偏態、峰態和波動率在市場動盪期間對 VaR 影響的演變：基於 Cornish-Fisher 展開式的觀察

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摘要

本研究探討偏態、峰態與波動率在 2007-2008 年金融危機期間對風險值（Value-at-Risk, VaR）的動態影響。透過採用 Cornish-Fisher 展開式，我們將高階動差納入 VaR 框架中，使得能夠對全球主要股票市場的崩潰與脆弱風險貢獻進行詳細分解。我們的分析顯示，這些風險組成部分在危機前後的相對重要性發生顯著變化。Cornish-Fisher VaR 模型在市場動盪期間的表現持續優於傳統 Gaussian 和 EWMA VaR 模型，這一點在大多數指數的概似比(likelihood ratio)檢定結果中得到證實。儘管在危機後波動率成為 VaR 的主要貢獻因素，但偏態和峰態效應的大小和方向呈現多樣化的變化，反映了市場對極端事件風險認知的動態調整。透過貢獻值、敏感度和影響份額的分解，全面闡釋了各風險組成部分如何影響 VaR。值得注意的是，我們發現一個潛在的市場動盪早期警示信號：在危機前，偏態和峰態的短期影響份額在多個市場中開始低於其長期平均水準，而波動率的短期影響份額則開始高於其長期平均水準。

本研究的發現對股價指數期貨市場具有重要意義。除了在 VaR 估計中納入高階動差可以更準確地反映極端事件風險外，危機前的早期警示信號也使期貨交易者能夠預先調整部位或避險策略以因應市場動盪。此外，我們的風險分解方法提供了市場風險來源的深入分析，有助於在期貨交易中發展適應性風險管理策略。本研究透過對這些風險因素在市場危機前後對 VaR 影響的探討，為文獻做出貢獻，從而為現貨和期貨市場的風險評估和管理提供參考。

關鍵詞：風險值、偏態、峰態、Cornish-Fisher 展開式、次貸危機

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The Evolving Role of Skewness, Kurtosis, and Volatility in Shaping VaR During Market Turbulence: Insights from the Cornish-Fisher Expansion

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Abstract

This study examines the dynamic roles of skewness, kurtosis, and volatility in shaping Value-at-Risk (VaR) during the 2007-2008 financial crisis. Employing the Cornish-Fisher expansion, we integrate higher-order moments into the VaR framework, enabling a detailed decomposition of risk contributions across major global stock markets. Our analysis reveals significant shifts in the relative importance of these risk components before and after the crisis. The Cornish-Fisher VaR model consistently outperforms traditional Gaussian and EWMA VaR models during periods of market turbulence, as evidenced by improved likelihood ratio test results across most indices. While volatility emerges as the dominant contributor to VaR in the post-crisis period, the magnitude and direction of skewness and kurtosis effects exhibit heterogeneous changes, reflecting dynamic adjustments in market perceptions of extreme event risk. Our decomposition, encompassing Contribution Values, Sensitivity, and Impact Shares, provides a comprehensive elucidation of how each risk component influences VaR. Notably, we identify a potential early warning signal for market turbulence: prior to the crisis, short-term Impact Shares of skewness and kurtosis start to fall below their long-term averages across multiple markets, while the

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short-term Impact Share of volatility start to rise above its long-term average. These findings have important implications for futures markets. The incorporation of higher-order moments in VaR estimation allows for more accurate reflection of extreme event risk, particularly relevant for futures contracts on stock indices. The identified pre-crisis pattern can serve as an early warning indicator, enabling futures traders to adjust positions or hedging strategies in anticipation of market turbulence. Furthermore, our risk decomposition methodology provides a nuanced analysis of market risk sources, facilitating the development of adaptive risk management strategies in futures trading. This study contributes to the literature by offering insights into how these risk factors recalibrate VaR estimates during market crises, thereby informing more robust risk assessment and management practices in both spot and futures markets.

Keywords: Value-at-Risk, Skewness, Kurtosis, Cornish-Fisher Expansion, Subprime Mortgage Crisis

I. Introduction

Value-at-Risk (VaR) remains a cornerstone of financial risk management, quantifying potential losses within a defined confidence interval over a specified time horizon. However, conventional VaR models, which predominantly rely on volatility as the primary risk measure under the assumption of normally distributed asset returns, often underestimate risk during periods of market turmoil. This limitation became starkly evident during the 2007-2008 Subprime Mortgage Crisis, underscoring the critical need for more robust risk assessment methods that fully capture the complexities of financial markets.

When evaluating VaR as a comprehensive measure of total risk, it is crucial to consider the impact of higher moments such as skewness and kurtosis. These moments serve as indicators of distinct risk types that go beyond mere deviations from normality. Negative skewness, linked to crash risk, signals an elevated probability of extreme negative outcomes in the left tail of the return distribution. This suggests a likelihood of abrupt and severe market downturns triggered by rare, unpredictable "black swan" events, such as significant economic recessions, policy shifts, or financial crises (Chen et al., 2001). High kurtosis, associated with fragility risk, reflects an increased frequency of extreme values in the return distribution. This implies that markets, while appearing stable, remain susceptible to shocks from systemic risks, including economic slowdowns or inflation spikes ("grey rhino" events) (Hallett and Anthony, 1997). High kurtosis indicates concentrated distributions with elevated probabilities of extreme events, highlighting the market's vulnerability to significant shocks.

Analyzing the proportion of these risks within the overall risk framework before and after financial crises offers a nuanced perspective on market dynamics. This approach enhances our understanding of how these factors contribute to total risk beyond volatility alone. Our study diverges from previous works primarily focused on VaR model accuracy by providing an examination of risk dynamics through the relative contributions of skewness, kurtosis, and volatility to VaR, potentially identifying market crash precursors.

Recent research has emphasized the roles of skewness and kurtosis in forecasting financial market volatility. Studies by Mei et al. (2017) and Gkillas et al. (2019) demonstrate the efficacy of realized skewness and kurtosis in predicting volatility across major indices and currency markets. Bonato et al. (2021) extend this concept to international REITs, underscoring the effectiveness of kurtosis as a predictor of extreme market movements. Rehman (2024) explores these higher moments in emerging markets, finding support for kurtosis in forecasting volatility, while the predictive power of skewness is limited. These findings suggest that kurtosis captures crucial tail risk and extreme price movement information, making it an important addition to conventional volatility models.

Gabrielsen et al. (2015) highlight that variations in skewness and kurtosis often correspond to sharp changes during periods of high volatility, significantly impacting risk management and VaR estimations. Lux and Marchesi (2000) observe that kurtosis in time-series data is frequently linked to volatility clustering, underscoring the importance of incorporating higher moments in portfolio construction. Including skewness and kurtosis in volatility forecasting models better captures market behaviors under varying risk conditions, especially in emerging markets characterized by higher volatility and lower market efficiency (Gkillas et al., 2019; Mei et al., 2017).

