

## Appendix A.

### A.1 The optimal equilibrium solutions under strategy $\mathcal{N}$

As  $\frac{d^2 \Pi_A^{\mathcal{N}}}{dq_A^2} = \frac{d^2 \Pi_B^{\mathcal{N}}}{dq_B^2} = -2$ , then solving  $\frac{d \Pi_A^{\mathcal{N}}}{dq_A} = 0$  and  $\frac{d \Pi_B^{\mathcal{N}}}{dq_B} = 0$ , we can get that  $q_A^{\mathcal{N}} = \frac{1}{2}(a_A - p_{rA} - c_A \varepsilon_A)$  and  $q_B^{\mathcal{N}} = \frac{1}{2}(a_B - p_{rB} - c_B \varepsilon_B)$ ,  $\Pi_A^{\mathcal{N}} = \frac{1}{4}(a_A - p_{rA} - c_A \varepsilon_A)^2$  and  $\Pi_B^{\mathcal{N}} = \frac{1}{4}(a_B - p_{rB} - c_B \varepsilon_B)^2$ ,  $E^{\mathcal{N}} = \frac{1}{2}e(\varepsilon_A(a_A - p_{rA} - c_A \varepsilon_A) + \varepsilon_B(a_B - p_{rB} - c_B \varepsilon_B))$ .  $\square$

### A.2 The optimal equilibrium solutions under strategy $\mathcal{C}$

1.  $q_B$

As  $\frac{d^2 \Pi_B^{\mathcal{C}}}{dq_B^2} = -2$ , then solving  $\frac{d \Pi_B^{\mathcal{C}}}{dq_B} = 0$ , we can get that  $q_B = \frac{1}{2}(a_B - w - c_d - c_B \varepsilon_B)$ .

2.  $q_A$

As  $\frac{d^2 \Pi_A^{\mathcal{C}}}{dq_A^2} = -2$ . The unconstraint solution for this problem is  $q_A = \frac{1}{2}(a_A - p_{rA} - c_A \varepsilon_A)$ . Substituting this unconstraint solution into the constraint  $\varepsilon q_A > q_B$ , identifies a cutoff value of  $a_B - c_d - a_A \varepsilon_A + p_{rA} \varepsilon_A + c_A \varepsilon_A^2 - c_B \varepsilon_B$ . Then, we know that when  $w \geq a_B - c_d - a_A \varepsilon_A + p_{rA} \varepsilon_A + c_A \varepsilon_A^2 - c_B \varepsilon_B$ , the unconstraint solution is optimal. When  $w < a_B - c_d - a_A \varepsilon_A + p_{rA} \varepsilon_A + c_A \varepsilon_A^2 - c_B \varepsilon_B$ , the production quantity of manufacturer A is  $q_A = \frac{a_B - w - c_d - c_B \varepsilon_B}{2 \varepsilon_A}$ .

3.  $w$

Depending on  $w \geq a_B - c_d - a_A \varepsilon_A + p_{rA} \varepsilon_A + c_A \varepsilon_A^2 - c_B \varepsilon_B$  and  $w < a_B - c_d - a_A \varepsilon_A + p_{rA} \varepsilon_A + c_A \varepsilon_A^2 - c_B \varepsilon_B$ , we propose two sub-optimization problems (Sub-problem 1 and Sub-problem 2). The optimal solution for the original problem of manufacturer A is thus the maximum of these two sub-problems.

Sub-problem 1: When  $w \geq a_B - c_d - a_A \varepsilon_A + p_{rA} \varepsilon_A + c_A \varepsilon_A^2 - c_B \varepsilon_B$

As  $\frac{d^2 \Pi_A^{\mathcal{C}}}{dw^2} = -1$ . The unconstraint solution for this problem is  $w^{c1} = \frac{1}{2}(a_B - c_A - c_d - c_B \varepsilon_B)$ . Substituting this unconstraint solution into the constraint identifies two cutoff values of  $c_{d0}$  and  $c_{d1}$ , where  $c_{d0} = a_B - c_B \varepsilon_B + c_A$ ,  $c_{d1} = a_B - c_B \varepsilon_B - 2(a_A - p_{rA})\varepsilon_A + c_A(1 + 2\varepsilon_A^2)$ , when  $c_{d1} \leq c_d \leq c_{d0}$ , the unconstraint solution is

optimal, and  $\Pi_A^{c1} = \frac{1}{8}((a_B + c_A - c_d - c_B \varepsilon_B)^2 + 2(a_A - p_{rA} - c_A \varepsilon_A)^2)$ . When  $c_d < c_{d1}$ , the wholesale price of the by-product is  $w^{c2} = a_B - c_d - a_A \varepsilon_A + p_{rA} \varepsilon_A + c_A \varepsilon_A^2 - c_B \varepsilon_B$ , and  $\Pi_A^{c2} = -\frac{1}{4}(a_A - p_{rA} - c_A \varepsilon_A)((a_A - p_{rA})(-1 + 2\varepsilon_A^2) - \varepsilon_A(2(a_B - c_d - c_B \varepsilon_B) + c_A(1 + 2\varepsilon_A^2)))$ .

Sub-problem 2: When  $w < a_B - c_d - a_A \varepsilon_A + p_{rA} \varepsilon_A + c_A \varepsilon_A^2 - c_B \varepsilon_B$

As  $\frac{d^2 \Pi_A^c}{dw^2} = -1 - \frac{1}{2\varepsilon_A^2}$ . The unconstraint solution for this problem is  $w^{c3} = \frac{a_B(1+\varepsilon_A^2) - c_d(1+\varepsilon_A^2) - c_B(1+\varepsilon_A^2)\varepsilon_B - (a_A - p_{rA})\varepsilon_A}{1+2\varepsilon_A^2}$ . Substituting this unconstraint solution into the constraint identifies a cutoff value of  $c_{d1}$ , then when  $c_d < c_{d1}$  the unconstraint solution is optimal, and  $\Pi_A^{c3} = \frac{(a_A - p_{rA} + \varepsilon_A(a_B - c_d - c_B \varepsilon_B))^2}{4+8\varepsilon_A^2}$ . When  $c_{d1} \leq c_d$ , the wholesale price of the by-product is  $w^{c4} = a_B - c_d - a_A \varepsilon_A + p_{rA} \varepsilon_A + c_A \varepsilon_A^2 - c_B \varepsilon_B$ , and  $\Pi_A^{c4} = -\frac{1}{4}(a_A - p_{rA} - c_A \varepsilon_A)((a_A - p_{rA})(-1 + 2\varepsilon_A^2) - \varepsilon_A(2(a_B - c_d - c_B \varepsilon_B) + c_A(1 + 2\varepsilon_A^2)))$ .

Then, when  $c_{d1} \leq c_d \leq c_{d0}$ , compare  $\Pi_A^{c1}$  and  $\Pi_A^{c4}$ . We can get that  $\Pi_A^{c1} - \Pi_A^{c4} = \frac{1}{8}(a_B - c_d - 2(a_A - p_{rA})\varepsilon_A + c_A(1 + 2\varepsilon_A^2) - c_B \varepsilon_B)^2 > 0$ , the optimal equilibrium solution is  $w^{c1} = \frac{1}{2}(a_B - c_A - c_d - c_B \varepsilon_B)$ ,  $\Pi_B^{c1} = \frac{1}{16}(a_B + c_A - c_d - c_B \varepsilon_B)^2$ ,  $E^{c1} = \frac{1}{4}e(2\varepsilon_A(a_A - p_{rA} - c_A \varepsilon_A) - (a_B + c_A - c_d - c_B \varepsilon_B)(1 - \varepsilon_B))$ . When  $c_d < c_{d1}$ , compare  $\Pi_A^{c2}$  and  $\Pi_A^{c3}$ . We can get that  $\Pi_A^{c3} - \Pi_A^{c2} = \frac{\varepsilon_A^2(a_B - c_d - 2(a_A - p_{rA})\varepsilon_A + c_A(1 + 2\varepsilon_A^2) - c_B \varepsilon_B)^2}{4+8\varepsilon_A^2} > 0$ , the optimal equilibrium solution is  $w^{c3} = \frac{(a_B - c_d - c_B \varepsilon_B)(1 + \varepsilon_A^2) - (a_A - p_{rA})\varepsilon_A}{1+2\varepsilon_A^2}$ ,  $\Pi_B^{c3} = \frac{\varepsilon_A^2(a_A - p_{rA} + \varepsilon_A(a_B - c_d - c_B \varepsilon_B))^2}{4(1+2\varepsilon_A^2)^2}$ ,  $E^{c3} = \frac{e\varepsilon_A \varepsilon_B(a_A - p_{rA} + \varepsilon_A(a_B - c_d - c_B \varepsilon_B))}{2+4\varepsilon_A^2}$ . When  $c_d > c_{d0}$ , the optimal equilibrium solution is  $w^{c4} = a_B - c_d - a_A \varepsilon_A + p_{rA} \varepsilon_A + c_A \varepsilon_A^2 - c_B \varepsilon_B$ ,  $\Pi_B^{c4} = \frac{1}{4}\varepsilon_A^2(a_A - p_{rA} - c_A \varepsilon_A)^2$ ,  $E^{c4} = \frac{1}{2}e\varepsilon_A(a_A - p_{rA} - c_A \varepsilon_A)\varepsilon_B$ . As  $\frac{d\Pi_A^{c4}}{dc_d} = -\frac{1}{2}\varepsilon_A(a_A - p_{rA} - c_A \varepsilon_A) < 0$  and  $\Pi_A^{c4}|_{c_d=c_{d0}} = -\frac{1}{4}(a_A - p_{rA} - c_A \varepsilon_A)^2(2\varepsilon_A^2 - 1) < 0$ , manufacturer A is not willing to produce any products for sale, so we omit this case in our study.  $\square$

### A.3 The optimal equilibrium solutions under strategy $\mathcal{S}$ if $s$ is exogenous

The proof is similar to that of A.2, and the optimal equilibrium solutions are summarized in Table A.1, where  $s_1 = -a_B + c_d + c_B\varepsilon_B + 2(a_A - p_{rA})\varepsilon_A - c_A(1 + 2\varepsilon_A^2)$ .

**Table A.1**

The optimal equilibrium solutions under strategy  $\mathcal{S}$  if  $s$  is exogenous

	$s < s_1$	$s > s_1$
$q_A^{\mathcal{S}}$	$\frac{1}{2}(a_A - p_{rA} - c_A\varepsilon_A)$	$\frac{a_A - p_{rA} + \varepsilon_A(s + a_B - c_d - c_B\varepsilon_B)}{2 + 4\varepsilon_A^2}$
$q_B^{\mathcal{S}}$	$\frac{1}{4}(s + a_B + c_A - c_d - c_B\varepsilon_B)$	$\frac{\varepsilon_A(a_A - p_{rA} + \varepsilon_A(s + a_B - c_d - c_B\varepsilon_B))}{2 + 4\varepsilon_A^2}$
$w^{\mathcal{S}}$	$\frac{1}{2}(s + a_B - c_A - c_d - c_B\varepsilon_B)$	$\frac{-(a_A - p_{rA})\varepsilon_A + (s + a_B - c_d - c_B\varepsilon_B)(1 + \varepsilon_A^2)}{1 + 2\varepsilon_A^2}$
$\Pi_A^{\mathcal{S}}$	$\frac{1}{8}((s + a_B + c_A - c_d - c_B\varepsilon_B)^2 + 2(a_A - p_{rA} - c_A\varepsilon_A)^2)$	$\frac{(a_A - p_{rA} + \varepsilon_A(s + a_B - c_d - c_B\varepsilon_B))^2}{4 + 8\varepsilon_A^2}$
$\Pi_B^{\mathcal{S}}$	$\frac{1}{16}(s + a_B + c_A - c_d - c_B\varepsilon_B)^2$	$\frac{\varepsilon_A^2(a_A - p_{rA} + \varepsilon_A(s + a_B - c_d - c_B\varepsilon_B))^2}{4(1 + 2\varepsilon_A^2)^2}$
$E^{\mathcal{S}}$	$\frac{1}{4}e(2(a_A - p_{rA})\varepsilon_A + c_A(-1 - 2\varepsilon_A^2 + \varepsilon_B) + (a_B - c_d + s - c_B\varepsilon_B)(-1 + \varepsilon_B))$	$\frac{e\varepsilon_A\varepsilon_B(a_A - p_{rA} + \varepsilon_A(s + a_B - c_d - c_B\varepsilon_B))}{2 + 4\varepsilon_A^2}$

□

### A.4 The optimal equilibrium solutions under strategy $\mathcal{S}$ if $s$ is endogenous

The proof is similar to that of A.2, and the optimal equilibrium solutions are summarized in Table A.2, where  $e_1 = \frac{2\varepsilon_A(-a_A + p_{rA} + c_A\varepsilon_A)}{-1 + \varepsilon_B}$ ,  $c_{d6} = e + a_B + c_A - (e + c_B)\varepsilon_B$ ,  $c_{d7} = e + a_B - 4(a_A - p_{rA})\varepsilon_A + c_A(1 + 4\varepsilon_A^2) - (e + c_B)\varepsilon_B$ ,  $c_{d8} = -e + a_B + c_A + (e - c_B)\varepsilon_B$ . As  $c_{d6} > c_{d0}$ , we just consider the case where  $c_d \leq c_{d0}$ .

