

Operation Research Final Project

Mask Distribution

Group 7

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1 Background and Motivation

Due to the outbreak of COVID-19, people around the globe require plenty of virus prevention tools such as, masks, disinfectant alcohol and swabs. Of which, masks are in short supply due to the need by both healthcare workers and the general population. Stockpiling efforts by governments and hospitals further increased demand towards masks, causing a global severe shortage on face masks. The masks shortage has ended up as a national security issue. Even in the developed nations, such as Europe Union or the United States, there are huge excess demand in contrast to the low supply of masks. In R.O.C.(Taiwan), although there is no infectious spread like Mainland China or the United States, we still face the problem of not having enough mask to provide for our citizens. Hence, the government initially implemented a policy called "7-day-2-mask" for each people in February 2. The government intervention has resulted in public fear and anxiety, and even stimulated the demand for masks, which caused many chaos and hoarded the masks. As the masks manufacturing lines expanded, two months later, in April 9, the government released the masks regulation to "14-day-9-mask". However, the public anxieties and complain still existed. Having witness this situation, our group decided to take a deep look in the distribution of mask, to see if we can come up with a better solution to provide the masks rather than merely equality.

We examine the problem from two perspectives, in this project they are named as the Provider Problem and the Utility Problem. The former is from providers' aspect in which we assume that all the transportation cost for each factory to each town are homogeneous. The latter is from public's aspect. For public aspect, we assume the safety confidence utility for each town and maximize the sum of entire town in Taiwan Province. The reason for measuring

safety confidence utility in the unit of town is that in the threat of COVID-19, people tends to not travel far from their place of residence and rarely leaves home except for necessary affairs. In that sense, the people in the town cares about the mask coverage of the town for the reason that the more the coverage is, the more secure for the isolation between individuals. Moreover, the density of the town is also the matter of concern of the residents in the town for that the higher the density is, the more frequently of contacts between individuals in town if the resident are outdoors. Hence, the project examines both the distribution solution of masks and compare the result with the arbitrary equality of the existing policies of the government.

2 Data Process

2.1 Data Source

1. District data - <https://data.gov.tw/dataset/7441> in SHP format.

The district data is gathered from open access database provided by government. With the assistance of shapefile with geographic attribute table inside it, we are able to visualize Taiwan separated by district, with applying other accompanied methods like choropleth.

2. Population data - <https://www.ris.gov.tw>

The population data used in this project generates from data provided by Department of Household Registration, Ministry of the Interior. From the latest population data released in 2019, it provides population of all district with different age and gender in a excel format. By merging different genders into a single data set, with separating age into below and above 12 years old and binding with other info like town id and density, the population data is ready, which is shown as Figure 1.