To address the limitations of conventional VaR models, this study employs the Cornish-Fisher expansion, which integrates skewness and kurtosis into the quantile function of the standard normal distribution. This method enhances VaR's ability to reflect the asymmetric and heavy-tailed nature of real-world return distributions.

Notably, Cornish-Fisher VaR uniquely enables the decomposition of total risk into contributions from volatility, skewness, and kurtosis, unlike Gaussian VaR and EWMA VaR, which serve as benchmarks for performance comparison under varying market conditions.

Our research investigates the evolving roles of skewness, kurtosis, and volatility in shaping VaR during periods of market turbulence, with a specific focus on the 2007-2008 Subprime Mortgage Crisis. The analysis spans major global stock markets, examining how these risk factors influenced VaR estimates before and after the crisis. Empirical results are primarily based on Cornish-Fisher VaR and are compared with Gaussian VaR and EWMA VaR models to assess relative performance under changing conditions.

The study's design centers on estimating the influences of volatility (σ_t), skewness (s_t), and kurtosis (k_t) on VaR, observing variations before and after the 2007-2008 financial crisis. This approach enables the identification of potential precursors to market crashes, offering insights into the dynamics of risk factors leading to significant market turbulence.

Our methodology employs three distinct approaches to estimate VaR: conventional Gaussian VaR, EWMA VaR, and Cornish-Fisher VaR. We first apply a series of likelihood ratio tests to validate these models, then focus on Cornish-Fisher VaR for an in-depth analysis of skewness, volatility, and kurtosis contributions.

The comprehensive framework developed quantifies the individual impacts of volatility, skewness, and kurtosis on VaR, involving Contribution Values, Sensitivities, and Impact Shares. Key findings reveal shifts in the contributions of skewness, kurtosis, and volatility to VaR during the crisis. Pre- and post-crisis analyses indicate that while volatility remains a primary factor, the roles of skewness and kurtosis fluctuate, illustrating the interplay of risk components in turbulent markets. The study also explores the sensitivity of VaR to these higher-order moments, providing insights into their importance across different phases of market stress.

Additionally, the research includes rigorous backtesting of the VaR models to evaluate predictive accuracy and robustness under extreme conditions. Backtesting results, including VaR Failure Rate and Likelihood Ratio tests (LRuc, LRind, LRcc), offer an evaluation of model performance, highlighting the need for adaptive models capable of addressing evolving risk landscapes.

This study advances the literature by detailing how skewness, kurtosis, and volatility recalibrate VaR estimates during market crises. Utilizing the Cornish-Fisher expansion, the research refines risk assessment approaches, informing the development of more resilient risk management strategies in anticipation of future financial shocks. The findings emphasize the essential role of higher-order moments in risk modeling and highlight the necessity of continuously refining VaR methodologies to capture the complexities of global financial markets during periods of extreme volatility.

The structure of this paper is as follows: Section 2 details the methodology, including the modified VaR calculation using the Cornish-Fisher expansion and the sensitivity analysis approach. Section 3 presents the empirical results, comparing different VaR models across various market conditions, with a focus on the year preceding and following the 2007-2008 crisis. Finally, Section 4 concludes with the key takeaways and implications for risk management practices in light of our findings on the evolving roles of volatility, skewness, and kurtosis in shaping VaR during periods of market turbulence.

II. Methodology

1. Value-at-Risk (VaR) and Limitations of the Conventional VaR Model

Value-at-Risk (VaR) is a widely used measure in financial risk management, estimating the potential loss of a portfolio over a specified time period at a given confidence level α. It is formally defined as:

$$\alpha = Prob(R_t < -VaR_{\alpha,t}) \tag{1}$$

where R_t represents the return at time t.

The conventional Gaussian VaR (relative to the mean) is expressed as:

$$VaR_{\alpha,t} = -z_\alpha \sigma_t \tag{2}$$

where σ_t is the volatility for a portfolio; z_α represents the Gaussian quantile at the given confidence level α. However, the assumption of normally distributed returns in the Gaussian VaR may lead to underestimation of risk during periods of market stress characterized by skewness (asymmetries) and kurtosis (heavy tails) in return distributions (Pedrosa & Roll, 1998; Alizadeh & Gabrielsen, 2013).

Another conventional method, the Exponentially Weighted Moving Average (EWMA) VaR, incorporates time-varying volatility into Equation (2) by applying exponentially declining weights to historical returns. The volatility σ_t in the EWMA model is updated recursively using the formula:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)R_{t-1}^2 \tag{3}$$

where λ is the decay factor, typically set close to 1 (e.g., 0.94), which controls the rate at which older data is de-emphasized. This approach allows the model to react more quickly to recent market changes, providing a more adaptive measure of risk. However, while EWMA VaR captures the time-varying nature of volatility, it does not account for skewness and kurtosis in return distributions, similar to the Gaussian VaR.

Consequently, EWMA may still underestimate risk during periods of market stress when returns exhibit significant deviations from normality.

2. Cornish-Fisher VaR

To address the limitations of conventional VaR models, we employ the Cornish-Fisher expansion, which modifies the quantile by incorporating skewness (s_t) and kurtosis (k_t) adjustments. Using the third-order expansion (Maillard, 2018; Amédée-Manesme et al., 2019), the adjusted quantile is:

$$z_{\alpha,t}^{CF} = z_\alpha + \frac{1}{6}(z_\alpha^2 - 1)s_t + \frac{1}{24}(z_\alpha^3 - 3z_\alpha)(k_t - 3) - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha)s_t^2 \tag{4}$$

The Cornish-Fisher VaR is then calculated as:

$$VaR_{\alpha,t}^{CF} = -z_{\alpha,t}^{CF}\sigma_t \tag{5}$$

This adjustment allows the VaR calculation to reflect the asymmetry and fat tails observed in empirical return distributions, providing a more accurate assessment of risk during periods of market turbulence.

3. Decomposition of Cornish-Fisher VaR

To quantify the individual contributions and impacts of volatility, skewness, and kurtosis to VaR, we perform a total differentiation of the adjusted VaR formula:

$$dVaR_{\alpha,t}^{CF} = -\sigma_t dz_{\alpha,t}^{CF} - z_{\alpha,t}^{CF} d\sigma_t \tag{6}$$

Substituting the expression for $dz_{\alpha,t}^{CF}$, we obtain:

$$dVaR_{\alpha,t}^{CF} = -\sigma_t \left[\left(\frac{1}{6}(z_\alpha^2 - 1) - \frac{1}{18}(2z_\alpha^3 - 5z_\alpha)s_t \right) ds_t + \frac{1}{24}(z_\alpha^3 - 3z_\alpha) dk_t \right] - z_{\alpha,t}^{CF}\, d\sigma_t \tag{7}$$

This decomposition isolates the sensitivity of VaR to changes in each component, allowing us to derive the contribution values and impact shares of each component on VaR.