**Table A.2**The optimal equilibrium solutions under strategy  $\mathcal{S}$  if  $s$  is endogenous

	$\langle q_A, w, q_B, s \rangle^\mathcal{S}$
$EqS1^\mathcal{S}$	$\langle \frac{1}{2}(a_A - p_{rA} - c_A \varepsilon_A), \frac{1}{4}(e + a_B - 3c_A - c_d - (e + c_B)\varepsilon_B), \frac{1}{8}(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B), \frac{1}{2}(e - a_B - c_A + c_d + (-e + c_B)\varepsilon_B) \rangle$
$EqS2^\mathcal{S}$	$\langle \frac{1}{2}(a_A - p_{rA} - c_A \varepsilon_A), (a_A - p_{rA})\varepsilon_A - c_A(1 + \varepsilon_A^2), \frac{1}{2}\varepsilon_A(a_A - p_{rA} - c_A \varepsilon_A), 2\varepsilon_A(a_A - p_{rA} - c_A \varepsilon_A) - (a_B - c_d - c_B \varepsilon_B + c_A) \rangle$
$EqS3^\mathcal{S}$	$\langle \frac{1}{2}(a_A - p_{rA} - c_A \varepsilon_A), \frac{1}{2}(a_B - c_A - c_d - c_B \varepsilon_B), \frac{1}{4}(a_B + c_A - c_d - c_B \varepsilon_B), 0 \rangle$
$EqS4^\mathcal{S}$	$\langle \frac{a_A - p_{rA} + \varepsilon_A(a_B - c_d - c_B \varepsilon_B)}{2 + 4\varepsilon_A^2}, \frac{(a_B - c_d - c_B \varepsilon_B)(1 + \varepsilon_A^2) - (a_A - p_{rA})\varepsilon_A}{1 + 2\varepsilon_A^2}, \frac{\varepsilon_A(a_A - p_{rA} + \varepsilon_A(a_B - c_d - c_B \varepsilon_B))}{2 + 4\varepsilon_A^2}, 0 \rangle$

Moreover, the optimal choice of the triplets of  $\langle q_A, w, q_B, s \rangle^\mathcal{S}$  can be characterized as follows

Condition	$\langle q_A, w, q_B, s \rangle^\mathcal{S}$
$c_d \leq c_{d1} \Rightarrow$	$EqS4^\mathcal{S}$
$\delta_B > 1 \quad c_{d1} \leq c_d \leq c_{d0} \Rightarrow$	$EqS3^\mathcal{S}$
$\delta_B < 1 \text{ and } e < e_1 \quad c_{d1} \leq c_d \leq c_{d8} \Rightarrow$	$EqS3^\mathcal{S}$
$\delta_B < 1 \text{ and } e_1 < e \quad c_{d1} \leq c_d \leq \min\{c_{d0}, c_{d7}\} \Rightarrow$	$EqS2^\mathcal{S}$
$\delta_B < 1 \text{ and } e < e_1 \quad c_{d8} \leq c_d \leq c_{d0} \Rightarrow$	$EqS1^\mathcal{S}$
$\delta_B < 1 \text{ and } e_1 < e \quad \min\{c_{d0}, c_{d7}\} \leq c_d \leq c_{d0} \Rightarrow$	$EqS1^\mathcal{S}$

Furthermore, the total profits of both manufacturers, the benefit of the government, and the environmental impact under strategy  $\mathcal{S}$ ,  $\Pi_A^\mathcal{S}$ ,  $\Pi_B^\mathcal{S}$ ,  $G^\mathcal{S}$  and  $E^\mathcal{S}$  in each of the possible equilibrium solutions can also be solved.  $\square$

### A.5 The optimal equilibrium solutions under strategy $\mathcal{T}$ and $\mathcal{NT}$ if $t$ is exogenous

The proof is similar to that of A.2, and the optimal equilibrium solutions are summarized in Tables A.3 and A.4, where  $t_1 = \frac{-a_B + c_d + 2(a_A - p_{rA})\varepsilon_A - c_A(1 + 2\varepsilon_A^2) + c_B \varepsilon_B}{1 + 2\varepsilon_A^2 - \varepsilon_B}$ .

**Table A.3**The optimal equilibrium solutions under strategy  $\mathcal{T}$  if  $t$  is exogenous

	$t < t_1$	$t > t_1$
$q_A^{\mathcal{T}}$	$\frac{1}{2}(a_A - p_{rA} - (t + c_A)\varepsilon_A)$	$\frac{a_A - p_{rA} + \varepsilon_A(a_B - c_d - (t + c_B)\varepsilon_B)}{2 + 4\varepsilon_A^2}$
$q_B^{\mathcal{T}}$	$\frac{1}{4}(t + a_B + c_A - c_d - (t + c_B)\varepsilon_B)$	$\frac{\varepsilon_A(a_A - p_{rA} + \varepsilon_A(a_B - c_d - (t + c_B)\varepsilon_B))}{2 + 4\varepsilon_A^2}$
$w^{\mathcal{T}}$	$\frac{1}{2}(-t + a_B - c_A - c_d - (t + c_B)\varepsilon_B)$	$\frac{(a_B(1 + \varepsilon_A^2) - (a_A - p_{rA})\varepsilon_A - c_d(1 + \varepsilon_A^2) - (t + c_B)(1 + \varepsilon_A^2)\varepsilon_B)}{1 + 2\varepsilon_A^2}$
$\Pi_A^{\mathcal{T}}$	$\frac{1}{8}((t + a_B + c_A - c_d - (t + c_B)\varepsilon_B)^2 + 2(a_A - p_{rA} - (t + c_A)\varepsilon_A)^2)$	$\frac{(a_A - p_{rA} + \varepsilon_A(a_B - c_d - (t + c_B)\varepsilon_B))^2}{4 + 8\varepsilon_A^2}$
$\Pi_B^{\mathcal{T}}$	$\frac{1}{16}(t + a_B + c_A - c_d - (t + c_B)\varepsilon_B)^2$	$\frac{\varepsilon_A^2(a_A - p_{rA} + \varepsilon_A(a_B - c_d - (t + c_B)\varepsilon_B))^2}{4(1 + 2\varepsilon_A^2)^2}$
$E^{\mathcal{T}}$	$\frac{1}{4}e(c_d + 2(a_A - p_{rA})\varepsilon_A + (a_B - c_B\varepsilon_B - t\varepsilon_B)(-1 + \varepsilon_B) - c_d\varepsilon_B + (t + c_A)(-1 - 2\varepsilon_A^2 + \varepsilon_B))$	$\frac{e\varepsilon_A\varepsilon_B(a_A - p_{rA} + \varepsilon_A(a_B - c_d - (t + c_B)\varepsilon_B))}{2 + 4\varepsilon_A^2}$

**Table A.4**The optimal equilibrium solutions under strategy  $\mathcal{NT}$  if  $t$  is exogenous

$q_A^{\mathcal{NT}}$	$q_B^{\mathcal{NT}}$	$\Pi_A^{\mathcal{NT}}$	$\Pi_B^{\mathcal{NT}}$	$E^{\mathcal{NT}}$
$\frac{1}{2}(a_A - p_{rA} - (t + c_A)\varepsilon_A)$	$\frac{1}{2}(a_B - p_{rB} - (t + c_B)\varepsilon_B)$	$\frac{1}{4}(a_A - p_{rA} - (t + c_A)\varepsilon_A)^2$	$\frac{1}{4}(a_B - p_{rB} - (t + c_B)\varepsilon_B)^2$	$\frac{1}{2}e(\varepsilon_A(a_A - p_{rA} - (t + c_A)\varepsilon_A) + \varepsilon_B(a_B - p_{rB} - (t + c_B)\varepsilon_B))$

□

**A.6 The optimal equilibrium solutions under strategy  $\mathcal{T}$  and  $\mathcal{NT}$  if  $t$  is endogenous**

The proof is similar to that of A.2, and the optimal equilibrium solutions are summarized in Tables A.5 and A.6, where  $e_4 = -c_A + \frac{a_A - p_{rA}}{\varepsilon_A}$ ,  $e_5 = -c_B + \frac{2(a_B - p_{rB})}{\varepsilon_B} + \frac{-\varepsilon_A(a_A - p_{rA} + (-c_A + c_B)\varepsilon_A) + (-a_B + p_{rB})\varepsilon_B}{\varepsilon_A^2 + \varepsilon_B^2}$ ,  $e_6 = -\frac{(a_A - p_{rA})(\varepsilon_A^2 + 2\varepsilon_B^2) + \varepsilon_A(\varepsilon_B(-a_B + p_{rB} + c_B\varepsilon_B) - c_A(\varepsilon_A^2 + 2\varepsilon_B^2))}{\varepsilon_A(\varepsilon_A^2 + \varepsilon_B^2)}$ ,  $c_{d10} = a_B - \frac{(a_A - p_{rA})(-1 + \varepsilon_B)}{\varepsilon_A} + (c_A - c_B)\varepsilon_B$ ,  $c_{d11} = a_B + \frac{a_A - p_{rA}}{\varepsilon_A} - (e + c_B)\varepsilon_B$ ,  $c_{d12} = -e +$

$$a_B - c_A + c_A \varepsilon_B - c_B \varepsilon_B - \frac{(a_A - p_{rA} - (e + c_A) \varepsilon_A)(2\varepsilon_A^2 + 1 - \varepsilon_B)}{\sqrt{(1 + 2\varepsilon_A^2)(2\varepsilon_A^2 + (-1 + \varepsilon_B)^2)}} \left( \sqrt{2} + \frac{\varepsilon_B^2}{\sqrt{2}(1 + 2\varepsilon_A^2 - \varepsilon_B)} + \frac{4(1 - \varepsilon_B + \varepsilon_B^2)}{\sqrt{2}(-1 + 2\varepsilon_A^2 + \varepsilon_B)} \right) -$$

$$\frac{(2(a_A - p_{rA})\varepsilon_A + e(-1 + \varepsilon_B) + c_A(-1 + \varepsilon_B))(-2 + \varepsilon_B)}{-1 + 2\varepsilon_A^2 + \varepsilon_B}, \quad c_{d13} = e + a_B + c_A + \frac{2\varepsilon_A(-a_A + p_{rA} + (e + c_A)\varepsilon_A)(-1 + \varepsilon_B)}{4\varepsilon_A^2 + (-1 + \varepsilon_B)^2} -$$

$$(e + c_B)\varepsilon_B.$$

**Table A.5**

The optimal equilibrium solutions under strategy  $\mathcal{T}$  if  $t$  is endogenous

	$\langle q_A, w, q_B, t \rangle^T$
$EqS1^T$	$\left( \frac{e + c_d + 2(a_A - p_{rA})\varepsilon_A + 2e\varepsilon_A^2 + a_B(-1 + \varepsilon_B) + (-2e + c_B - c_d)\varepsilon_B + (e - c_B)\varepsilon_B^2 + c_A(-1 - 2\varepsilon_A^2 + \varepsilon_B)}{2(2\varepsilon_A^2 + (-1 + \varepsilon_B)^2)}, \right.$ $\frac{(-e - 2\varepsilon_A(a_A - p_{rA} + e\varepsilon_A) - c_d(3 + 4\varepsilon_A^2) + c_A(1 + 2\varepsilon_A^2 - 3\varepsilon_B)(-1 + \varepsilon_B) + (e - 3c_B)\varepsilon_B - 2(-2c_d + \varepsilon_A(a_A - p_{rA} + (e + 2c_B)\varepsilon_A))\varepsilon_B + (e + 4c_B - c_d)\varepsilon_B^2 - (e + c_B)\varepsilon_B^3 + a_B(3 + 4\varepsilon_A^2 - 4\varepsilon_B + \varepsilon_B^2))}{4(2\varepsilon_A^2 + (-1 + \varepsilon_B)^2)}, \left. \right)$ $\frac{1}{8}(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B + \frac{2\varepsilon_A(-(a_A - p_{rA})(-1 + \varepsilon_B) + \varepsilon_A(a_B - c_d + (c_A - c_B)\varepsilon_B))}{2\varepsilon_A^2 + (-1 + \varepsilon_B)^2}),$ $\frac{e + c_d + 2(a_A - p_{rA})\varepsilon_A + 2e\varepsilon_A^2 + a_B(-1 + \varepsilon_B) + (-2e + c_B - c_d)\varepsilon_B + (e - c_B)\varepsilon_B^2 + c_A(-1 - 2\varepsilon_A^2 + \varepsilon_B)}{2(2\varepsilon_A^2 + (-1 + \varepsilon_B)^2)}$
$EqS2^T$	$\left( \frac{a_A - p_{rA} + \varepsilon_A(a_B - c_d - (e + c_B)\varepsilon_B)}{4 + 8\varepsilon_A^2}, \frac{-(a_A - p_{rA})(1 + 3\varepsilon_A^2) + (\varepsilon_A + \varepsilon_A^3)(a_B - c_d - (e + c_B)\varepsilon_B)}{2(\varepsilon_A + 2\varepsilon_A^3)}, \right.$ $\left. \frac{\varepsilon_A(a_A - p_{rA} + \varepsilon_A(a_B - c_d - (e + c_B)\varepsilon_B))}{4 + 8\varepsilon_A^2}, \frac{a_A - p_{rA} + \varepsilon_A(a_B - c_d + (e - c_B)\varepsilon_B)}{2\varepsilon_A\varepsilon_B} \right)$
$EqS3^T$	$\left\langle \frac{1}{2} \left( a_A - p_{rA} + \frac{\varepsilon_A(-a_B + c_d + (-c_A + c_B)\varepsilon_B)}{-1 + \varepsilon_B} \right), \frac{-a_B + c_d + (-c_A + c_B)\varepsilon_B}{-1 + \varepsilon_B}, 0, \frac{a_B + c_A - c_d - c_B\varepsilon_B}{-1 + \varepsilon_B} \right\rangle$
$EqS4^T$	$\left\langle 0, -\frac{a_A}{\varepsilon_A}, 0, \frac{a_A - p_{rA} + \varepsilon_A(a_B - c_d - c_B\varepsilon_B)}{\varepsilon_A\varepsilon_B} \right\rangle$

Moreover, the optimal choice of the triplets of  $\langle q_A, w, q_B, t \rangle^T$  can be characterized as follows

	Condition		$\langle q_A, w, q_B, t \rangle^T$
	$c_d \leq c_{d11}$	$\Rightarrow$	$EqS2^T$
$e_4 < e$	$c_{d11} \leq c_d \leq \min \{c_{d0}, c_{d10}\}$	$\Rightarrow$	$EqS4^T$
	$\min \{c_{d0}, c_{d10}\} \leq c_d \leq c_{d0}$	$\Rightarrow$	$EqS3^T$
	$c_d \leq c_{d12}$	$\Rightarrow$	$EqS2^T$
$e < e_4$	$c_{d12} \leq c_d \leq \min \{c_{d0}, c_{d13}\}$	$\Rightarrow$	$EqS1^T$
	$\min \{c_{d0}, c_{d13}\} \leq c_d \leq c_{d0}$	$\Rightarrow$	$EqS3^T$

Furthermore, the total profits of both manufacturers, the environmental impact, and the benefit of the government under  $\mathcal{T}$ ,  $\Pi_A^{\mathcal{T}}$ ,  $\Pi_B^{\mathcal{T}}$ ,  $E^{\mathcal{T}}$  and  $G^{\mathcal{T}}$  in each of the possible equilibrium solutions can also be solved.

