Turtle Fan Club (Letao Zhang, Haoming Yu, Lian Huang) 的 icpc 模板

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目录

1 杂项

1.1 fastio

```
static char buf[1000000],*p1=buf,*p2=buf;
#define getchar() p1==p2&&(p2=(p1=buf)+fread(buf,1,1000000,stdin),p1==p2)?EOF:*p1++
inline int read(){int x=0,f=1;char c=getchar();while(c<'0'||c>'9'){if(c=='-')f=-1;c=getchar();}
    while(c>='0'&&c<='9'){x=(x<<3)+(x<<1)+c-48;c=getchar();}return x*f;}
inline void write(int x){static char buf[20];static int len=-1;if(x<0)putchar('-'),x=-x;do buf[++
    len]=x%10,x/=10;while(x);while(len>=0)putchar(buf[len--]+48);}
```

1.2 取模优化

1.3 i24

2 dp

2.1 决策单调性

2.1.1 SMAWK

求 $n \times m$ 矩阵每行最小值。矩阵满足对于任意 $x < y$，x 行最小值位置 $\leq y$ 行最小值位置。

cf 的交互格式，没有实际用过。

$4(n+m)$ 次。

```
int n,m,a[10][10];
map<pii,int> mp;
int pre[maxn],suf[maxn];
vector<int> reduce(vector<int> x,vector<int> y){
    int n=x.size(),m=y.size();
    for(int i=0;i<m-1;i++)pre[y[i+1]]=y[i],suf[y[i]]=y[i+1];
    pre[y[0]]=0,suf[0]=y[0],suf[y[m-1]]=y[m-1];
    auto del=[&](int u){suf[pre[u]]=suf[u],pre[suf[u]]=pre[u];};
    for(int i=0,j=y[0],t=n+1;t<=m;){
        if(ask(x[i],j)>ask(x[i],suf[j])){
            del(j);t++;
            if(i)i--,j=pre[j];
            else j=suf[j];
        }
        else{
            if(i==n-1)del(suf[j]),t++;
            else i++,j=suf[j];
        }
    }
    y.clear();
    for(int i=0,j=suf[0];i<n;i++,j=suf[j])y.pb(j);
    return y;
}
int p[maxn];
void smawk(vector<int> x,vector<int> y){
    y=reduce(x,y);
    if(x.size()==1){
        p[x[0]]=y[0];
        return ;
    }
    vector<int> z;
    for(int i=0;i<x.size();i++)if(!(i&1))z.pb(x[i]);
    smawk(z,y);
    for(int i=1;i<x.size();i+=2){
        int l=lower_bound(y.begin(),y.end(),p[x[i-1]])-y.begin();
        int r=(i==x.size()-1?y.size()-1:lower_bound(y.begin(),y.end(),p[x[i+1]])-y.begin());
        p[x[i]]=y[l];
        for(int j=l+1;j<=r;j++)if(ask(x[i],y[j])<ask(x[i],p[x[i]]))p[x[i]]=y[j];
    }
}
void work(){
    n=read();m=read();
    vector<int> x(n),y(m);
    for(int i=1;i<=n;i++)x[i-1]=i;
    for(int i=1;i<=m;i++)y[i-1]=i;
    smawk(x,y);
    int ans=inf;for(int i=1;i<=n;i++)ans=min(ans,ask(i,p[i]));
```

```
    printf("! %d\n",ans);fflush(stdout);
}
```

2.1.2 LARSCH

基于魔改的分治，可以在线，$O(n \log n)$，支持类莫队计算贡献，常数小，码量小。

```
struct ds{
    int l,r,ans;
    ds(){l=1,r=0;}
    ll que(int ql,int qr){
        while(r<qr)r++;
        while(l>ql)l--;
        while(r>qr)r--;
        while(l<ql)l++;
        return ans;
    }
}a[2];
void upd(int j,int i,int op){
    int nw=dp[j-1]+a[op].que(j,i)+w;
    if(nw<dp[i])dp[i]=nw,p[i]=j;
}
void sovle(int l,int r){
    if(l==r)return ;
    int mid=l+r>>1;
    for(int i=p[l];i<=p[r];i++)upd(i,mid,0);
    sovle(l,mid);
    for(int i=l;i<=mid;i++)upd(i,r,1);
    sovle(mid+1,r);
}
```

3 ds

3.1 线段树

3.1.1 zkw

3.1.2 楼房重建

```
int mn[maxn<<2],ans[maxn<<2];
int query(int nd,int l,int r,int w){
    if(w<mn[nd])return (r-l+1)*w;
    if(l==r)return mn[nd];
    if(mn[ls]<=w)return query(ls,l,mid,w)+ans[rs];
    else return (mid-l+1)*w+query(rs,mid+1,r,w);
}
void up(int nd,int l,int r){
    mn[nd]=min(mn[ls],mn[rs]);
    ans[rs]=query(rs,mid+1,r,mn[ls]);
}
void build(int nd,int l,int r){
    if(l==r){
        mn[nd]=a[l];
        return ;
    }
    build(ls,l,mid),build(rs,mid+1,r);
    up(nd,l,r);
}
int query(int nd,int l,int r,int ql,int qr,int &w){
    if(l>=ql&&r<=qr){
        int res=query(nd,l,r,w);
        w=min(w,mn[nd]);
        return res;
    }
    int res=0;
    if(ql<=mid)res+=query(ls,l,mid,ql,qr,w);
    if(qr>mid)res+=query(rs,mid+1,r,ql,qr,w);
    return res;
}
```

3.1.3 beats

3.1.4 合并分裂

```
int merge(int u,int v,int l,int r){
    if(!u||!v)return u|v;
    if(l==r){tree[u]+=tree[v];clr(v);return u;}
    lc[u]=merge(lc[u],lc[v],l,mid);
    rc[u]=merge(rc[u],rc[v],mid+1,r);
    tree[u]=tree[lc[u]]+tree[rc[u]];clr(v);
    return u;
}
int split(int nd,int l,int r,ll k){
    if(!nd)return 0;
    int u=newnode();
    if(k>tree[ls])rc[u]=split(rs,mid+1,r,k-tree[ls]);
    else rc[u]=rs,rs=0;
```

```
    if(k<tree[ls])lc[u]=split(ls,l,mid,k);
    tree[nd]=tree[ls]+tree[rs],tree[u]=tree[lc[u]]+tree[rc[u]];
    return u;
}
```

3.1.5 KTT

3.2 手写 STL

3.2.1 bitset

```
#define ull unsigned long long
ull pw[65];
struct bs{
    vector<ull> a;
    int len,n;
    void init(int _n){
        n=_n,len=(n+63)/64;a.resize(len+1,0);
    }
    void set0(int x){a[x>>6]&=~pw[x&63];}
    void set1(int x){a[x>>6]|=pw[x&63];}
    bool operator[](int x){return (a[x>>6]>>(x&63))&1;}
    bs operator<<(int x)const{
        bs res;res.init(n);
        int y=x>>6,z=x&63;
        ull lst=0;
        for(int i=0;i+y<res.len;i++){
            res.a[i+y]=lst|(a[i]<<z);
            if(z)lst=a[i]>>(64ll-z);
        }
        return res;
    }
}f;
```

3.2.2 哈希表

```
struct hsh_table{
    int head[maxn],tot;
    struct nd{
        int nxt;ull key;int val;
    }e[maxn];
    inline int hsh(ull u){return u%maxn;}
    inline int &operator[](ull key){
        int u=hsh(key);
        for(int i=head[u];i;i=e[i].nxt){
            if(e[i].key==key)return e[i].val;
        }
        e[++tot]={head[u],key,0};head[u]=tot;
        return e[tot].val;
    }
}mp;
```

3.2.3 deque

```
vector<int> st,ed;
void push_front(int x){st.pb(x);}
void push_back(int x){ed.pb(x);}
void rebuild(){

}
void rebuildfront(){
    int pos=(ed.size()+1)/2;
    for(int i=0;i<pos;i++)st.pb(ed[i]);
    reverse(st.begin(),st.end());
    reverse(ed.begin(),ed.end());
    for(int i=1;i<=pos;i++)ed.pop_back();
    reverse(ed.begin(),ed.end());
    rebuild();
}
void rebuildback(){
    int pos=(st.size()+1)/2;
    for(int i=0;i<pos;i++)ed.pb(st[i]);
    reverse(ed.begin(),ed.end());
    reverse(st.begin(),st.end());
    for(int i=1;i<=pos;i++)st.pop_back();
    reverse(st.begin(),st.end());
    rebuild();
}
int front(){
    if(!st.size())rebuildfront();
    return st.back();
}
int back(){
    if(!ed.size())rebuildback();
    return ed.back();
}
void pop_front(){
    if(!st.size())rebuildfront();
    st.pop_back();
}
void pop_back(){
    if(!ed.size())rebuildback();
    ed.pop_back();
}
int size(){
    return st.size()+ed.size();
}
```