3.1 Contribution Values

The contribution values of each component on VaR are quantified as follows:

(1) The Skewness Contribution ($dVaR_s$) quantifies how changes in skewness influence VaR, reflecting the impact of asymmetry in return distributions. The formula is given by:

$$dVaR_s = -\sigma_t \left[\left(\frac{1}{6}(z_\alpha^2 - 1) - \frac{1}{18}(2z_\alpha^3 - 5z_\alpha)s_t \right) ds_t \right] \tag{8}$$

This equation shows that the impact of changes in skewness on VaR is influenced by the current levels of volatility (σ_t) and skewness (s_t).

(2) The Kurtosis Contribution ($dVaR_k$) captures the effect of kurtosis, highlighting the sensitivity of VaR to the presence of heavy tails. The formula is given by:

$$dVaR_k = -\sigma_t \left[\frac{1}{24}(z_\alpha^3 - 3z_\alpha) dk_t \right] \tag{9}$$

This equation indicates that the impact of changes in kurtosis on VaR is related to the current level of volatility (σ_t).

(3) The Volatility Contribution ($dVaR_\sigma$) represents the impact of volatility on VaR. The formula is given by:

$$dVaR_\sigma = -z_{\alpha,t}^{CF}\, d\sigma_t \tag{10}$$

This shows that the impact of volatility on VaR is related to the current levels of skewness (s_t) and kurtosis (k_t) through their influence on $z_{\alpha,t}^{CF}$.

3.2 Sensitivity

Sensitivity reflects the change of VaR with respect to the unit change in each risk component. The sensitivities (β) are given by:

$$\beta_s = -\sigma_t \left[\left(\frac{1}{6}(z_\alpha^2 - 1) - \frac{1}{18}(2z_\alpha^3 - 5z_\alpha)s_t\right)\right] \tag{11}$$

$$\beta_k = -\sigma_t \left[\frac{1}{24}(z_\alpha^3 - 3z_\alpha)\right] \tag{12}$$

$$\beta_\sigma = -z_{\alpha,t}^{CF} \tag{13}$$

These sensitivity measures provide insight into how changes in each risk component affect the overall VaR estimate.

3.3 Impact Shares

Impact Share (IS) provides a comparative analysis of the relative importance of each component:

$$IS_i = \frac{|dVaR_i|}{\sum_{i=s,k,\sigma}|dVaR_i|} \tag{14}$$

where i represents skewness (s), kurtosis (k), or volatility (σ). This metric helps identify the proportional influence of each risk component on VaR, allowing for a comprehensive understanding of their relative contributions to overall risk.

4. Backtesting and Model Validation

To evaluate the predictive accuracy and robustness of each VaR model under extreme conditions, we conduct rigorous backtesting. The backtesting results are assessed using the following metrics:

(1) Failure Rate: This measure represents the proportion of times actual losses exceed the VaR estimate. It provides a straightforward assessment of the model's accuracy in predicting extreme losses.

(2) Likelihood Ratio Tests: We employ three tests proposed by Kupiec (1995) and Christoffersen (1998): a) Unconditional Coverage Test (LRuc): This test examines whether the actual failure rate matches the expected rate based on the chosen confidence level. b) Independence Test (LRind): This test assesses whether VaR violations occur independently over time or if there are clusters of violations that might indicate model misspecification. c) Conditional Coverage Test (LRcc): This test combines the unconditional coverage and independence tests to provide a comprehensive evaluation of the model's performance.

These backtesting procedures allow us to compare the performance of the Cornish-Fisher VaR model against the conventional Gaussian and EWMA VaR models, providing insight into their relative strengths and weaknesses under various market conditions.

III. Empirical Results

1. Data and Sample Selection

Our study examines a diverse set of major global stock market indices, representing key markets from different regions including Europe, the Americas, Asia-Pacific, and emerging markets. This selection provides a comprehensive view of market conditions to examine the performance of various VaR models under a range of scenarios. Table 1 presents the sample of indices along with their relevant descriptions, including trading hours and the markets they represent.

[Table 1]

The study employs weekly data from these indices, focusing on the period surrounding the 2007–2008 Subprime Mortgage Crisis. We set September 30, 2007, as the event date, marking the onset of significant market turmoil. Our analysis covers an event window comprising 52 weeks before and after this date, totaling 104 weeks.

To capture the dynamic nature of market parameters, we calculate volatility (σ_t), skewness (s_t), and kurtosis (k_t) using a 26-week rolling window. This approach balances responsiveness to changing market conditions with the need for statistical reliability.

We set the VaR confidence level at 5% to ensure sufficient failure samples for backtesting within the event window. Three VaR models are estimated for comparison:

1. Gaussian VaR: Based on the assumption of normally distributed returns.
2. EWMA VaR: Incorporates time-varying volatility using an exponentially

weighted moving average.

3. Cornish-Fisher VaR: Adjusts for skewness and kurtosis using the Cornish-Fisher expansion.

2. VaR Model Evaluation

Table 2 presents the results of our VaR model evaluation across the various global stock indices. The analysis primarily focuses on the VaR failure rates and the likelihood ratio tests (LRuc, LRind, LRcc) to assess the models' predictive accuracy during periods of market turbulence.

[Table 2]

The results indicate that the Cornish-Fisher model generally performs well, with p-values in the LRuc, LRind, and LRcc tests mostly above the 0.05 significance level. This suggests that it adequately captures the expected failure rate and the independence of violations across most indices. The model's incorporation of skewness and kurtosis appears to improve the accuracy of VaR estimates during periods of market stress.

The Gaussian model, which assumes normally distributed returns, shows mixed results. While it meets the expected failure rate for some indices, its p-values in the LRind and LRcc tests often approach the 0.05 threshold, highlighting potential limitations in accurately predicting risk when market conditions deviate from normality.

The EWMA model displays significant variability, with several indices showing p-values below the 0.05 threshold in the LRuc, LRind, and LRcc tests. This suggests that while the model captures time-varying volatility, it may struggle with violations of independence and accuracy under extreme conditions.

Overall, the Cornish-Fisher model appears to offer a more robust risk assessment by accounting for skewness and kurtosis, which are critical during market downturns. These findings emphasize the importance of integrating higher moments into risk models to enhance their performance under volatile market conditions.