**Table A.6**

The optimal equilibrium solutions under strategy  $\mathcal{NT}$  if  $t$  is endogenous

	$e < \min \{e_5, e_6\}$	$e > e_5$	$e > e_6$
$q_A^{NT}$	$\frac{1}{2} (a_A - p_{rA} - \frac{\varepsilon_A(a_A \varepsilon_A - p_{rA} \varepsilon_A + (e + c_A) \varepsilon_A^2 + \varepsilon_B(a_B - p_{rB} + (e + 2c_A - c_B) \varepsilon_B))}{2(\varepsilon_A^2 + \varepsilon_B^2)})$	$\frac{1}{2} (a_A - p_{rA} - \varepsilon_A(c_A - c_B + \frac{a_B - p_{rB}}{\varepsilon_B}))$	0
$q_B^{NT}$	$\frac{1}{2} (a_B - p_{rB} - \frac{\varepsilon_B(a_A \varepsilon_A - p_{rA} \varepsilon_A + (e - c_A + 2c_B) \varepsilon_A^2 + \varepsilon_B(a_B - p_{rB} + (e + c_B) \varepsilon_B))}{2(\varepsilon_A^2 + \varepsilon_B^2)})$	0	$\frac{1}{2} (a_B - p_{rB} + (c_A - c_B - \frac{a_A - p_{rA}}{\varepsilon_A}) \varepsilon_B)$
$t^{NT}$	$\frac{a_A \varepsilon_A - p_{rA} \varepsilon_A + (e - c_A) \varepsilon_A^2 + \varepsilon_B(a_B - p_{rB} + (e - c_B) \varepsilon_B)}{2(\varepsilon_A^2 + \varepsilon_B^2)}$	$-c_B + \frac{a_B - p_{rB}}{\varepsilon_B}$	$-c_A + \frac{a_A - p_{rA}}{\varepsilon_A}$
$\Pi_A^{NT}$	$\left( \frac{(e + c_A) \varepsilon_A^3 + \varepsilon_A \varepsilon_B(a_B - p_{rB} + (e + 2c_A - c_B) \varepsilon_B) - (a_A - p_{rA})(\varepsilon_A^2 + 2\varepsilon_B^2)}{16(\varepsilon_A^2 + \varepsilon_B^2)^2} \right)^2$	$\frac{(a_A \varepsilon_B - p_{rA} \varepsilon_B + \varepsilon_A(-a_B + p_{rB} + (-c_A + c_B) \varepsilon_B))^2}{4\varepsilon_B^2}$	0
$\Pi_B^{NT}$	$\left( \frac{(2(-a_B + p_{rB}) \varepsilon_A^2 + \varepsilon_A(a_A - p_{rA} + (e - c_A + 2c_B) \varepsilon_A) \varepsilon_B) + (-a_B + p_{rB}) \varepsilon_B^2 + (e + c_B) \varepsilon_B^3}{16(\varepsilon_A^2 + \varepsilon_B^2)^2} \right)^2$	0	$\frac{(a_A \varepsilon_B - p_{rA} \varepsilon_B + \varepsilon_A(-a_B + p_{rB} + (-c_A + c_B) \varepsilon_B))^2}{4\varepsilon_A^2}$
$E^{NT}$	$-\frac{1}{4} e(-a_A \varepsilon_A + p_{rA} \varepsilon_A + (e + c_A) \varepsilon_A^2 + \varepsilon_B(-a_B + p_{rB} + (e + c_B) \varepsilon_B))$	$\frac{1}{2} e \varepsilon_A(a_A - p_{rA} - \varepsilon_A(c_A - c_B + \frac{a_B - p_{rB}}{\varepsilon_B}))$	$\frac{1}{2} e \varepsilon_B(a_B - p_{rB} + (c_A - c_B - \frac{a_A - p_{rA}}{\varepsilon_A}) \varepsilon_B)$



Furthermore, the benefit of the government under  $\mathcal{NT}$ ,  $G^{\mathcal{NT}}$  in each of the possible equilibrium solutions can also be solved.  $\square$

### A.7 The optimal equilibrium solutions under strategy $\mathcal{S}'$

The proof is similar to that of A.2, and the optimal equilibrium solutions are summarized in Table A.7, where  $e_{s1}$ ,  $e_{s2}$ ,  $e_{s3}$ ,  $c_{ds1}$ ,  $c_{ds2}$ ,  $c_{ds3}$ ,  $c_{ds4}$ ,  $c_{ds5}$  are given in Appendix B.

**Table A.7**

The optimal equilibrium solutions under strategy  $\mathcal{S}'$

	$\langle q_A, w, q_B, s \rangle^{\mathcal{S}'}$
$EqS1^{\mathcal{S}'}$	$\langle \frac{1}{2}(a_A - p_{rA} - c_A \varepsilon_A), 2e + 2a_B + c_A - 2c_d - 2(e + c_B)\varepsilon_B, e + a_B + c_A - c_d - (e + c_B)\varepsilon_B, 4e + 3a_B + 3c_A - 3c_d - (4e + 3c_B)\varepsilon_B \rangle$
$EqS2^{\mathcal{S}'}$	$\langle \frac{1}{2}(a_A - p_{rA} - c_A \varepsilon_A), (a_A - p_{rA})\varepsilon_A - c_A(1 + \varepsilon_A^2), \frac{1}{2}\varepsilon_A(a_A - p_{rA} - c_A \varepsilon_A), -(a_B - c_d - c_B \varepsilon_B + c_A) + 2\varepsilon_A(a_A - p_{rA} - c_A \varepsilon_A) \rangle$
$EqS3^{\mathcal{S}'}$	$\langle \frac{1}{2}(a_A - p_{rA} - c_A \varepsilon_A), \frac{1}{2}(a_B - c_A - c_d - c_B \varepsilon_B), \frac{1}{4}(a_B + c_A - c_d - c_B \varepsilon_B), 0 \rangle$
$EqS4^{\mathcal{S}'}$	$\langle \frac{a_A - p_{rA} + \varepsilon_A(a_B - c_d - (e + c_B)\varepsilon_B)}{1 + \varepsilon_A^2}, 2a_B + \frac{a_A - p_{rA}}{\varepsilon_A} - 2(c_d + (e + c_B)\varepsilon_B), \frac{\varepsilon_A(a_A - p_{rA} + \varepsilon_A(a_B - c_d - (e + c_B)\varepsilon_B))}{1 + \varepsilon_A^2}, \frac{(a_A - p_{rA} + (a_B - c_d)\varepsilon_A)(1 + 3\varepsilon_A^2) - \varepsilon_A(2e + c_B + (4e + 3c_B)\varepsilon_A^2)\varepsilon_B}{\varepsilon_A + \varepsilon_A^3} \rangle$
$EqS5^{\mathcal{S}'}$	$\langle -\frac{(a_A - p_{rA})\varepsilon_A - a_B(1 + \varepsilon_A^2) + c_d(1 + \varepsilon_A^2) + c_B(1 + \varepsilon_A^2)\varepsilon_B}{1 + 2\varepsilon_A^2}, \frac{\varepsilon_A(a_A - p_{rA} + \varepsilon_A(a_B - c_d - c_B \varepsilon_B))}{2 + 4\varepsilon_A^2}, 0 \rangle$

Moreover, the optimal choice of the triplets of  $\langle q_A, w, q_B, s \rangle^{S'}$  can be characterized as follows

Condition		$\langle q_A, w, q_B, s \rangle^{S'}$
$e < e_{s3}$	$c_d \leq c_{ds5} \Rightarrow$	$EqS4^{S'}$
	$c_{ds5} \leq c_d \leq \min \{c_{ds2}, c_{d0}\} \Rightarrow$	$EqS1^{S'}$
	$\min\{c_{ds2}, c_{d0}\} \leq c_d \leq c_{d0} \Rightarrow$	$EqS3^{S'}$
$e_{s3} < e < e_{s2}$	$c_d \leq c_{ds3} \Rightarrow$	$EqS4^{S'}$
	$c_{ds3} \leq c_d \leq c_{ds1} \Rightarrow$	$EqS2^{S'}$
	$c_{ds1} \leq c_d \leq \min \{c_{ds2}, c_{d0}\} \Rightarrow$	$EqS1^{S'}$
	$\min\{c_{ds2}, c_{d0}\} \leq c_d \leq c_{d0} \Rightarrow$	$EqS3^{S'}$
$e_{s2} < e < e_{s1}$ and $1 < \varepsilon_B$ or $e_{s2} < e$ and $\varepsilon_B < 1$	$c_d \leq c_{ds4} \Rightarrow$	$EqS4^{S'}$
	$c_{ds4} \leq c_d \leq c_{d1} \Rightarrow$	$EqS5^{S'}$
	$c_{d1} \leq c_d \leq c_{ds1} \Rightarrow$	$EqS2^{S'}$
	$c_{ds1} \leq c_d \leq \min \{c_{ds2}, c_{d0}\} \Rightarrow$	$EqS1^{S'}$
	$\min\{c_{ds2}, c_{d0}\} \leq c_d \leq c_{d0} \Rightarrow$	$EqS3^{S'}$
$e_{s1} < e$ and $1 < \varepsilon_B$	$c_d \leq c_{ds4} \Rightarrow$	$EqS4^{S'}$
	$c_{ds4} \leq c_d \leq c_{d1} \Rightarrow$	$EqS5^{S'}$
	$c_{d1} \leq c_d \leq c_{d0} \Rightarrow$	$EqS3^{S'}$

Furthermore, the total profits of both manufacturers, the benefit of the government, and the environmental impact of the strategy  $S'$ ,  $\Pi_A^{S'}$ ,  $\Pi_B^{S'}$ ,  $G^{S'}$  and  $E^{S'}$  in each of the possible equilibriums can be solved.  $\square$

### A.8 The optimal equilibrium solutions under strategy $\mathcal{NT}'$ and $\mathcal{T}'$

The proof is similar to that of A.2, and the optimal equilibrium solutions are summarized in Tables A.8 and A.9, where  $e_{t1}$ ,  $e_{t2}$ ,  $e_{t3}$ ,  $e_{t4}$ ,  $e_{t5}$ ,  $c_{dt1}$ ,  $c_{dt2}$ ,  $c_{dt3}$ ,  $c_{dt4}$ ,  $c_{dt5}$  are given in Appendix B.

**Table A.8**

The optimal equilibrium solutions under strategy  $\mathcal{NT}'$

	$e < e_{t3}$	$e_{t3} < e < \min \{e_{t1}, e_{t2}\}$	$e > e_{t1}$	$e > e_{t2}$
$q_A^{NJ'}$	$\frac{1}{2}(a_A - p_{rA} - c_A \varepsilon_A)$	$\frac{1}{2} \left( a_A - p_{rA} + \frac{\varepsilon_A((a_A - p_{rA})\varepsilon_A - 2(e + c_A)\varepsilon_A^2 + \varepsilon_B(a_B - p_{rB} - (2e + c_A + c_B)\varepsilon_B))}{\varepsilon_A^2 + \varepsilon_B^2} \right)$	$\frac{1}{2}(a_A - p_{rA} - \varepsilon_A(c_A - c_B + \frac{a_B - p_{rB}}{\varepsilon_B}))$	0
$q_B^{NJ'}$	$\frac{1}{2}(a_B - p_{rB} - c_B \varepsilon_B)$	$\frac{1}{2} \left( a_B - p_{rB} + \frac{\varepsilon_B((a_A - p_{rA})\varepsilon_A - (2e + c_A + c_B)\varepsilon_A^2 + \varepsilon_B(a_B - p_{rB} - 2(e + c_B)\varepsilon_B))}{\varepsilon_A^2 + \varepsilon_B^2} \right)$	0	$\frac{1}{2}(a_B - p_{rB} + (c_A - c_B - \frac{a_A - p_{rA}}{\varepsilon_A})\varepsilon_B)$
$t^{NJ'}$	0	$\frac{-(a_A - p_{rA})\varepsilon_A + (2e + c_A)\varepsilon_A^2 + \varepsilon_B(-a_B + p_{rB} + (2e + c_B)\varepsilon_B)}{\varepsilon_A^2 + \varepsilon_B^2}$	$-c_B + \frac{a_B - p_{rB}}{\varepsilon_B}$	$-c_A + \frac{a_A - p_{rA}}{\varepsilon_A}$
$\Pi_A^{NJ'}$	$\frac{1}{4}(a_A - p_{rA} - c_A \varepsilon_A)^2$	$\left( \frac{2(e + c_A)\varepsilon_A^3 + \varepsilon_A \varepsilon_B(-a_B + p_{rB} + (2e + c_A + c_B)\varepsilon_B)}{(2e + c_A + c_B)\varepsilon_B} - (a_A - p_{rA})(2\varepsilon_A^2 + \varepsilon_B^2) \right)^2 \frac{1}{4(\varepsilon_A^2 + \varepsilon_B^2)^2}$	$\frac{((a_A - p_{rA})\varepsilon_B + \varepsilon_A(-a_B + p_{rB} + (2e + c_A + c_B)\varepsilon_B))^2}{4\varepsilon_B^2}$	0
$\Pi_B^{NJ'}$	$\frac{1}{4}(a_B - p_{rB} - c_B \varepsilon_B)^2$	$\left( \frac{(-a_B + p_{rB})\varepsilon_A^2 + \varepsilon_A(-a_A + p_{rA} + (2e + c_A + c_B)\varepsilon_A)\varepsilon_B}{-2(-a_B + p_{rB})\varepsilon_B^2 + 2(e + c_B)\varepsilon_B^3} + 2(\varepsilon_A^2 + \varepsilon_B^2) \right)^2 \frac{1}{4(\varepsilon_A^2 + \varepsilon_B^2)^2}$	0	$\frac{((a_A - p_{rA})\varepsilon_B + \varepsilon_A(-a_B + p_{rB} + (2e + c_A + c_B)\varepsilon_B))^2}{4\varepsilon_A^2}$
$E^{NJ'}$	$\frac{1}{2}e(\varepsilon_A(a_A - p_{rA} - c_A \varepsilon_A) + \varepsilon_B(a_B - p_{rB} - c_B \varepsilon_B))$	$-e(-(a_A - p_{rA})\varepsilon_A + (e + c_A)\varepsilon_A^2 + \varepsilon_B(-a_B + p_{rB} + (e + c_B)\varepsilon_B))$	$\frac{1}{2}e\varepsilon_A(a_A - p_{rA} - \varepsilon_A(c_A - c_B + \frac{a_B - p_{rB}}{\varepsilon_B}))$	$\frac{1}{2}e\varepsilon_B(a_B - p_{rB} + (c_A - c_B - \frac{a_A - p_{rA}}{\varepsilon_A})\varepsilon_B)$