4 graph

4.1 树

4.1.1 毛毛虫剖分

```
int n,q,k=3;
vector<int> e[maxn];
int siz[maxn],son[maxn],fa[maxn];
int dfn[maxn],st[18][maxn],tim;
void dfs(int u){
    siz[u]=1;
    st[0][dfn[u]=++tim]=fa[u];
    vector<int> tmp;
    for(int v:e[u])if(v!=fa[u]){
        fa[v]=u;dfs(v);siz[u]+=siz[v];
        tmp.pb(v);
    }
    e[u]=tmp;
    sort(e[u].begin(),e[u].end(),[&](int u,int v){return siz[u]>siz[v];});
    if(e[u].size())son[u]=e[u][0];
}
int id[maxn],idx;
void downid(int u,int d){
    if(!d){
        if(!id[u])id[u]=++idx;
        return ;
    }
    for(int v:e[u])downid(v,d-1);
}
void dfsid(int u){
    vector<int> path;
    for(int x=u;x;x=son[x])path.pb(x);
    for(int i=0;i<=k;i++){
        for(int u:path)downid(u,i);
    }
    reverse(path.begin(),path.end());
    for(int u:path){
        for(int v:e[u])if(v!=son[u])dfsid(v);
    }
}
void merge(vector<pii> &u,vector<pii> v){
    for(auto p:v)u.pb(p);
}
void reinit(vector<pii> &u){
    sort(u.begin(),u.end());
    vector<pii> nw;
    for(auto[l,r]:u){
        if(nw.size()&&nw.back().se+1==l)nw.back().se=r;
        else nw.pb({l,r});
    }
    u=nw;
}
vector<pii> sub[maxn],kson[maxn][maxk],bro[maxn][maxk];
void dfsup(int u){
    sub[u]={{id[u],id[u]}},kson[u][0]={{id[u],id[u]}};
    for(int v:e[u]){
```

```
        dfsup(v);
        merge(sub[u],sub[v]);
        for(int i=0;i<=k;i++)bro[v][i]=kson[u][i];
        for(int i=1;i<=k;i++)merge(kson[u][i],kson[v][i-1]),reinit(kson[u][i]);
    }
    if(e[u].size()){
        vector<pii> tmp[maxk];
        for(int ii=e[u].size()-1;~ii;ii--){
            int v=e[u][ii];
            for(int i=1;i<=k;i++)merge(bro[v][i],tmp[i]),reinit(bro[v][i]);
            for(int i=1;i<=k;i++)merge(tmp[i],kson[v][i-1]),reinit(tmp[i]);
        }
    }
    reinit(sub[u]);
}
vector<pii> kans[maxn][maxk],kdis[maxn][maxk];
void dfsdw(int u){
    for(int i=0;i<=k;i++){
        kans[u][i]=kans[fa[u]][i];
        merge(kans[u][i],kson[u][i]);
        reinit(kans[u][i]);
    }
    for(int i=0;i<=k;i++){
        for(int j=0;j<=i;j++)merge(kdis[u][i],kson[u][j]);
        for(int j=1,x=u;j<=k&&x;x=fa[x],j++){
            for(int k=0;k<=i-j;k++)merge(kdis[u][i],bro[x][k]);
        }
        reinit(kdis[u][i]);
    }
    for(int v:e[u])dfsdw(v);
}
vector<pii> getsub(int u){return sub[u];}
vector<pii> gettp(int u,int tp,int k){
    vector<pii> a=kans[u][k],b=kans[tp][k],nw;
    for(int i=0,l=0;i<a.size();i++){
        while(l<b.size()&&b[l].se<a[i].fi)l++;
        int r=l;while(r<b.size()&&b[r].se<=a[i].se)r++;
        if(l==r)nw.pb(a[i]);
        else{
            int lst=a[i].fi;
            for(int j=l;j<r;j++){
                if(lst<b[j].fi)nw.pb({lst,b[j].fi-1});
                lst=b[j].se+1;
            }
            if(lst<=a[i].se)nw.pb({lst,a[i].se});
        }
        l=r;
    }
    reinit(nw);
    return nw;
}
vector<pii> getpath(int u,int v,int k){
    int tp=lca(u,v);
    vector<pii> a=kdis[tp][k];
    merge(a,gettp(u,tp,k));
    merge(a,gettp(v,tp,k));
    reinit(a);
```

```
    return a;
}
void work(){
    dfs(1);
    lca_init();
    dfsid(1);
    dfsup(1);
    dfsdw(1);
}
```

4.1.2 Prüfer 序

```
void fa2prufer() {
    for (int i = 1; i < n; ++i) {
        scanf("%lld", &f[i]);
        ++du[f[i]];
    }
    for (int i = 1, j = 1; i <= n - 2; ++i, ++j) {
        while (du[j]) {
            ++j;
        }
        p[i] = f[j];
        while (i <= n - 2 && !--du[p[i]] && p[i] < j) {
            p[i + 1] = f[p[i]];
            ++i;
        }
    }
}

void prufer2fa() {
    for (int i = 1; i <= n - 2; ++i) {
        scanf("%lld", &p[i]);
        ++du[p[i]];
    }
    p[n - 1] = n;
    for (int i = 1, j = 1; i < n; ++i, ++j) {
        while (du[j]) {
            ++j;
        }
        f[j] = p[i];
        while (i < n && !--du[p[i]] && p[i] < j) {
            f[p[i]] = p[i + 1];
            ++i;
        }
    }
}
```

4.1.3 prufer 序

4.1.4 并查集启发式分裂

```
set<int> e[maxn],id[maxn<<1];
int fa[maxn],idx;
ll del[maxn*3];
struct node{
```

```
    int u,fa;
    set<int>::iterator it;
};
void add(int u,int v){
    e[u].insert(v),e[v].insert(u);
    int uu=fa[u],vv=fa[v];
    sum+=1ll*id[uu].size()*id[vv].size();
    if(id[uu].size()<id[vv].size())swap(u,v),swap(uu,vv);
    for(int i:id[vv])fa[i]=uu,id[uu].insert(i);id[vv].clear();
}
void del(int u,int v){
    e[u].erase(v),e[v].erase(u);
    vector<int> pos[2];
    queue<node> que[2];
    pos[0].pb(u),pos[1].pb(v);
    if(e[u].size())que[0].push({u,0,e[u].begin()});
    if(e[v].size())que[1].push({v,0,e[v].begin()});
    while(que[0].size()&&que[1].size()){
        int o=pos[1].size()<pos[0].size();
        auto[u,fa,it]=que[o].front();que[o].pop();
        int v=(*it);
        if(v!=fa){
            pos[o].pb(v);
            if(e[v].size())que[o].push({v,u,e[v].begin()});
        }
        it++;
        if(it==e[u].end())continue;
        if((*it)==fa)it++;
        if(it==e[u].end())continue;
        que[o].push({u,fa,it});
    }
    if(!que[0].size()&&(que[1].size()||pos[0].size()<pos[1].size())){
        swap(u,v),swap(pos[0],pos[1]);
    }
    del[qq]+=1ll*pos[1].size()*(id[fa[u]].size()-pos[1].size());
    ++idx;
    for(int i:pos[1])id[fa[u]].erase(i),id[idx].insert(i),fa[i]=idx;
}
```

4.1.5 Maintaining mst with online edge insertions —no LCT needed

比 lct 快 10 倍。

```
mt19937 rnd(time(0));
struct weightdsu{
    int fa[maxn],rd[maxn];pii val[maxn];
    int siz[maxn];
    weightdsu(int n){
        for(int i=1;i<=n;i++)fa[i]=i,rd[i]=rnd(),val[i]={inf,inf};
    }
    int fd(int u,pii w={inf-1,inf}){
        while(val[u]<=w){
            while(val[fa[u]]<=val[u]){
                // siz[fa[u]]-=siz[u];
                fa[u]=fa[fa[u]];
            }
            u=fa[u];
```

```
        }
        return u;
    }
    void del(int u){
        // if(fa[u]==u)return ;
        // del(fa[u]);siz[fa[u]]-=siz[u];
    }
    int ins(int u,pii w={inf-1,inf}){
        while(val[u]<=w){
            // siz[fa[u]]+=siz[u];
            u=fa[u];
        }
        return u;
    }
    void merge(int u,int v,pii w){
        del(u),del(v);
        while(u!=v){
            u=ins(u,w),v=ins(v,w);
            if(rd[u]<rd[v])swap(u,v);
            swap(fa[u],v),swap(val[u],w);
        }
        ins(u);
    }
    pii max_path(int u,int v){
        int uu=fd(u),vv=fd(v);
        if(uu!=vv)return {inf,-1};
        if(val[u]>val[v])swap(u,v);
        while(fa[u]!=v){
            u=fa[u];
            if(val[u]>val[v])swap(u,v);
        }
        return val[u];
    }
    pii del_path(int u,int v){
        int uu=fd(u),vv=fd(v);
        if(uu!=vv)return {inf,-1};
        if(val[u]>val[v])swap(u,v);
        while(fa[u]!=v){
            u=fa[u];
            if(val[u]>val[v])swap(u,v);
        }
        v=u;
        while(fa[v]!=v){
            // siz[fa[v]]-=siz[v];
            v=fa[v];
        }
        fa[u]=u;
        pii res={inf,inf};swap(res,val[u]);
        return res;
    }
    pii add_edge(int u,int v,pii w){
        pii res=del_path(u,v);
        if(res<=w)swap(res,w);
        merge(u,v,w);
        return res;
    }
};
```