3. Decomposition of Cornish-Fisher VaR

Given the observed performance of the Cornish-Fisher VaR model in capturing skewness and kurtosis, we proceed to decompose the Cornish-Fisher VaR to investigate the individual contributions of volatility, skewness, and kurtosis. This decomposition allows us to quantify the impact of each factor on the overall VaR estimate, providing insights into how each component influences risk prediction under different market conditions, particularly during periods of market stress.

3.1 Contribution Values

Table 3 presents the contribution values of volatility ($dVaR_\sigma$), skewness ($dVaR_s$), and kurtosis ($dVaR_k$) to the Cornish-Fisher VaR before and after the financial crisis.

[Table 3]

Our analysis reveals that volatility emerged as the primary contributor to VaR across most market indices, with notable increases observed post-crisis. For example, the contribution value of volatility in the AEX index rose from -0.186 to 3.269, though this change was not statistically significant (p = 0.348). The only instance approaching statistical significance was observed in the TA125 index, where the change in volatility's contribution value (t = 1.863, p = 0.065) suggests that volatility shifts may have a more pronounced influence on VaR in certain markets.

The changes in skewness contribution values illustrate the potential role of negative skewness in affecting VaR, which is often associated with increased crash risk. However, most results did not achieve statistical significance. For instance, the skewness contribution value in the GSPC index decreased from 0.137 to -0.565, with a p-value of 0.404, indicating insufficient statistical evidence to confirm a significant effect on total risk.

Similarly, the contribution values of kurtosis reflect the potential role of extreme events in influencing VaR. In the SSMI index, for example, the kurtosis contribution value increased from 0.018 to 0.065, yet this change was not statistically significant (p = 0.669), suggesting that the variations in kurtosis contribution values to VaR remain inconclusive across the sampled indices.

In summary, while the contribution values of volatility, skewness, and kurtosis to VaR exhibited some variations before and after the crisis, the majority of these changes were not statistically significant. This implies that, despite observed shifts in these risk components, their influence on VaR requires further verification and exploration from alternative perspectives.

3.2 Sensitivity Analysis

Table 4 presents the results of our sensitivity analysis, showing how changes in volatility, skewness, and kurtosis affect the Cornish-Fisher VaR estimates.

[Table 4]

Our analysis reveals that the sensitivity of volatility (β_σ) to Cornish-Fisher VaR generally decreased significantly before and after the financial crisis. This reflects a reduced role of volatility in VaR during the post-crisis period, suggesting that the influence of market volatility on risk values has diminished following the crisis. For instance, the KLSE index exhibited a notable decrease in volatility sensitivity from 1.826 to 1.719, representing one of the most significant changes observed. In contrast, the AEX and FTSE indices showed no significant difference in volatility sensitivity before and after the crisis, indicating that the influence of volatility on VaR remained relatively stable in these markets.

The sensitivity of skewness (β_s) became significantly more negative across most markets, indicating an increased effect of negative skewness on VaR. This aligns with the phenomenon where heightened negative skewness leads to higher risk values. The HSI index, in particular, saw its skewness sensitivity decrease from -0.635 to -1.165, the largest change among the indices. This suggests a substantial increase in the market's sensitivity to negative skewness and an elevated perception of crash risk in Hong Kong. Conversely, the GDAXI index exhibited the smallest change in skewness sensitivity, from -0.642 to -0.725, reflecting a relatively limited increase in sensitivity to negative skewness and demonstrating varied responses to skewness changes across different markets.

The sensitivity of kurtosis (β_k) also became significantly more negative in most markets. This indicates that as kurtosis rises (i.e., the distribution becomes more concentrated around the center), its effect on risk values decreases. Higher kurtosis, combined with a negative sensitivity, implies that markets perceive a reduced likelihood of extreme events, resulting in a lower VaR. For example, the HSI index's kurtosis sensitivity declined from -0.044 to -0.083, marking the largest observed change and reflecting a significant post-crisis increase in sensitivity to kurtosis changes, alongside a reduced perception of extreme event risk. Meanwhile, the TA125 index displayed the smallest change in kurtosis sensitivity, from -0.044 to -0.053, indicating a relatively stable response to extreme loss risk, despite the significant change. This highlights the variability in how markets respond to extreme event risks.

3.3 Impact Shares

Table 5 presents the impact shares of volatility, skewness, and kurtosis on the Cornish-Fisher VaR before and after the financial crisis.

[Table 5]

Our analysis reveals that, for most markets, the impact shares of skewness (IS_s) and kurtosis (IS_k) were relatively high before the crisis, indicating that crash risk (reflected by skewness) and fragility risk (reflected by kurtosis) exerted a substantial influence on the VaR. However, post-crisis, these impact shares significantly declined. For instance, in the GSPC index, the impact share of skewness decreased from 40.65% to 22.19%, and kurtosis from 5.53% to 1.97%. This shift suggests that after markets experienced and adapted to the stress conditions, the attention towards the influence of extreme negative outcomes and the frequency of extreme events diminished.

Conversely, the impact share of volatility (IS_σ) was lower before the crisis but increased significantly afterward, highlighting volatility's emerging prominence as the primary source of risk in the post-crisis environment. For example, in the GSPC index, the impact share of volatility rose from 53.82% to 71.83%, the largest change among the indices. This increase reflects a market behavior where, amid heightened uncertainty and increased volatility, there is a stronger emphasis on the influence of volatility in risk assessments, positioning it as the principal factor in determining VaR.

Additionally, some markets, such as the GDAXI and TA125 indices, exhibited relatively stable changes in impact shares across the risk components before and after the crisis, indicating a consistent response to risk elements. This stability might suggest inherent market structures and risk management strategies that maintain a uniform approach to risk evaluation despite changes in market conditions.

Overall, these results underscore the dynamic adjustments in market risk perceptions and valuations of different risk sources before and after the crisis. Notably, the increasing importance of volatility in the post-crisis period contrasts with the declining roles of skewness and kurtosis. This adjustment reflects the market's adaptability and the shifting focus in risk evaluation, highlighting the evolving nature of risk management practices and the application of VaR models in turbulent financial environments.

Before the crisis, an intriguing pattern emerged: the short-term impact shares of skewness (IS_s) and kurtosis (IS_k) (proxied by a 20-week moving average) began to fall below their long-term averages (proxied by a 30-week moving average), while the short-term impact share of volatility (IS_σ) rose above its long-term average. This consistent pattern, observed across multiple global markets, suggests that changes in the relative impacts of these risk components may serve as early warning signals for impending market turbulence. Figure 1 illustrates this pattern in selected markets, and similar trends were observed in many other markets as well.