**Table A.9**

The optimal equilibrium solutions under strategy  $\mathcal{T}'$

	$\langle q_A, w, q_B, t \rangle^{\mathcal{T}'}$
$EqS1^{\mathcal{T}'}$	$\left( \frac{a_A - p_{rA} - \varepsilon_A(c_A + (4e + 3a_B - 3c_d - 4(a_A - p_{rA})\varepsilon_A + 8e\varepsilon_A^2) + c_A(3 + 4\varepsilon_A^2) - 8e\varepsilon_B - 3(a_B + c_A + c_B - c_d)\varepsilon_B + (4e + 3c_B)\varepsilon_B^2}{2(4\varepsilon_A^2 + (-1 + \varepsilon_B)^2)}, \right.$ $\left( \frac{-2c_A + c_d + 2(a_A - p_{rA})\varepsilon_A - 2(2c_A + c_d)\varepsilon_A^2 + (c_A + c_B + c_d + 2(a_A - p_{rA})\varepsilon_A - 2(2e + c_A + c_B)\varepsilon_A^2)\varepsilon_B + (2e + c_A + c_B - 2c_d)\varepsilon_B^2}{-2(e + c_B)\varepsilon_B^3 + 2e(-1 - 2\varepsilon_A^2 + \varepsilon_B) + a_B(2\varepsilon_A^2 + (-1 + \varepsilon_B)(1 + 2\varepsilon_B))} \right)$ $\left( \frac{e + a_B + c_A - c_d - (e + c_B)\varepsilon_B + (\varepsilon_A((a_A - p_{rA})(-1 + \varepsilon_B) + \varepsilon_A(-2e - 3a_B + 3c_d + (2e + 3c_B)\varepsilon_B - c_A(2 + \varepsilon_B))))}{4\varepsilon_A^2 + (-1 + \varepsilon_B)^2}, \right.$ $\left. \frac{4e + 3a_B - 3c_d - 4(a_A - p_{rA})\varepsilon_A + 8e\varepsilon_A^2 + c_A(3 + 4\varepsilon_A^2) - 8e\varepsilon_B - 3(a_B + c_A + c_B - c_d)\varepsilon_B + (4e + 3c_B)\varepsilon_B^2}{4\varepsilon_A^2 + (-1 + \varepsilon_B)^2} \right)$
$EqS2^{\mathcal{T}'}$	$\left( \frac{-(a_A - p_{rA})(-1 + \varepsilon_B) + \varepsilon_A(a_B - c_d + (c_A - c_B)\varepsilon_B)}{2 + 4\varepsilon_A^2 - 2\varepsilon_B}, \right.$ $\left( \frac{(-(a_A - p_{rA})\varepsilon_A + a_B(1 + \varepsilon_A^2) - c_d(1 + \varepsilon_A^2) + (-(a_A - p_{rA})\varepsilon_A + c_A(1 + \varepsilon_A^2) - c_B(1 + \varepsilon_A^2))\varepsilon_B)}{1 + 2\varepsilon_A^2 - \varepsilon_B}, \right)$ $\left( \frac{\varepsilon_A(-(a_A - p_{rA})(-1 + \varepsilon_B) + \varepsilon_A(a_B - c_d + (c_A - c_B)\varepsilon_B))}{2 + 4\varepsilon_A^2 - 2\varepsilon_B}, \right.$ $\left. \frac{-a_B + c_d + 2(a_A - p_{rA})\varepsilon_A - c_A(1 + 2\varepsilon_A^2) + c_B\varepsilon_B}{1 + 2\varepsilon_A^2 - \varepsilon_B} \right)$
$EqS3^{\mathcal{T}'}$	$\left\langle \frac{1}{2}(a_A - p_{rA} - c_A\varepsilon_A), \frac{1}{2}(a_B - c_A - c_d - c_B\varepsilon_B), \frac{1}{4}(a_B + c_A - c_d - c_B\varepsilon_B), 0 \right\rangle$
$EqS4^{\mathcal{T}'}$	$\left\langle \frac{1}{2} \left( a_A - p_{rA} + \frac{\varepsilon_A(-a_B + c_d + (-c_A + c_B)\varepsilon_B)}{-1 + \varepsilon_B} \right), \frac{-a_B + c_d + (-c_A + c_B)\varepsilon_B}{-1 + \varepsilon_B}, \right.$ $\left. 0, \frac{a_B + c_A - c_d - c_B\varepsilon_B}{-1 + \varepsilon_B} \right\rangle$
$EqS5^{\mathcal{T}'}$	$\left( \frac{a_A + \varepsilon_A(a_B - c_d - (e + c_B)\varepsilon_B)}{1 + \varepsilon_A^2}, 2a_B + \frac{a_A - p_{rA}}{\varepsilon_A} - 2(c_d + (e + c_B)\varepsilon_B), \right.$ $\left( \frac{\varepsilon_A(a_A - p_{rA} + \varepsilon_A(a_B - c_d - (e + c_B)\varepsilon_B))}{1 + \varepsilon_A^2}, \right)$ $\left( \frac{(-(a_A - p_{rA} + (a_B - c_d)\varepsilon_A)(1 + 3\varepsilon_A^2) + \varepsilon_A(2e + c_B + (4e + 3c_B)\varepsilon_A^2)\varepsilon_B)}{\varepsilon_A(1 + \varepsilon_A^2)\varepsilon_B} \right)$
$EqS6^{\mathcal{T}'}$	$\left( \frac{a_A - p_{rA} + \varepsilon_A(a_B - c_d - c_B\varepsilon_B)}{2 + 4\varepsilon_A^2}, -\frac{(a_A - p_{rA})\varepsilon_A - (a_B - c_d - c_B\varepsilon_B)(1 + \varepsilon_A^2)}{1 + 2\varepsilon_A^2}, \right.$ $\left( \frac{\varepsilon_A(a_A - p_{rA} + \varepsilon_A(a_B - c_d - c_B\varepsilon_B))}{2 + 4\varepsilon_A^2}, 0 \right)$
$EqS7^{\mathcal{T}'}$	$\left\langle 0, -\frac{a_A - p_{rA}}{\varepsilon_A}, 0, \frac{a_A - p_{rA} + \varepsilon_A(a_B - c_d - c_B\varepsilon_B)}{\varepsilon_A\varepsilon_B} \right\rangle$

Moreover, the optimal choice of the triplets of  $\langle q_A, w, q_B, t \rangle^{T'}$  can be characterized as follows

Condition		$\langle q_A, w, q_B, t \rangle^{T'}$
$e < \min \{e_4, e_{t4}, e_{t5}\}$	$c_d \leq c_{d1}$	$\Rightarrow EqS6^{T'}$
	$c_{d1} \leq c_d \leq c_{d0}$	$\Rightarrow EqS3^{T'}$
$e_{t5} < e < \min \{e_4, e_{t4}\}$	$c_d \leq c_{d1}$	$\Rightarrow EqS6^{T'}$
	$c_{d1} \leq c_d \leq c_{dt2}$	$\Rightarrow EqS3^{T'}$
	$c_{dt2} \leq c_d \leq c_{dt5}$	$\Rightarrow EqS1^{T'}$
	$c_{dt5} \leq c_d \leq c_{d0}$	$\Rightarrow EqS4^{T'}$
$\min\{e_4, e_{t4}\} < e < e_{t4}$	$c_d \leq c_{d1}$	$\Rightarrow EqS6^{T'}$
	$c_{d1} \leq c_d \leq c_{dt2}$	$\Rightarrow EqS3^{T'}$
	$c_{dt2} \leq c_d \leq c_{dt1}$	$\Rightarrow EqS1^{T'}$
	$c_{dt2} \leq c_d \leq c_{dt3}$	$\Rightarrow EqS2^{T'}$
	$c_{dt3} \leq c_d \leq c_{d10}$	$\Rightarrow EqS5^{T'}$
	$c_{d10} \leq c_d \leq c_{d9}$	$\Rightarrow EqS7^{T'}$
	$c_{d9} \leq c_d \leq c_{d0}$	$\Rightarrow EqS4^{T'}$
$\min\{e_4, e_{t4}\} < e < e_4$	$c_d \leq c_{d1}$	$\Rightarrow EqS6^{T'}$
	$c_{d1} \leq c_d \leq c_{dt1}$	$\Rightarrow EqS2^{T'}$
	$\varepsilon_B < 1, c_{dt1} \leq c_d \leq c_{dt2}$	$\Rightarrow EqS1^{T'}$
	$1 < \varepsilon_B, c_{dt1} \leq c_d \leq c_{dt5}$	$\Rightarrow EqS1^{T'}$
	$\varepsilon_B < 1, c_{dt2} \leq c_d \leq c_{d0}$	$\Rightarrow EqS3^{T'}$
$\max\{e_4, e_{t4}\} < e < e_2$	$1 < \varepsilon_B, c_{dt5} \leq c_d \leq c_{d0}$	$\Rightarrow EqS4^{T'}$
	$c_d \leq c_{d1}$	$\Rightarrow EqS6^{T'}$
	$c_{d1} \leq c_d \leq c_{dt3}$	$\Rightarrow EqS2^{T'}$
	$c_{dt3} \leq c_d \leq c_{d10}$	$\Rightarrow EqS5^{T'}$
	$c_{d10} \leq c_d \leq c_{d9}$	$\Rightarrow EqS7^{T'}$
$e_2 < e$	$c_{d9} \leq c_d \leq c_{d0}$	$\Rightarrow EqS4^{T'}$
	$c_d \leq c_{dt4}$	$\Rightarrow EqS6^{T'}$
	$c_{dt4} \leq c_d \leq c_{d10}$	$\Rightarrow EqS5^{T'}$
	$c_{d10} \leq c_d \leq c_{d9}$	$\Rightarrow EqS7^{T'}$
	$c_{d9} \leq c_d \leq c_{d0}$	$\Rightarrow EqS4^{T'}$

Furthermore, the total profits of both manufacturers, the benefit of the government, and

the environmental impact of the strategy  $\mathcal{T}'$ ,  $\Pi_A^{\mathcal{T}'}$ ,  $\Pi_B^{\mathcal{T}'}$ ,  $G^{\mathcal{T}'}$ , and  $E^{\mathcal{T}'}$  in each of the possible equilibriums can also be solved.  $\square$

**Corollary 1.** When  $c_{d1} \leq c_d \leq c_{d0}$ ,  $w^{c1}|_{c_d=c_{d0}} = -c_A < 0$ ,  $w^{c1}|_{c_d=c_{d1}} = (a_A - p_{rA})\varepsilon_A - c_A(1 + \varepsilon_A^2)$ , then if  $a_A \geq \frac{c_A(1+\varepsilon_A^2)}{\varepsilon_A} + p_{rA}$ ,  $w^{c1}|_{c_d=c_{d1}} \geq 0$ ; otherwise,  $w^{c1}|_{c_d=c_{d1}} < 0$ . Therefore, if  $a_A \geq \frac{c_A(1+\varepsilon_A^2)}{\varepsilon_A} + p_{rA}$  and  $a_B - c_A - c_B\varepsilon_B \leq c_d \leq c_{d0}$  or  $a_A \leq \frac{c_A(1+\varepsilon_A^2)}{\varepsilon_A} + p_{rA}$  and  $c_{d1} \leq c_d \leq c_{d0}$ ,  $w^{c1} \leq 0$ . When  $c_d < c_{d1}$ ,  $w^{c3}|_{c_d=c_{d1}} = (a_A - p_{rA})\varepsilon_A - c_A(1 + \varepsilon_A^2)$ , then if  $a_A \leq \frac{c_A(1+\varepsilon_A^2)}{\varepsilon_A} + p_{rA}$ ,  $w^{c3}|_{c_d=c_{d1}} \leq 0$ ; otherwise,  $w^{c3}|_{c_d=c_{d1}} > 0$ . Therefore, if  $a_A \leq \frac{c_A(1+\varepsilon_A^2)}{\varepsilon_A} + p_{rA}$  and  $a_B - \frac{(a_A - p_{rA})\varepsilon_A}{1+\varepsilon_A^2} - c_B\varepsilon_B \leq c_d \leq c_{d1}$ ,  $w^{c3} \leq 0$ .  $\square$

**Lemma 1.** When  $c_{d1} \leq c_d \leq c_{d0}$ ,  $\Pi_A^{c1} - \Pi_A^{\mathcal{N}} = \frac{1}{8}(a_B + c_A - c_d - c_B\varepsilon_B)^2 > 0$ . When  $c_d < c_{d1}$ , as  $\frac{d^2(\Pi_A^{c3} - \Pi_A^{\mathcal{N}})}{d(c_d)^2} = \frac{\varepsilon_A^2}{2+4\varepsilon_A^2} > 0$ ,  $\frac{d(\Pi_A^{c3} - \Pi_A^{\mathcal{N}})}{dc_d}|_{c_d=c_{d1}} = -\frac{1}{2}\varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) < 0$ , and  $\Pi_A^{c3} - \Pi_A^{\mathcal{N}}|_{c_d=c_{d1}} = \frac{1}{2}\varepsilon_A^2(a_A - p_{rA} - c_A\varepsilon_A)^2 > 0$ , then  $\Pi_A^{c3} > \Pi_A^{\mathcal{N}}$ .  $\square$

**Lemma 2.** When  $c_{d1} \leq c_d \leq c_{d0}$ , we can get that  $\frac{d^2(\Pi_B^{c1} - \Pi_B^{\mathcal{N}})}{d(c_d)^2} = \frac{1}{8} > 0$ ,  $\frac{d(\Pi_B^{c1} - \Pi_B^{\mathcal{N}})}{dc_d}|_{c_d=c_{d1}} = -\frac{1}{4}\varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) < 0$ , and  $\Pi_B^{c1} - \Pi_B^{\mathcal{N}}|_{c_d=c_{d1}} = \frac{1}{16}(4\varepsilon_A^2(a_A - p_{rA} - c_A\varepsilon_A)^2 - 4(a_B - p_{rB} - c_B\varepsilon_B)^2)$ . When  $a_A \geq a_{A1}$ ,  $a_{A1} = \frac{a_B - p_{rB} - c_B\varepsilon_B + c_A\varepsilon_A^2 + p_{rA}\varepsilon_A}{\varepsilon_A}$ ,  $\Pi_B^{c1} - \Pi_B^{\mathcal{N}}|_{c_d=c_{d1}} \geq 0$ , as  $\Pi_B^{c1} - \Pi_B^{\mathcal{N}}|_{c_d=c_{d0}} = -\frac{1}{4}(a_B - p_{rB} - c_B\varepsilon_B)^2 < 0$ , then if  $c_{d1} \leq c_d \leq c_{d2}$ ,  $\Pi_B^{c1} \geq \Pi_B^{\mathcal{N}}$ , where  $c_{d2} = -a_B + c_A + 2p_{rB} + c_B\varepsilon_B$ ; if  $c_{d2} < c_d \leq c_{d0}$ ,  $\Pi_B^{c1} < \Pi_B^{\mathcal{N}}$ . When  $a_A < a_{A1}$ ,  $\Pi_B^{c1} - \Pi_B^{\mathcal{N}}|_{c_d=c_{d1}} < 0$ , then if  $c_{d1} \leq c_d \leq c_{d0}$ ,  $\Pi_B^{c1} < \Pi_B^{\mathcal{N}}$ .