id	A		B		C		D		E		F		G		H		I		J		K		L		M		N		O	
	TOWNID	TOWNCODE	COUNTYNAME	TOWNNAME	TOWNENG	COUNTYID	sum_children	sum_adult	density_100	z(density)	total pop																			
1	A01	63000010	台北市	松山區	Songshan District	A	23729	187068	22286.0096	3.403969	0.57203965	210797																		
2	A02	63000030	台北市	大安區	Dalan District	A	36839	281481	27282.8412	4.280305	0.709319154	318320																		
3	A03	63000050	台北市	中正區	Zhongzheng District	A	20262	141530	20981.4515	3.175156	0.538543786	161732																		
4	A05	63000070	台北市	萬華區	Wanhua District	A	16326	172657	21872.5786	3.296371	0.566287248	188883																		
5	A09	63000090	台北市	大同區	Datong District	A	13152	114739	22754.2022	3.486074	0.586595988	127861																		
6	A10	63000040	台北市	中山區	Zhongshan District	A	22648	212662	16862.1776	2.452887	0.402788107	239510																		
7	A11	63000080	台北市	文山區	Wanshan District	A	29051	248615	8759.38462	1.02279	0.223468461	277666																		
8	A13	63000090	台北市	南港區	Nangang District	A	12827	108666	5592.56309	0.478139	0.143481681	121893																		
9	A14	63000100	台北市	內湖區	Neihu District	A	33304	259178	9112.81972	1.093548	0.230974624	260492																		
10	A15	63000110	台北市	士林區	Shilin District	A	28346	260273	4622.46786	0.305997	0.1185662298	288619																		
11	A16	63000120	台北市	北投區	Beitou District	A	26995	231335	4513.35408	0.28688	0.1137548293	258330																		
12	A17	63000020	台北市	信義區	Xinyi District	A	21232	204191	20142.6898	3.028044	0.517098915	229423																		
13	B01	66000010	台中市	中區	Central District	B	2303	16207	20958.7641	3.171177	0.507951383	18810																		
14	B02	66000020	台中市	東區	East District	B	7188	68033	8164.44995	0.927216	0.2084602803	76221																		
15	B03	66000030	台中市	南區	South District	B	13765	115519	18364.8286	2.716251	0.4713692462	126284																		
16	B04	66000040	台中市	西區	West District	B	13608	103706	20270.8024	3.050464	0.502091442	118114																		
17	B05	66000050	台中市	北區	North District	B	14436	136361	21283.0085	3.228045	0.5462857115	150797																		
18	B06	66000060	台中市	西屯區	Xitun District	B	28171	207191	5897.91225	0.494816	0.1461681508	235362																		
19	B07	66000070	台中市	南屯區	Nantun District	B	21836	150529	5388.97242	0.442186	0.134841587	177365																		
20	B08	66000080	台中市	北屯區	Beitun District	B	32853	257074	4382.84048	0.263969	0.1124042228	289907																		
21	B09	66000090	台中市	豐原區	Fengyuan District	B	16571	148474	4051.98558	0.205941	0.103919189	168845																		
22	B10	66000100	台中市	東勢區	Dongshi District	B	4024	45136	431.952234	-0.428987	0.0198722546	49146																		
23	B11	66000110	台中市	大甲區	Dajia District	B	8550	68154	1328.81174	-0.271669	0.0339791487	76704																		
24	B12	66000120	台中市	清水區	Qingshui District	B	9271	77293	1346.71635	-0.268529	0.03485716371	86564																		
25	B13	66000130	台中市	沙鹿區	Shalu District	B	12574	81627	2289.78675	-0.153129	0.0684688786	94451																		
26	B14	66000140	台中市	梧棲區	Wupqi District	B	7022	51837	3492.52329	0.107819	0.0864682651	58889																		
27	B15	66000150	台中市	后寮區	Houli District	B	8626	48481	924.3026	-0.342815	0.0281248464	54307																		
28	B16	66000160	台中市	神岡區	Shenggang District	B	8678	58395	1875.58457	-0.178652	0.0470004878	60273																		
29	B17	66000170	台中市	潭子區	Tanzi District	B	11913	98115	4188.13653	0.229996	0.1074323865	100028																		
30	B18	66000180	台中市	大雅區	Daya District	B	11337	84663	2937.37601	0.070453	0.07529491824	96000																		
31	B19	66000190	台中市	新社區	Xinshe District	B	1861	21894	353.346254	-0.441702	0.0093808261	23855																		
32	B20	66000200	台中市	石碇區	Shide District	B	1505	43495	879.374666	-0.368279	0.004400000000	44924																		

Figure 1: The population data

3. Factory data - <https://serv.gcis.nat.gov.tw/Fidbweb/index.jsp>

The mask factory data is gathered from Department of Industrial Development Bureau, Ministry of Economic Affairs, with related news. By those resource provided, we are capable of listing out all the factories with their number of machines, location, latitude and longitude for further discussions, which is displayed as Figure 2.