The reduction in the short-term impact shares of skewness and kurtosis indicates a market underestimation of the likelihood of extreme negative returns and the frequency of tail events, which are typically associated with increased fragility. This implies that, prior to the crisis, investors may not have fully recognized or responded to the growing systemic vulnerabilities, potentially missing critical signals of looming market stress. Conversely, the rise in the short-term impact share of volatility reflects an increasing market focus on immediate price fluctuations and short-term shocks, indicating a heightened state of anxiety and uncertainty in the market environment.

As the crisis unfolded, this pattern underwent significant changes. Much like a reservoir releasing floodwaters, the impacts of skewness and kurtosis on VaR significantly diminished, as if the contributions of these components rapidly receded. Simultaneously, the impact of volatility surged dramatically, akin to the intense surface waves that follow a deluge. This observation underscores the market's acute focus on volatility during the crisis, reflecting an environment characterized by high uncertainty and reactive short-term behaviors.

These observations further highlight that, during the crisis, the contributions of skewness and kurtosis to VaR weakened, while the contribution of volatility increased. This reflects a notable shift in market risk preferences and risk assessments amid severe volatility and uncertainty. This shift supports the necessity of incorporating higher-order moments into VaR models, not merely as static indicators but as dynamic measures responsive to changing market conditions.

Overall, these findings provide valuable insights into the dynamics of market stress evolution and underscore the importance of monitoring the relative changes in the impacts of skewness, kurtosis, and volatility. Notably, the simultaneous decline in the short-term impact shares of skewness and kurtosis, combined with the rise in the short-term impact of volatility, may serve as key indicators of impending market adjustments or crises.

[Figure 1]

IV. Conclusion

This study examines the impact of skewness, kurtosis, and volatility on Value-at-Risk (VaR) during the 2007-2008 Subprime Mortgage Crisis. We employ the

Cornish-Fisher expansion to incorporate higher-order moments into the VaR framework, enabling a decomposition of risk contributions across major global stock markets.

Empirical results demonstrate that the Cornish-Fisher VaR model outperforms traditional Gaussian and EWMA VaR models during periods of market stress, as evidenced by improved likelihood ratio test results across most indices. This underscores the necessity of incorporating skewness and kurtosis in risk modeling when market conditions deviate from normality.

The study reveals changes in risk component dynamics before and after the crisis. Volatility emerged as the primary contributor to VaR for most market indices post-crisis, although these changes were not statistically significant. For instance, in the AEX index, the volatility contribution value increased from -0.186 to 3.269 ($p = 0.348$). Concurrently, the impact of skewness and kurtosis diminished post-crisis. In the GSPC index, the skewness contribution value decreased from 0.137 to -0.565 ($p = 0.404$).

Sensitivity analysis indicates that post-crisis, volatility sensitivity decreased, while skewness and kurtosis sensitivities became more negative across most markets. The HSI index exhibited the largest change in skewness sensitivity, decreasing from -0.635 to -1.165. This suggests increased market sensitivity to negative skewness and reduced perception of extreme event risk following the crisis.

Impact share analysis reveals that pre-crisis, skewness and kurtosis had higher impact shares in most markets. Post-crisis, the impact share of volatility increased, while those of skewness and kurtosis decreased. In the GSPC index, the impact share of skewness decreased from 40.65% to 22.19%, and kurtosis from 5.53% to 1.97%, while volatility's impact share increased.

The study identifies a potential market turbulence indicator. Prior to the crisis, short-term impact shares of skewness and kurtosis fell below their long-term averages across multiple markets, while volatility's short-term impact share rose above its long-term average. This pattern may serve as a predictive indicator of market stress.

These findings have implications for risk management practices, particularly in futures markets. Incorporating skewness and kurtosis into the VaR framework allows for more accurate reflection of extreme event risk during market stress. For futures traders, this risk assessment method can help identify signals of increased market volatility, informing hedging strategies and risk adjustments. The observed pre-crisis pattern of declining short-term impact shares of skewness and kurtosis, coupled with rising short-term impact shares of volatility, may serve as a warning indicator of market turbulence, aiding futures traders in position or hedging strategy adjustments.

The risk decomposition method proposed in this study, quantifying contribution values, sensitivities, and impact shares of volatility, skewness, and kurtosis to VaR, provides a deeper analysis of market risk sources. These analytical results can be used to assess current market risk levels and as a reference for future risk predictions, facilitating timely responses to increased market volatility. This detailed risk component decomposition can assist futures market traders in understanding market risk dynamics, enhancing asset allocation stability, and developing adaptive risk management strategies.

In conclusion, the improved VaR model proposed in this study advances risk assessment methodology and provides actionable insights for practical risk management in futures markets. These findings prompt a reconsideration of conventional risk assessment and management approaches, driving the development of more precise risk management tools. Future research could extend to other market crisis events, explore cross-market dependencies, and incorporate additional risk factors to provide a more comprehensive understanding of market risk dynamics.