When  $c_d < c_{d1}$ , we can get that  $\frac{d^2(\Pi_B^{c3} - \Pi_B^{\mathcal{N}})}{d(c_d)^2} = \frac{\varepsilon_A^4}{2(1+2\varepsilon_A^2)^2} > 0$ ,  $\frac{d(\Pi_B^{c3} - \Pi_B^{\mathcal{N}})}{dc_d}|_{c_d=c_{d1}} = -\frac{\varepsilon_A^3(a_A - p_{rA} - c_A\varepsilon_A)}{2+4\varepsilon_A^2} < 0$ , and  $\Pi_B^{c3} - \Pi_B^{\mathcal{N}}|_{c_d=c_{d1}} = \frac{1}{4}(\varepsilon_A^2(a_A - p_{rA} - c_A\varepsilon_A)^2 - (a_B - p_{rB} - c_B\varepsilon_B)^2)$ . When  $a_A \geq a_{A1}$ ,  $\Pi_B^{c3} - \Pi_B^{\mathcal{N}}|_{c_d=c_{d1}} \geq 0$ , then  $\Pi_B^{c3} - \Pi_B^{\mathcal{N}} \geq 0$ . When  $a_A < a_{A1}$ ,  $\Pi_B^{c3} - \Pi_B^{\mathcal{N}}|_{c_d=c_{d1}} < 0$ , then if  $c_d \leq c_{d4}$ ,  $\Pi_B^{c3} \geq \Pi_B^{\mathcal{N}}$ , where  $c_{d4} =$

$\frac{(a_A - p_{rA})\varepsilon_A - a_B(1 + \varepsilon_A^2) + p_{rB}(1 + 2\varepsilon_A^2) + c_B(1 + \varepsilon_A^2)\varepsilon_B}{\varepsilon_A^2}$ ; if  $c_{d4} < c_d < c_{d1}$ ,  $\Pi_B^{C3} < \Pi_B^N$ .  $\square$

**Lemma 3.** When  $c_{d1} \leq c_d \leq c_{d0}$ , we can get that  $\frac{d(E^{C1} - E^N)}{dc_d} = \frac{1}{4}e(1 - \varepsilon_B)$  and  $E^{C1} - E^N|_{c_d=c_{d0}} = -\frac{1}{2}e\varepsilon_B(a_B - p_{rB} - c_B\varepsilon_B) < 0$ . Then, if  $\varepsilon_B \geq 1$ ,  $\frac{d(E^{C1} - E^N)}{dc_d} \leq 0$ ; otherwise,  $\frac{d(E^{C1} - E^N)}{dc_d} > 0$ . (i) When  $\varepsilon_B \geq 1$ ,  $E^{C1} - E^N|_{c_d=c_{d1}} = \frac{1}{2}e(\varepsilon_A(-1 + \varepsilon_B)(a_A - p_{rA} - c_A\varepsilon_A)) - \varepsilon_B(a_B - p_{rB} - c_B\varepsilon_B)$ , we can derive that if  $a_A \geq a_{A2}$ , where  $a_{A2} = p_{rA} + c_A\varepsilon_A + \frac{\varepsilon_B(a_B - p_{rB} - c_B\varepsilon_B)}{\varepsilon_A(-1 + \varepsilon_B)}$ ,  $E^{C1} - E^N|_{c_d=c_{d1}} \geq 0$ , then if  $c_{d1} \leq c_d \leq c_{d3}$ , where  $c_{d3} = \frac{(a_B - c_B\varepsilon_B)(1 + \varepsilon_B) + c_A(1 - \varepsilon_B) - 2p_{rB}\varepsilon_B}{1 - \varepsilon_B}$ ,  $E^{C1} - E^N \geq 0$ , otherwise, if  $c_{d3} < c_d \leq c_{d0}$ ,  $E^{C1} - E^N < 0$ . If  $a_A < a_{A2}$ ,  $E^{C1} - E^N|_{c_d=c_{d1}} < 0$ ,  $E^{C1} - E^N < 0$ . (ii) When  $\varepsilon_B < 1$ ,  $E^{C1} - E^N|_{c_d=c_{d1}} = \frac{1}{2}e(\varepsilon_A(-1 + \varepsilon_B)(a_A - p_{rA} - c_A\varepsilon_A)) - \varepsilon_B(a_B - p_{rB} - c_B\varepsilon_B) < 0$ , we can derive that if  $c_{d1} \leq c_d \leq c_{d0}$ ,  $E^{C1} - E^N \leq 0$ .

When  $c_d < c_{d1}$ , we can get that  $\frac{d(E^{C3} - E^N)}{dc_d} = -\frac{e\varepsilon_A^2\varepsilon_B}{2 + 4\varepsilon_A^2} < 0$ , and  $E^{C3} - E^N|_{c_d=c_{d1}} = \frac{1}{2}e(\varepsilon_A(-1 + \varepsilon_B)(a_A - p_{rA} - c_A\varepsilon_A)) - \varepsilon_B(a_B - p_{rB} - c_B\varepsilon_B)$ . (i) When  $\varepsilon_B \geq 1$ , we can derive that if  $a_A \geq a_{A2}$ ,  $E^{C3} - E^N|_{c_d=c_{d1}} \geq 0$ , then if  $c_d \leq c_{d1}$ ,  $E^{C3} - E^N \geq 0$ . If  $a_A < a_{A2}$ ,  $E^{C3} - E^N|_{c_d=c_{d1}} < 0$ , then if  $c_d \leq c_{d5}$ ,  $E^{C3} - E^N \geq 0$ , otherwise, if  $c_{d5} < c_d < c_{d1}$ ,  $E^{C3} - E^N < 0$ , where  $c_{d5} = \frac{-(a_A - p_{rA} - c_A\varepsilon_A)(\varepsilon_A + 2\varepsilon_A^3) + ((a_A - p_{rA})\varepsilon_A - a_B(1 + \varepsilon_A^2) + p_{rB}(1 + 2\varepsilon_A^2))\varepsilon_B + c_B(1 + \varepsilon_A^2)\varepsilon_B^2}{\varepsilon_A^2\varepsilon_B}$ . (ii) When  $\varepsilon_B < 1$ ,  $E^{C3} - E^N|_{c_d=c_{d1}} < 0$ , we can derive that if  $c_{d5} < c_d < c_{d1}$ ,  $E^{C3} - E^N < 0$ , otherwise, if  $c_d \leq c_{d5}$ ,  $E^{C3} - E^N \geq 0$ .  $\square$

**Theorem 1.** This result can be directly derived from Lemmas 1, 2, and 3.  $\square$

**Corollary 2.** This result can be directly derived based on Theorem 1.  $\square$

**Theorem 2.** When  $s < s_1$ ,

$$(i) \quad \Pi_A^S - \Pi_A^N = \frac{1}{8}(s + a_B + c_A - c_d - c_B\varepsilon_B)^2 > 0;$$

$$(ii) \quad \frac{d^2(\Pi_B^S - \Pi_B^N)}{ds^2} = \frac{1}{8} > 0 \quad ; \quad \Pi_B^S - \Pi_B^N|_{s=0} = \frac{1}{16}((a_B + c_A - c_d - c_B\varepsilon_B)^2 - 4(a_B - p_{rB} - c_B\varepsilon_B)^2) \quad , \quad (a_B + c_A - c_d - c_B\varepsilon_B) - 2(a_B - p_{rB} - c_B\varepsilon_B) < 0 \quad ;$$

$\frac{d(\Pi_B^S - \Pi_B^N)}{ds} \Big|_{s=s_1} = \frac{1}{4} \varepsilon_A (a_A - p_{rA} - c_A \varepsilon_A) > 0$  ;  $\Pi_B^S - \Pi_B^N \Big|_{s=s_1} = \frac{1}{16} (4\varepsilon_A^2 (a_A - p_{rA} - c_A \varepsilon_A)^2 - 4(a_B - p_{rB} - c_B \varepsilon_B)^2)$ , then if  $a_A < a_{A1}$ ,  $\Pi_B^S - \Pi_B^N \Big|_{s=s_1} < 0$ , otherwise, if  $a_A > a_{A1}$ ,  $\Pi_B^S - \Pi_B^N \Big|_{s=s_1} > 0$ ; By solving  $\Pi_B^S - \Pi_B^N = 0$ , we can get two solutions that satisfy  $a_B - c_A + c_d - 2p_{rB} - c_B \varepsilon_B > -3a_B - c_A + c_d + 2p_{rB} + 3c_B \varepsilon_B$ , and  $-3a_B - c_A + c_d + 2p_{rB} + 3c_B \varepsilon_B < 0$  if  $c_d < c_{d0}$ ; then, we can derive that when  $a_A < a_{A1}$ ,  $\Pi_B^S - \Pi_B^N < 0$ , when  $a_A > a_{A1}$ , if  $s_2 < s < s_1$ ,  $\Pi_B^S - \Pi_B^N > 0$ , if  $s < s_2$ ,  $\Pi_B^S - \Pi_B^N < 0$ , where  $s_2 = a_B - c_A + c_d - 2p_{rB} - c_B \varepsilon_B$ ;  
 (iii)  $\frac{d(E^S - E^N)}{ds} = \frac{1}{4} e(-1 + \varepsilon_B)$ , if  $\varepsilon_B > 1$ ,  $\frac{d(E^S - E^N)}{ds} > 0$ , otherwise,  $\frac{d(E^S - E^N)}{ds} < 0$ ; only consider the case where  $\Pi_B^S > \Pi_B^N$ , i.e., when  $a_A > a_{A1}$ , and  $s_2 < s < s_1$ ,  $E^S - E^N \Big|_{s=s_2} = -\frac{1}{2} e(a_B - p_{rB} - c_B \varepsilon_B) < 0$ , (a) when  $\varepsilon_B > 1$ ,  $E^S - E^N \Big|_{s=s_1} = \frac{1}{2} e((a_A - p_{rA} - c_A \varepsilon_A) \varepsilon_A (-1 + \varepsilon_B) - \varepsilon_B (a_B - p_{rB} - c_B \varepsilon_B))$ , then if  $a_{A1} < a_A < a_{A2}$ ,  $E^S - E^N \Big|_{s=s_1} < 0$ ,  $E^S - E^N < 0$ , otherwise, if  $a_A > a_{A2}$ ,  $E^S - E^N \Big|_{s=s_1} > 0$ , if  $s_2 < s < s_3$ ,  $E^S - E^N < 0$ , if  $s_3 < s < s_1$ ,  $E^S - E^N > 0$ , where  $s_3 = \frac{c_A - c_d + a_B(1 + \varepsilon_B) - \varepsilon_B(c_A - c_d + 2p_{rB} + c_B(1 + \varepsilon_B))}{-1 + \varepsilon_B}$ ; (b) when  $\varepsilon_B < 1$ ,  $E^S - E^N \Big|_{s=s_1} < 0$ ,  $E^S - E^N < 0$ .

When  $s > s_1$ ,

- (i)  $\frac{d^2(\Pi_A^S - \Pi_A^N)}{ds^2} = \frac{\varepsilon_A^2}{2 + 4\varepsilon_A^2} > 0$  ;  $\frac{d(\Pi_A^S - \Pi_A^N)}{ds} \Big|_{s=s_1} = \frac{1}{2} \varepsilon_A (a_A - p_{rA} - c_A \varepsilon_A) > 0$  ;  $\Pi_A^S - \Pi_A^N \Big|_{s=s_1} = \frac{1}{2} \varepsilon_A^2 (a_A - p_{rA} - c_A \varepsilon_A)^2 > 0$ , then  $\Pi_A^S > \Pi_A^N$ ;  
 (ii)  $\frac{d^2(\Pi_B^S - \Pi_B^N)}{ds^2} = \frac{\varepsilon_A^4}{2(1 + 2\varepsilon_A^2)^2} > 0$  ;  $\frac{d(\Pi_B^S - \Pi_B^N)}{ds} \Big|_{s=s_1} = \frac{\varepsilon_A^3 (a_A - p_{rA} - c_A \varepsilon_A)}{2 + 4\varepsilon_A^2} > 0$  ;  $\Pi_B^S - \Pi_B^N \Big|_{s=s_1} = \frac{1}{4} (\varepsilon_A^2 (a_A - p_{rA} - c_A \varepsilon_A)^2 - (a_B - p_{rB} - c_B \varepsilon_B)^2)$ , then if  $a_A < a_{A1}$ ,  $\Pi_B^S - \Pi_B^N \Big|_{s=s_1} < 0$ , otherwise, if  $a_A > a_{A1}$ ,  $\Pi_B^S - \Pi_B^N \Big|_{s=s_1} > 0$ ; then, we can derive that when  $a_A > a_{A1}$ ,  $\Pi_B^S - \Pi_B^N > 0$ , when  $a_A < a_{A1}$ , if  $s_1 < s < s_4$ ,  $\Pi_B^S - \Pi_B^N < 0$ , if  $s > s_4$ ,  $\Pi_B^S - \Pi_B^N > 0$ , where  $s_4 = c_d - 2p_{rB} - c_B \varepsilon_B - \frac{p_{rB} + (a_A - p_{rA}) \varepsilon_A - a_B(1 + \varepsilon_A^2) + c_B \varepsilon_B}{\varepsilon_A^2}$ ;  
 (iii)  $\frac{d(E^S - E^N)}{ds} = \frac{e \varepsilon_A^2 \varepsilon_B}{2 + 4\varepsilon_A^2} > 0$ ; only consider the case where  $\Pi_B^S > \Pi_B^N$ , (a) when  $a_A <$



$a_{A1}$  and  $s > s_4$ , as  $E^S - E^N|_{s=s_4} = -\frac{1}{2}e\varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) < 0$ , then if  $s_4 < s < s_5$ ,  $E^S - E^N < 0$ , otherwise, if  $s > s_5$ ,  $E^S - E^N > 0$ , where  $s_5 = \frac{(a_A - p_{rA} - c_A\varepsilon_A)(\varepsilon_A + 2\varepsilon_A^3) + (a_B - p_{rB} - (a_A - p_{rA})\varepsilon_A + (a_B + c_d - 2p_{rB})\varepsilon_A^2)\varepsilon_B - c_B(1 + \varepsilon_A^2)\varepsilon_B^2}{\varepsilon_A^2\varepsilon_B}$ ; (b) when  $a_A > a_{A1}$ ,  $E^S - E^N|_{s=s_1} = \frac{1}{2}e(a_A - p_{rA} - c_A\varepsilon_A)\varepsilon_A(-1 + \varepsilon_B) - \varepsilon_B(a_B - p_{rB} - c_B\varepsilon_B)$ ; Then if  $\varepsilon_B < 1$ ,  $E^S - E^N|_{s=s_1} < 0$ , and if  $s_1 < s < s_5$ ,  $E^S - E^N < 0$ , if  $s > s_5$ ,  $E^S - E^N > 0$ ; Otherwise, if  $\varepsilon_B > 1$ ,  $E^S - E^N|_{s=s_1} > E^S - E^N|_{s=s_1|a_A=a_{A1}} = -\frac{1}{2}e(a_B - p_{rB} - c_B\varepsilon_B)$ , if  $a_{A1} < a_A < a_{A2}$ ,  $E^S - E^N|_{s=s_1} < 0$ , then if  $s_1 < s < s_5$ ,  $E^S - E^N < 0$ , if  $s > s_5$ ,  $E^S - E^N > 0$ , otherwise, if  $a_A > a_{A2}$ ,  $E^S - E^N|_{s=s_1} > 0$ ,  $E^S - E^N > 0$ .  $\square$

**Corollary 3 and Corollary 4.** The results can be directly derived based on the equilibrium solution in Appendix A.4.  $\square$

**Lemma 4.** We hereafter use  $EqSk$  to represent  $EqSk^S$ , where  $k = 1, 2, 3, 4$ .

(1) When  $\varepsilon_B > 1$ , then

(i) If  $a_A < a_{A1}$ , we need to compare  $c_{d4} < c_d < c_{d0}$ .

When  $c_{d4} < c_d < c_{d1}$ ,  $\Pi_A^S(EqS4) = \Pi_A^{c3} > \Pi_A^N$ , when  $c_{d1} < c_d < c_{d0}$ ,  $\Pi_A^S(EqS3) = \Pi_A^{c1} > \Pi_A^N$ .

(ii) If  $a_A > a_{A1}$ , we need to compare  $c_{d2} < c_d < c_{d0}$ . Then,  $\Pi_A^S(EqS3) = \Pi_A^{c1} > \Pi_A^N$ .

(2) When  $\varepsilon_B < 1$  and  $e < e_1$ , then

(i) If  $a_A < a_{A1}$ , we need to compare  $c_{d4} < c_d < c_{d0}$ .

When  $c_{d4} < c_d < c_{d1}$ ,  $\Pi_A^S(EqS4) = \Pi_A^{c3} > \Pi_A^N$ , when  $c_{d1} < c_d < c_{d8}$ ,  $\Pi_A^S(EqS3) = \Pi_A^{c1} > \Pi_A^N$ ; when  $c_{d8} < c_d < c_{d0}$ ,  $\Pi_A^S(EqS1) - \Pi_A^N = \frac{1}{32}(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B)^2 > 0$ .

(ii) If  $a_A > a_{A1}$ , we need to compare  $c_{d2} < c_d < c_{d0}$ .