4.2 连通性

4.2.1 边双

```
int dfn[maxn],lw[maxn],idx;
int st[maxn],tp;
vector<int> g[maxn];
int scct;
bool vis[maxn];
void tar(int u,int fl){
    dfn[u]=lw[u]=++idx;st[++tp]=u;
    for(int i=head[u];i;i=e[i].nxt){
        int v=e[i].to;
        if(i==(fl^1))continue;
        if(!dfn[v]){
            tar(v,i);
            lw[u]=min(lw[u],lw[v]);
        }
        else lw[u]=min(lw[u],dfn[v]);
    }
    if(lw[u]==dfn[u]){
        g[++scct].push_back(st[tp]);
        while(st[tp--]!=u){
            g[scct].push_back(st[tp]);
        }
    }
}
```

4.2.2 点双

```
vector<int> e[maxn],g[maxn];
int dfn[maxn],idx,lw[maxn];
int st[maxn],tp;
void tar(int u){
    dfn[u]=lw[u]=++idx;st[++tp]=u;
    for(int v:e[u]){
        if(!dfn[v]){
            tar(v);
            lw[u]=min(lw[u],lw[v]);
            if(lw[v]>=dfn[u]){
                g[++num].push_back(st[tp]);
                while(st[tp--]!=v){
                    g[num].push_back(st[tp]);
                }
                g[num].push_back(u);
            }
        }
        else lw[u]=min(lw[u],dfn[v]);
    }
}
```

4.2.3 双极定向

```
int n,m,s,t;
pii g[maxn];
```

```
int lw[maxn],dfn[maxn],idx,fa[maxn];
vector<int> id;
bool vis[maxn];
bool dfs(int u){
    dfn[u]=lw[u]=++idx;vis[u]=1;
    bool fl=u==t;
    for(int i=head[u];i;i=e[i].nxt){
        int v=e[i].to;
        if(!vis[v]){
            fa[v]=u;fl|=dfs(v);
            lw[u]=min(lw[u],lw[v]);
        }
        else lw[u]=min(lw[u],dfn[v]);
    }
    if(fl)id.pb(u);
    return fl;
}
queue<int> q;
int d[maxn];
vector<int> a[maxn];
int st[maxn],tp,rnk[maxn];
void dfs1(int u){
    if(vis[u])return ;vis[u]=1;
    st[++tp]=u;
    for(int v:a[dfn[u]])dfs1(v);
}
void work(){
    n=read();m=read();s=read();t=read();
    for(int i=1;i<=n;i++)head[i]=0;tot=0;
    for(int i=1;i<=m;i++){
        int u=read(),v=read();
        add(u,v),add(v,u);
        g[i]={u,v};
    }
    idx=0;id.clear();
    for(int i=1;i<=n;i++)vis[i]=0;
    fa[s]=0;dfs(s);
    for(int i=1;i<=n;i++)d[i]=0;
    for(int i:id)d[i]++;
    for(int i=1;i<=n;i++)d[fa[i]]++;
    for(int i=1;i<=n;i++)if(!d[i])q.push(i);
    for(int i=1;i<=n;i++)a[i].clear();
    while(!q.empty()){
        int u=q.front();q.pop();
        a[lw[u]].pb(u),a[dfn[fa[u]]].pb(u);
        d[fa[u]]--;
        if(!d[fa[u]])q.push(fa[u]);
    }
    tp=0;
    for(int i=1;i<=n;i++)vis[i]=0;
    while(id.size())dfs1(id.back()),id.pop_back();
    if(st[1]!=s||st[tp]!=t){puts("No");return ;}
    check();
}
```

4.2.4 广义串并联图

```
map<int,int> mp[maxn];
int d[maxn];
queue<int> q;
void add(int u,int v,int w){
    if(mp[u].find(v)!=mp[u].end())ans=max(ans,mp[u][v]+w);
    else mp[u][v]=-inf,d[u]++;
    mp[u][v]=max(mp[u][v],w);
}
void work(){
    n=read();m=read();
    for(int i=1;i<=m;i++){
        int u=read(),v=read(),w=read();
        add(u,v,w),add(v,u,w);
    }
    for(int i=1;i<=n;i++)if(d[i]<=2)q.push(i);
    while(!q.empty()){
        int u=q.front();q.pop();
        if(!d[u])continue;
        else if(d[u]==1){
            int v=(*mp[u].begin()).fi;
            mp[u].erase(v),mp[v].erase(u),d[u]--,d[v]--;
            if(d[v]<=2)q.push(v);
        }
        else if(d[u]==2){
            int v1=(*mp[u].begin()).fi,v2=(*--mp[u].end()).fi;
            int w1=(*mp[u].begin()).se,w2=(*--mp[u].end()).se;
            add(v1,v2,w1+w2),add(v2,v1,w1+w2);
            mp[u].erase(v1),mp[u].erase(v2),mp[v1].erase(u),mp[v2].erase(u),d[u]-=2,d[v1]--,d[v2
                ]--;
            if(d[v1]<=2)q.push(v1);
            if(d[v2]<=2)q.push(v2);
        }
    }
    printf("%lld\n",ans);
}
```

4.3 流

4.3.1 预留推进

4.3.2 原始对偶

```
int h[maxn];bool vis[maxn];
void spfa(){
    queue<int> q;
    for(int i=1;i<=n;i++)h[i]=inf,vis[i]=0;
    h[s]=0,vis[s]=1,q.push(s);
    while(!q.empty()){
        int u=q.front();q.pop();vis[u]=0;
        for(int i=head[u];i;i=e[i].nxt){
            int v=e[i].to;
            if(e[i].w&&h[v]>h[u]+e[i].c){
                h[v]=h[u]+e[i].c;
                if(!vis[v])vis[v]=1,q.push(v);
```

```
            }
        }
    }
}
int dis[maxn],pre[maxn],id[maxn];
bool dij(){
    priority_queue<pii> q;
    for(int i=1;i<=n;i++)dis[i]=inf,vis[i]=0;
    dis[s]=0;q.push({0,s});
    while(!q.empty()){
        int u=q.top().se;q.pop();
        if(vis[u])continue;vis[u]=1;
        for(int i=head[u];i;i=e[i].nxt){
            int v=e[i].to,val=e[i].c+h[u]-h[v];
            if(e[i].w&&dis[v]>dis[u]+val){
                dis[v]=dis[u]+val,pre[v]=u,id[v]=i;
                q.push({-dis[v],v});
            }
        }
    }
    return dis[t]!=inf;
}
void work(){
    n=read();m=read();s=read();t=read();
    for(int i=1;i<=m;i++){
        int u=read(),v=read(),w=read(),c=read();
        add(u,v,w,c);
    }
    spfa();
    while(dij()){
        for(int i=1;i<=n;i++)h[i]+=dis[i];
        int mn=inf;
        for(int u=t;u!=s;u=pre[u])mn=min(mn,e[id[u]].w);
        flow+=mn;
        for(int u=t;u!=s;u=pre[u]){
            e[id[u]].w-=mn,e[id[u]^1].w+=mn;
            ans+=e[id[u]].c*mn;
        }
    }
    printf("%lld %lld\n",flow,ans);
}
```

4.3.3 k 正则二分图完美匹配

单次期望 $O(n \log n)$。

```
mt19937 rnd(1);
int id[maxn];
int st[maxn*maxn],tp;
int to[maxn<<1];
bool vis[maxn<<1];
int que[maxn*maxn],tl;
vector<vector<int>> color(vector<vector<int>> e){
    vector<vector<int>> ans;
    int n=e.size()-1,d=e[1].size();
    for(int i=1;i<=n;i++)id[i]=i;
    for(int i=1;i<=n;i++){
```

```
        for(int j=0;j<d;j++)e[i][j]+=n;
    }
    while(d){
        vector<int> res;
        shuffle(id+1,id+n+1,rnd);
        for(int i=1;i<=2*n;i++)to[i]=0;
        for(int i=1;i<=n;i++){
            int x=id[i],y=0;st[++tp]=x;
            while(!y||to[y]){
                while(!y||y==to[x])y=e[x][rnd()%d];
                st[++tp]=y,x=to[y],st[++tp]=x;
            }
            tp--;
            while(tp){
                if(vis[st[tp]]){
                    while(tl&&que[tl]!=st[tp])vis[que[tl]]=0,tl--;
                    vis[que[tl]]=0,tl--;
                }
                vis[st[tp]]=1,que[++tl]=st[tp];
                tp--;
            }
            for(int j=1;j<=tl;j++)vis[que[j]]=0;
            while(tl){
                to[que[tl-1]]=que[tl],to[que[tl]]=que[tl-1];
                tl-=2;
            }
        }
        for(int i=1;i<=n;i++){
            res.pb(to[i]-n);
            for(int j=0;j<d;j++)if(e[i][j]==to[i]){swap(e[i][j],e[i][d-1]);break;}
        }
        ans.pb(res);
        d--;
    }
    return ans;
}
```

4.3.4 一般图最大匹配

4.4 杂项

4.4.1 欧拉回路

4.4.2 四元环计数

```
vector<int> e[maxn],g[maxn];
int d[maxn],cnt[maxn],ans;
void work(){
    n=read();m=read();
    for(int i=1;i<=m;i++){
        int u=read(),v=read();
        e[u].push_back(v),e[v].push_back(u);
        d[u]++,d[v]++;
    }
    for(int u=1;u<=n;u++){
        for(int v:e[u]){
```

```
            if(d[u]>d[v]||(d[u]==d[v]&&u>v))g[u].push_back(v);
        }
    }
    for(int i=1;i<=n;i++){
        for(int j:g[i]){
            for(int k:e[j])if(d[i]>d[k]||(d[i]==d[k]&&i>k))ans+=cnt[k]++;
        }
        for(int j:g[i]){
            for(int k:e[j])cnt[k]=0;
        }
    }
    printf("%lld\n",ans);
}
```

