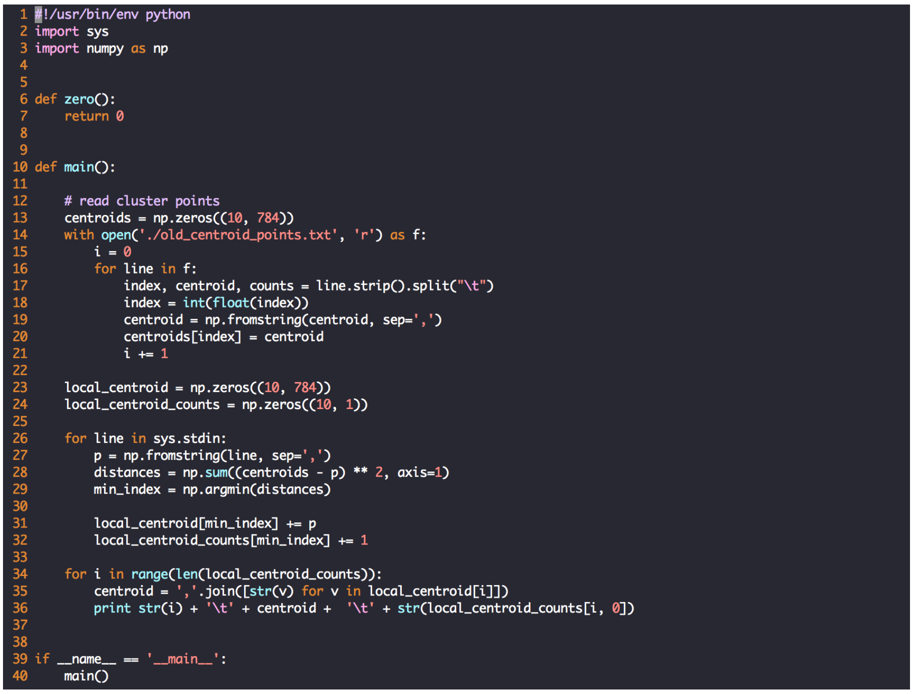
Reducer:
Merge partial sum and image number of same cluster_id, divided sum by number to get mean of this cluster_id

Code and Result:

- mapper1. py: mapper
- reducer1.py: reducer
- format_output.py: get desire output format
- cal_error.py: calculate error between consecutive iterations
- run.sh: run the program in an iterative manner
- error.txt: track the error changing
- ori_centroid_points.txt, old_centroid_points.txt, new_centroid_points.txt: tmp files
- res_a: result of part_a

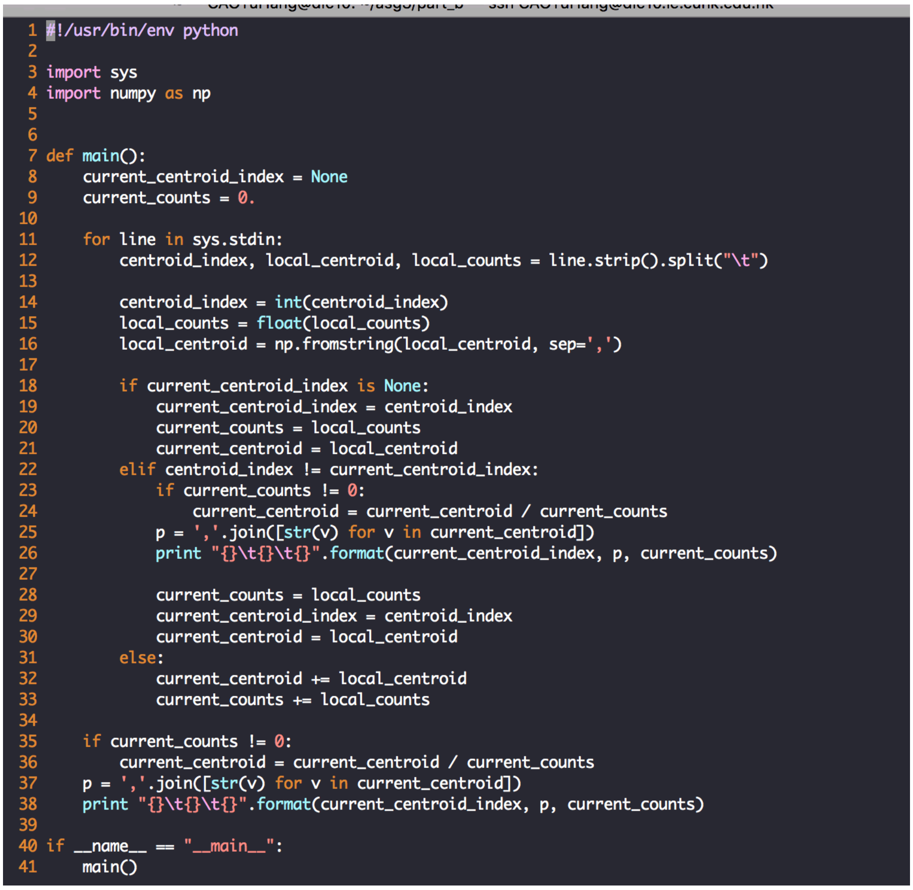
Code:

Mapper:



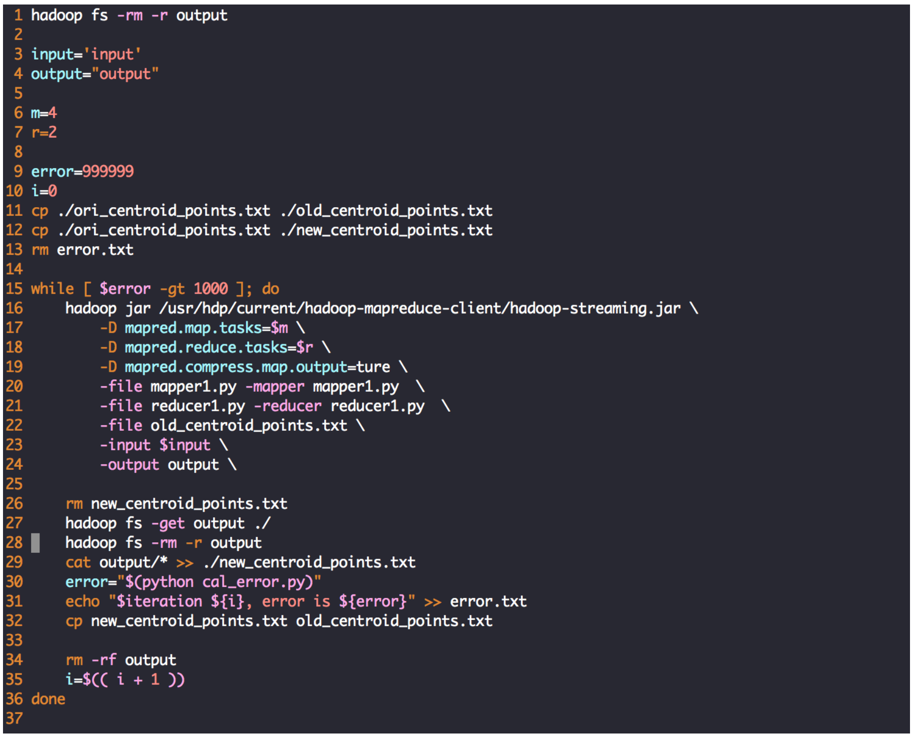
```python
#!/usr/bin/env python
import sys
import numpy as np


def zero():
    return 0


def main():

    # read cluster points
    centroids = np.zeros((10, 784))
    with open('./old_centroid_points.txt', 'r') as f:
        i = 0
        for line in f:
            index, centroid, counts = line.strip().split("\t")
            index = int(float(index))
            centroid = np.fromstring(centroid, sep=',')
            centroids[index] = centroid
            i += 1

    local_centroid = np.zeros((10, 784))
    local_centroid_counts = np.zeros((10, 1))

    for line in sys.stdin:
        p = np.fromstring(line, sep=',')
        distances = np.sum((centroids - p) ** 2, axis=1)
        min_index = np.argmin(distances)

        local_centroid[min_index] += p
        local_centroid_counts[min_index] += 1

    for i in range(len(local_centroid_counts)):
        centroid = ','.join([str(v) for v in local_centroid[i]])
        print str(i) + '\t' + centroid +  '\t' + str(local_centroid_counts[i, 0])


if __name__ == '__main__':
    main()
```