序	Factory	A	B	C	D	E	F	G	H	I
序	Factory	Location	latitude	longitude	產能(個/日)	Adult Mask	Child Mask			
1	台灣康匠製鞋股份有限公司	桃園市桃園區	24.971583	121.3183584	30	1	1			
2	中國衛生材料工業中心股份有限公司	彰化縣彰化市	24.0404369	120.6162033	10	1	1			
3	永聯股份有限公司(獨家)	新北市鶯歌區	24.9750048	121.3372133	10	1	1			
4	恆大股份有限公司台灣製造廠(台南)	台南市白河區	23.359416	120.422647	10	1	0			
5	華新隆寧工業股份有限公司田中廠	彰化縣田中鎮	23.947919	120.559461	10	1	0			
6	康軒書企業股份有限公司口寮廠	台南市將軍區	23.200149	120.130691	9	1	0			
7	康軒利實業有限公司七堵廠	基隆市七堵區	25.095131	121.70644	7	1	1			
8	昇達興業股份有限公司彰化	彰化縣田中鎮	23.946365	120.571282	6	1	1			
9	宜達興材料科技股份有限公司(彰化)	彰化縣田中鎮	23.847559	120.569428	6	1	1			
10	德安達工業股份有限公司	台南市學甲區	23.243293	120.16366	6	1	1			
11	普達興由亞人有限公司台灣分公司(宜蘭)	宜蘭縣五結鄉	24.046559	121.837366	5	1	0			
12	康廷企業有限公司(龜山)	桃園市龜山區	25.045064	121.3773392	5	1	1			
13	淨新科技股份有限公司(高雄)	高雄市三民區	22.6552419	120.3156663	5	1	1			
14	威利利實業有限公司	新北市林口區	25.024502	121.3891711	4	1	1			
15	加利利有限公司(八德/新北)	新北市八德區	25.1347353	121.3623913	4	1	1			
16	奕威有限公司(新竹)	新竹縣新豐鄉	24.8801008	120.9849256	4	1	1			
17	裕豐實業股份有限公司(鹿港)	鹿港鎮外子市	23.465454	120.2509801	4	1	0			
18	千禧康企業股份有限公司(彰化)	桃園市平鎮區	24.5861448	121.2591573	1	1	0 小廠			
19	南達生物科技股份有限公司(汐止)	新北市汐止區	25.0588305	121.6338917	1	1	1 小廠			
20	怡實有限公司(桂林)	彰化縣鹿港鎮	24.5965568	121.4192584	1	1	0 小廠			
21	台灣聯發股份有限公司(三寶)	彰化市三寶區	25.0699229	121.4362539	1	1	1 小廠			
22	宏春實業有限公司(苗栗/高雄)	高雄市中區	22.637439	120.3035734	1	1	0 小廠			
23	聯竹生物科技股份有限公司(台南)	台南市六甲區	23.224941	120.3393155	1	1	0 小廠			
24	司宏電腦科技股份有限公司(鹿林)	雲林縣四湖鄉	23.795827	120.445392	1	1	0 小廠			
25	欣邦有限公司(宜蘭)	宜蘭縣冬山鄉	24.626747	121.804869	1	1	0 小廠			
26	台灣美紙企業有限公司(彰化)	彰化縣鹿港鎮	24.081179	120.411918	1	1	1 小廠			
27	約谷利有限公司	新北市中和區	25.003234	121.402285	1	1	0 小廠			
28	台灣精強有限公司	台南市永寧區	23.029612	120.227387	1	1	0 小廠			
29	健康天使股份有限公司	彰化縣田尾鄉	23.884328	120.526522	1	1	0 小廠			
30	裕豐生實業股份有限公司	彰化縣福心鄉	23.951341	120.548551	1	1	0 小廠			
31	宏達興有限公司	台中市烏日區	24.046591	120.654111	1	1	0 小廠			
32	康軒利實業股份有限公司(彰化)	彰化市南區	24.0451763	120.505154	1	1	0 小廠			

Figure 2: The factory data

2.2 Data Process

2.2.1 Generate the Variables of Concern

Given all source data needed, the following step is to generate variables of concern. Since the vision of this project is to find some better mask allocation solution to deal with this situation, the distance between each factory and district matters. However, the transportation time and cost may differ between transportation time, factories and districts, which is case-by-case sensitive. Therefore, we try to evaluate the transportation cost by using some intuitive way, which is the concept of Euclidean Distance, to generate the comparison. In practice, we utilize the formula of The Great Circle Distance provided by Professor Wen¹ in lecture of Spatial Analysis, to deal with the distance calculation. After the calculation process, the result is shown as Figure 5