5 geometry

```
struct node {
    ll x, y;
    node(ll _x = 0, ll _y = 0) : x(_x), y(_y) {}

    inline ll len() {
        return x * x + y * y;
    }

    inline ldb dis() {
        return sqrtl(x * x + y * y);
    }

    // 极角排序用（从 x 轴负半轴（不含）开始进行逆时针排序）
    inline int reg() {
        if (x < 0 && y < 0) {
            return 1;
        } else if (x == 0 && y < 0) {
            return 2;
        } else if (x > 0 && y < 0) {
            return 3;
        } else if (x >= 0 && y == 0) {
            return 4;
        } else if (x > 0 && y > 0) {
            return 5;
        } else if (x == 0 && y > 0) {
            return 6;
        } else if (x < 0 && y > 0) {
            return 7;
        } else {
            return 8;
        }
    }
};

typedef vector<node> conv;

inline node operator + (const node &a, const node &b) {
    return node(a.x + b.x, a.y + b.y);
}

inline node operator - (const node &a, const node &b) {
    return node(a.x - b.x, a.y - b.y);
}

inline bool operator < (const node &a, const node &b) {
    return a.x < b.x || (a.x == b.x && a.y < b.y);
}

inline bool operator == (const node &a, const node &b) {
    return a.x == b.x && a.y == b.y;
}

inline ll operator * (const node &a, const node &b) {
    return a.x * b.y - a.y * b.x;
}
```

```
inline ll dot(const node &a, const node &b) {
    return a.x * b.x + a.y * b.y;
}

// 检查是否为凸包
inline bool checkcon(conv a) {
    int n = (int)a.size(), p = 0;
    if ((a[1] - a[0]) * (a[2] - a[0]) < 0) {
        reverse(a.begin(), a.end());
    }
    for (int i = 1; i < n; ++i) {
        if (a[i] < a[p]) {
            p = i;
        }
    }
    rotate(a.begin(), a.begin() + p, a.end());
    for (int i = 1; i <= n - 2; ++i) {
        if ((a[i] - a[0]) * (a[i + 1] - a[i]) <= 0) {
            return 0;
        }
    }
    for (int i = 0; i < n; ++i) {
        int j = (i == n - 1 ? 0 : i + 1);
        int k = (j == n - 1 ? 0 : j + 1);
        if ((a[j] - a[i]) * (a[k] - a[j]) <= 0) {
            return 0;
        }
    }
    return 1;
}

// 建凸包
inline conv makecon(conv a) {
    int n = (int)a.size();
    sort(a.begin(), a.end());
    vector<int> stk(n + 1);
    int top = 0;
    for (int i = 0; i < n; ++i) {
        while (top >= 2 && (a[stk[top]] - a[stk[top - 1]]) * (a[i] - a[stk[top - 1]]) <= 0) {
            --top;
        }
        stk[++top] = i;
    }
    conv b;
    for (int i = 1; i < top; ++i) {
        b.pb(a[stk[i]]);
    }
    top = 0;
    for (int i = n - 1; ~i; --i) {
        while (top >= 2 && (a[stk[top]] - a[stk[top - 1]]) * (a[i] - a[stk[top - 1]]) <= 0) {
            --top;
        }
        stk[++top] = i;
    }
    for (int i = 1; i < top; ++i) {
        b.pb(a[stk[i]]);
```

```
    }
    return b;
}

// 凸包直径
inline ll diam(conv a) {
    int n = (int)a.size(), j = 2;
    if (n == 2) {
        return (a[0] - a[1]).len();
    }
    ll ans = 0;
    for (int i = 0; i < n; ++i) {
        ans = max(ans, (a[i] - a[(i + 1) % n]).len());
        while ((a[(i + 1) % n] - a[i]) * (a[j] - a[(i + 1) % n]) < (a[(i + 1) % n] - a[i]) * (a[(j
            + 1) % n] - a[(i + 1) % n])) {
            j = (j + 1) % n;
        }
        ans = max({ans, (a[i] - a[j]).len(), (a[(i + 1) % n] - a[j]).len()});
    }
    return ans;
}

// P 到 AB 所在直线距离
inline ldb pointdis(node a, node b, node p) {
    if (dot(p - a, b - a) <= 0) {
        return (p - a).dis();
    } else if (dot(p - b, a - b) <= 0) {
        return (p - b).dis();
    } else {
        return abs((p - a) * (p - b)) / (a - b).dis();
    }
}

// P 是否在线段 AB 上
inline bool onseg(node a, node b, node p) {
    if ((p - a) * (p - b) != 0) {
        return 0;
    } else {
        return dot(p - a, p - b) <= 0;
    }
}

// 线段 AB 和线段 CD 是否相交（含端点）
inline bool seginter(node a, node b, node c, node d) {
    auto sgn = [&](ll x) -> int {
        return x > 0 ? 1 : (x < 0 ? -1 : 0);
    };
    int o1 = sgn((b - a) * (c - a)), o2 = sgn((b - a) * (d - a)), o3 = sgn((d - c) * (a - c)), o4
        = sgn((d - c) * (b - c));
    return ((o1 * o2 < 0 && o3 * o4 < 0) || onseg(a, b, c) || onseg(a, b, d) || onseg(c, d, a) ||
        onseg(c, d, b));
}

// 线段 AB 到线段 CD 距离
inline ldb segdis(node a, node b, node c, node d) {
    return seginter(a, b, c, d) ? 0 : min({pointdis(a, b, c), pointdis(a, b, d), pointdis(c, d, a)
        , pointdis(c, d, b)});
```

```
}
```

6 math

6.1 筛

6.1.1 PN 筛

定义

Powerful Number（以下简称 PN）筛类似于杜教筛，或者说是杜教筛的一个扩展，可以拿来求一些积性函数的前缀和。

要求：

- 存在一个函数 g 满足：
  - g 是积性函数。
  - g 易求前缀和。
  - 对于质数 p，$g(p)=f(p)$。

假设现在要求积性函数 f 的前缀和 $F(n)=\sum_{i=1}^{n}f(i)$。

Powerful Number

定义：对于正整数 n，记 n 的质因数分解为 $n=\prod_{i=1}^{m}p_i^{c_i}$。$n$ 是 PN 当且仅当 $\forall 1\le i\le m, c_i>1$。

性质 1：所有 PN 都可以表示成 a^2b^3 的形式。

证明：若 c_i 是偶数，则将 $p_i^{c_i}$ 合并进 a^2 里；若 c_i 为奇数，则先将 p_i^3 合并进 b^3 里，再将 $p_i^{c_i-3}$ 合并进 a^2 里。

性质 2：n 以内的 PN 至多有 $O(\sqrt{n})$ 个。

证明：考虑枚举 a，再考虑满足条件的 b 的个数，有 PN 的个数约等于

$$\int_1^{\sqrt{n}}\sqrt[3]{\frac{n}{x^2}}\mathrm{d}x=O(\sqrt{n})$$

那么如何求出 n 以内所有的 PN 呢？线性筛找出 $\sqrt{n}$ 内的所有素数，再 DFS 搜索各素数的指数即可。由于 n 以内的 PN 至多有 $O(\sqrt{n})$ 个，所以至多搜索 $O(\sqrt{n})$ 次。

PN 筛

首先，构造出一个易求前缀和的积性函数 g，且满足对于素数 p，$g(p)=f(p)$。记 $G(n)=\sum_{i=1}^{n}g(i)$。

然后，构造函数 $h=f/g$，这里的 / 表示狄利克雷卷积除法。根据狄利克雷卷积的性质可以得知 h 也为积性函数，因此 $h(1)=1$。$f=g*h$，这里 $*$ 表示狄利克雷卷积。

对于素数 p，$f(p)=g(1)h(p)+g(p)h(1)=h(p)+g(p)\implies h(p)=0$。根据 $h(p)=0$ 和 h 是积性函数可以推出对于非 PN 的数 n 有 $h(n)=0$，即 h 仅在 PN 处取有效值。

现在，根据 $f=g*h$ 有

$$\begin{aligned}F(n)&=\sum_{i=1}^{n}f(i)\\&=\sum_{i=1}^{n}\sum_{d|i}h(d)g\left(\frac{i}{d}\right)\\&=\sum_{d=1}^{n}\sum_{i=1}^{\lfloor\frac{n}{d}\rfloor}h(d)g(i)\\&=\sum_{d=1}^{n}h(d)\sum_{i=1}^{\lfloor\frac{n}{d}\rfloor}g(i)\\&=\sum_{d=1}^{n}h(d)G\left(\left\lfloor\frac{n}{d}\right\rfloor\right)\\&=\sum_{\substack{d=1\\d\text{ is PN}}}^{n}h(d)G\left(\left\lfloor\frac{n}{d}\right\rfloor\right)\end{aligned}$$