Reducer:



```python
#!/usr/bin/env python

import sys
import numpy as np


def main():
    current_centroid_index = None
    current_counts = 0.

    for line in sys.stdin:
        centroid_index, local_centroid, local_counts = line.strip().split("\t")

        centroid_index = int(centroid_index)
        local_counts = float(local_counts)
        local_centroid = np.fromstring(local_centroid, sep=',')

        if current_centroid_index is None:
            current_centroid_index = centroid_index
            current_counts = local_counts
            current_centroid = local_centroid
        elif centroid_index != current_centroid_index:
            if current_counts != 0:
                current_centroid = current_centroid / current_counts
            p = ','.join([str(v) for v in current_centroid])
            print "{}\t{}\t{}".format(current_centroid_index, p, current_counts)

            current_counts = local_counts
            current_centroid_index = centroid_index
            current_centroid = local_centroid
        else:
            current_centroid += local_centroid
            current_counts += local_counts

    if current_counts != 0:
        current_centroid = current_centroid / current_counts
    p = ','.join([str(v) for v in current_centroid])
    print "{}\t{}\t{}".format(current_centroid_index, p, current_counts)

if __name__ == "__main__":
    main()
```

run.sh:



```
hadoop fs -rm -r output

input='input'
output="output"

m=4
r=2

error=999999
i=0
cp ./ori_centroid_points.txt ./old_centroid_points.txt
cp ./ori_centroid_points.txt ./new_centroid_points.txt
rm error.txt

while [ $error -gt 1000 ]; do
    hadoop jar /usr/hdp/current/hadoop-mapreduce-client/hadoop-streaming.jar \
        -D mapred.map.tasks=$m \
        -D mapred.reduce.tasks=$r \
        -D mapred.compress.map.output=ture \
        -file mapper1.py -mapper mapper1.py  \
        -file reducer1.py -reducer reducer1.py  \
        -file old_centroid_points.txt \
        -input $input \
        -output output \

    rm new_centroid_points.txt
    hadoop fs -get output ./
    hadoop fs -rm -r output
    cat output/* >> ./new_centroid_points.txt
    error="$(python cal_error.py)"
    echo "$iteration ${i}, error is ${error}" >> error.txt
    cp new_centroid_points.txt old_centroid_points.txt

    rm -rf output
    i=$(( i + 1 ))
done

```

Result:

Centroid 0:

[0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.015,0.02,0.022,0.023,0.024,0.012,0.047,0.096,0.156,0.045,0.001,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.006,0.073,0.167,0.296,0.303,0.334,0.452,0.517,0.48,0.503,0.338,0.2,0.146,0.104,0.041,0.007,0.026,0.012,0.0,0.0,0.0,0.0,0.0,0.004,0.001,0.011,0.035,0.035,0.218,0.654,1.375,2.081,2.613,2.946,3.555,3.694,3.461,2.859,2.602,2.516,1.971,1.167,0.718,0.395,0.2,0.093,0.011,0.012,0.0,0.0,0.0,0.015,0.009,0.066,0.117,0.34,1.069,2.242,4.131,6.768,10.634,17.906,27.925,36.744,40.644,39.351,36.134,29.892,23.333,15.524,8.293,3.666,1.511,0.59,0.117,0.031,0.012,0.0,0.0,0.01,0.025,0.08,0.464,1.177,2.526,4.417,6.702,10.926,18.681,33.345,53.565,71.533,81.112,80.723,72.96,59.566,43.459,28.303,15.05,6.92,3.161,1.309,0.419,0.076,0.01,0.0,0.0,0.025,0.058,0.108,0.733,2.043,4.049,6.496,9.254,14.22,23.555,41.133,66.433,91.607,106.937,107.129,95.066,75.394,51.407,31.49,16.636,8.453,4.606,2.47,0.978,0.183,0.024,0.0,0.0,0.04,0.031,0.188,1.177,2.91,5.074,7.658,10.397,15.547,24.817,42.767,71.697,102.611,123.503,124.323,107.632,79.708,50.943,28.649,15.614,9.024,6.221,4.37,1.939,0.289,0.038,0.0,0.0,0.02,0.024,0.304,1.505,3.307,5.194,7.184,9.655,13.653,21.939,39.631,72.587,111.316,138.326,137.28,112.695,77.528,44.767,23.554,14.033,9.843,8.213,6.015,2.992,0.458,0.019,0.0,0.008,0.012,0.032,0.319,1.443,2.989,4.009,5.412,7.074,10.358,17.648,36.702,75.888,124.421,156.189,148.242,111.548,68.703,36.124,18.873,12.185,9.965,8.335,6.018,2.792,0.486,0.058,0.0,0.014,0.006,0.026,0.261,1.072,2.154,2.647,3.415,5.181,8.55,16.3,38.507,85.77,144.835,176.351,152.387,100.252,51.773,23.499,12.036,8.465,6.853,5.541,3.783,1.698,0.382,0.028,0.0,0.006,0.004,0.028,0.204,0.652,1.334,1.723,2.555,4.858,9.386,19.433,44.198,100.502,169.401,190.869,145.071,77.797,31.001,11.314,5.678,3.912,3.164,2.541,1.619,0.691,0.148,0.011,0.0,0.0,0.007,0.028,0.189,0.509,0.92,1.213,2.368,5.747,12.798,23.985,50.412,120.686,193.496,196.861,128.169,51.532,14.394,4.679,2.461,1.871,1.556,1.155,0.697,0.285,0.107,0.021,0.003,0.0,0.002,0.02,0.191,0.439,0.742,1.324,3.042,7.782,16.172,27.795,60.785,149.74,211.614,195.142,103.614,29.73,7.137,3.178,2.095,1.704,1.24

```
1,1.086,0.828,0.355,0.151,0.025,0.0,0.0,0.001,0.026,0.108,0.525,0.936,1.713,4.086,9.42,17.445,3
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0.523,218.151,160.129,57.204,16.531,7.433,4.353,3.16,2.645,2.043,1.533,0.984,0.424,0.199,0.02
6,0.0,0.004,0.015,0.052,0.289,1.376,2.866,4.071,6.253,11.065,24.096,57.787,123.895,192.8,203.2
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04,0.046,0.424,2.395,5.226,6.651,8.672,15.82,35.871,77.725,136.007,185.538,177.294,109.008,4
6.66,20.179,9.735,5.699,4.422,3.781,3.18,2.79,2.156,1.229,0.331,0.013,0.0,0.002,0.005,0.04,0.54
5,3.336,7.663,10.03,13.117,25.459,52.884,94.828,140.645,171.911,152.434,95.562,45.98,20.655,
10.059,6.388,5.49,4.734,4.115,3.445,2.411,1.043,0.211,0.029,0.002,0.008,0.028,0.106,0.652,3.65
3,9.154,13.998,21.764,40.015,70.79,107.878,140.25,155.038,133.023,87.28,45.754,20.973,10.756
,7.073,5.792,4.851,3.896,3.067,2.028,0.755,0.108,0.016,0.0,0.0,0.014,0.112,0.652,3.334,8.994,16.
892,30.669,54.756,84.699,113.797,133.733,139.221,119.334,82.35,46.611,22.735,12.615,8.279,6.
282,4.748,3.46,2.388,1.64,0.587,0.113,0.005,0.003,0.001,0.029,0.079,0.586,2.843,8.463,18.535,3
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36,2.604,1.674,0.977,0.345,0.101,0.0,0.003,0.001,0.023,0.048,0.512,2.616,7.84,18.617,37.307,60.
411,84.317,100.159,107.997,106.635,93.98,68.803,41.603,20.753,10.181,5.798,3.919,2.702,1.607
,0.912,0.361,0.12,0.026,0.0,0.0,0.0,0.0,0.054,0.591,2.269,6.464,15.178,29.707,47.722,64.133,74.2
78,77.983,77.182,68.904,50.679,29.692,13.566,5.657,2.977,1.957,1.245,0.821,0.482,0.177,0.046,
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6.781,9.343,3.926,1.604,0.813,0.477,0.367,0.28,0.141,0.047,0.014,0.002,0.0,0.0,0.0,0.0,0.005,0.1
28,0.194,0.352,0.853,1.473,2.292,3.185,3.897,4.764,5.113,3.585,2.007,0.926,0.367,0.173,0.124,0.
109,0.082,0.057,0.024,0.004,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.003,0.037,0.069,0.082,0.104,0.182,0.
405,0.664,0.827,0.467,0.172,0.095,0.036,0.046,0.028,0.026,0.028,0.008,0.003,0.0,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.012,0.026,0.036,0.009,0.0,0.0,0.006,0.02,0.0,0.0,0.0,0.0,0.
0,0.0,0.0,0.0,0.0], 9781.0
Centroid 1:
[0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0
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0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.016,0.046,0.04,0.047,0.076,0.079,0.064,0.083
,0.253,0.438,0.435,0.365,0.235,0.173,0.052,0.04,0.005,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.002,0.014,0.00
3,0.066,0.204,0.576,1.508,3.451,6.886,13.326,20.995,28.784,33.922,34.204,30.496,24.046,16.898
,11.597,7.685,4.664,2.053,0.645,0.14,0.0,0.0,0.0,0.0,0.023,0.049,0.117,0.217,0.601,1.507,3.839,1
0.412,24.204,48.605,80.335,109.989,133.143,145.179,144.975,131.627,108.802,80.434,54.781,34
.894,20.901,10.145,3.138,0.628,0.01,0.0,0.001,0.029,0.051,0.104,0.189,0.668,1.788,5.638,15.387,
37.455,75.377,122.137,164.269,190.912,202.276,205.166,203.234,192.903,171.185,136.25,96.73
4,61.801,36.509,18.783,6.587,1.155,0.043,0.0,0.055,0.015,0.027,0.176,0.41,1.462,5.056,14.868,3
6.732,76.956,130.808,176.326,199.041,197.628,187.306,177.897,177.312,181.271,176.688,152.3
63,111.519,71.331,41.708,20.577,7.016,1.18,0.098,0.0,0.031,0.0,0.11,0.251,0.765,2.916,9.489,25.
724,59.431,110.091,159.274,182.479,170.228,140.423,116.505,107.542,117.084,140.632,155.178
,140.61,104.088,65.311,35.473,16.428,5.65,1.059,0.093,0.0,0.033,0.035,0.19,0.469,1.169,4.252,1
2.676,34.179,73.778,123.692,157.019,152.519,115.589,78.35,59.55,65.302,91.541,124.989,140.4
45,123.091,86.244,51.053,26.176,11.687,4.129,0.741,0.075,0.0,0.017,0.04,0.197,0.501,1.386,4.66
3,15.067,38.646,78.664,124.409,145.138,124.909,83.04,52.548,48.908,70.155,106.214,135.002,1
32.974,102.73,64.459,34.742,16.493,7.326,2.602,0.412,0.041,0.0,0.012,0.031,0.142,0.431,1.291,4
.345,14.563,38.426,79.145,121.659,140.097,123.331,89.038,69.548,78.644,107.957,136.38,140.7