Table 1. Sample of Major Global Stock Market Indices and Descriptions

Ticker	**Index Name**	**Country/Region**	**Description & Trading Hours**
AEX	AEX Index	Netherlands	Tracks the performance of the 25 largest companies on Euronext Amsterdam. Trading hours: 9:00 AM - 5:30 PM CET.
AXJO	S&P/ASX 200 Index	Australia	Represents the 200 largest stocks listed on the Australian Securities Exchange. Trading hours: 10:00 AM - 4:00 PM AEST.
BSESN	SENSEX (BSE Sensex)	India	Comprises 30 financially sound and well-established companies on the Bombay Stock Exchange. Trading hours: 9:15 AM - 3:30 PM IST.
BVSP	Ibovespa Index	Brazil	Reflects the performance of the most traded stocks on the B3 (Brasil Bolsa Balcão). Trading hours: 10:00 AM - 5:00 PM BRT.
FCHI	CAC 40 Index	France	Includes the 40 largest and most liquid stocks on Euronext Paris. Trading hours: 9:00 AM - 5:30 PM CET.
FTSE	FTSE 100 Index	United Kingdom	Comprises the 100 most highly capitalized companies listed on the London Stock Exchange. Trading hours: 8:00 AM - 4:30 PM GMT.
GDAXI	DAX Index	Germany	Tracks the 40 largest companies listed on the Frankfurt Stock Exchange. Trading hours: 9:00 AM - 5:30 PM CET.
GSPC	S&P 500 Index	United States	Measures the stock performance of 500 large companies listed on stock exchanges in the U.S. Trading hours: 9:30 AM - 4:00 PM ET.
HSI	Hang Seng Index	Hong Kong	Represents the largest companies listed on the Hong Kong Stock Exchange. Trading hours: 9:30 AM - 4:00 PM HKT (with a 1-hour lunch break).
JKSE	Jakarta Composite Index	Indonesia	Tracks the performance of all listed stocks on the Indonesia Stock Exchange. Trading hours: 9:00 AM - 4:00 PM WIB (with a lunch break).
KLSE	FTSE Bursa Malaysia KLCI Index	Malaysia	Reflects the performance of the top 30 companies on the Bursa Malaysia. Trading hours: 9:00 AM - 5:00 PM MYT (with a lunch break).
KS11	KOSPI Index	South Korea	Comprises all common stocks traded on the Korea Exchange (KRX). Trading hours: 9:00 AM - 3:30 PM KST.
MXX	IPC Index	Mexico	Represents the 35 most liquid stocks on the Mexican Stock Exchange (Bolsa Mexicana de Valores). Trading hours: 8:30 AM - 3:00 PM CST.
N225	Nikkei 225 Index	Japan	Tracks the top 225 blue-chip companies listed on the Tokyo Stock Exchange. Trading hours: 9:00 AM - 3:00 PM JST (with a lunch break).
NZ50	NZX 50 Index	New Zealand	Represents the top 50 companies listed on the New Zealand Exchange. Trading hours: 10:00 AM - 4:45 PM NZST.
SSMI	Swiss Market Index (SMI)	Switzerland	Measures the performance of the 20 largest and most liquid stocks on the SIX Swiss Exchange. Trading hours: 9:00 AM - 5:30 PM CET.
STI	Straits Times Index	Singapore	Tracks the performance of the top 30 companies listed on the Singapore Exchange. Trading hours: 9:00 AM - 5:00 PM SGT (with a lunch break).
TA125	TA-125 Index	Israel	Comprises the 125 largest companies listed on the Tel Aviv Stock Exchange. Trading hours: 9:00 AM - 5:25 PM IST.
TWII	TSEC Weighted Index	Taiwan	Measures the performance of all listed stocks on the Taiwan Stock Exchange. Trading hours: 9:00 AM - 1:30 PM TST.

Table 2. VaR Model Evaluation for Global Stock Indices

index	Model	Failure Rate	LRuc	LRind	LRcc
AEX	Cornish-Fisher	0.079	0.189	0.214	0.195
	Gaussian	0.079	0.189	0.214	0.195
	EWMA	0.096	0.042	0.947	0.127
AXJO	Cornish-Fisher	0.088	0.093	0.245	0.124
	Gaussian	0.088	0.093	0.245	0.124
	EWMA	0.088	0.093	0.888	0.242
BSESN	Cornish-Fisher	0.061	0.589	0.419	0.623
	Gaussian	0.079	0.189	0.723	0.397
	EWMA	0.070	0.350	0.564	0.547
BVSP	Cornish-Fisher	0.088	0.093	0.888	0.242
	Gaussian	0.096	0.042	0.947	0.127
	EWMA	0.096	0.042	0.947	0.127
FCHI	Cornish-Fisher	0.079	0.189	0.723	0.397
	Gaussian	0.079	0.189	0.723	0.397
	EWMA	0.105	0.018	0.788	0.057
FTSE	Cornish-Fisher	0.061	0.589	0.338	0.547
	Gaussian	0.061	0.589	0.338	0.547
	EWMA	0.070	0.350	0.272	0.353
GDAXI	Cornish-Fisher	0.088	0.093	0.245	0.124
	Gaussian	0.088	0.093	0.245	0.124
	EWMA	0.105	0.018	0.491	0.047
GSPC	Cornish-Fisher	0.079	0.189	0.723	0.397
	Gaussian	0.079	0.189	0.723	0.397
	EWMA	0.088	0.093	0.245	0.124
HSI	Cornish-Fisher	0.061	0.589	0.338	0.547
	Gaussian	0.070	0.350	0.272	0.353
	EWMA	0.079	0.189	0.214	0.195
JKSE	Cornish-Fisher	0.061	0.589	0.338	0.547
	Gaussian	0.079	0.189	0.723	0.397
	EWMA	0.088	0.093	0.888	0.242
KLSE	Cornish-Fisher	0.070	0.350	0.564	0.547
	Gaussian	0.070	0.350	0.564	0.547
	EWMA	0.079	0.189	0.157	0.155
KS11	Cornish-Fisher	0.079	0.189	0.214	0.195
	Gaussian	0.088	0.093	0.165	0.093
	EWMA	0.079	0.189	0.214	0.195
MXX	Cornish-Fisher	0.061	0.589	0.419	0.623
	Gaussian	0.061	0.589	0.419	0.623
	EWMA	0.070	0.350	0.093	0.158
N225	Cornish-Fisher	0.079	0.189	0.723	0.397
	Gaussian	0.079	0.189	0.723	0.397
	EWMA	0.070	0.350	0.564	0.547
NZ50	Cornish-Fisher	0.079	0.189	0.723	0.397
	Gaussian	0.079	0.189	0.723	0.397
	EWMA	0.096	0.042	0.357	0.083
SSMI	Cornish-Fisher	0.088	0.093	0.165	0.093
	Gaussian	0.088	0.093	0.165	0.093
	EWMA	0.096	0.042	0.125	0.039
STI	Cornish-Fisher	0.070	0.350	0.093	0.158
	Gaussian	0.079	0.189	0.157	0.155
	EWMA	0.096	0.042	0.947	0.127
TA125	Cornish-Fisher	0.096	0.042	0.357	0.083
	Gaussian	0.105	0.018	0.491	0.047
	EWMA	0.123	0.002	0.300	0.006
TWII	Cornish-Fisher	0.070	0.350	0.272	0.353
	Gaussian	0.070	0.350	0.272	0.353
	EWMA	0.079	0.189	0.214	0.195

Note:

1. *Shaded cells indicate p-values below 0.05, meaning the model fails the test at the 5% level.*
2. *LRuc, LRind, and LRcc are p-values.*
3. *LRuc: Tests if the actual failure rate matches the expected rate.*
4. *LRind: Tests if failures occur independently over time.*
5. *LRcc: Combines LRuc and LRind to test both accuracy and independence of failures.*