$O(\sqrt{n})$ 找出所有 PN，计算出所有 h 的有效值。对于 h 有效值的计算，只需要计算出所有 $h(p^c)$ 处的值，就可以根据 h 为积性函数推出 h 的所有有效值。现在对于每一个有效值 d，计算 $h(d)G\left(\left\lfloor\frac{n}{d}\right\rfloor\right)$ 并累加即可得到 $F(n)$。

下面考虑计算 $h(p^c)$，一共有两种方法：一种是直接推出 $h(p^c)$ 仅与 p,c 有关的计算公式，再根据公式计算 $h(p^c)$；另一种是根据 $f=g*h$ 有 $f(p^c)=\sum_{i=0}^{c}g(p^i)h(p^{c-i})$，移项可得 $h(p^c)=f(p^c)-\sum_{i=1}^{c}g(p^i)h(p^{c-i})$，现在就可以枚举素数 p 再枚举指数 c 求解出所有 $h(p^c)$。

过程

1. 构造 g
2. 构造快速计算 G 的方法
3. 计算 $h(p^c)$
4. 搜索 PN，过程中累加答案
5. 得到结果

对于第 3 步，可以直接根据公式计算，可以使用枚举法预处理打表，也可以搜索到了再临时推。

```
ll n, pr[maxn / 5], tot, pw[999], ans;
bool vis[maxn];
int phi[maxn], a[maxn];
vector<ll> h[100100];

inline void init() {
    phi[1] = 1;
    for (int i = 2; i <= N; ++i) {
        if (!vis[i]) {
            pr[++tot] = i;
            phi[i] = i - 1;
        }
        for (int j = 1; j <= tot && i * pr[j] <= N; ++j) {
            vis[i * pr[j]] = 1;
            if (i % pr[j] == 0) {
                phi[i * pr[j]] = phi[i] * pr[j];
                break;
            }
            phi[i * pr[j]] = phi[i] * (pr[j] - 1);
```

```
        }
    }
    for (int i = 1; i <= N / 2; ++i) {
        a[i] = (a[i - 1] + phi[i * 2]) % mod;
    }
    for (int i = 1; i <= N; ++i) {
        phi[i] = (phi[i] + phi[i - 1]) % mod;
    }
}

unordered_map<ll, ll> mp, M;

ll dfs(ll n) {
    if (n <= N) {
        return phi[n];
    }
    if (mp.find(n) != mp.end()) {
        return mp[n];
    }
    ll ans = (n % mod) * ((n + 1) % mod) % mod * inv2 % mod;
    for (ll i = 2, j; i <= n; i = j + 1) {
        j = n / (n / i);
        ans = (ans - (j - i + 1) % mod * dfs(n / i) % mod) % mod;
    }
    return mp[n] = ans;
}

ll calc(ll n) {
    if (n <= N / 2) {
        return a[n];
    }
    if (M.find(n) != M.end()) {
        return M[n];
    }
    return M[n] = (dfs(n) + calc(n / 2)) % mod;
}

inline ll G(ll n) {
    return (dfs(n) + calc(n / 2) * 2 % mod) % mod;
}

void dfs(int d, ll x, ll y) {
    if (pr[d] * pr[d] > n / x) {
        ans = (ans + y * G(n / x) % mod) % mod;
        return;
    }
    int k = 0;
    while (x <= n) {
        if (k != 1) {
            dfs(d + 1, x, y * h[d][k] % mod);
        }
        x *= pr[d];
        ++k;
    }
}

void solve() {
```

```
    scanf("%lld", &n);
    for (int i = 1; i <= tot && pr[i] * pr[i] <= n; ++i) {
        int cnt = 0;
        ll x = n;
        while (x) {
            ++cnt;
            x /= pr[i];
        }
        pw[0] = 1;
        for (int j = 1; j <= cnt + 3; ++j) {
            pw[j] = pw[j - 1] * pr[i] % mod;
        }
        ll p = 1;
        for (int j = 0; p <= n; ++j, p *= pr[i]) {
            if (j == 0) {
                h[i].pb(1);
            } else if (j == 1) {
                h[i].pb(0);
            } else {
                ll t = pr[i] ^ j;
                for (int k = 0; k < j; ++k) {
                    ll g = pw[j - k - 1] * (pr[i] - 1) % mod;
                    if (pr[i] == 2) {
                        g = g * 3 % mod;
                    }
                    t = (t - g * h[i][k] % mod + mod) % mod;
                }
                h[i].pb(t);
            }
        }
    }
    dfs(1, 1, 1);
    printf("%lld\n", ans);
}
```

6.2 矩阵

6.2.1 高斯消元（模 2）

```
const int maxn = 5050;

int n, m, b[maxn], ans[maxn];
bitset<maxn> a[maxn];

void solve() {
    n = read();
    m = read();
    for (int i = 1; i <= m; ++i) {
        for (int j = 1; j <= n + 1; ++j) {
            int x = read();
            if (x) {
                a[i].set(j);
            }
        }
    }
    mems(b, -1);
```

```
    for (int i = 1, r = 1; i <= n; ++i) {
        int p = -1;
        for (int j = r; j <= m; ++j) {
            if (a[j].test(i)) {
                p = j;
                break;
            }
        }
        if (p == -1) {
            continue;
        }
        if (p != r) {
            swap(a[p], a[r]);
        }
        for (int j = 1; j <= m; ++j) {
            if (j != r && a[j].test(i)) {
                a[j] ^= a[r];
            }
        }
        ++r;
    }
    for (int i = 1; i <= m; ++i) {
        if (a[i].any()) {
            ans[a[i]._Find_first()] = a[i].test(n + 1);
        }
    }
    for (int i = 1; i <= n; ++i) {
        pc('0' + ans[i]);
        pc(" \n"[i == n]);
    }
}
```

6.2.2 行列式

```
inline int det() {
    int ans = 1;
    for (int i = 1; i <= n; ++i) {
        for (int j = i; j <= n; ++j) {
            if (a[j][i]) {
                if (j != i) {
                    swap(a[j], a[i]);
                    fix(ans = -ans);
                }
                break;
            }
        }
        if (!a[i][i]) {
            return 0;
        }
        ans = 1LL * ans * a[i][i] % mod;
        int t = qpow(a[i][i], mod - 2);
        for (int j = i; j <= n; ++j) {
            a[i][j] = 1LL * a[i][j] * t % mod;
        }
        for (int j = 1; j <= n; ++j) {
            if (i == j) {
```

```
                continue;
            }
            for (int k = i + 1; k <= n; ++k) {
                fix(a[j][k] -= 1LL * a[j][i] * a[i][k] % mod);
            }
        }
    }
    return ans;
}
```

6.2.3 特征多项式

A 是 n 阶方阵，若对于数 x，存在向量 α，$A\alpha = x\alpha$，则 x 为 A 的特征值，α 为特征向量。
即 $(xI - A)\alpha = 0$，即 $det(xI - A) = 0$。
$det(xI - A)$ 为关于 x 的至多 n 次多项式，称为特征多项式。

```
int f[maxn][maxn];
void work(){
    for(int i=1;i<=n;i++){
        for(int j=1;j<=n;j++)a[i][j]=read();
    }
    for(int i=1;i<n;i++){
        int p=-1;for(int j=i+1;j<=n;j++)if(a[j][i])p=j;
        if(p==-1)continue;
        for(int j=1;j<=n;j++)swap(a[p][j],a[i+1][j]);
        for(int j=1;j<=n;j++)swap(a[j][p],a[j][i+1]);
        for(int j=i+2;j<=n;j++){
            int val=a[j][i]*ksm(a[i+1][i])%mod;
            for(int k=1;k<=n;k++)(a[j][k]+=mod-val*a[i+1][k]%mod)%=mod;
            for(int k=1;k<=n;k++)(a[k][i+1]+=val*a[k][j])%=mod;
        }
    }
    f[0][0]=1;
    for(int i=1;i<=n;i++){
        for(int k=0;k<i;k++)(f[i][k+1]+=f[i-1][k])%=mod,(f[i][k]+=mod-a[i][i]*f[i-1][k]%mod)%=mod;
        for(int j=1;j<i;j++){
            int val=a[j][i];for(int k=j+1;k<=i;k++)val=val*a[k][k-1]%mod;
            for(int k=0;k<=j-1;k++)(f[i][k]+=mod-val*f[j-1][k]%mod)%=mod;
        }
    }
    for(int i=0;i<=n;i++)printf("%lld ",f[n][i]);
}
```

6.3 poly

6.3.1 fft

```
struct cp{
    db a,b;
    cp(db u=0,db v=0){a=u,b=v;}
    cp operator+(const cp&tmp)const{return {a+tmp.a,b+tmp.b};}
    cp operator-(const cp&tmp)const{return {a-tmp.a,b-tmp.b};}
    cp operator*(const cp&tmp)const{return {a*tmp.a-b*tmp.b,a*tmp.b+b*tmp.a};}
};
const db pi=acos(-1);
```

```
int to[maxn<<3];
void fft(vector<cp> &a,int flag){
    int n=a.size();
    for(int i=0;i<n;i++)if(i<to[i])swap(a[i],a[to[i]]);
    for(int l=2;l<=n;l<<=1){
        cp bas=cp(cos(2*pi/l),flag*sin(2*pi/l));
        int k=l>>1;
        for(int i=0;i<n;i+=l){
            cp mul=cp(1,0);
            for(int j=i;j<i+k;j++){
                cp val=mul*a[j+k];
                a[j+k]=a[j]-val,a[j]=a[j]+val;
                mul=mul*bas;
            }
        }
    }
    if(flag==-1){
        for(int i=0;i<n;i++)a[i].a/=n,a[i].b/=n;
    }
}
```