98,114.766,75.567,40.716,19.444,8.303,3.388,1.055,0.199,0.005,0.0,0.002,0.004,0.092,0.238,0.98
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53.013,197.395,212.76,206.796,194.254,183.252,176.792,178.192,184.855,188.019,176.322,139.
856,90.077,45.675,19.14,6.05,1.271,0.153,0.0,0.0,0.0,0.0,0.057,0.296,1.295,6.015,19.905,49.861,
98.199,151.993,193.752,214.255,220.103,217.245,211.469,203.689,190.688,165.277,126.617,81.
937,43.557,18.881,7.344,2.218,0.647,0.121,0.0,0.0,0.0,0.0,0.0,0.146,0.393,1.595,5.652,16.128,37.
881,72.732,113.0,148.982,172.672,180.509,172.732,153.273,123.479,88.112,56.843,30.653,14.69
3,6.166,2.545,0.913,0.269,0.089,0.0,0.0,0.0,0.0,0.0,0.081,0.104,0.248,0.854,2.285,5.232,13.41,25.
196,39.051,51.708,59.465,58.032,48.63,36.197,23.909,14.116,7.96,4.136,2.038,1.091,0.519,0.171
,0.065,0.0,0.0,0.0,0.0,0.0,0.027,0.079,0.085,0.12,0.275,0.465,0.95,2.108,4.031,6.193,7.894,8.274,
7.243,5.412,3.663,2.357,1.794,1.087,0.733,0.428,0.361,0.123,0.018,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,
0.0,0.013,0.099,0.196,0.436,0.813,1.156,1.161,0.954,0.83,0.497,0.338,0.58,0.543,0.402,0.233,0.0
83,0.01,0.001,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.027,0.081,0.047,0.083,0.054,0.00
6,0.0,0.0,0.076,0.197,0.11,0.029,0.005,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0
,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.047,0.032,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0
,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0], 3137.0
Centroid 9:
[0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0
.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.
0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.002,0.046,0.11,0.253,0.488,0.884,1.459,1.974,2.302,2.363,2.
384,2.093,1.676,1.341,0.998,0.697,0.462,0.374,0.14,0.0,0.0,0.0,0.0,0.0,0.0,0.005,0.011,0.019,0.28
7,1.402,3.504,7.582,14.196,23.576,36.052,48.436,58.897,65.345,65.378,58.736,46.881,33.471,22.
409,14.174,8.952,5.677,3.237,1.639,0.595,0.068,0.0,0.0,0.0,0.0,0.012,0.306,2.001,6.673,15.424,2
9.914,52.111,81.136,113.198,144.258,165.859,176.057,173.053,160.209,136.841,105.681,72.837,
46.936,28.733,17.168,9.414,4.909,2.13,0.473,0.068,0.0,0.0,0.001,0.027,1.174,5.505,14.56,31.653,
58.513,94.389,133.802,170.023,192.805,199.701,198.688,195.868,190.973,178.672,151.686,113.
206,73.326,43.333,24.23,12.885,6.79,2.903,0.85,0.137,0.0,0.0,0.004,0.101,2.002,8.573,20.683,43.
057,73.917,109.479,142.848,161.607,161.233,149.318,138.627,137.693,146.937,154.965,147.423
,119.747,80.534,44.947,22.968,11.068,5.275,2.447,0.721,0.081,0.0,0.0,0.0,0.101,2.428,9.483,21.0
23,41.973,68.797,96.502,115.259,116.519,100.433,79.506,69.292,75.63,96.928,119.517,126.86,1
10.222,75.401,40.012,18.771,8.241,3.492,1.556,0.493,0.006,0.0,0.0,0.0,0.032,1.744,7.135,16.815,
33.751,55.182,76.272,86.87,81.621,63.677,45.527,42.626,58.082,85.337,109.506,117.345,99.94,6
5.636,33.539,14.822,6.031,2.298,0.738,0.174,0.0,0.0,0.0,0.0,0.031,1.205,4.447,11.505,24.892,43.
256,62.326,74.273,72.797,62.31,54.827,62.054,84.477,108.723,121.617,114.334,87.929,54.24,26.
941,11.067,4.459,1.476,0.352,0.12,0.001,0.0,0.0,0.0,0.04,0.59,2.704,8.484,19.781,37.034,58.44,7
7.649,88.69,93.364,101.503,117.08,134.789,144.664,138.755,115.26,79.936,47.417,23.357,9.169,
3.148,1.217,0.35,0.102,0.0,0.0,0.0,0.0,0.007,0.352,1.687,6.213,16.817,34.523,61.079,91.153,118.
738,140.29,156.428,170.113,176.996,174.613,158.88,126.677,88.268,55.094,28.365,11.122,3.439
,1.277,0.316,0.074,0.0,0.0,0.0,0.0,0.0,0.165,1.241,4.559,13.926,31.313,60.487,97.239,133.051,15
9.01,176.034,182.552,182.295,176.114,163.794,142.374,113.666,78.654,44.626,19.621,6.698,2.2
43,0.608,0.123,0.004,0.01,0.016,0.0,0.0,0.09,0.831,3.485,10.074,24.857,50.261,83.284,113.261,1
33.451,144.361,146.397,145.459,144.422,145.566,145.56,136.333,109.581,70.01,34.708,13.55,4.
24,1.12,0.208,0.047,0.007,0.005,0.0,0.035,0.176,0.697,2.68,7.044,16.583,34.485,56.678,75.844,8
6.566,90.27,88.903,86.355,90.559,105.387,127.749,142.329,133.027,96.076,52.023,23.531,7.669,
1.829,0.332,0.097,0.012,0.0,0.0,0.093,0.457,1.379,3.041,6.075,11.838,23.189,36.884,46.048,49.7
24,47.945,42.658,38.891,43.807,61.043,97.458,135.871,144.288,115.315,68.192,32.615,12.2,3.05
1,0.457,0.125,0.024,0.0,0.018,0.225,1.066,2.906,5.394,8.802,14.004,22.454,31.348,36.159,35.168
,30.201,22.717,16.756,17.948,32.913,73.808,126.673,149.499,125.328,79.268,40.156,16.006,4.02
```
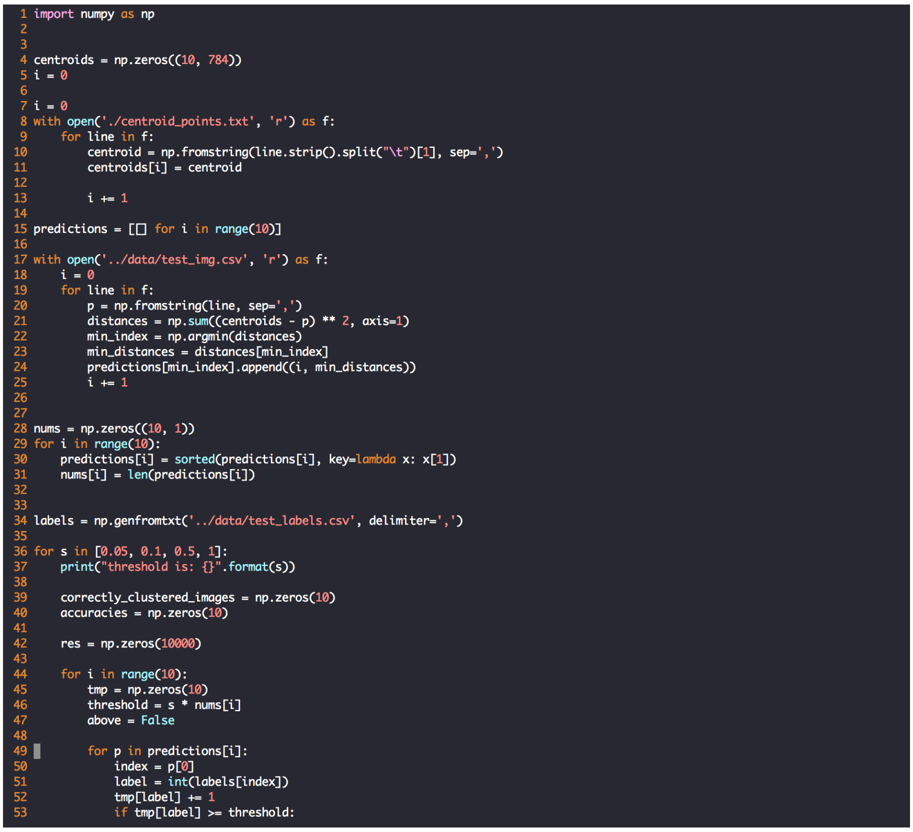
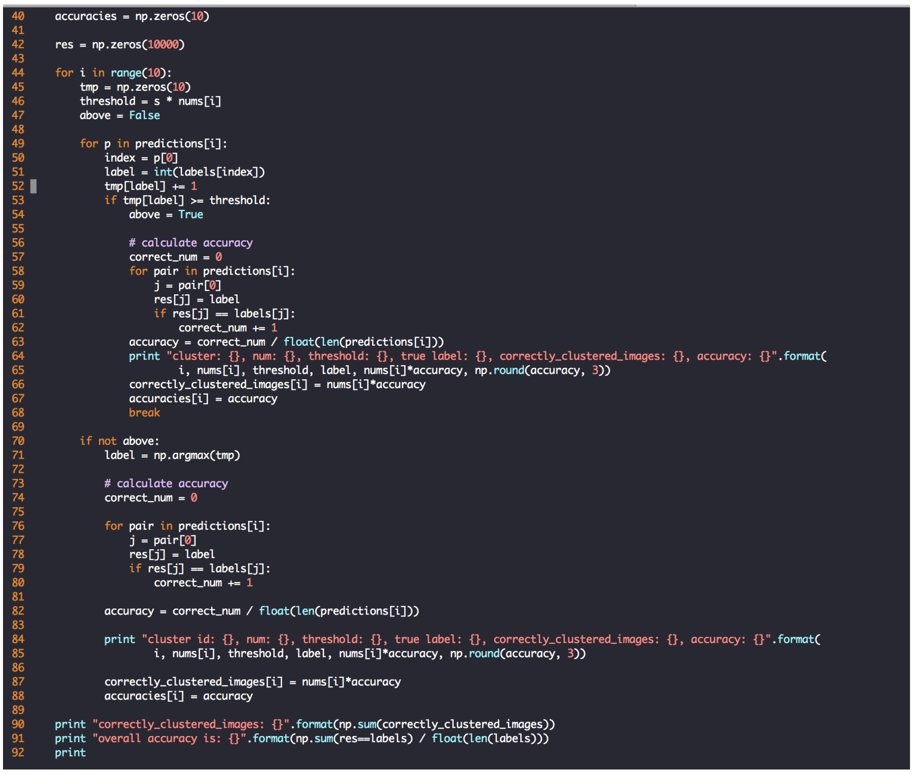

```
4,0.484,0.05,0.001,0.0,0.032,0.429,1.885,5.459,10.196,15.919,23.39,31.22,37.44,39.442,34.318,2
6.319,16.558,9.903,9.65,25.22,69.871,128.302,154.567,130.211,83.857,44.366,17.581,4.426,0.53
1,0.053,0.0,0.0,0.055,0.601,2.734,8.302,16.926,26.899,37.804,48.794,55.2,53.737,44.048,31.225,
19.204,13.821,19.024,43.741,93.43,145.928,161.911,129.605,82.342,42.363,15.732,3.79,0.578,0.
101,0.0,0.0,0.069,0.737,3.489,10.256,22.018,36.686,54.588,73.452,84.951,84.905,72.762,55.93,4
5.165,45.105,60.24,94.913,140.22,172.261,162.296,118.053,69.473,32.831,11.656,2.84,0.428,0.0
82,0.0,0.0,0.049,0.753,3.348,9.645,21.741,40.983,66.023,94.82,119.291,130.873,128.566,118.638
,112.971,118.401,138.099,165.344,185.768,178.309,140.865,90.407,47.774,20.3,6.825,1.709,0.39
,0.03,0.0,0.0,0.006,0.532,2.364,7.057,16.498,34.002,59.417,93.326,129.653,160.41,180.37,188.94
2,191.968,196.014,201.703,199.848,180.057,141.04,93.147,52.353,24.174,9.378,2.858,0.743,0.20
3,0.001,0.0,0.0,0.0,0.211,1.133,3.506,8.324,18.742,35.412,60.585,91.982,127.566,160.76,184.222
,194.745,191.716,175.985,147.66,109.869,71.412,40.833,19.35,7.815,2.782,0.814,0.176,0.055,0.0
11,0.0,0.0,0.0,0.025,0.32,0.946,2.482,5.829,10.878,19.3,31.737,46.614,63.296,77.063,83.884,79.9
79,67.613,50.238,32.54,18.302,8.833,3.623,1.361,0.427,0.154,0.074,0.031,0.0,0.0,0.0,0.0,0.0,0.03
5,0.118,0.245,0.553,0.973,1.577,2.441,3.335,4.011,4.7,5.084,4.82,4.137,3.072,1.929,1.03,0.448,0.
143,0.008,0.001,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.002,0.026,0.028,0.035,
0.026,0.004,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,
0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0], 7285.0
```