Table 3. Contribution values of each component on the Cornish-Fisher VaR

Index	Comp.	Pre	Post	Diff.	t-stat.	p-value
AEX	$dVaR_\sigma$	-0.186	3.269	3.455	0.943	0.348
	$dVaR_s$	0.034	-0.185	-0.218	-0.246	0.806
	$dVaR_k$	0.009	0.056	0.047	0.485	0.629
AXJO	$dVaR_\sigma$	2.499	1.337	-1.162	-0.318	0.751
	$dVaR_s$	-0.396	-0.235	0.161	0.135	0.893
	$dVaR_k$	0.098	0.060	-0.038	-0.181	0.857
BSESN	$dVaR_\sigma$	-4.212	4.180	8.392	1.320	0.190
	$dVaR_s$	-0.564	-0.067	0.497	0.474	0.636
	$dVaR_k$	0.088	0.036	-0.053	-0.232	0.817
BVSP	$dVaR_\sigma$	0.392	0.578	0.185	0.037	0.971
	$dVaR_s$	0.058	-0.238	-0.296	-0.689	0.492
	$dVaR_k$	-0.004	0.034	0.038	0.548	0.585
FCHI	$dVaR_\sigma$	0.625	2.115	1.491	0.495	0.621
	$dVaR_s$	0.028	-0.196	-0.224	-0.338	0.736
	$dVaR_k$	0.015	0.029	0.015	0.140	0.889
FTSE	$dVaR_\sigma$	0.681	1.858	1.177	0.385	0.701
	$dVaR_s$	-0.134	-0.047	0.087	0.078	0.938
	$dVaR_k$	0.032	0.016	-0.016	-0.102	0.919
GDAXI	$dVaR_\sigma$	0.187	0.184	-0.003	-0.001	0.999
	$dVaR_s$	0.046	-0.271	-0.317	-0.406	0.686
	$dVaR_k$	0.003	0.034	0.030	0.256	0.799
GSPC	$dVaR_\sigma$	0.948	1.603	0.655	0.224	0.823
	$dVaR_s$	0.137	-0.565	-0.702	-0.839	0.404
	$dVaR_k$	0.021	0.047	0.026	0.175	0.861
HSI	$dVaR_\sigma$	4.872	2.137	-2.735	-0.543	0.588
	$dVaR_s$	-0.273	-0.087	0.185	0.341	0.734
	$dVaR_k$	0.059	0.025	-0.033	-0.342	0.733
JKSE	$dVaR_\sigma$	3.811	0.313	-3.498	-0.370	0.712
	$dVaR_s$	-0.360	-0.224	0.137	0.094	0.925
	$dVaR_k$	0.086	0.028	-0.058	-0.260	0.795
KLSE	$dVaR_\sigma$	5.316	-0.443	-5.759	-0.935	0.352
	$dVaR_s$	-1.064	-0.252	0.811	0.875	0.384
	$dVaR_k$	0.210	0.026	-0.183	-0.828	0.410
KS11	$dVaR_\sigma$	3.945	-1.752	-5.697	-1.011	0.315
	$dVaR_s$	-0.169	-0.216	-0.047	-0.056	0.955
	$dVaR_k$	0.052	0.007	-0.045	-0.271	0.787
MXX	$dVaR_\sigma$	-4.048	-0.263	3.785	0.697	0.488
	$dVaR_s$	-0.198	-0.079	0.119	0.149	0.882
	$dVaR_k$	-0.038	0.017	0.055	0.507	0.614
N225	$dVaR_\sigma$	1.192	0.198	-0.993	-0.241	0.810
	$dVaR_s$	0.141	-0.236	-0.377	-0.429	0.669
	$dVaR_k$	-0.003	0.059	0.062	0.463	0.644
NZ50	$dVaR_\sigma$	1.048	2.077	1.029	0.360	0.720
	$dVaR_s$	0.214	-0.346	-0.561	-0.513	0.609
	$dVaR_k$	0.037	0.043	0.006	0.040	0.968
SSMI	$dVaR_\sigma$	0.071	2.386	2.316	0.712	0.478
	$dVaR_s$	-0.012	-0.318	-0.306	-0.389	0.698
	$dVaR_k$	0.018	0.065	0.047	0.429	0.669
STI	$dVaR_\sigma$	3.077	0.698	-2.379	-0.504	0.615
	$dVaR_s$	-0.159	-0.088	0.071	0.052	0.958
	$dVaR_k$	0.164	0.014	-0.149	-0.902	0.369
TA125	$dVaR_\sigma$	-3.991	4.661	8.652	1.863	0.065
	$dVaR_s$	0.134	0.138	0.004	0.005	0.996
	$dVaR_k$	-0.039	0.013	0.052	0.485	0.629
TWII	$dVaR_\sigma$	1.366	3.074	1.708	0.326	0.745
	$dVaR_s$	0.217	-0.247	-0.463	-0.714	0.477
	$dVaR_k$	0.008	0.040	0.032	0.300	0.765

Note:

Pre and Post values were multiplied by 100 for readability, thus Diff values were also scaled up.

Table 4. Sensitivity of each component on the Cornish-Fisher VaR

Index	Comp.	Pre	Post	Diff.	t-stat.	p-value
AEX	β_σ	1.703	1.704	0.001	0.152	0.880
	β_s	-0.592	-0.761	-0.169	-9.176	0.000
	β_k	-0.042	-0.054	-0.012	-9.464	0.000
AXJO	β_σ	1.788	1.686	-0.102	-6.987	0.000
	β_s	-0.468	-0.742	-0.275	-18.031	0.000
	β_k	-0.031	-0.053	-0.022	-19.673	0.000
BSESN	β_σ	1.762	1.706	-0.056	-6.227	0.000
	β_s	-0.748	-1.190	-0.441	-12.307	0.000
	β_k	-0.052	-0.084	-0.033	-12.568	0.000
BVSP	β_σ	1.743	1.710	-0.033	-10.247	0.000
	β_s	-0.905	-1.107	-0.202	-12.589	0.000
	β_k	-0.063	-0.078	-0.015	-13.463	0.000
FCHI	β_σ	1.739	1.712	-0.027	-5.576	0.000
	β_s	-0.607	-0.749	-0.141	-9.138	0.000
	β_k	-0.042	-0.053	-0.011	-9.587	0.000
FTSE	β_σ	1.727	1.712	-0.014	-1.256	0.212
	β_s	-0.510	-0.672	-0.163	-11.573	0.000
	β_k	-0.035	-0.047	-0.012	-12.656	0.000
GDAXI	β_σ	1.752	1.721	-0.031	-5.600	0.000
	β_s	-0.642	-0.725	-0.083	-4.920	0.000
	β_k	-0.044	-0.051	-0.007	-5.555	0.000
GSPC	β_σ	1.741	1.715	-0.026	-2.278	0.025
	β_s	-0.440	-0.703	-0.263	-15.640	0.000
	β_k	-0.030	-0.050	-0.019	-16.465	0.000
HSI	β_σ	1.760	1.691	-0.069	-15.294	0.000
	β_s	-0.635	-1.165	-0.530	-22.846	0.000
	β_k	-0.044	-0.083	-0.039	-22.941	0.000
JKSE	β_σ	1.784	1.730	-0.054	-11.461	0.000
	β_s	-0.810	-1.204	-0.394	-11.839	0.000
	β_k	-0.055	-0.084	-0.029	-12.549	0.000
KLSE	β_σ	1.826	1.719	-0.107	-11.572	0.000
	β_s	-0.605	-0.813	-0.208	-6.392	0.000
	β_k	-0.040	-0.057	-0.017	-7.201	0.000
KS11	β_σ	1.789	1.704	-0.084	-10.375	0.000
	β_s	-0.632	-0.995	-0.363	-10.324	0.000
	β_k	-0.043	-0.071	-0.028	-10.794	0.000
MXX	β_σ	1.751	1.693	-0.058	-10.951	0.000
	β_s	-0.779	-0.868	-0.089	-3.209	0.002
	β_k	-0.054	-0.062	-0.008	-4.220	0.000
N225	β_σ	1.726	1.711	-0.015	-2.024	0.046
	β_s	-0.603	-0.854	-0.251	-14.837	0.000
	β_k	-0.042	-0.060	-0.018	-15.838	0.000
NZ50	β_σ	1.727	1.706	-0.021	-1.729	0.087
	β_s	-0.343	-0.596	-0.253	-17.145	0.000
	β_k	-0.024	-0.042	-0.019	-16.639	0.000
SSMI	β_σ	1.743	1.709	-0.034	-4.351	0.000
	β_s	-0.525	-0.705	-0.180	-10.485	0.000
	β_k	-0.036	-0.050	-0.014	-11.106	0.000
STI	β_σ	1.752	1.684	-0.067	-6.992	0.000
	β_s	-0.610	-0.909	-0.299	-12.499	0.000
	β_k	-0.042	-0.065	-0.023	-13.812	0.000
TA125	β_σ	1.769	1.723	-0.046	-6.456	0.000
	β_s	-0.639	-0.758	-0.119	-4.298	0.000
	β_k	-0.044	-0.053	-0.009	-4.599	0.000
TWII	β_σ	1.743	1.719	-0.024	-3.002	0.003
	β_s	-0.519	-0.990	-0.471	-16.231	0.000
	β_k	-0.036	-0.070	-0.034	-16.901	0.000