¹Wen T.H., Professor, Department of Geography, National Taiwan University

	A01	A02	A03	A05	A09	A10	A11	A13	A14	A15	A16	A17	B01
1	台灣遠東新藥股份有限公司	26065.21048	23531.32	21428.72	19169.2	22172.35	24728.14	25878.2	30271.28	30322.33	29019.36	28061.19	26396.47
2	中國民生材料生產中心股份有限公司	147591.8802	143388.8	142274.8	146564.5	144936.4	148847.1	142291.5	148834.5	151256.7	152211.6	152162.8	145588
3	永源股份有限公司(實業)	24161.19047	21954.96	19504	17295.52	20316.25	22857.68	23652.74	28351.98	28434.19	27260.5	26461.63	24458.49
4	恆大建設股份有限公司(台灣製造商/台灣)	220630.0095	218967	214349	214785.6	218901.9	220799	219087.8	221429.2	224932.8	228799.2	227206.6	218849
5	聯新機織工業股份有限公司(中國)	167535.2992	167073.2	162820.2	161203	162392.2	167262.9	162233.2	168666.9	171763.3	172614.5	173162.6	165634.6
6	康寧安全玻璃股份有限公司(臺灣)	251822.8553	248113.7	24740.7	245660.2	249726.9	251703.8	246823.5	252891.6	256171.6	257022.9	257721.6	250133.3
7	康寧安全玻璃股份有限公司(中國)	15502.50966	18103.41	20066.01	22319.55	19796.03	17188.17	17827.08	17145.97	11572.67	16048.81	19808.64	15356.23
8	齊魯鋼鐵股份有限公司(中國)	167852.4888	163884.4	162598.8	161326.8	165378.8	167264.3	162436.5	168763.1	171878.3	173146.7	173218.3	165845.9
9	寶德豐材料科技股份有限公司(彰化)	167542.6401	163760.8	162627.6	161210.4	165266.5	167270.2	162340.5	168864.2	171770.7	173021.9	173190	168841.9
10	德安工業股份有限公司	246073.5113	242284.1	241391.6	238811.7	243878.8	243854.9	240674.9	247042.9	250288	251655.2	251877	244283.9
11	寶德豐材料科技股份有限公司(台灣分公司/實業)	53880.34562	51641.4	52642.49	54689.76	56761.1	55877.26	46454.99	48962.51	54460.76	60544.39	64360.51	50416.01
12	德昌公司(中國)	16712.5334	17646.18	14984.66	13496.75	13746.31	16528.81	21961.63	25647.78	21932.52	19177.15	17330.35	19790.73
13	康寧科技股份有限公司(高雄)	294840.2431	290925.4	290481.4	289190.4	293344.7	295003.9	288487.5	294861.8	298755.3	301133.4	302026	292559.6
14	康寧科技股份有限公司	16235.1173	15986.11	15465.26	15077.67	11784.71	14209.56	20051.55	21960.56	19554.27	16126.44	14395.59	13416.09
15	康寧科技股份有限公司(江蘇/蘇州)	19529.86976	20166.95	17906.85	16453.82	15346.12	17284.06	23205.96	25409.38	21921.95	17021.02	13738.53	22286.39
16	康寧公司(廈門)	60816.86962	56396.61	56272.02	53996.5	56711.84	58117.79	60487.35	65134.95	64875.12	62802.95	60915.05	61263.27
17	康寧實業股份有限公司(廣東/廣州)	220397.7418	216816.4	215679.7	214055	218098	220119.5	215176.9	221534	224626.5	225853.6	226976.3	218694.6
18	千禧康寧玻璃股份有限公司(湖北)	345619.9732	31782.13	30558.23	27977.34	31549.81	33952.79	32814.6	38184.11	39291.18	36884.4	38375.7	34452.36
19	康寧生物科技股份有限公司(沙市)	7683.135008	8792.19	11873.45	14161.45	12190.5	9718.095	9848.65	3506.138	5016.497	11173.72	15330.99	7008.79
20	康寧實業股份有限公司(蘇州)	15621.33361	12971.04	10903.24	8675.778	12087.65	14469.53	15840.47	19716.59	19962.35	19508	19518.45	15837.81
21	台灣康寧玻璃股份有限公司(三重)	12384.13442	11688.44	8587.867	7438.469	7701.396	10334.83	16361.66	17839.05	15863.86	13265.25	52198.47	14327.41
22	宏普實業有限公司(蘇州/蘇州)	297153.2734	293238.7	292772.3	291496.2	296563.3	297314.7	290805	297120	301070.2	303442.7	304330.8	294875.1
23	康寧實業股份有限公司(台灣)	237985.0965	234120.8	233496.7	231945.1	236081.7	237917.3	232187.3	238568.2	242084.7	243868.1	244365.5	235997.4
24	康寧實業股份有限公司(蘇州)	178256.3444	175516.8	174498.4	172815.6	178823.7	178966.5	174285.4	189017.3	183253.3	184546.5	184551.3	177671.4
25	沙坪有限公司(實業)	54082.65474	51902.69	53383.06	54363.39	56703.85	55698.36	46389.77	49442.35	54991.73	60928.6	64633.69	50581.72
26	台灣康寧實業股份有限公司(彰化)	158814.8346	152893.9	153990.7	152091.2	158871.1	158148.8	154815.3	160973.2	163184.9	163362.1	162818.1	157692.5
27	約安科技股份有限公司	6058.142787	6724.729	4232.677	2928.241	7034.4	8603.722	6324.621	12325.16	13425.04	14762.2	16252.82	8905.405
28	台灣康寧實業有限公司	262433.2677	258955.3	257863.2	256439	260527.7	262376.1	259619.2	262978.9	266524.3	268335.8	268843	260429
29	康寧天德股份有限公司	166076.3222	163234.4	162219.1	160543.1	164592.2	166824.5	161969.7	168331.4	171243.2	172982.5	172307.9	165398.5
30	康寧實業股份有限公司(中國)	168799.4971	166004.7	165300.7	167293.2	165999.7	166294.9	161214.1	164051.1	165025.8	164681.9	163243.3	162643.2
31	宏普實業有限公司	144832.7523	141100.3	141006.1	138363.1	142390.4	144467	139622.1	146243.9	149107.4	150114	150135.9	142370.3
32	康寧實業股份有限公司(蘇州)	975868.6873	974593.6	975951.4	974863.6	974863.6	975851.7	975107.6	974478.6	974506.7	975196	974564.6	974864.6