As  $c_{d2} - c_{d8} = e - 2a_B + 2p_{rB} - (e - 2c_B)\varepsilon_B < c_{d2} - c_{d8}|_{e=e_1} = 2(-a_B + p_{rB} + \varepsilon_A(a_A - c_A\varepsilon_A) + c_B\varepsilon_B)$ ,  $c_{d2} - c_{d8}|_{e=e_1} > c_{d2} - c_{d8}|_{e=e_1|a_A=a_{A1}} = 0$ , so there exists a threshold value of  $e_2$ ,  $e_2 = \frac{2(-a_B + p_{rB} + c_B\varepsilon_B)}{-1 + \varepsilon_B}$ , if  $e < e_2$ ,  $c_{d2} - c_{d8} < 0$ , else if  $e > e_2$ ,  $c_{d2} - c_{d8} > 0$ .

If  $e < e_2$ , when  $c_{d2} < c_d < c_{d8}$ ,  $\Pi_A^S(EqS3) = \Pi_A^{c1} > \Pi_A^N$ ; when  $c_{d8} < c_d < c_{d0}$ ,

$\Pi_A^S(EqS1) - \Pi_A^N = \frac{1}{32}(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B)^2 > 0$ . Otherwise, if  $e > e_2$ ,

when  $c_{d2} < c_d < c_{d0}$ ,  $\Pi_A^S(EqS1) - \Pi_A^N > 0$ .

(3) When  $\varepsilon_B < 1$  and  $e > e_1$ , then

(i) If  $a_A < a_{A1}$ , we need to compare  $c_{d4} < c_d < c_{d0}$ .  $c_{d7} - c_{d0} = e - 4(a_A - p_{rA})\varepsilon_A + 4c_A\varepsilon_A^2 - e\varepsilon_B$ , then if  $e > 2e_1$ ,  $c_{d7} > c_{d0}$ ; else if  $e < 2e_1$ ,  $c_{d7} < c_{d0}$ .

If  $e_1 < e < 2e_1$ , when  $c_{d4} < c_d < c_{d1}$ ,  $\Pi_A^S(EqS4) = \Pi_A^{c3} > \Pi_A^N$ , when  $c_{d1} < c_d < c_{d7}$ ,  $\Pi_A^S(EqS2) - \Pi_A^N = \frac{1}{2}\varepsilon_A^2(a_A - p_{rA} - c_A\varepsilon_A)^2 > 0$ ; when  $c_{d7} < c_d < c_{d0}$ ,

$\Pi_A^S(EqS1) - \Pi_A^N = \frac{1}{32}(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B)^2 > 0$ .

If  $e > 2e_1$ , when  $c_{d4} < c_d < c_{d1}$ ,  $\Pi_A^S(EqS4) = \Pi_A^{c3} > \Pi_A^N$ , when  $c_{d1} < c_d < c_{d0}$ ,  $\Pi_A^S(EqS2) - \Pi_A^N = \frac{1}{2}\varepsilon_A^2(a_A - p_{rA} - c_A\varepsilon_A)^2 > 0$ .

(ii) If  $a_A > a_{A1}$ , we need to compare  $c_{d2} < c_d < c_{d0}$ .  $\frac{d(c_{d2} - c_{d7})}{de} = -1 + \varepsilon_B < 0$ ,

$c_{d2} - c_{d7} < c_{d2} - c_{d7}|_{e=e_1} = 2(-a_B + p_{rB} + \varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) + c_B\varepsilon_B)$ , as  $c_{d2} - c_{d7}|_{e=e_1} > c_{d2} - c_{d7}|_{e=e_1|a_A=a_{A1}} = 0$ , then there exists a threshold value of  $e_3$ ,

where  $e_3 = \frac{2(-a_B + p_{rB} + 2\varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) + c_B\varepsilon_B)}{1 - \varepsilon_B}$ , then if  $e_1 < e < e_3$ ,  $c_{d7} < c_{d2} < c_{d0}$ ,

else if  $e_3 < e < 2e_1$ ,  $c_{d2} < c_{d7} < c_{d0}$ , else if  $e > 2e_1$ ,  $c_{d2} < c_{d0} < c_{d7}$ .

If  $e_1 < e < e_3$ , when  $c_{d2} < c_d < c_{d0}$ ,  $\Pi_A^S(EqS1) - \Pi_A^N = \frac{1}{32}(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B)^2 > 0$ .

If  $e_1 < e < 2e_1$ , when  $c_{d2} < c_d < c_{d7}$ ,  $\Pi_A^S(EqS2) - \Pi_A^N = \frac{1}{2}\varepsilon_A^2(a_A - p_{rA} - c_A\varepsilon_A)^2 > 0$ ; when  $c_{d7} < c_d < c_{d0}$ ,  $\Pi_A^S(EqS1) - \Pi_A^N = \frac{1}{32}(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B)^2 > 0$ .

If  $e > 2e_1$ , when  $c_{d2} < c_d < c_{d0}$ ,  $\Pi_A^S(EqS2) - \Pi_A^N = \frac{1}{2}\varepsilon_A^2(a_A - p_{rA} - c_A\varepsilon_A)^2 > 0$ .

0. □

### Lemma 5.

(1) When  $\varepsilon_B > 1$ , then

(i) If  $a_A < a_{A1}$ , we need to compare  $c_{d4} < c_d < c_{d0}$ .

When  $c_{d4} < c_d < c_{d1}$ ,  $\Pi_B^S(EqS4) = \Pi_B^{c3} < \Pi_B^N$ , when  $c_{d1} < c_d < c_{d0}$ ,  $\Pi_B^S(EqS3) = \Pi_B^{c1} < \Pi_B^N$ .

(ii) If  $a_A > a_{A1}$ , we need to compare  $c_{d2} < c_d < c_{d0}$ . Then,  $\Pi_B^S(EqS3) = \Pi_B^{c1} < \Pi_B^N$ .

(2) When  $\varepsilon_B < 1$  and  $e < e_1$ , then

(i) If  $a_A < a_{A1}$ , we need to compare  $c_{d4} < c_d < c_{d0}$ .

When  $c_{d4} < c_d < c_{d1}$ ,  $\Pi_B^S(EqS4) = \Pi_B^{c3} < \Pi_B^N$ , when  $c_{d1} < c_d < c_{d8}$ ,  $\Pi_B^S(EqS3) = \Pi_B^{c1} < \Pi_B^N$ ; when  $c_{d8} < c_d < c_{d0}$ ,  $\frac{d^2(\Pi_B^S(EqS1) - \Pi_B^N)}{dc_d^2} = \frac{1}{32} > 0$ ,  $\frac{d(\Pi_B^S(EqS1) - \Pi_B^N)}{dc_d}|_{c_d=c_{d0}} = \frac{1}{32}e(-1 + \varepsilon_B) < 0$ ,  $\Pi_B^S(EqS1) - \Pi_B^N|_{c_d=c_{d8}} = \frac{1}{64}(4e^2(-1 + \varepsilon_B)^2 - 16(-a_B + p_{rB} + c_B\varepsilon_B)^2)$ , as  $2e(1 - \varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B) < 2e(1 - \varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B)|_{e=e_1} = 4(-a_B + p_{rB} + \varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) + c_B\varepsilon_B) < 4(-a_B + p_r + \varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) + c_B\varepsilon_B)|_{a_A=a_{A1}} = 0$ , so  $\Pi_B^S(EqS1) - \Pi_B^N|_{c_d=c_{d8}} < 0$ ,  $\Pi_B^S(EqS1) - \Pi_B^N < 0$ .

(ii) If  $a_A > a_{A1}$ , we need to compare  $c_{d2} < c_d < c_{d0}$ .

If  $e < e_2$ , when  $c_{d2} < c_d < c_{d8} < c_{d0}$ ,  $\Pi_B^S(EqS3) = \Pi_B^{c1} < \Pi_B^N$ ; when  $c_{d8} < c_d < c_{d0}$ ,  $\Pi_B^S(EqS1) - \Pi_B^N = \frac{1}{64}(-16(-a_B + p_{rB} + c_B\varepsilon_B)^2 + (e + a_B + c_A - c_d - (e + c_B)\varepsilon_B)^2)$ , as  $(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B) < (e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B)|_{c_d=c_{d8}} = 2e - 4a_B + 4p_{rB} - 2(e - 2c_B)\varepsilon_B < 2e - 4a_B + 4p_{rB} - 2(e - 2c_B)\varepsilon_B|_{e=e_2} = 0$ , then  $\Pi_B^S(EqS1) - \Pi_B^N < 0$ .

If  $e > e_2$ , when  $c_{d2} < c_d < c_{d0}$ ,  $\Pi_B^S(EqS1) - \Pi_B^N = \frac{1}{64}(-16(-a_B + p_{rB} + c_B\varepsilon_B)^2 + (e + a_B + c_A - c_d - (e + c_B)\varepsilon_B)^2)$ , as  $(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B)|_{c_d=c_{d2}} = e - 2a_B + 2p_{rB} - (e - 2c_B)\varepsilon_B > e - 2a_B + 2p_{rB} - (e - 2c_B)\varepsilon_B|_{e=e_2} = 0$ , then  $\Pi_B^S(EqS1) - \Pi_B^N|_{c_d=c_{d2}} > 0$ .  $(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B)|_{c_d=c_{d0}} = e - 4a_B + 4p_{rB} - (e - 4c_B)\varepsilon_B > e - 4a_B + 4p_{rB} - (e - 4c_B)\varepsilon_B|_{e=e_2} = 2(-a_B + p_{rB} + c_B\varepsilon_B)$ , then there exists a threshold value of  $2e_2$ , if  $e > 2e_2$ ,  $(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B)|_{c_d=c_{d0}} > 0$ , then  $\Pi_B^S(EqS1) - \Pi_B^N > 0$ , else if  $e_2 < e < 2e_2$ ,  $(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B)|_{c_d=c_{d0}} < 0$ , and there exists a threshold value of  $c_{d9}$ , where  $c_{d9} = e - 3a_B + c_A + 4p_{rB} - (e - 3c_B)\varepsilon_B$ , if  $c_d < c_{d9}$ ,  $\Pi_B^S(EqS1) - \Pi_B^N > 0$ , else if  $c_{d9} < c_d < c_{d0}$ ,  $\Pi_B^S(EqS1) - \Pi_B^N < 0$ .  $2e_2 -$

$$e_1 = \frac{-4a_B + 4p_{rB} + 2\varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) + 4c_B\varepsilon_B}{-1 + \varepsilon_B} < 2e_2 - e_1|_{a_A=a_{A1}} = \frac{2(-a_B + p_{rB} + c_B\varepsilon_B)}{-1 + \varepsilon_B} > 0 \quad ,$$

then there exists a threshold value of  $a_{A3}$ , where  $a_{A3} = \frac{2a_B - 2p_{rB} + c_A\varepsilon_A^2 - 2c_B\varepsilon_B}{\varepsilon_A}$ , if  $a_{A1} < a_A < a_{A3}$ ,  $2e_2 > e_1$ ; else if  $a_A > a_{A3}$ ,  $2e_2 < e_1$ . We can find that  $c_{d9} > c_{d0}$  if  $e > 2e_2$ .

(3) When  $\varepsilon_B < 1$  and  $e > e_1$ , then

(i) If  $a_A < a_{A1}$ , we need to compare  $c_{d4} < c_d < c_{d0}$ .

If  $e_1 < e < 2e_1$ , when  $c_{d4} < c_d < c_{d1}$ ,  $\Pi_B^S(EqS4) = \Pi_B^{c3} < \Pi_B^N$ , when  $c_{d1} < c_d < c_{d7}$ ,  $\Pi_B^S(EqS2) - \Pi_B^N = \frac{1}{4}(\varepsilon_A^2(a_A - p_{rA} - c_A\varepsilon_A)^2 - (-a_B + p_{rB} + c_B\varepsilon_B)^2)$ , as  $\varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) - (a_B - p_{rB} - c_B\varepsilon_B) < \varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) - (a_B - p_{rB} - c_B\varepsilon_B)|_{a_A=a_{A1}} = 0$ , then  $\Pi_B^S(EqS2) - \Pi_B^N < 0$ ; when  $c_{d7} < c_d < c_{d0}$ ,  $\Pi_B^S(EqS1) - \Pi_B^N = \frac{1}{64}((e + a_B + c_A - c_d - (e + c_B)\varepsilon_B)^2 - 16(-a_B + p_{rB} + c_B\varepsilon_B)^2)$ , as  $(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B) < (e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B)|_{c_d=c_{d7}} = 4(-a_B + p_{rB} + \varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) + c_B\varepsilon_B) < 4(-a_B + p_{rB} + \varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) + c_B\varepsilon_B)|_{a_A=a_{A1}} = 0$ , then  $\Pi_B^S(EqS1) - \Pi_B^N < 0$ .

If  $e > 2e_1$ , when  $c_{d4} < c_d < c_{d1}$ ,  $\Pi_B^S(EqS4) = \Pi_B^{c3} < \Pi_B^N$ , when  $c_{d1} < c_d < c_{d0}$ ,  $\Pi_B^S(EqS2) - \Pi_B^N = \frac{1}{4}(\varepsilon_A^2(a_A - c_A\varepsilon_A)^2 - (-a_B + p_r + c_B\varepsilon_B)^2) < 0$ .

(ii) If  $a_A > a_{A1}$ , we need to compare  $c_{d2} < c_d < c_{d0}$ .

If  $e_1 < e < e_3$ , when  $c_{d2} < c_d < c_{d0}$ ,  $\Pi_B^S(EqS1) - \Pi_B^N = \frac{1}{64}((e + a_B + c_A - c_d - (e + c_B)\varepsilon_B)^2 - 16(-a_B + p_{rB} + c_B\varepsilon_B)^2)$ , as  $(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B) > (e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B)|_{c_d=c_{d0}} = e - 4a_B + 4p_{rB} - (e - 4c_B)\varepsilon_B$ ,  $e - 4a_B + 4p_{rB} - (e - 4c_B)\varepsilon_B|_{e=e_1} = -4a_B + 4p_{rB} + 2\varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) + 4c_B\varepsilon_B > 0$  if  $a_A > a_{A3}$ , otherwise,  $e - 4a_B + 4p_{rB} - (e - 4c_B)\varepsilon_B|_{e=e_1} < 0$ ; then if  $a_A > a_{A3}$ ,  $(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B) > (e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B)|_{c_d=c_{d0}} > 0$ ,  $\Pi_B^S(EqS1) > \Pi_B^N$ ; if  $a_{A1} < a_A < a_{A3}$ , there exists a threshold value of  $a_{A4}$ , where  $a_{A4} = \frac{3a_B - 3p_{rB} + 2c_A\varepsilon_A^2 - 3c_B\varepsilon_B}{2\varepsilon_A}$ ,  $e - 4a_B + 4p_{rB} - (e -$

$4c_B)\varepsilon_B|_{e=e_3} = -6a_B + 6p_{rB} + 4\varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) + 6c_B\varepsilon_B > 0$  if  $a_{A4} < a_A < a_{A3}$ , otherwise,  $e - 4a_B + 4p_{rB} - (e - 4c_B)\varepsilon_B|_{e=e_3} = -6a_B + 6p_{rB} + 4\varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) + 6c_B\varepsilon_B < 0$ ; then when  $a_{A1} < a_A < a_{A4}$ ,  $(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B)|_{c_d=c_{d0}} < 0$ , if  $c_{d2} < c_d < c_{d9}$ ,  $\Pi_B^S(EqS1) - \Pi_B^N > 0$ , if  $c_{d9} < c_d < c_{d0}$ ,  $\Pi_B^S(EqS1) - \Pi_B^N < 0$ ; when  $a_{A4} < a_A < a_{A3}$ , if  $e < 2e_2$ ,  $(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B)|_{c_d=c_{d0}} < 0$ , if  $c_{d2} < c_d < c_{d9}$ ,  $\Pi_B^S(EqS1) - \Pi_B^N > 0$ , if  $c_{d9} < c_d < c_{d0}$ ,  $\Pi_B^S(EqS1) - \Pi_B^N < 0$ , if  $2e_2 < e < e_3$ ,  $\Pi_B^S(EqS1) - \Pi_B^N > 0$ .