6.3.2 ntt

```
int gg=3,invg=ksm(gg);
int to[maxn<<3];
vector<int> ntt(vector<int> a,int flag){
    int n=a.size();
    for(int i=0;i<n;i++)if(i<to[i])swap(a[i],a[to[i]]);
    for(int len=2;len<=n;len<<=1){
        int bas=ksm(flag==1?gg:invg,(mod-1)/len),l=len>>1;
        for(int i=0;i<n;i+=len){
            int mul=1;
            for(int j=i;j<i+l;j++){
                int val=mul*a[j+l]%mod;
                inc(a[j+l]=a[j],mod-val);
                inc(a[j],val);
                mul=mul*bas%mod;
            }
        }
    }
    if(flag==-1){
        int inv=ksm(n);
        for(int i=0;i<n;i++)a[i]=a[i]*inv%mod;
    }
    return a;
}
vector<int> mul(vector<int> a,vector<int> b){
    int n=a.size()-1,m=b.size()-1;int k=1;
    while(k<n+m+1)k<<=1;
    vector<int> f(k),g(k);
    for(int i=0;i<=n;i++)f[i]=a[i];
    for(int i=0;i<=m;i++)g[i]=b[i];
    for(int i=0;i<k;i++)to[i]=(to[i>>1]>>1)|((i&1)?(k>>1):0);
    f=ntt(f,1),g=ntt(g,1);
    for(int i=0;i<k;i++)f[i]=f[i]*g[i]%mod;
    f=ntt(f,-1);f.resize(n+m+1);
```

```
    return f;
}
```

6.3.3 mtt

```
const int B=(1<<15)-1;
int calc(db x){return (long long)(x+0.5)%_mod;}
vector<int> mul(vector<int> a,vector<int> b){
    int n=a.size()-1,m=b.size()-1,k=1;
    while(k<n+m+1)k<<=1;
    for(int i=0;i<k;i++)to[i]=(to[i>>1]>>1)|((i&1)?(k>>1):0);
    vector<cp> f(k),g(k),h(k);
    for(int i=0;i<=n;i++)f[i]=cp(a[i]&B,0),g[i]=cp(a[i]>>15,0);
    for(int i=0;i<=m;i++)h[i]=cp(b[i]&B,b[i]>>15);
    fft(f,1),fft(g,1),fft(h,1);
    for(int i=0;i<k;i++)f[i]=f[i]*h[i],g[i]=g[i]*h[i];
    fft(f,-1),fft(g,-1);
    vector<int> ans(n+m+1);
    for(int i=0;i<=n+m;i++)ans[i]=(1ll*calc(f[i].a)+(1ll*(calc(f[i].b)+calc(g[i].a))<<15ll)%_mod
        +(1ll*calc(g[i].b)<<30ll)%_mod)%_mod;
    return ans;
}
```

6.3.4 ni ln exp

$O(n \log^2 n)$ 版本。

```
vector<int> f,g;
void cdqni(int l,int r){
    if(r-l+1<=64){
        for(int i=l;i<=r;i++){
            for(int j=l;j<i;j++)inc(g[i],1ll*g[j]*f[i-j]%mod);
            g[i]=1ll*(mod-g[i])*g[0]%mod;
        }
        return ;
    }
    if(l==r){g[l]=1ll*(mod-g[l])*g[0]%mod;return ;}
    int mid=l+r>>1;
    cdqni(l,mid);
    vector<int> ff(mid-l+1),gg(r-l+1);
    for(int i=l;i<=mid;i++)ff[i-l]=g[i];
    for(int i=0;i<=r-l;i++)gg[i]=f[i];
    ff=poly::mul(ff,gg);
    for(int i=mid+1;i<=r;i++)inc(g[i],ff[i-l]);
    cdqni(mid+1,r);
}
vector<int> ni(vector<int> a){
    int n=a.size()-1;
    f.resize(n+1),g.resize(n+1);
    for(int i=0;i<=n;i++)f[i]=a[i],g[i]=0;
    g[0]=ksm(f[0]);for(int i=1;i<=n;i++)inc(g[i],1ll*g[0]*f[i]%mod);
    cdqni(1,n);
    return g;
}
void cdqln(int l,int r){
    if(r-l+1<=64){
```

```
        for(int i=l;i<=r;i++){
            for(int j=l;j<i;j++)inc(g[i],1ll*g[j]*j%mod*f[i-j]%mod);
            g[i]=1ll*::ni[i]*(1ll*f[i]*i%mod-g[i]+mod)%mod;
        }
        return ;
    }
    if(l==r){g[l]=1ll*::ni[l]*(1ll*f[l]*l%mod-g[l]+mod)%mod;return ;}
    int mid=l+r>>1;
    cdqln(l,mid);
    vector<int> ff(mid-l+1),gg(r-l+1);
    for(int i=l;i<=mid;i++)ff[i-l]=1ll*g[i]*i%mod;
    for(int i=0;i<=r-l;i++)gg[i]=f[i];
    ff=poly::mul(ff,gg);
    for(int i=mid+1;i<=r;i++)inc(g[i],ff[i-l]);
    cdqln(mid+1,r);
}
vector<int> ln(vector<int> a){
    int n=a.size()-1;
    f.resize(n+1);g.resize(n+1);
    for(int i=0;i<=n;i++)f[i]=a[i],g[i]=0;
    f[0]=1,g[0]=0;cdqln(1,n);
    return g;
}
void cdqexp(int l,int r){
    if(r-l+1<=64){
        for(int i=l;i<=r;i++){
            for(int j=l;j<i;j++)inc(g[i],1ll*g[j]*f[i-j]%mod);
            g[i]=1ll*::ni[i]*g[i]%mod;
        }
        return ;
    }
    if(l==r){g[l]=1ll*::ni[l]*g[l]%mod;return ;}
    int mid=l+r>>1;
    cdqexp(l,mid);
    vector<int> ff(mid-l+1),gg(r-l+1);
    for(int i=l;i<=mid;i++)ff[i-l]=g[i];
    for(int i=0;i<=r-l;i++)gg[i]=f[i];
    ff=poly::mul(ff,gg);
    for(int i=mid+1;i<=r;i++)inc(g[i],ff[i-l]);
    cdqexp(mid+1,r);
}
vector<int> exp(vector<int> a){
    int n=a.size()-1;
    f.resize(n+1);g.resize(n+1);
    for(int i=0;i<=n;i++)f[i]=1l*a[i]*i%mod,g[i]=0;
    f[0]=0,g[0]=1;cdqexp(0,n);
    return g;
}
```