# Q1 (b):

code:



```python
import numpy as np


centroids = np.zeros((10, 784))
i = 0

i = 0
with open('./centroid_points.txt', 'r') as f:
    for line in f:
        centroid = np.fromstring(line.strip().split("\t")[1], sep=',')
        centroids[i] = centroid

        i += 1

predictions = [[] for i in range(10)]

with open('../data/test_img.csv', 'r') as f:
    i = 0
    for line in f:
        p = np.fromstring(line, sep=',')
        distances = np.sum((centroids - p) ** 2, axis=1)
        min_index = np.argmin(distances)
        min_distances = distances[min_index]
        predictions[min_index].append((i, min_distances))
        i += 1


nums = np.zeros((10, 1))
for i in range(10):
    predictions[i] = sorted(predictions[i], key=lambda x: x[1])
    nums[i] = len(predictions[i])


labels = np.genfromtxt('../data/test_labels.csv', delimiter=',')

for s in [0.05, 0.1, 0.5, 1]:
    print("threshold is: {}".format(s))

    correctly_clustered_images = np.zeros(10)
    accuracies = np.zeros(10)

    res = np.zeros(10000)

    for i in range(10):
        tmp = np.zeros(10)
        threshold = s * nums[i]
        above = False

        for p in predictions[i]:
            index = p[0]
            label = int(labels[index])
            tmp[label] += 1
            if tmp[label] >= threshold:
```



```python
    accuracies = np.zeros(10)

    res = np.zeros(10000)

    for i in range(10):
        tmp = np.zeros(10)
        threshold = s * nums[i]
        above = False

        for p in predictions[i]:
            index = p[0]
            label = int(labels[index])
            tmp[label] += 1
            if tmp[label] >= threshold:
                above = True

                # calculate accuracy
                correct_num = 0
                for pair in predictions[i]:
                    j = pair[0]
                    res[j] = label
                    if res[j] == labels[j]:
                        correct_num += 1
                accuracy = correct_num / float(len(predictions[i]))
                print "cluster: {}, num: {}, threshold: {}, true label: {}, correctly_clustered_images: {}, accuracy: {}".format(
                        i, nums[i], threshold, label, nums[i]*accuracy, np.round(accuracy, 3))
                correctly_clustered_images[i] = nums[i]*accuracy
                accuracies[i] = accuracy
                break

        if not above:
            label = np.argmax(tmp)

            # calculate accuracy
            correct_num = 0

            for pair in predictions[i]:
                j = pair[0]
                res[j] = label
                if res[j] == labels[j]:
                    correct_num += 1

            accuracy = correct_num / float(len(predictions[i]))

            print "cluster id: {}, num: {}, threshold: {}, true label: {}, correctly_clustered_images: {}, accuracy: {}".format(
                    i, nums[i], threshold, label, nums[i]*accuracy, np.round(accuracy, 3))

            correctly_clustered_images[i] = nums[i]*accuracy
            accuracies[i] = accuracy

    print "correctly_clustered_images: {}".format(np.sum(correctly_clustered_images))
    print "overall accuracy is: {}".format(np.sum(res==labels) / float(len(labels)))
    print
```

| **Table. 1.** | **The Accuracy** | **of Clustering** | **Performance** | **with** | **x = 5%** |
|---|---|---|---|---|---|
| Cluster Number | # images in the entire cluster | # of images considered (m) when determining the cluster label | Major Label of central images | # correctly clustered images | Classification Accuracy (%) |
| 0 | 1075 | 54 | 8 | 574 | 0.534 |
| 1 | 799 | 40 | 2 | 712 | 0.891 |
| 2 | 666 | 33 | 6 | 357 | 0.536 |
| 3 | 970 | 49 | 7 | 360 | 0.371 |
| 4 | 1285 | 64 | 3 | 668 | 0.52 |
| 5 | 1597 | 80 | 1 | 1109 | 0.694 |
| 6 | 1307 | 65 | 7 | 484 | 0.37 |
| 7 | 806 | 40 | 0 | 757 | 0.939 |
| 8 | 883 | 45 | 9 | 295 | 0.334 |
| 9 | 612 | 30 | 6 | 548 | 0.895 |
| Total Set | 10000 | 500 | NA | 5864 | 0.5864 |

| **Table. 2.** | **The Accuracy** | **of Clustering** | **Performance** | **with** | **x = 10%** |
|---|---|---|---|---|---|
| Cluster Number | # images in the entire cluster | # of images considered (m) when determining the cluster label | Major Label of central images | # correctly clustered images | Classification Accuracy (%) |
| 0 | 1075 | 108 | 8 | 574 | 0.534 |
| 1 | 799 | 80 | 2 | 712 | 0.891 |
| 2 | 666 | 66 | 6 | 357 | 0.536 |
| 3 | 970 | 98 | 7 | 360 | 0.371 |
| 4 | 1285 | 128 | 3 | 668 | 0.52 |
| 5 | 1597 | 160 | 1 | 1109 | 0.694 |
| 6 | 1307 | 130 | 9 | 458 | 0.35 |
| 7 | 806 | 80 | 0 | 757 | 0.939 |
| 8 | 883 | 90 | 9 | 295 | 0.334 |
| 9 | 612 | 60 | 6 | 548 | 0.895 |
| Total Set | 10000 | 500 | NA | 5838 | 0.5838 |

| Table. 3. | The Accuracy | of Clustering | Performance | with | x = 50% |
|---|---|---|---|---|---|
| Cluster Number | # images in the entire cluster | # of images considered (m) when determining the cluster label | Major Label of central images | # correctly clustered images | Classification Accuracy (%) |
| 0 | 1075 | 537 | 8 | 574 | 0.534 |
| 1 | 799 | 400 | 2 | 712 | 0.891 |
| 2 | 666 | 333 | 6 | 357 | 0.536 |
| 3 | 970 | 485 | 7 | 360 | 0.371 |
| 4 | 1285 | 643 | 3 | 668 | 0.52 |
| 5 | 1597 | 799 | 1 | 1109 | 0.694 |
| 6 | 1307 | 653 | 7 | 484 | 0.37 |
| 7 | 806 | 403 | 0 | 757 | 0.939 |
| 8 | 883 | 441 | 4 | 393 | 0.445 |
| 9 | 612 | 306 | 6 | 548 | 0.895 |
| Total Set | 10000 | 500 | NA | 5962 | 0.5962 |

| Table. 4. | The Accuracy | of Clustering | Performance | with | x = 100% |
|---|---|---|---|---|---|
| Cluster Number | # images in the entire cluster | # of images considered (m) when determining the cluster label | Major Label of central images | # correctly clustered images | Classification Accuracy (%) |
| 0 | 1075 | 537 | 8 | 574 | 0.534 |
| 1 | 799 | 400 | 2 | 712 | 0.891 |
| 2 | 666 | 333 | 6 | 357 | 0.536 |
| 3 | 970 | 485 | 7 | 360 | 0.371 |
| 4 | 1285 | 643 | 3 | 668 | 0.52 |
| 5 | 1597 | 799 | 1 | 1109 | 0.694 |
| 6 | 1307 | 653 | 7 | 484 | 0.37 |
| 7 | 806 | 403 | 0 | 757 | 0.939 |
| 8 | 883 | 441 | 4 | 393 | 0.445 |
| 9 | 612 | 306 | 6 | 548 | 0.895 |
| Total Set | 10000 | 500 | NA | 5962 | 0.5962 |

Best x: 50% and 100%

Explain:

1. Firstly observe that x=5% is better than x=10%, we rely more on the points that has smaller distance, in some sense we can reduce the noise (points that far away from centroid) influence. Let’s look cluster 6, when x=5%, it choose 7 as its label; when x=10%, it choose 9 as its label; however, in cluster 6, 7 is of 37% and 9 is of 35%, although the difference is not so big, but it really shows that sometimes choose nearby points has higher accuracy.

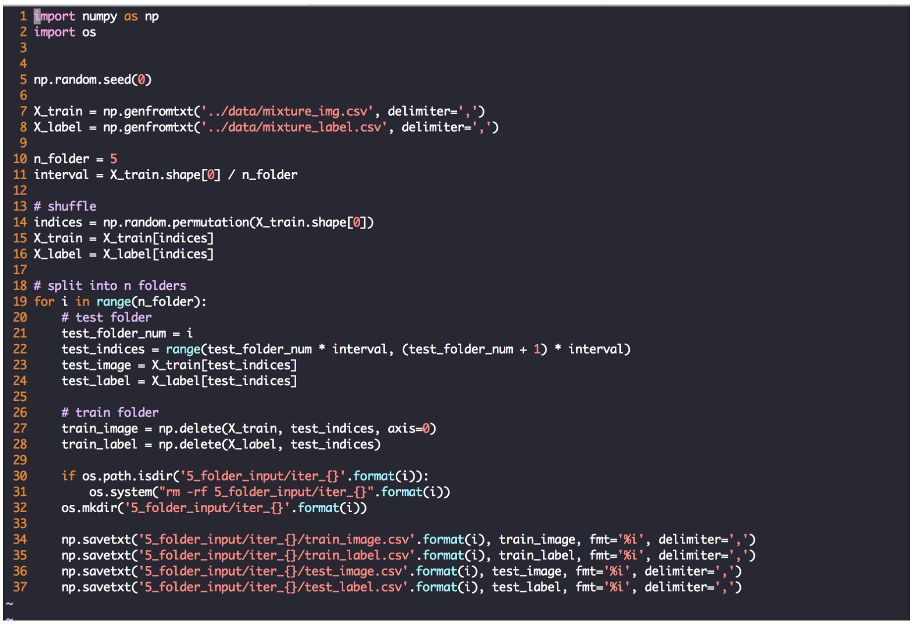
2. Secondly we observe x=50% and x=100% is better than x=5% and x=10%. In fact this is the phenomenon of “curse of dimensionality”, as dimension increase, the data points will concentrated more and more in a then-shell near the surface.
   Let’s look at cluster 8, in cluster 5% and cluster 10%, we can find the true label is 9; in cluster 50% and cluster 100%, we can find the true label is 4. While 9 is only of 0.334% and 4 is of 0.445%, although label 4 is more similar to centroid, but in the cluster it is minor; although label 9 is far away from centroid, but it is majority in the cluster