If  $e_3 < e < 2e_1$ , when  $c_{d2} < c_d < c_{d7}$ ,  $\Pi_B^S(EqS2) - \Pi_B^N = \frac{1}{4}(\varepsilon_A^2(a_A - p_{rA} - c_A\varepsilon_A)^2 - (a_B - p_{rB} - c_B\varepsilon_B)^2)$ , as  $\varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) - (a_B - p_{rB} - c_B\varepsilon_B) > \varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) - (a_B - p_{rB} - c_B\varepsilon_B)|_{a_A=a_{A1}} = 0$ , then  $\Pi_B^S(EqS2) - \Pi_B^N > 0$ ; when  $c_{d7} < c_d < c_{d0}$ ,  $\Pi_B^S(EqS1) - \Pi_B^N = \frac{1}{64}((e + a_B + c_A - c_d - (e + c_B)\varepsilon_B)^2 - 16(-a_B + p_{rB} + c_B\varepsilon_B)^2)$ , as  $(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B) > (e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B)|_{c_d=c_{d0}} = e - 4a_B + 4p_{rB} - (e - 4c_B)\varepsilon_B$ ,  $e - 4a_B + 4p_{rB} - (e - 4c_B)\varepsilon_B|_{e=e_3} = -6a_B + 6p_{rB} + 4\varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) + 6c_B\varepsilon_B > 0$  if  $a_A > a_{A4}$ , otherwise,  $e - 4a_B + 4p_{rB} - (e - 4c_B)\varepsilon_B|_{e=e_3} < 0$ ; then if  $a_A > a_{A4}$ ,  $(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B) > (e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B)|_{c_d=c_{d0}} > 0$ ,  $\Pi_B^S(EqS1) > \Pi_B^N$ ; if  $a_{A1} < a_A < a_{A4}$ ,  $e - 4a_B + 4p_{rB} - (e - 4c_B)\varepsilon_B|_{e=2e_1} = 4(-a_B + p_{rB} + \varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) + c_B\varepsilon_B) > 4(-a_B + p_{rB} + \varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) + c_B\varepsilon_B)|_{a_A=a_{A1}} > 0$ , then if  $e_3 < e < 2e_2$ ,  $(e + a_B + c_A - c_d - (e + c_B)\varepsilon_B) - 4(a_B - p_{rB} - c_B\varepsilon_B)|_{c_d=c_{d0}} < 0$ , if  $c_{d7} < c_d < c_{d9}$ ,  $\Pi_B^S(EqS1) - \Pi_B^N > 0$ , if  $c_{d9} < c_d < c_{d0}$ ,  $\Pi_B^S(EqS1) - \Pi_B^N < 0$ , if  $2e_2 < e < 2e_1$ ,  $\Pi_B^S(EqS1) - \Pi_B^N > 0$ .

If  $e > 2e_1$ , when  $c_{d2} < c_d < c_{d0}$ ,  $\Pi_B^S(EqS2) - \Pi_B^N = \frac{1}{4}(\varepsilon_A^2(a_A - p_{rA} - c_A\varepsilon_A)^2 - (a_B - p_{rB} - c_B\varepsilon_B)^2)$ , as  $\varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) - (a_B - p_{rB} - c_B\varepsilon_B) > \varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) - (a_B - p_{rB} - c_B\varepsilon_B)|_{a_A=a_{A1}} = 0$ , then  $\Pi_B^S(EqS2) - \Pi_B^N > 0$ .  $\square$

**Lemma 6.**

(1) When  $\varepsilon_B > 1$ , then

(i) If  $a_A < a_{A1}$ , we need to compare  $c_{d4} < c_d < c_{d0}$ .

When  $c_{d4} < c_d < c_{d1}$ ,  $E^S(EqS4) = E^{c3} < E^N$ , when  $c_{d1} < c_d < c_{d0}$ ,  $E^S(EqS3) = E^{c1} < E^N$ .

(ii) If  $a_A > a_{A1}$ , we need to compare  $c_{d2} < c_d < c_{d0}$ . Then,  $E^S(EqS3) = E^{c1} < E^N$ .

(2) When  $\varepsilon_B < 1$  and  $e < e_1$ , then

(i) If  $a_A < a_{A1}$ , we need to compare  $c_{d4} < c_d < c_{d0}$ .

When  $c_{d4} < c_d < c_{d1}$ ,  $E^S(EqS4) = E^{c3} < E^N$ , when  $c_{d1} < c_d < c_{d8}$ ,  $E^S(EqS3) = E^{c1} < E^N$ ; when  $c_{d8} < c_d < c_{d0}$ ,  $\frac{d(E^S(EqS1)-E^N)}{dc_d} = \frac{1}{8}e(1-\varepsilon_B) > 0$ ,

$$E^S(EqS1) - E^N|_{c_d=c_{d0}} = -\frac{1}{8}e(e + \varepsilon_B(-2e + 4a_B - 4p_{rB} + (e - 4c_B)\varepsilon_B)) < 0,$$

$$E^S(EqS1) - E^N < 0.$$

(ii) If  $a_A > a_{A1}$ , we need to compare  $c_{d2} < c_d < c_{d0}$ .

If  $e < e_2$ , when  $c_{d2} < c_d < c_{d8}$ ,  $E^S(EqS3) = E^{c1} < E^N$ ; when  $c_{d8} < c_d < c_{d0}$ ,  $\frac{d(E^S(EqS1)-E^N)}{dc_d} = \frac{1}{8}e(1-\varepsilon_B) > 0$ ,  $E^S(EqS1) - E^N|_{c_d=c_{d0}} = -\frac{1}{8}e(e + \varepsilon_B(-2e + 4a_B - 4p_{rB} + (e - 4c_B)\varepsilon_B)) < 0$ ,  $E^S(EqS1) - E^N < 0$ . Similarly, if  $e > e_2$ , when  $c_{d2} < c_d < c_{d0}$ ,  $E^S(EqS1) - E^N < 0$ .

(3) When  $\varepsilon_B < 1$  and  $e > e_1$ , then

(i) If  $a_A < a_{A1}$ , we need to compare  $c_{d4} < c_d < c_{d0}$ .

If  $e_1 < e < 2e_1$ , when  $c_{d4} < c_d < c_{d1}$ ,  $E^S(EqS4) = E^{c3} < E^N$ , when  $c_{d1} < c_d < c_{d7}$ ,  $E^S(EqS2) - E^N = \frac{1}{2}e((a_A - p_{rA} - c_A\varepsilon_A)\varepsilon_A(-1 + \varepsilon_B) - \varepsilon_B(a_B - p_{rB} - c_B\varepsilon_B)) < 0$ ; when  $c_{d7} < c_d < c_{d0}$ ,  $\frac{d(E^S(EqS1)-E^N)}{dc_d} = \frac{1}{8}e(1-\varepsilon_B) > 0$ ,  $E^S(EqS1) - E^N|_{c_d=c_{d0}} = -\frac{1}{8}e(e(1-\varepsilon_B)^2 + 4\varepsilon_B(a_B - p_{rB} - c_B\varepsilon_B)) < 0$ .

If  $e > 2e_1$ , when  $c_{d4} < c_d < c_{d1}$ ,  $E^S(EqS4) = E^{c3} < E^N$ , when  $c_{d1} < c_d < c_{d0}$ ,  $E^S(EqS2) - E^N = \frac{1}{2}e((a_A - p_{rA} - c_A\varepsilon_A)\varepsilon_A(-1 + \varepsilon_B) - \varepsilon_B(a_B - p_{rB} - c_B\varepsilon_B)) < 0$ .

(ii) If  $a_A > a_{A1}$ , we need to compare  $c_{d2} < c_d < c_{d0}$ .

If  $e_1 < e < e_3$ , when  $c_{d2} < c_d < c_{d0}$ ,  $\frac{d(E^S(EqS1)-E^N)}{dc_d} = \frac{1}{8}e(1-\varepsilon_B) > 0$ ,

$$E^S(EqS1) - E^N < E^S(EqS1) - E^N|_{c_d=c_{d0}} = -\frac{1}{8}e(e(1 - \varepsilon_B)^2 + 4\varepsilon_B(a_B - p_{rB} - c_B\varepsilon_B)) < 0.$$

If  $e_1 < e < 2e_1$ , when  $c_{d2} < c_d < c_{d7}$ ,  $E^S(EqS2) - E^N = \frac{1}{2}e((a_A - p_{rA} - c_A\varepsilon_A)\varepsilon_A(-1 + \varepsilon_B) - \varepsilon_B(a_B - p_{rB} - c_B\varepsilon_B)) < 0$  ; when  $c_{d7} < c_d < c_{d0}$ ,

$$\frac{d(E^S(EqS1) - E^N)}{dc_d} = \frac{1}{8}e(1 - \varepsilon_B) > 0, \quad E^S(EqS1) - E^N < E^S(EqS1) - E^N|_{c_d=c_{d0}} = -\frac{1}{8}e(e(1 - \varepsilon_B)^2 + 4\varepsilon_B(a_B - p_{rB} - c_B\varepsilon_B)) < 0.$$

If  $e > 2e_1$ , when  $c_{d2} < c_d < c_{d0}$ ,  $E^S(EqS2) - E^N = \frac{1}{2}e((a_A - p_{rA} - c_A\varepsilon_A)\varepsilon_A(-1 + \varepsilon_B) - \varepsilon_B(a_B - p_{rB} - c_B\varepsilon_B)) < 0.$  □

**Theorem 3.** The result can be directly derived from Lemmas 4, 5, 6. □

**Theorem 4.** The proof is similar to that of Theorem 1. □

**Corollary 5.** The result can be derived directly based on the optimal equilibrium solutions in Appendix A.6. □

**Theorem 5.** The proof is similar to that of Theorem 1. □

**Corollary 6 and Corollary 7.** The results can be directly derived based on the optimal solutions in Appendix A.7. □

**Lemma 7.** The proof is similar to that of Lemma 4. □

**Lemma 8.** The proof is similar to that of Lemma 5. □

**Lemma 9.** The proof is similar to that of Lemma 6. □

**Theorem 6.** The result can be directly derived from Lemmas 7, 8, and 9. □

**Corollary 8 and Corollary 9.** The result can be derived directly based on the optimal equilibrium solutions in Appendix A.8. □

**Lemma 10.** The proof is similar to that of Lemma 6. □

**Corollary 10.** The result can be derived directly. □

## Appendix B.

$c_{d0}$	$a_B - c_B \varepsilon_B + c_A$
$c_{d1}$	$a_B - c_B \varepsilon_B - 2(a_A - p_{rA})\varepsilon_A + c_A(1 + 2\varepsilon_A^2)$
$c_{d2}$	$-a_B + c_A + 2p_{rB} + c_B \varepsilon_B$
$c_{d3}$	$\frac{(a_B - c_B \varepsilon_B)(1 + \varepsilon_B) + c_A(1 - \varepsilon_B) - 2p_{rB} \varepsilon_B}{1 - \varepsilon_B}$
$c_{d4}$	$\frac{(a_A - p_{rA})\varepsilon_A - a_B(1 + \varepsilon_A^2) + p_{rB}(1 + 2\varepsilon_A^2) + c_B(1 + \varepsilon_A^2)\varepsilon_B}{\varepsilon_A^2}$
$c_{d5}$	$\frac{-(a_A - p_{rA} - c_A \varepsilon_A)(\varepsilon_A + 2\varepsilon_A^3) + ((a_A - p_{rA})\varepsilon_A - (a_B - c_B \varepsilon_B)(1 + \varepsilon_A^2) + p_{rB}(1 + 2\varepsilon_A^2))\varepsilon_B}{\varepsilon_A^2 \varepsilon_B}$
$c_{d6}$	$e + a_B + c_A - (e + c_B)\varepsilon_B$
$c_{d7}$	$e + a_B - 4(a_A - p_{rA})\varepsilon_A + c_A(1 + 4\varepsilon_A^2) - (e + c_B)\varepsilon_B$
$c_{d8}$	$-e + a_B + c_A + (e - c_B)\varepsilon_B$
$c_{d9}$	$e - 3a_B + c_A + 4p_{rB} - (e - 3c_B)\varepsilon_B$
$c_{d10}$	$a_B - \frac{(a_A - p_{rA})(-1 + \varepsilon_B)}{\varepsilon_A} + (c_A - c_B)\varepsilon_B$
$c_{d11}$	$a_B + \frac{a_A - p_{rA}}{\varepsilon_A} - (e + c_B)\varepsilon_B$
$c_{d12}$	$-e + a_B - c_A + c_A \varepsilon_B - c_B \varepsilon_B - \frac{(a_A - p_{rA} - (e + c_A)\varepsilon_A)(2\varepsilon_A^2 + 1 - \varepsilon_B)}{\sqrt{(1 + 2\varepsilon_A^2)(2\varepsilon_A^2 + (-1 + \varepsilon_B)^2)}}(\sqrt{2}$ $+ \frac{\varepsilon_B^2}{\sqrt{2}(1 + 2\varepsilon_A^2 - \varepsilon_B)} + \frac{4(1 - \varepsilon_B + \varepsilon_B^2)}{\sqrt{2}(-1 + 2\varepsilon_A^2 + \varepsilon_B)})$ $- \frac{(2(a_A - p_{rA})\varepsilon_A + e(-1 + \varepsilon_B) + c_A(-1 + \varepsilon_B))(-2 + \varepsilon_B)}{-1 + 2\varepsilon_A^2 + \varepsilon_B}$
$c_{d13}$	$e + a_B + c_A + \frac{2\varepsilon_A(-a_A + p_{rA} + (e + c_A)\varepsilon_A)(-1 + \varepsilon_B)}{4\varepsilon_A^2 + (-1 + \varepsilon_B)^2} - (e + c_B)\varepsilon_B$
$c_{d14}$	$\frac{1}{\varepsilon_A^2 \varepsilon_B}(-2(a_A - p_{rA} - c_A \varepsilon_A)(\varepsilon_A + 2\varepsilon_A^3) + (a_A \varepsilon_A - p_{rA} \varepsilon_A - a_B(2 + 3\varepsilon_A^2) + p_{rB}(2 + 4\varepsilon_A^2))\varepsilon_B$ $+ (2c_B - (e - 3c_B)\varepsilon_A^2)\varepsilon_B^2)$
$c_{ds1}$	$e + a_B - \frac{(a_A - p_{rA})\varepsilon_A}{2} + \frac{1}{2}c_A(2 + \varepsilon_A^2) - (e + c_B)\varepsilon_B$
$c_{ds2}$	$\frac{4e}{3} + a_B + c_A - \frac{1}{3}(4e + 3c_B)\varepsilon_B$
$c_{ds3}$	$a_B + \frac{1}{2}((a_A - p_{rA})(\frac{1}{\varepsilon_A} - \varepsilon_A) + c_A(1 + \varepsilon_A^2) - 2(e + c_B)\varepsilon_B)$
$c_{ds4}$	$a_B + \frac{a_A - p_{rA}}{\varepsilon_A} - \frac{(2e + c_B + (4e + 3c_B)\varepsilon_A^2)\varepsilon_B}{1 + 3\varepsilon_A^2}$
$c_{ds5}$	$\frac{1}{2}(-2(a_A - p_{rA})\varepsilon_A + (a_A - p_{rA} - (2e + c_A)\varepsilon_A)\sqrt{1 + \varepsilon_A^2} +$