Figure 3: The Great Circle Distance between factories and districts

2.2.2 Data Normalization

In the utility problem, we tried to design a function for evaluating the performance of different solution. However, the relative data input is given in different unit, which means it may affect the weight of different data and leads to distortion in the result. Thus, the data of density and population are normalized within the range of 0 and 1. Given the series of data, X , for each data i , the normalized value Z_i is,

$$Z_i = \frac{X_i - \max(X)}{\max(X) - \min(X)}$$

2.3 Obtaining Data Matrix

To obtain the distance matrix between the factories and each district, we used R to import the District data in SHP format using the `readOGR()` function in `rgdal` R package. The factory data is also imported using the `read.csv()` function in `readr` R Package and converted to an sf object using `st_as_sf()` function is `sf` R Package. Then we plot the factory and District data to ensure that the coordinates are correct. After that, we extract the coordinates of the factory and District data (centroid) into a matrix and use the `dism()` function from `geosphere` R Package to obtain the distance matrix. The distance calculated is based on the geodesic using WGS84 ellipsoid. The resulting distance matrix is output as a csv file for later use to solve the model.

3 Problem Description

3.1 Provider Problem

The Provide problem is to minimize the total transportation cost for the summation of delivery from factory i to town j . The unit of the transportation is mask times unit transportation cost per meter. The parameters include

1. M : the number of Factory
2. N : the number of District
3. $D_{i,j}$: the distance between Factory i and District j for $i = 1, \dots, M, j = 1, \dots, N$
4. P_j^A be the number of adult population in District j for $j = 1, \dots, N$
5. P_j^Y be the number of children population in District j for $j = 1, \dots, N$
6. C : the cost of delivering one mask per meter
7. S_i be the production capacity of Factory i for $i = 1, \dots, M$
8. K be the maximum production of the factory