## Q1 (c):

code: (5_folder and 10_folder are almost same, just modify 5 to 10)

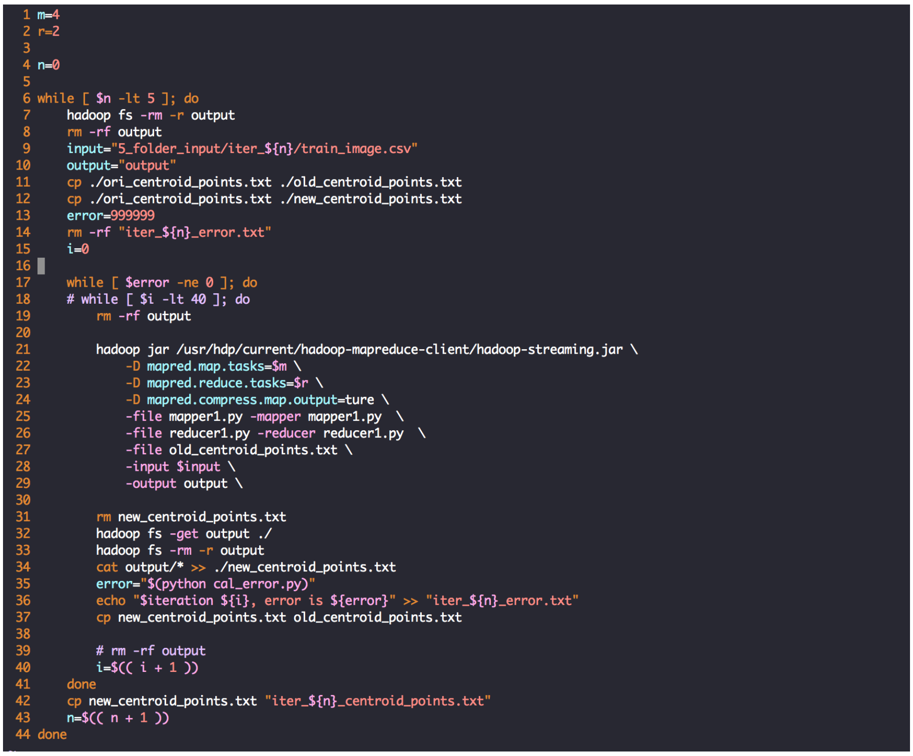
mapper and reducer are same with Q1 (a)

get_data.py: split total data (mixture with training and testing data) into 5 folder, loop over 5 folder, every time choose 1 folder as test set and 4 for train set



```python
import numpy as np
import os


np.random.seed(0)

X_train = np.genfromtxt('../data/mixture_img.csv', delimiter=',')
X_label = np.genfromtxt('../data/mixture_label.csv', delimiter=',')

n_folder = 5
interval = X_train.shape[0] / n_folder

# shuffle
indices = np.random.permutation(X_train.shape[0])
X_train = X_train[indices]
X_label = X_label[indices]

# split into n folders
for i in range(n_folder):
    # test folder
    test_folder_num = i
    test_indices = range(test_folder_num * interval, (test_folder_num + 1) * interval)
    test_image = X_train[test_indices]
    test_label = X_label[test_indices]

    # train folder
    train_image = np.delete(X_train, test_indices, axis=0)
    train_label = np.delete(X_label, test_indices)

    if os.path.isdir('5_folder_input/iter_{}'.format(i)):
        os.system("rm -rf 5_folder_input/iter_{}".format(i))
    os.mkdir('5_folder_input/iter_{}'.format(i))

    np.savetxt('5_folder_input/iter_{}/train_image.csv'.format(i), train_image, fmt='%i', delimiter=',')
    np.savetxt('5_folder_input/iter_{}/train_label.csv'.format(i), train_label, fmt='%i', delimiter=',')
    np.savetxt('5_folder_input/iter_{}/test_image.csv'.format(i), test_image, fmt='%i', delimiter=',')
    np.savetxt('5_folder_input/iter_{}/test_label.csv'.format(i), test_label, fmt='%i', delimiter=',')
~
```

run.sh: run 5 experiments for the corresponding 5 dataset (different test and train folder combination)



```bash
m=4
r=2

n=0

while [ $n -lt 5 ]; do
    hadoop fs -rm -r output
    rm -rf output
    input="5_folder_input/iter_${n}/train_image.csv"
    output="output"
    cp ./ori_centroid_points.txt ./old_centroid_points.txt
    cp ./ori_centroid_points.txt ./new_centroid_points.txt
    error=999999
    rm -rf "iter_${n}_error.txt"
    i=0

    while [ $error -ne 0 ]; do
    # while [ $i -lt 40 ]; do
        rm -rf output

        hadoop jar /usr/hdp/current/hadoop-mapreduce-client/hadoop-streaming.jar \
            -D mapred.map.tasks=$m \
            -D mapred.reduce.tasks=$r \
            -D mapred.compress.map.output=ture \
            -file mapper1.py -mapper mapper1.py  \
            -file reducer1.py -reducer reducer1.py  \
            -file old_centroid_points.txt \
            -input $input \
            -output output \

        rm new_centroid_points.txt
        hadoop fs -get output ./
        hadoop fs -rm -r output
        cat output/* >> ./new_centroid_points.txt
        error="$(python cal_error.py)"
        echo "$iteration ${i}, error is ${error}" >> "iter_${n}_error.txt"
        cp new_centroid_points.txt old_centroid_points.txt

        # rm -rf output
        i=$(( i + 1 ))
    done
    cp new_centroid_points.txt "iter_${n}_centroid_points.txt"
    n=$(( n + 1 ))
done
```

The result is tested on x=100%, which is best x in part (b)

| Testing set | Classification Accuracy |
|---|---|
| Part 1 | 0.5901 |
| Part 2 | 0.5849 |
| Part 3 | 0.5959 |
| Part 4 | 0.5889 |
| Part 5 | 0.5916 |
| Average | 0.59028 |

| Testing set | Classification Accuracy |
|---|---|
| Part 1 | 0.5946 |
| Part 2 | 0.5844 |
| Part 3 | 0.5861 |
| Part 4 | 0.5874 |
| Part 5 | 0.5907 |
| Part 6 | 0.6014 |
| Part 7 | 0.5984 |
| Part 8 | 0.5819 |
| Part 9 | 0.5873 |
| Part 10 | 0.5986 |
| Average | 0.59108 |

# Q2 (a):

Input: $X$ : $N \times D$ data matrix

1. Initialize the parameters: $q_k, \pi_k$ ($Q$ is $K \times D$ matrix, $\pi$ is $K \times 1$ matrix)
   (can randomly initialize or using the centroids from k-means)

2. E Step: For each data point $x_n$, determine its assignment score to each Bernoulli k

$$\gamma\left(z_{nk}\right)=\pi_k p\left(x_n \mid q_k\right) / \left( \sum_{j=1}^{K} \pi_j p\left(x_n \mid q_j\right) \right)$$

$$p\left(x_n \mid q_k\right)=\prod_{i=1}^{D} q_{ki}{}^{x_{ni}} \left(1-q_{ki}\right)^{\left(1-x_{ni}\right)}$$

3. M Step: For each Bernoulli k, update parameters using new $\gamma\left(z_{nk}\right)$

$$q_k = \frac{\sum_{n=1}^{N} \gamma\left(z_{nk}\right) x_n}{\sum_{n=1}^{N} \gamma\left(z_{nk}\right)} \qquad \pi_k = \frac{\sum_{n=1}^{N} \gamma\left(z_{nk}\right)}{N}$$