6.4 另一份 poly

```
const int maxn = (1 << 22) + 50;
const int mod = 998244353, G = 3;

inline void fix(int &x) {
    x += ((x >> 31) & mod);
```

```
}

inline int qpow(int b, int p) {
    int res = 1;
    while (p) {
        if (p & 1) {
            res = 1ULL * res * b % mod;
        }
        b = 1ULL * b * b % mod;
        p >>= 1;
    }
    return res;
}

typedef vector<int> poly;

int r[maxn];

inline void NTT(poly &a, int o) {
    int n = (int)a.size();
    for (int i = 0; i < n; ++i) {
        if (i < r[i]) {
            swap(a[i], a[r[i]]);
        }
    }
    static int pw[maxn];
    for (int k = 1; k < n; k <<= 1) {
        int wn = qpow(o ? G : qpow(G, mod - 2), (mod - 1) / (k << 1));
        pw[0] = 1;
        for (int i = 1; i < k; ++i) {
            pw[i] = 1ULL * pw[i - 1] * wn % mod;
        }
        for (int i = 0; i < n; i += (k << 1)) {
            for (int j = 0; j < k; ++j) {
                int x = a[i + j], y = 1ULL * a[i + j + k] * pw[j] % mod;
                fix(a[i + j] = x + y - mod);
                fix(a[i + j + k] = x - y);
            }
        }
    }
    if (!o) {
        int inv = qpow(n, mod - 2);
        for (int i = 0; i < n; ++i) {
            a[i] = 1ULL * a[i] * inv % mod;
        }
    }
}

inline poly operator * (poly a, poly b) {
    NTT(a, 1);
    NTT(b, 1);
    int n = (int)a.size();
    for (int i = 0; i < n; ++i) {
        a[i] = 1ULL * a[i] * b[i] % mod;
    }
    NTT(a, 0);
    return a;
```

```
}

inline poly mul(poly a, poly b) {
    int n = (int)a.size() - 1, m = (int)b.size() - 1, k = 0;
    if (min(n, m) <= 100) {
        poly res(n + m + 1);
        for (int i = 0; i <= n + m; ++i) {
            ull x = 0;
            for (int j = max(i - m, 0); j <= i && j <= n; ++j) {
                x += 1ULL * a[j] * b[i - j];
                if (!(j & 15)) {
                    x %= mod;
                }
            }
            res[i] = x % mod;
        }
        return res;
    }
    while ((1 << k) <= n + m) {
        ++k;
    }
    for (int i = 1; i < (1 << k); ++i) {
        r[i] = (r[i >> 1] >> 1) | ((i & 1) << (k - 1));
    }
    poly A(1 << k), B(1 << k);
    for (int i = 0; i <= n; ++i) {
        A[i] = a[i];
    }
    for (int i = 0; i <= m; ++i) {
        B[i] = b[i];
    }
    poly res = A * B;
    res.resize(n + m + 1);
    return res;
}

poly inv(poly &a, int m) {
    if (m == 1) {
        return poly(1, qpow(a[0], mod - 2));
    }
    poly b = inv(a, m >> 1), c(m), res(m);
    for (int i = 0; i < m; ++i) {
        c[i] = a[i];
        if (i < (m >> 1)) {
            res[i] = b[i] * 2 % mod;
        }
    }
    c = mul(c, mul(b, b));
    for (int i = 0; i < m; ++i) {
        fix(res[i] -= c[i]);
    }
    return res;
}

inline poly inv(poly a) {
    int n = (int)a.size() - 1;
    int t = __lg(n + 1);
```

```
    if ((1 << t) < n + 1) {
        ++t;
    }
    poly b(1 << t);
    for (int i = 0; i <= n; ++i) {
        b[i] = a[i];
    }
    b = inv(b, 1 << t);
    b.resize(n + 1);
    return b;
}

inline poly der(poly a) {
    int n = (int)a.size() - 1;
    poly res(n);
    for (int i = 1; i <= n; ++i) {
        res[i - 1] = 1ULL * a[i] * i % mod;
    }
    return res;
}

inline poly itg(poly a) {
    int n = (int)a.size() - 1;
    poly res(n + 2), I(n + 2);
    I[1] = 1;
    for (int i = 2; i <= n + 1; ++i) {
        I[i] = 1ULL * (mod - mod / i) * I[mod % i] % mod;
    }
    for (int i = 1; i <= n + 1; ++i) {
        res[i] = 1ULL * a[i - 1] * I[i] % mod;
    }
    return res;
}

inline poly ln(poly a) {
    int n = (int)a.size() - 1;
    poly res = itg(mul(der(a), inv(a)));
    res.resize(n + 1);
    return res;
}

poly exp(poly &a, int m) {
    if (m == 1) {
        poly res(1, 1);
        return res;
    }
    poly b = exp(a, m >> 1);
    b.resize(m);
    poly c = ln(b), d(m);
    for (int i = 0; i < m; ++i) {
        fix(d[i] = a[i] - c[i]);
    }
    fix(d[0] += 1 - mod);
    b.resize(m >> 1);
    poly res = mul(b, d);
    res.resize(m);
    return res;
```

```
}

inline poly exp(poly a) {
    int n = (int)a.size() - 1;
    int t = __lg(n + 1);
    if ((1 << t) < n + 1) {
        ++t;
    }
    poly b(1 << t);
    for (int i = 0; i <= n; ++i) {
        b[i] = a[i];
    }
    b = exp(b, 1 << t);
    b.resize(n + 1);
    return b;
}

inline poly pmod(poly a, poly b) {
    int n = (int)a.size() - 1, m = (int)b.size() - 1;
    if (n < m) {
        return a;
    }
    poly c = a, d = b;
    reverse(c.begin(), c.end());
    c.resize(n - m + 1);
    reverse(d.begin(), d.end());
    d.resize(n - m + 1);
    poly q = mul(c, inv(d));
    q.resize(n - m + 1);
    reverse(q.begin(), q.end());
    q = mul(q, b);
    poly res(m);
    for (int i = 0; i < m; ++i) {
        fix(res[i] = a[i] - q[i]);
    }
    return res;
}

inline poly multipoint(poly a, poly b) {
    int n = (int)a.size() - 1, m = (int)b.size() - 1;
    vector<poly> F(1 << (__lg(m + 1) + 2));
    auto dfs = [&](auto &self, int rt, int l, int r) -> void {
        if (l == r) {
            F[rt] = poly(2);
            fix(F[rt][0] = -b[l]);
            F[rt][1] = 1;
            return;
        }
        int mid = (l + r) >> 1;
        self(self, rt << 1, l, mid);
        self(self, rt << 1 | 1, mid + 1, r);
        F[rt] = mul(F[rt << 1], F[rt << 1 | 1]);
    };
    dfs(dfs, 1, 0, m);
    for (int i = 1; i < (int)F.size(); ++i) {
        reverse(F[i].begin(), F[i].end());
    }
```

```
    reverse(a.begin(), a.end());
    F[1].resize(n);
    poly q = mul(a, inv(F[1])), ans(m + 1);
    q.resize(n);
    auto dfs2 = [&](auto &self, int rt, int l, int r, poly A) -> void {
        if ((int)A.size() > r - l + 1) {
            A.erase(A.begin(), A.end() - (r - l + 1));
        }
        if (l == r) {
            ans[l] = (a[n] + 1ULL * A[0] * b[l]) % mod;
            return;
        }
        int mid = (l + r) >> 1;
        poly a = mul(A, F[rt << 1 | 1]);
        a.resize(A.size());
        self(self, rt << 1, l, mid, a);
        a = mul(A, F[rt << 1]);
        a.resize(A.size());
        self(self, rt << 1 | 1, mid + 1, r, a);
    };
    dfs2(dfs2, 1, 0, m, q);
    return ans;
}

inline int recur(int m, poly a, poly b) {
    int n = (int)a.size();
    if (m < n) {
        return a[m];
    }
    poly f(n + 1), g(2);
    g[1] = f[n] = 1;
    for (int i = 0; i < n; ++i) {
        fix(f[i] = -b[n - i]);
    }
    auto dfs = [&](auto &self, int k) -> poly {
        if (k < n) {
            poly res(k + 1);
            res[k] = 1;
            return res;
        }
        auto res = self(self, k >> 1);
        res = mul(res, res);
        if (k & 1) {
            res.insert(res.begin(), 0);
        }
        res = pmod(res, f);
        return res;
    };
    auto res = dfs(dfs, m);
    int ans = 0;
    for (int i = 0; i < n; ++i) {
        fix(ans += 1ULL * a[i] * res[i] % mod - mod);
    }
    return ans;
}
```

6.5 集合幂级数

6.5.1 FWT

```
void fwtor(int *a,int n,int fl=1){
    for(int l=2;l<=n;l<<=1){
        int k=l>>1;
        for(int i=0;i<n;i+=l){
            for(int j=i;j<i+k;j++)(a[j+k]+=a[j]*fl)%=mod;
        }
    }
}
void fwtand(int *a,int n,int fl=1){
    for(int l=2;l<=n;l<<=1){
        int k=l>>1;
        for(int i=0;i<n;i+=l){
            for(int j=i;j<i+k;j++)(a[j]+=a[j+k]*fl)%=mod;
        }
    }
}
void fwtxor(int *a,int n,int fl=1){
    for(int l=2;l<=n;l<<=1){
        int k=l>>1;
        for(int i=0;i<n;i+=l){
            for(int j=i;j<i+k;j++){
                int u=a[j],v=a[j+k];
                a[j]=(u+v)*fl%mod,a[j+k]=(u+mod-v)*fl%mod;
            }
        }
    }
}
```

6.5.2 子集卷积 ln exp

```
void fmt(int *a,int n,int w=1){
    for(int i=0;i<n;i++){
        for(int s=0;s<(1<<n);s++)if(s&(1<<i))(a[s]+=a[s^(1<<i)]*w)%=mod;
    }
}
int ff[maxn+1][1<<maxn],gg[maxn+1][1<<maxn],hh[1<<maxn],ni[maxn+1];
void xormul(int *a,int *b,int *c,int n){
    for(int i=0;i<=n;i++){
        for(int s=0;s<(1<<n);s++)ff[i][s]=gg[i][s]=0;
    }
    for(int s=0;s<(1<<n);s++)ff[__builtin_popcount(s)][s]=a[s];
    for(int s=0;s<(1<<n);s++)gg[__builtin_popcount(s)][s]=b[s];
    for(int i=0;i<=n;i++)fmt(ff[i],n,1);
    for(int i=0;i<=n;i++)fmt(gg[i],n,1);
    for(int s=0;s<(1<<n);s++){
        for(int i=0;i<=n;i++){
            hh[i]=0;
            for(int j=0;j<=i;j++)(hh[i]+=ff[j][s]*gg[i-j][s])%=mod;
        }
        for(int i=0;i<=n;i++)ff[i][s]=hh[i];
    }
    /*ln
```

```
        for(int s=0;s<(1<<n);s++){
            for(int i=0;i<n;i++){
                hh[i]=ff[i+1][s]*(i+1)%mod;
                for(int j=1;j<=i;j++)(hh[i]+=mod-ff[j][s]*hh[i-j]%mod)%=mod;
            }
            for(int i=1;i<=n;i++)ff[i][s]=hh[i-1]*ni[i]%mod;
        }
    */
    /*exp
        for(int s=0;s<(1<<n);s++){
            for(int i=0;i<n;i++){
                hh[i]=ff[i+1][s]*(i+1)%mod;
                for(int j=1;j<=i;j++)(hh[i]+=ff[j][s]*j%mod*hh[i-j]%mod*ni[i-j+1])%=mod;
            }
            for(int i=1;i<=n;i++)ff[i][s]=hh[i-1]*ni[i]%mod;
        }
    */
    for(int i=0;i<=n;i++)fmt(ff[i],n,mod-1);
    for(int s=0;s<(1<<n);s++)c[s]=ff[__builtin_popcount(s)][s];
}
```