The variables include

1. α_i be 0 if Factory i cannot produce children mask and 1 if Factory i can produce children mask for $i = 1, \dots, M$. The formal definition is

$$\begin{cases} \sum_{j=1}^N X_{i,j}^Y = 0, & \text{if } \alpha = 0 \\ \sum_{j=1}^N X_{i,j}^Y \geq 0, & \text{otherwise} \end{cases}$$

2. $X_{i,j}^A$: the number of adult mask produced by Factory i for District j for $i = 1, \dots, M, j = 1, \dots, N$
3. $X_{i,j}^Y$: the number of children mask produced by Factory i for District j for $i = 1, \dots, M, j = 1, \dots, N$

The complete linear model formulation is illustrated below

$$\begin{aligned}
\min \quad & \sum_{i=1}^M \sum_{j=1}^N (X_{i,j}^A + X_{i,j}^Y) D_{i,j} C \\
\text{s.t.} \quad & \sum_{j=1}^N X_{i,j}^A + X_{i,j}^Y = S_i \quad \forall i = 1, \dots, M \quad (\text{Factory Limit}) \\
& \sum_{i=1}^M X_{i,j}^A \leq P_j^A \quad \forall j = 1, \dots, N \quad (\text{Adult Requirement}) \\
& \sum_{i=1}^M X_{i,j}^Y \leq P_j^Y \quad \forall j = 1, \dots, N \quad (\text{Children Requirement}) \\
& \sum_{j=1}^N X_{i,j}^Y \leq K \alpha_i \quad \forall i = 1, \dots, M \quad (\text{Children Production Capability}) \\
& X_{i,j}^A \geq 0 \quad \forall i = 1, \dots, M, j = 1, \dots, N \\
& X_{i,j}^Y \geq 0 \quad \forall i = 1, \dots, M, j = 1, \dots, N \\
& \alpha_i \in \{0, 1\} \quad \forall i = 1, \dots, M
\end{aligned}$$

3.2 Utility Problem

The Utility problem is to maximize the safety confidence score of all the towns in Taiwan Province. The project assumes the utility function, $U_j(\cdot)$ as the measurement of the safety confidence score of all the residences in town j . The reason is that during the period of COVID-19, people tend not to leave far away from their place of residence. All of the necessary affairs are done within the neighborhood. Thus, the project assumes that all the residence in town j shares the same utility of safety. Meanwhile, the utility function form is proposed on the basis of public thoughts and rationales. Those considerations includes,

1. Density: People in Taiwan complains the "7-day-2-mask" or "14-day-9-mask" about that the policies ignores the difference between each town, especially the crowd are difference between town and town. For those live in town that has relatively high density, they need to have wear masks more frequently, just in case of the highly infected possibility from massive contact opportunity. Thus, many complains that the urban areas with high population density should have relatively more mask per day in contrast to the rural area that has lower population density. As a result, the utility function should make the density of the town positively affects the utility, i.e., the denser, the higher need of masks.

2. Coverage: The coverage of the masks of the town must positively effects the utility without a doubt. That is, the more masks, the more safety the town is, thus results in high utility in safety confidence.
3. Population: Given all others equal, the more the population of the town j is, the safer the town gets the masks. Hence, the population should positively effects the utility.
4. Diminishing Marginal Returns: As a basic idea in the utility function(and many other functions in Economics), the rational function must be assume to be exhibited with diminishing marginal returns. Hence, the utility function of the town's safety confidence score should exist the property of diminishing marginal returns. Specifically speaking, the value of the second partial derivatives should be negative for the three factors mentioned above.
5. Equal Weight of Factors: From the basic viewpoint, the project assumes that the three factors are equally important. In order to equalize the effect of the three factors, all of the factors, r_j , P_j , d_j are normalized within the range of 0 and 1. Meanwhile, considering both diminishing marginal return and equal effects, the project assumes the utility function as the cubic root of the product of the three factors.