	$2(a_B + (e + c_A)(1 + \varepsilon_A^2) - (e + c_B)\varepsilon_B))$
$c_{ds6}$	$\frac{p_r + 2(a_A - p_{rA})\varepsilon_A + a_B(-1 + \varepsilon_A^2) + c_B\varepsilon_B + \varepsilon_A^2(p_{rB} - (2e + c_B)\varepsilon_B)}{2\varepsilon_A^2}$
$c_{ds7}$	$\frac{1}{2}(2e + a_B + 2c_A + p_{rB} - (2e + c_B)\varepsilon_B)$
$c_{ds8}$	$\frac{(-(a_A - p_{rA} - c_A\varepsilon_A)(\varepsilon_A + \varepsilon_A^3) + (-a_B + p_{rB} + 2(a_A - p_{rA})\varepsilon_A + (a_B + p_{rB})\varepsilon_A^2)\varepsilon_B + (c_B - (2e + c_B)\varepsilon_A^2)\varepsilon_B^2)}{2\varepsilon_A^2\varepsilon_B}$
$c_{ds9}$	$\frac{(-2(e + a_B + c_A) + (a_B + 2(2e + c_A + c_B) + p_{rB})\varepsilon_B - (2e + c_B)\varepsilon_B^2)}{(2(-1 + \varepsilon_B))}$
$c_{dt1}$	$\frac{(2e - 3(a_A - p_{rA})\varepsilon_A + 8e\varepsilon_A^2 + 8\varepsilon_A^3(-a_A + p_{rA} + e\varepsilon_A) + 2c_A(1 + 2\varepsilon_A^2)^2 + a_B(\varepsilon_A^2(5 - 3\varepsilon_B) + 2(-1 + \varepsilon_B)^2) - (2(3e + 2c_A + c_B) - 4(a_A - p_{rA})\varepsilon_A + (12e + 7c_A + 5c_B)\varepsilon_A^2)\varepsilon_B + (2(3e + c_A + 2c_B) - (a_A - p_{rA})\varepsilon_A + (4e + c_A + 3c_B)\varepsilon_A^2)\varepsilon_B^2 - 2(e + c_B)\varepsilon_B^3)}{\varepsilon_A^2(5 - 3\varepsilon_B) + 2(-1 + \varepsilon_B)^2}$
$c_{dt2}$	$\frac{4e}{3} + a_B + c_A + \frac{4\varepsilon_A(a_A - p_{rA} - (2e + c_A)\varepsilon_A)}{3(-1 + \varepsilon_B)} - \frac{1}{3}(4e + 3c_B)\varepsilon_B$
$c_{dt3}$	$\frac{((a_A - p_{rA} + a_B\varepsilon_A)(1 + 3\varepsilon_A^2) + (-(2e + 2a_B + c_A + c_B)\varepsilon_A - (4e + c_A + 3c_B)\varepsilon_A^3 + (a_A - p_{rA})(-1 + \varepsilon_A^2))\varepsilon_B + 2(e + c_B)\varepsilon_A\varepsilon_B^2)}{\varepsilon_A(1 + 3\varepsilon_A^2 - 2\varepsilon_B)}$
$c_{dt4}$	$a_B + \frac{a_A - p_{rA}}{\varepsilon_A} - \frac{(2e + c_B + (4e + 3c_B)\varepsilon_A^2)\varepsilon_B}{1 + 3\varepsilon_A^2}$
$c_{dt5}$	$e + a_B + c_A + \frac{\varepsilon_A(a_A - p_{rA} - (e + c_A)\varepsilon_A)(-1 + \varepsilon_B)}{\varepsilon_A^2 + (-1 + \varepsilon_B)^2} - (e + c_B)\varepsilon_B$
$c_{dt6}$	$\frac{(-2(1 + 2\varepsilon_A^2)(e - (a_A - p_{rA})\varepsilon_A + 2e\varepsilon_A^2) + a_B(\varepsilon_A^2(-5 + \varepsilon_B) + (-2 + \varepsilon_B)(-1 + \varepsilon_B)^2) + (8e + 2c_B + p_{rB} - 4(a_A - p_{rA})\varepsilon_A + (5c_B + 4(4e + p_{rB}))\varepsilon_A^2)\varepsilon_B - (12e + 5c_B + 2p_{rB} - 2(a_A - p_{rA})\varepsilon_A + (8e + c_B)\varepsilon_A^2)\varepsilon_B^2 + (8e + 4c_B + p_{rB})\varepsilon_B^3 - (2e + c_B)\varepsilon_B^4 - c_A(4\varepsilon_A^4 - 2(-1 + \varepsilon_B)^3 + \varepsilon_A^2(-1 + \varepsilon_B)(-7 + 2\varepsilon_B)))}{(5\varepsilon_A^2 + 2(-1 + \varepsilon_B)^2)(-1 + \varepsilon_B)}$
$c_{dt7}$	$\frac{(-(a_A - p_{rA})\varepsilon_A(2\varepsilon_A^2 + (-1 + \varepsilon_B)^2) + c_A\varepsilon_A^2(1 + 2\varepsilon_A^2 + (-1 + \varepsilon_B)\varepsilon_B) + \varepsilon_B(p_{rB}(1 + 2\varepsilon_A^2 - \varepsilon_B) + c_B(1 + \varepsilon_A^2 - \varepsilon_B)\varepsilon_B + a_B(-1 - \varepsilon_A^2 + \varepsilon_B)))}{\varepsilon_A^2\varepsilon_B}$
$c_{dt8}$	$\frac{(-(a_A - p_{rA} - c_A\varepsilon_A)(\varepsilon_A + \varepsilon_A^3) + (-a_B + p_{rB} + 2(a_A - p_{rA})\varepsilon_A + (a_B + p_{rB})\varepsilon_A^2)\varepsilon_B + (c_B - (2e + c_B)\varepsilon_A^2)\varepsilon_B^2)}{2\varepsilon_A^2\varepsilon_B}$
$a_{A1}$	$\frac{a_B - p_{rB} - c_B\varepsilon_B + c_A\varepsilon_A^2 + p_{rA}\varepsilon_A}{\varepsilon_A}$
$a_{A2}$	$p_{rA} + c_A\varepsilon_A + \frac{\varepsilon_B(a_B - p_{rB} - c_B\varepsilon_B)}{\varepsilon_A(-1 + \varepsilon_B)}$
$a_{A3}$	$p_{rA} + \frac{2a_B - 2p_{rB} + c_A\varepsilon_A^2 - 2c_B\varepsilon_B}{\varepsilon_A}$
$a_{A4}$	$p_{rA} + \frac{3a_B - 3p_{rB} + 2c_A\varepsilon_A^2 - 3c_B\varepsilon_B}{2\varepsilon_A}$

$s_1$	$2(a_A - p_{rA})\varepsilon_A - c_A(1 + 2\varepsilon_A^2) - (a_B - c_d - c_B\varepsilon_B)$
$s_2$	$a_B - c_A + c_d - 2p_{rB} - c_B\varepsilon_B$
$s_3$	$\frac{c_A - c_d + a_B(1 + \varepsilon_B) - \varepsilon_B(c_A - c_d + 2p_{rB} + c_B(1 + \varepsilon_B))}{-1 + \varepsilon_B}$
$s_4$	$c_d - 2p_{rB} - c_B\varepsilon_B - \frac{p_{rB} + (a_A - p_{rA})\varepsilon_A - a_B(1 + \varepsilon_A^2) + c_B\varepsilon_B}{\varepsilon_A^2}$
$s_5$	$\frac{((a_A - p_{rA} - c_A\varepsilon_A)(\varepsilon_A + 2\varepsilon_A^3) + (a_B - p_{rB} - (a_A - p_{rA})\varepsilon_A + (a_B + c_d - 2p_{rB})\varepsilon_A^2)\varepsilon_B - c_B(1 + \varepsilon_A^2)\varepsilon_B^2)}{\varepsilon_A^2\varepsilon_B}$
$t_1$	$\frac{-a_B + c_d + 2(a_A - p_{rA})\varepsilon_A - c_A(1 + 2\varepsilon_A^2) + c_B\varepsilon_B}{1 + 2\varepsilon_A^2 - \varepsilon_B}$
$t_2$	$\frac{(-(a_A - p_{rA} - c_A\varepsilon_A)(\varepsilon_A + 2\varepsilon_A^3) + (-a_B + p_{rB} + (a_A - p_{rA})\varepsilon_A - (a_B + c_d - 2p_{rB})\varepsilon_A^2)\varepsilon_B + c_B(1 + \varepsilon_A^2)\varepsilon_B^2)}{\varepsilon_A^2\varepsilon_B^2}$
$t_3$	$\frac{-c_A + c_d - (a_B - c_B\varepsilon_B)(1 + \varepsilon_B) + \varepsilon_B(c_A - c_d + 2p_{rB})}{2\varepsilon_A^2 + (-1 + \varepsilon_B)^2}$
$t_4$	$\frac{a_B - p_{rB} - (a_A - p_{rA})\varepsilon_A + (a_B + c_d - 2p_{rB})\varepsilon_A^2}{(1 + \varepsilon_A^2)\varepsilon_B} - c_B$
$t_5$	$\frac{a_B - c_A + c_d - 2p_{rB} - c_B\varepsilon_B}{1 + \varepsilon_B}$
$e_1$	$\frac{2\varepsilon_A(-a_A + p_{rA} + c_A\varepsilon_A)}{-1 + \varepsilon_B}$
$e_2$	$\frac{2(-a_B + p_{rB} + c_B\varepsilon_B)}{-1 + \varepsilon_B}$
$e_3$	$\frac{2(-a_B + p_{rB} + 2\varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) + c_B\varepsilon_B)}{1 - \varepsilon_B}$
$e_4$	$-c_A + \frac{a_A - p_{rA}}{\varepsilon_A}$
$e_5$	$-c_B + \frac{2(a_B - p_{rB})}{\varepsilon_B} + \frac{-\varepsilon_A(a_A - p_{rA} + (-c_A + c_B)\varepsilon_A) + (-a_B + p_{rB})\varepsilon_B}{\varepsilon_A^2 + \varepsilon_B^2}$
$e_6$	$\frac{(a_A - p_{rA})(\varepsilon_A^2 + 2\varepsilon_B^2) + \varepsilon_A(\varepsilon_B(-a_B + p_{rB} + c_B\varepsilon_B) - c_A(\varepsilon_A^2 + 2\varepsilon_B^2))}{\varepsilon_A(\varepsilon_A^2 + \varepsilon_B^2)}$
$e_{s1}$	$\frac{3\varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A)}{2(-1 + \varepsilon_B)}$
$e_{s2}$	$\frac{(a_A - p_{rA} - c_A\varepsilon_A)(1 + 3\varepsilon_A^2)}{2\varepsilon_A\varepsilon_B}$
$e_{s3}$	$\frac{1}{2}(-c_A + \frac{a_A - p_{rA}}{\varepsilon_A})$
$e_{s4}$	$-\frac{(1 + 3\varepsilon_A^2)(-a_B + p_{rB} + c_B\varepsilon_B)}{2\varepsilon_A^2\varepsilon_B}$
$e_{s5}$	$\frac{-a_B + p_{rB} + c_B\varepsilon_B}{2(-1 + \varepsilon_B)}$

$e_{t1}$	$\frac{1}{2}(-2c_B + \frac{a_B - p_{rB}}{\varepsilon_B} + \frac{\varepsilon_A(a_A - p_{rA} + (-c_A + c_B)\varepsilon_A) + (a_B - p_{rB})\varepsilon_B}{\varepsilon_A^2 + \varepsilon_B^2})$
$e_{t2}$	$\frac{2\varepsilon_A^2(a_A - p_{rA} - c_A\varepsilon_A) + (a_B - p_{rB})\varepsilon_A\varepsilon_B + (a_A - p_{rA} - (c_A + c_B)\varepsilon_A)\varepsilon_B^2}{2\varepsilon_A(\varepsilon_A^2 + \varepsilon_B^2)}$
$e_{t3}$	$\frac{(a_A - p_{rA})\varepsilon_A - c_A\varepsilon_A^2 + \varepsilon_B(a_B - p_{rB} - c_B\varepsilon_B)}{2(\varepsilon_A^2 + \varepsilon_B^2)}$
$e_{t4}$	$\frac{\varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A)(-1 + 3\varepsilon_B)}{2(2\varepsilon_A^2 + (-1 + \varepsilon_B)^2)}$
$e_{t5}$	$\frac{\varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A)}{2\varepsilon_A^2 + (-1 + \varepsilon_B)^2}$
$e_{t6}$	$\frac{2\varepsilon_A(a_A - p_{rA} - c_A\varepsilon_A) - 3\varepsilon_B(-a_B + p_{rB} + c_B\varepsilon_B)}{2(2\varepsilon_A^2 + (-1 + \varepsilon_B)^2)}$
$e_{t7}$	$\frac{(c_A\varepsilon_A^2(\varepsilon_A^2(5 - 7\varepsilon_B) - 2(-1 + \varepsilon_B)^3) + (\varepsilon_A^2(5 - 3\varepsilon_B) + 2(-1 + \varepsilon_B)^2)\varepsilon_B(-a_B + p_{rB} + c_B\varepsilon_B) + (a_A - p_{rA})\varepsilon_A(2(-1 + \varepsilon_B)^3 + \varepsilon_A^2(-5 + 7\varepsilon_B)))}{2\varepsilon_A^2(2\varepsilon_A^2 + (-1 + \varepsilon_B)^2)\varepsilon_B}$
$e_{t8}$	$- \frac{((1 + 3\varepsilon_A^2 - 2\varepsilon_B)\varepsilon_B(-a_B + p_{rB} + c_B\varepsilon_B) - (a_A - p_{rA})\varepsilon_A(1 + 3\varepsilon_A^2 + 2(-1 + \varepsilon_B)\varepsilon_B) + c_A\varepsilon_A^2(1 + 3\varepsilon_A^2 + 2(-1 + \varepsilon_B)\varepsilon_B))}{2\varepsilon_A^2\varepsilon_B^2}$