6.5.3 多项式复合集合幂级数

6.6 杂项

6.6.1 插值

6.6.2 基于值域预处理的快速离散对数

复杂度 $O(\frac{mod^{\frac{3}{4}}}{\log^{\frac{1}{2}} mod}+q\log mod)$。

```
int B,bas,h0,n;
struct hsh_table{
}mp;
int bsgs(int v){
    int mul=v;
    for(int i=0;i<=mod/B;i++){
        if(mp.find(mul))return i*B+mp[mul];
        mul=1ll*mul*bas%mod;
    }
}
/*
mod=ba+c
log(a)=log(-c)-log(b)=log(mod-1)+log(c)-log(b)
log(a)=log(a-c)-log(b+1)
min(c,a-c)<=a/2
*/
int h[maxn];
int sovle(int a){
    int b=mod/a,c=mod%a;
    if(a<=n)return h[a];
    if(c<a-c)return inc(inc(h0,sovle(c)),(mod-1-h[b]));
    else return inc(sovle(a-c),mod-1-h[b+1]);
}
bool vis[maxn];
int pre[maxn],cnt;
int a[maxn],b[maxn];
```

```
void init(){
    n=sqrt(mod)+1;B=sqrt(1ll*mod*n/log2(n));
    // cout<<n<<" "<<B<<"\n";
    int mul=1;for(int i=0;i<B;i++){
        mp[mul]=i;
        mul=1ll*mul*g%mod;
    }
    bas=ksm(ksm(g,B));
    h0=bsgs(mod-1);
    h[1]=0;
    for(int i=2;i<=n;i++){
        if(!vis[i]){
            h[i]=bsgs(i);
            pre[++cnt]=i;
        }
        for(int j=1;j<=cnt&&1ll*i*pre[j]<=n;j++){
            vis[i*pre[j]]=0;
            h[i*pre[j]]=(h[i]+h[pre[j]])%(mod-1);
            if(i%pre[j]==0)break;
        }
    }
}
```

7 string

7.1 Manacher

```
for (int i = 1; i <= n; ++i) {
    s[i * 2 - 1] = '#';
    s[i * 2] = t[i];
}
s[n * 2 + 1] = '#';
int ans = 0;
for (int i = 1, mid = 0, r = 0; i <= n * 2 + 1; ++i) {
    if (i <= r) {
        f[i] = min(f[mid * 2 - i], r - i + 1);
    }
    while (i + f[i] <= n * 2 + 1 && i - f[i] >= 1 && s[i + f[i]] == s[i - f[i]]) {
        ++f[i];
    }
    if (i + f[i] - 1 > r) {
        mid = i;
        r = i + f[i] - 1;
    }
    ans = max(ans, f[i] - 1);
}
```

7.2 Z 函数

```
z[1] = n;
for (int i = 2, l = 0, r = 0; i <= n; ++i) {
    if (i <= r) {
        z[i] = min(z[i - l + 1], r - i + 1);
    }
    while (i + z[i] <= n && s[z[i] + 1] == s[i + z[i]]) {
        ++z[i];
    }
    if (i + z[i] - 1 > r) {
        l = i;
        r = i + z[i] - 1;
    }
}
```

7.3 Runs

```
const int maxn = 1000100;
const int logn = 22;

int n, m, sa[maxn], id[maxn], old[maxn << 1], h[maxn], cnt[maxn];
char s[maxn];
pii p[maxn];

struct SA {
    int f[logn][maxn], rk[maxn];
    char t[maxn];

    inline int qmin(int l, int r) {
```

```
    int k = __lg(r - l + 1);
    return min(f[k][l], f[k][r - (1 << k) + 1]);
}

inline int lcp(int x, int y) {
    if (x == y) {
        return n - x + 1;
    }
    if (t[x] != t[y]) {
        return 0;
    }
    if (t[x + 1] != t[y + 1]) {
        return 1;
    }
    if (t[x + 2] != t[y + 2]) {
        return 2;
    }
    x = rk[x];
    y = rk[y];
    if (x > y) {
        swap(x, y);
    }
    return qmin(x + 1, y);
}

inline void build() {
    int m = 127;
    for (int i = 1; i <= m; ++i) {
        cnt[i] = 0;
    }
    for (int i = 1; i <= n; ++i) {
        rk[i] = s[i];
        t[i] = s[i];
        ++cnt[rk[i]];
    }
    for (int i = 1; i <= m; ++i) {
        cnt[i] += cnt[i - 1];
    }
    for (int i = n; i; --i) {
        sa[cnt[rk[i]]--] = i;
    }
    for (int w = 1;; w <<= 1) {
        int tot = 0;
        for (int i = n - w + 1; i <= n; ++i) {
            id[++tot] = i;
        }
        for (int i = 1; i <= n; ++i) {
            old[i] = rk[i];
            if (sa[i] > w) {
                id[++tot] = sa[i] - w;
            }
        }
        for (int i = 1; i <= m; ++i) {
            cnt[i] = 0;
        }
        for (int i = 1; i <= n; ++i) {
            ++cnt[rk[id[i]]];
```

```
            }
            for (int i = 1; i <= m; ++i) {
                cnt[i] += cnt[i - 1];
            }
            for (int i = n; i; --i) {
                sa[cnt[rk[id[i]]]--] = id[i];
            }
            int p = 0;
            for (int i = 1; i <= n; ++i) {
                if (old[sa[i]] == old[sa[i - 1]] && old[sa[i] + w] == old[sa[i - 1] + w]) {
                    rk[sa[i]] = p;
                } else {
                    rk[sa[i]] = ++p;
                }
            }
            if (p == n) {
                break;
            }
            m = p;
        }
        h[1] = 0;
        for (int i = 1, k = 0; i <= n; ++i) {
            if (rk[i] == 1) {
                continue;
            }
            if (k) {
                --k;
            }
            while (i + k <= n && sa[rk[i] - 1] + k <= n && s[i + k] == s[sa[rk[i] - 1] + k]) {
                ++k;
            }
            h[rk[i]] = k;
        }
        for (int i = 1; i <= n; ++i) {
            f[0][i] = h[i];
        }
        for (int j = 1; (1 << j) <= n; ++j) {
            for (int i = 1; i + (1 << j) - 1 <= n; ++i) {
                f[j][i] = min(f[j - 1][i], f[j - 1][i + (1 << (j - 1))]);
            }
        }
    }
} A, B;

inline int lcp(int x, int y) {
    return A.lcp(x, y);
}

inline int lcs(int x, int y) {
    return B.lcp(n - x + 1, n - y + 1);
}

struct node {
    int l, r, k;
    node(int a = 0, int b = 0, int c = 0) : l(a), r(b), k(c) {}
} a[maxn << 1], b[maxn << 1];
```

```
void runs() {
    n = reads(s + 1);
    A.build();
    reverse(s + 1, s + n + 1);
    B.build();
    int m = 0;
    for (int k = 1; k * 2 <= n; ++k) {
        int tot = 0;
        for (int i = k + 1; i + k - 1 <= n; i += k) {
            int l = max(i - k, i - lcs(i - 1, i + k - 1)), r = min(i - 1, i + lcp(i, i + k) - k);
            if (l <= r) {
                if (!tot) {
                    p[++tot] = mkp(l, r);
                } else {
                    if (p[tot].scd == l - 1) {
                        p[tot].scd = r;
                    } else {
                        p[++tot] = mkp(l, r);
                    }
                }
            }
        }
        for (int i = 1; i <= tot; ++i) {
            int l = p[i].fst, r = p[i].scd;
            a[++m] = node(l, r + k * 2 - 1, k);
        }
    }
    sort(a + 1, a + m + 1, [&](const node &a, const node &b) {
        return a.l < b.l || (a.l == b.l && (a.r < b.r || (a.r == b.r && a.k < b.k)));
    });
    int tot = 0;
    for (int i = 1; i <= m; ++i) {
        if (!tot || !(a[i].l == b[tot].l && a[i].r == b[tot].r)) {
            b[++tot] = a[i];
        }
    }
}
```

7.4 后缀自动机

```
struct SAM {
    int lst, tot, fa[maxn], ch[maxn][26], len[maxn];

    inline void init() {
        lst = tot = 1;
    }

    inline void insert(int k, int c) {
        int u = ++tot, p = lst;
        sz[u] = 1;
        lst = u;
        len[u] = k;
        for (; p && !ch[p][c]; p = fa[p]) {
            ch[p][c] = u;
        }
        if (!p) {
```

```
            fa[u] = 1;
            return;
        }
        int q = ch[p][c];
        if (len[q] == len[p] + 1) {
            fa[u] = q;
            return;
        }
        int nq = ++tot;
        len[nq] = len[p] + 1;
        fa[nq] = fa[q];
        memcpy(ch[nq], ch[q], sizeof(ch[q]));
        fa[u] = fa[q] = nq;
        for (; p && ch[p][c] == q; p = fa[p]) {
            ch[p][c] = nq;
        }
    }
} sam;
```