4. Evaluate log likelihood. If likelihood or parameters converge, stop. Else go to step 2 (E step).

$$P\left(X\right)=\prod_{n=1}^{N} \sum_{k=1}^{K} \pi_k p\left(x_n \mid q_k\right)$$

Perform probabilistic clustering:

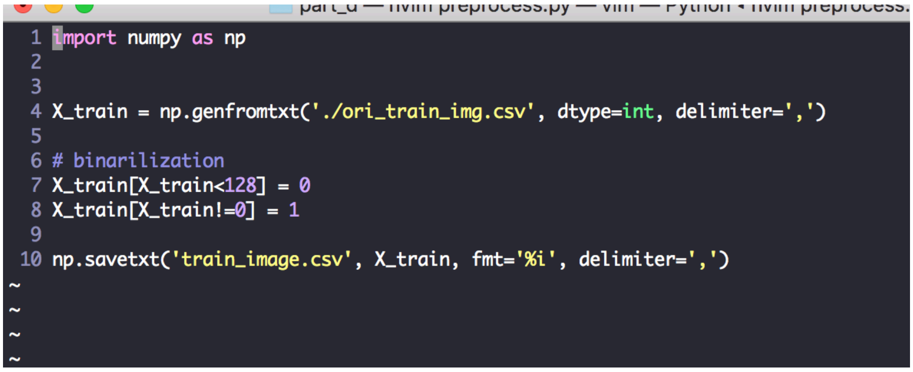
1. For each data point $x_n$, determine $p\left(x_n \mid q_k\right)$
2. Calculate $p\left(x_n, q_k\right)=p\left(x_n \mid q_k\right) \times \pi\left(k\right)$
3. Assign $x_n$ to cluster $k$ with maximum $p\left(x_n, q_k\right)$
   (Can also choose $k$ as probability distribution)

# Q2 (b):

Binarization:

Just set pixel < 128 to be 0, otherwise to be 1

code: preprocess.py

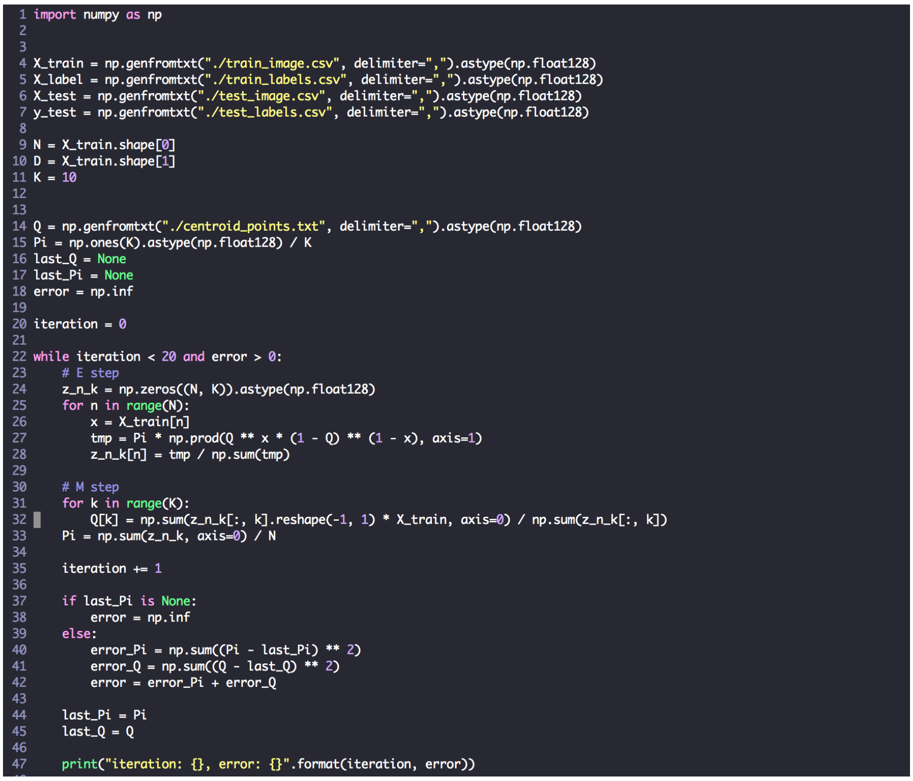

part_d — nvim preprocess.py — vim — Python ◂ nvim preprocess.

```python
import numpy as np


X_train = np.genfromtxt('./ori_train_img.csv', dtype=int, delimiter=',')

# binarilization
X_train[X_train<128] = 0
X_train[X_train!=0] = 1

np.savetxt('train_image.csv', X_train, fmt='%i', delimiter=',')
~
~
~
~
```

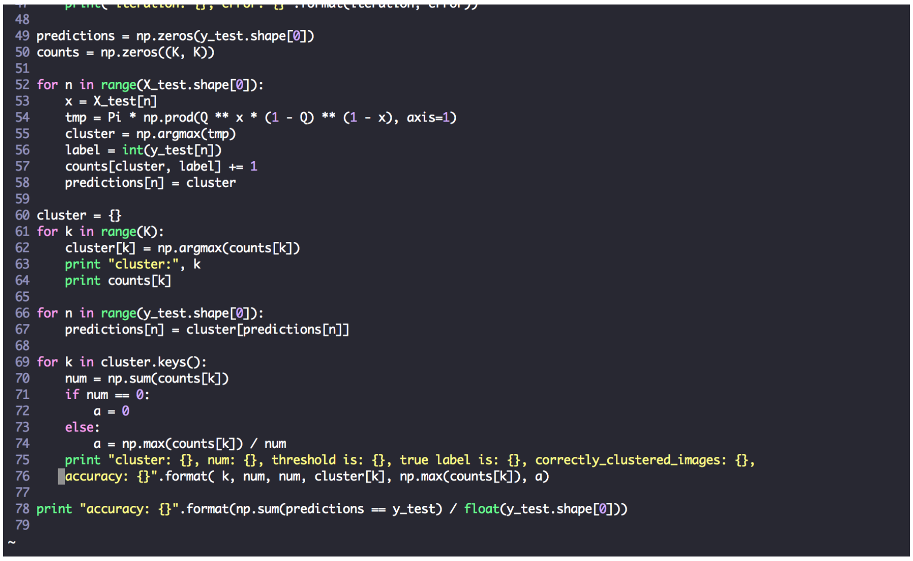
BMM and EM-algorithm:

I also use majority point method to choose true label, but just choose the label with largest number of points in a cluster to be its label.

code: BMM.py



```python
import numpy as np


X_train = np.genfromtxt("./train_image.csv", delimiter=",").astype(np.float128)
X_label = np.genfromtxt("./train_labels.csv", delimiter=",").astype(np.float128)
X_test = np.genfromtxt("./test_image.csv", delimiter=",").astype(np.float128)
y_test = np.genfromtxt("./test_labels.csv", delimiter=",").astype(np.float128)

N = X_train.shape[0]
D = X_train.shape[1]
K = 10


Q = np.genfromtxt("./centroid_points.txt", delimiter=",").astype(np.float128)
Pi = np.ones(K).astype(np.float128) / K
last_Q = None
last_Pi = None
error = np.inf

iteration = 0

while iteration < 20 and error > 0:
    # E step
    z_n_k = np.zeros((N, K)).astype(np.float128)
    for n in range(N):
        x = X_train[n]
        tmp = Pi * np.prod(Q ** x * (1 - Q) ** (1 - x), axis=1)
        z_n_k[n] = tmp / np.sum(tmp)

    # M step
    for k in range(K):
        Q[k] = np.sum(z_n_k[:, k].reshape(-1, 1) * X_train, axis=0) / np.sum(z_n_k[:, k])
    Pi = np.sum(z_n_k, axis=0) / N

    iteration += 1

    if last_Pi is None:
        error = np.inf
    else:
        error_Pi = np.sum((Pi - last_Pi) ** 2)
        error_Q = np.sum((Q - last_Q) ** 2)
        error = error_Pi + error_Q

    last_Pi = Pi
    last_Q = Q

    print("iteration: {}, error: {}".format(iteration, error))
```



```
    print("iteration: {}, error: {}".format(iteration, error))

predictions = np.zeros(y_test.shape[0])
counts = np.zeros((K, K))

for n in range(X_test.shape[0]):
    x = X_test[n]
    tmp = Pi * np.prod(Q ** x * (1 - Q) ** (1 - x), axis=1)
    cluster = np.argmax(tmp)
    label = int(y_test[n])
    counts[cluster, label] += 1
    predictions[n] = cluster

cluster = {}
for k in range(K):
    cluster[k] = np.argmax(counts[k])
    print "cluster:", k
    print counts[k]

for n in range(y_test.shape[0]):
    predictions[n] = cluster[predictions[n]]

for k in cluster.keys():
    num = np.sum(counts[k])
    if num == 0:
        a = 0
    else:
        a = np.max(counts[k]) / num
    print "cluster: {}, num: {}, threshold is: {}, true label is: {}, correctly_clustered_images: {},
    accuracy: {}".format( k, num, num, cluster[k], np.max(counts[k]), a)

print "accuracy: {}".format(np.sum(predictions == y_test) / float(y_test.shape[0]))

~
```