The utility function of the town j is described as,

$$U_j(r_j, P_j, d_j) = (P_j \cdot r_j \cdot d_j)^{\frac{1}{3}}$$

The entire non-linear model is exhibited below, all the notations are inherited from Section 3.1.

$$\begin{aligned}
\max \quad & \sum_{j=1}^N U_j \\
\text{s.t.} \quad & r_j^A = \frac{\sum_{i=1}^M X_{i,j}^A}{P_j^A} && \text{(Coverage of Adult)} \\
& r_j^Y = \frac{\sum_{i=1}^M X_{i,j}^Y}{P_j^Y} && \text{(Coverage of Children)} \\
& r_j = r_j^A + r_j^Y && \text{(Total Coverage of town } j) \\
& P_j = P_j^A + P_j^Y && \text{(Total Population in town } j) \\
& U_j = (P_j \cdot r_j \cdot d_j)^{\frac{1}{3}} && \text{(Utility of Town } j) \\
& \sum_{j=1}^N (x_{i,j}^A + \frac{1}{2} X_{i,j}^Y) = S_i, \quad \forall i = 1..M && \text{(Production Capacity)} \\
& \alpha_i = \{0, 1\} && \text{(Factory } i \text{ Capability of Making Children Mask)} \\
& \sum_{j=1}^N X_{i,j}^Y \leq 2S_i(\alpha_i), \quad \forall i = 1..M && \text{(Linearize } \alpha_i) \\
& \sum_{i=1}^M X_{i,j}^A \leq P_j^A && \text{(adult demand bound)} \\
& \sum_{i=1}^M X_{i,j}^Y \leq P_j^Y && \text{(children demand bound)} \\
& X_{i,j}^A \geq 0 \quad \forall i = 1, \dots, M, j = 1, \dots, N \\
& X_{i,j}^Y \geq 0 \quad \forall i = 1, \dots, M, j = 1, \dots, N
\end{aligned}$$

4 Performance Analysis

4.1 Provider Problem

1. Result

- We maximize the production of the mask to 16 million, and lower the travel distance by half comparing to the governments "14-day-9-mask" policy.

$$\begin{cases} \text{Our: 130 million (km) per day.} \\ \text{Gov: 348 million (km) per day.} \end{cases}$$

The calculation of governments travel distance is by making the coverage of every district regardless of adult or children equal to $\frac{9}{14}$, which is the "14-day-9-mask" policy, therefore we can get X_{ij}^A and X_{ij}^Y , hence we can get the travel distance.

- The district closer to the factories are the first to be satisfied with the demands of masks.

2. Benefit

- The production of face masks is maximized since we made all the factories to produce the mask at their most. So we can get more masks then what governments provided.
- We still manage to cut down the cost, while providing more masks to different in different districts.

3. Drawbacks

- We still don't have enough mask for everyone, since we can only produce 1.6 million mask per day, it is still lower than our total population which is approximately 2.3 million.
- We did not maximize the total utility, and it is not a fair allocation of masks because we first provide to the closest districts, leaving the places with no factories in there neighborhood with no masks provided. In the picture below we can see that the east side has no factory at all, this means that half of Taiwan may not be provided with masks.

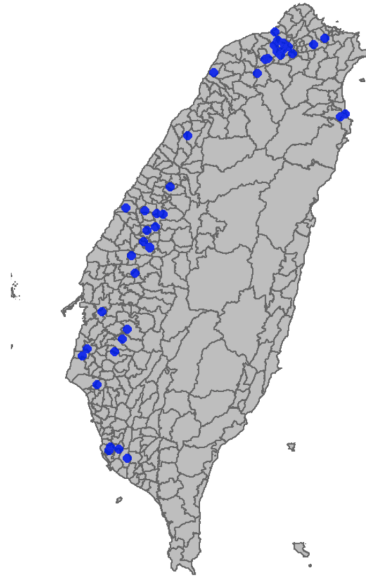


Figure 4: The Mask Factories in Taiwan Province

4.2 Utility Problem

1. Result

We provide a higher coverage of mask in high density and high population areas comparing to the government.