Results:

- I use the centroids from part a result.
- Assign cluster label that has largest number in this cluster.
- Iterations 20 times

| **Table. 1.** | **The Accuracy** | **of Clustering** | **Performance** | **with** | **x = 100%** |
|---|---|---|---|---|---|
| Cluster Number | # images in the entire cluster | # of images considered (m) when determining the cluster label | Major Label of central images | # correctly clustered images | Classification Accuracy (%) |
| 0 | 1200 | 1200 | 1 | 1042 | 0.868 |
| 1 | 1742 | 1742 | 6 | 848 | 0.487 |
| 2 | 0 | 0 | NA | NA | NA |
| 3 | 1131 | 1131 | 7 | 644 | 0.569 |
| 4 | 1606 | 1606 | 8 | 417 | 0.260 |
| 5 | 0 | 0 | NA | NA | NA |
| 6 | 0 | 0 | NA | NA | NA |
| 7 | 1825 | 1825 | 3 | 767 | 0.420 |
| 8 | 1635 | 1635 | 4 | 634 | 0.388 |
| 9 | 861 | 861 | 0 | 778 | 0.904 |
| Total Set | 10000 | 10000 | NA | 5130 | 0.5137 |

# Q3:

(a)

$$U=\begin{bmatrix} -0.31 & -0.28 \\ -0.27 & 0.22 \\ -0.23 & 0.47 \\ -0.36 & -0.47 \\ -0.70 & -0.23 \\ -0.40 & 0.62 \end{bmatrix}$$

$$S=\begin{bmatrix} 24.61 & 0 \\ 0 & 11.45 \end{bmatrix}$$

$$V^T=\begin{bmatrix} -0.27 & -0.51 & -0.37 & -0.22 & -0.43 & -0.35 & -0.41 \\ 0.69 & -0.45 & 0.42 & -0.37 & -0.03 & -0.07 & 0.01 \end{bmatrix}$$

Result is found by using “numpy package” of python

(b)

axis 1: $\begin{bmatrix} -0.27 & -0.51 & -0.37 & -0.22 & -0.43 & -0.35 & -0.41 \end{bmatrix}^T$

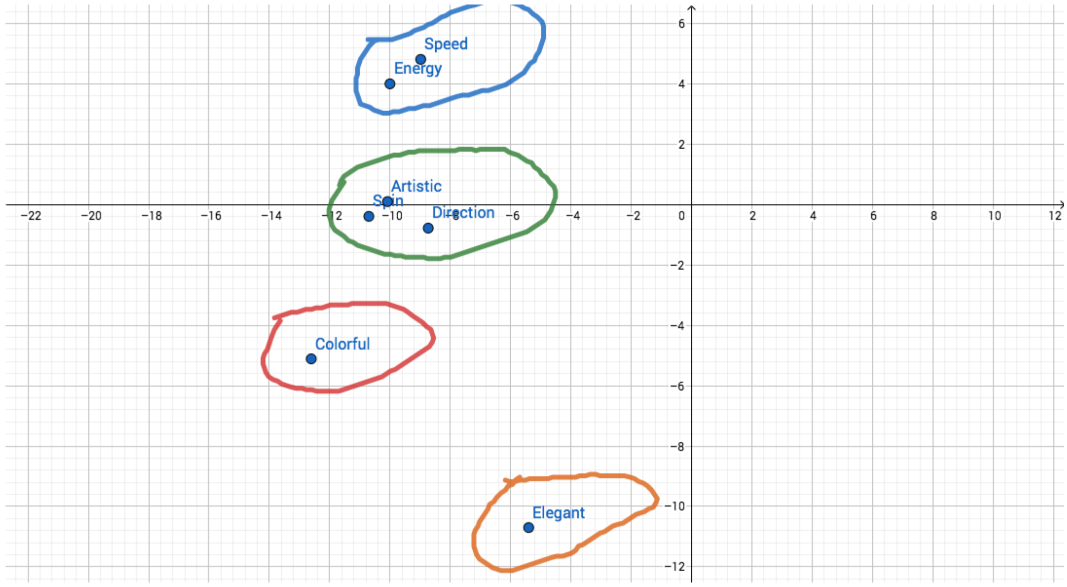
axis 2: $\begin{bmatrix} 0.69 & -0.45 & 0.42 & -0.37 & -0.03 & -0.07 & 0.01 \end{bmatrix}^T$

Coordinates of Doc under new 2-D system:

$$\begin{bmatrix} -0.27 & -0.51 & -0.37 & -0.22 & -0.43 & -0.35 & -0.41 \\ 0.69 & -0.45 & 0.42 & -0.37 & -0.03 & -0.07 & 0.01 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \\ 5 \\ 0 \\ 4 \\ 1 \\ 1 \end{bmatrix}=\begin{bmatrix} -5.68 \\ 5.37 \end{bmatrix}$$

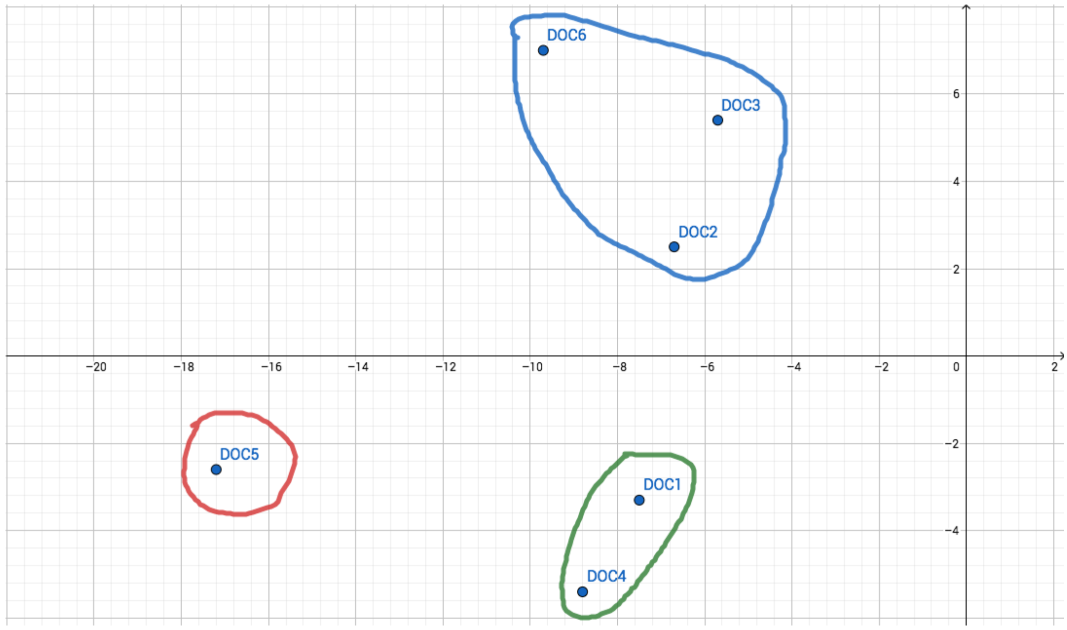
Coordinates of words:

| Energy | Colorful | Speed | Elegant | Spin | Direction | Artistic |
|---|---|---|---|---|---|---|
| [-6.74, 7.98] | [-12.61, -5.11] | [-8.98, 4.81] | [-5.4, -4.25] | [-10.7, -0.39] | [-8.73, -0.78] | [-10.08, 0.1] |



Coordinates of DOC:

| DOC1 | DOC2 | DOC3 | DOC4 | DOC5 | DOC6 |
|---|---|---|---|---|---|
| [-7.51, -3.25] | [-6.69, 2.46] | [-5.68, 5.37] | [-8.75, -5.39] | [-17.24, -2.64] | [-9.73, 7.02] |


DOC6
DOC3
DOC2
DOC5
DOC1
DOC4