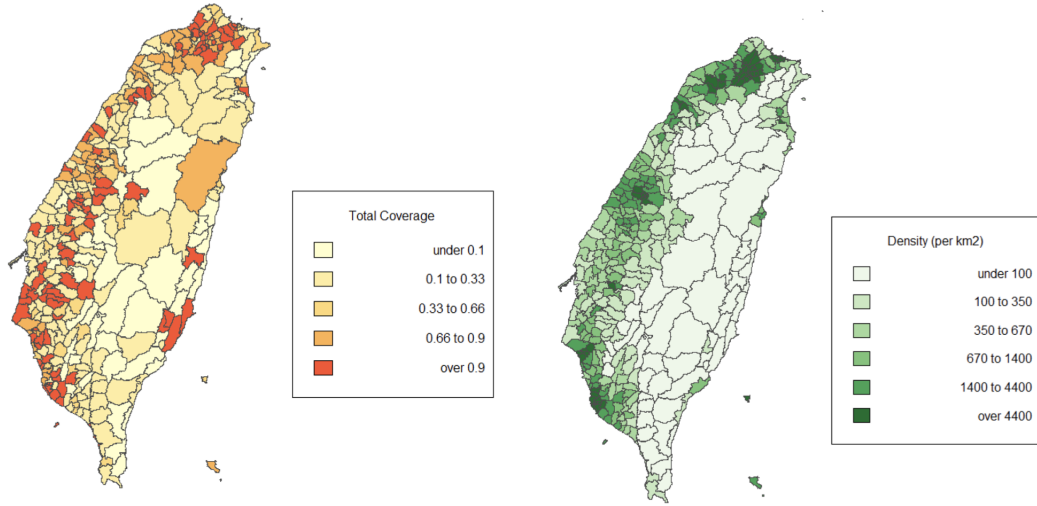


Figure 5: Comparison between the Coverage of the Mask and Density

2. Benefits

Large city with high population and high density has higher utility which means the coverage is higher. The risk of getting infected is lower, since more mask is provided.

3. Drawbacks

- The total travel distance is up to 579 million which is way higher than 130 million. This means that the cost will be super high even comparing to governments policy we are still 1.5 times higher.
- We sacrificed fairness in order to gain efficiency. So some districts may be left with no mask provided.

Conclusion and Future Works

4.3 Conclusion

Looking back at our formulation, not only did we out-performed governments policy, we also obtained some benefits.

1. The way of mask distribution is closer to public thoughts. This means that people are more willing to accept our policy, hence with better cooperation with the people, we can be more efficient at controlling the epidemic.
2. We lower people's frustration towards the disease. Since people with high risk of getting infected are provided with more masks, they should feel much safer and protected. Once feel secured, the fear of the COVID-19 no longer exists.
3. We maximize the social welfare. With all the benefits above, we can gain great control of the epidemic, and in this case the faster we can get back to our normal lives the higher social welfare we get.

4.4 Future Works

1. Application of Nations

The project is expected to expand the application situation not only considering our country but also the country in the globe. Recently, the COVID-19 disease is under well-control in R.O.C.(Taiwan) due to the border regulation and the 14 days quarantine policy. Nevertheless, the others countries, especially the United States and Brazil are still out of control. The masks distribution is indeed an existing crisis for them to distribute the goods and make the citizens safe and confident. Thus, the project is looking forward to expand to some foreign nations to aid in mask distribution.

2. Virus Prevention Stuffs of Concern

The current virus prevention stuff of concern is merely mask. However, there are other stuffs that are less severe but still important. Take protective clothing, alcohols, tissues for instance, the project can extend the virus prevention stuff of concern to other stuffs.

3. Rare Resource Reallocation

There are a lot of things that are not unlimited access for all of us. Most things are considered limited, or even rare. By using our formulation, we can distribute these resources in a better way, to those who are in need, instead of controlled by only few people. We might start with public goods provided by the governments, as for private goods the market itself should solve the problem.

4. Improvements

- Calculation of Population

The data we gathered is based on administrative district, but to make the model more

realistic, we should change it to living quarters. The administrative district gathers only the birth and the death population, we did not take the commuters in account, which is the most important cause of spreading the virus. The population flow should definitely be considered if we are trying to make the formulation more precise.

- Calculation of Travel Distance

The method we use to calculate the travel distance is designing the driver to visit only one district, we did this to simplify our formulation. So to let the model be more precise, we can design the route to be visiting multiple districts at once, also adding the capacity limit to the truck, last but not least the number of truck drivers should be limited too.