Table 5. Impact share of each component on the Cornish-Fisher VaR

Index	Comp.	Pre	Post	Diff.	t-stat.	p-value
AEX	IS_σ	60.71%	76.08%	15.37%	3.259	0.002
	IS_s	35.18%	22.09%	-13.09%	-2.778	0.007
	IS_k	4.11%	1.83%	-2.28%	-6.072	0.000
AXJO	IS_σ	53.39%	71.83%	18.44%	3.697	0.000
	IS_s	38.72%	25.68%	-13.05%	-2.655	0.009
	IS_k	7.89%	2.49%	-5.40%	-8.485	0.000
BSESN	IS_σ	71.35%	83.97%	12.62%	3.238	0.002
	IS_s	25.11%	14.94%	-10.17%	-2.723	0.008
	IS_k	3.54%	1.09%	-2.45%	-5.371	0.000
BVSP	IS_σ	74.44%	76.44%	2.00%	0.387	0.700
	IS_s	23.03%	22.57%	-0.46%	-0.092	0.927
	IS_k	2.53%	0.99%	-1.54%	-4.592	0.000
FCHI	IS_σ	67.56%	76.88%	9.32%	2.028	0.045
	IS_s	29.13%	21.62%	-7.51%	-1.663	0.099
	IS_k	3.31%	1.50%	-1.81%	-5.854	0.000
FTSE	IS_σ	60.28%	74.01%	13.73%	2.896	0.005
	IS_s	35.16%	24.21%	-10.95%	-2.309	0.023
	IS_k	4.57%	1.79%	-2.78%	-5.672	0.000
GDAXI	IS_σ	65.58%	64.23%	-1.35%	-0.248	0.805
	IS_s	30.96%	33.24%	2.29%	0.434	0.666
	IS_k	3.47%	2.53%	-0.94%	-2.224	0.028
GSPC	IS_σ	53.82%	75.84%	22.03%	4.665	0.000
	IS_s	40.65%	22.19%	-18.46%	-3.869	0.000
	IS_k	5.53%	1.97%	-3.57%	-5.496	0.000
HSI	IS_σ	70.53%	84.13%	13.60%	3.593	0.001
	IS_s	25.45%	14.69%	-10.76%	-2.960	0.004
	IS_k	4.02%	1.18%	-2.84%	-5.727	0.000
JKSE	IS_σ	68.62%	79.54%	10.92%	2.536	0.013
	IS_s	27.07%	18.58%	-8.50%	-2.077	0.040
	IS_k	4.31%	1.88%	-2.43%	-5.567	0.000
KLSE	IS_σ	61.75%	77.15%	15.41%	3.804	0.000
	IS_s	31.79%	20.36%	-11.43%	-3.032	0.003
	IS_k	6.46%	2.49%	-3.97%	-6.339	0.000
KS11	IS_σ	67.80%	78.81%	11.01%	2.620	0.010
	IS_s	27.85%	19.25%	-8.60%	-2.149	0.034
	IS_k	4.34%	1.94%	-2.41%	-3.719	0.000
MXX	IS_σ	73.23%	80.17%	6.94%	1.636	0.105
	IS_s	23.45%	18.39%	-5.06%	-1.254	0.213
	IS_k	3.32%	1.44%	-1.88%	-4.885	0.000
N225	IS_σ	64.56%	77.33%	12.77%	2.653	0.009
	IS_s	30.56%	20.91%	-9.65%	-2.075	0.041
	IS_k	4.88%	1.76%	-3.12%	-6.097	0.000
NZ50	IS_σ	50.22%	65.99%	15.78%	3.162	0.002
	IS_s	42.52%	30.61%	-11.91%	-2.398	0.018
	IS_k	7.27%	3.40%	-3.87%	-5.535	0.000
SSMI	IS_σ	63.03%	77.42%	14.38%	3.268	0.001
	IS_s	31.93%	20.19%	-11.74%	-2.773	0.007
	IS_k	5.03%	2.39%	-2.65%	-4.212	0.000
STI	IS_σ	66.75%	75.36%	8.61%	1.758	0.082
	IS_s	28.43%	23.28%	-5.15%	-1.072	0.286
	IS_k	4.83%	1.36%	-3.47%	-8.297	0.000
TA125	IS_σ	69.84%	75.39%	5.55%	1.265	0.209
	IS_s	26.27%	23.15%	-3.12%	-0.737	0.463
	IS_k	3.89%	1.46%	-2.43%	-5.482	0.000
TWII	IS_σ	65.47%	80.91%	15.44%	3.340	0.001
	IS_s	29.89%	17.50%	-12.40%	-2.855	0.005
	IS_k	4.64%	1.59%	-3.05%	-4.964	0.000

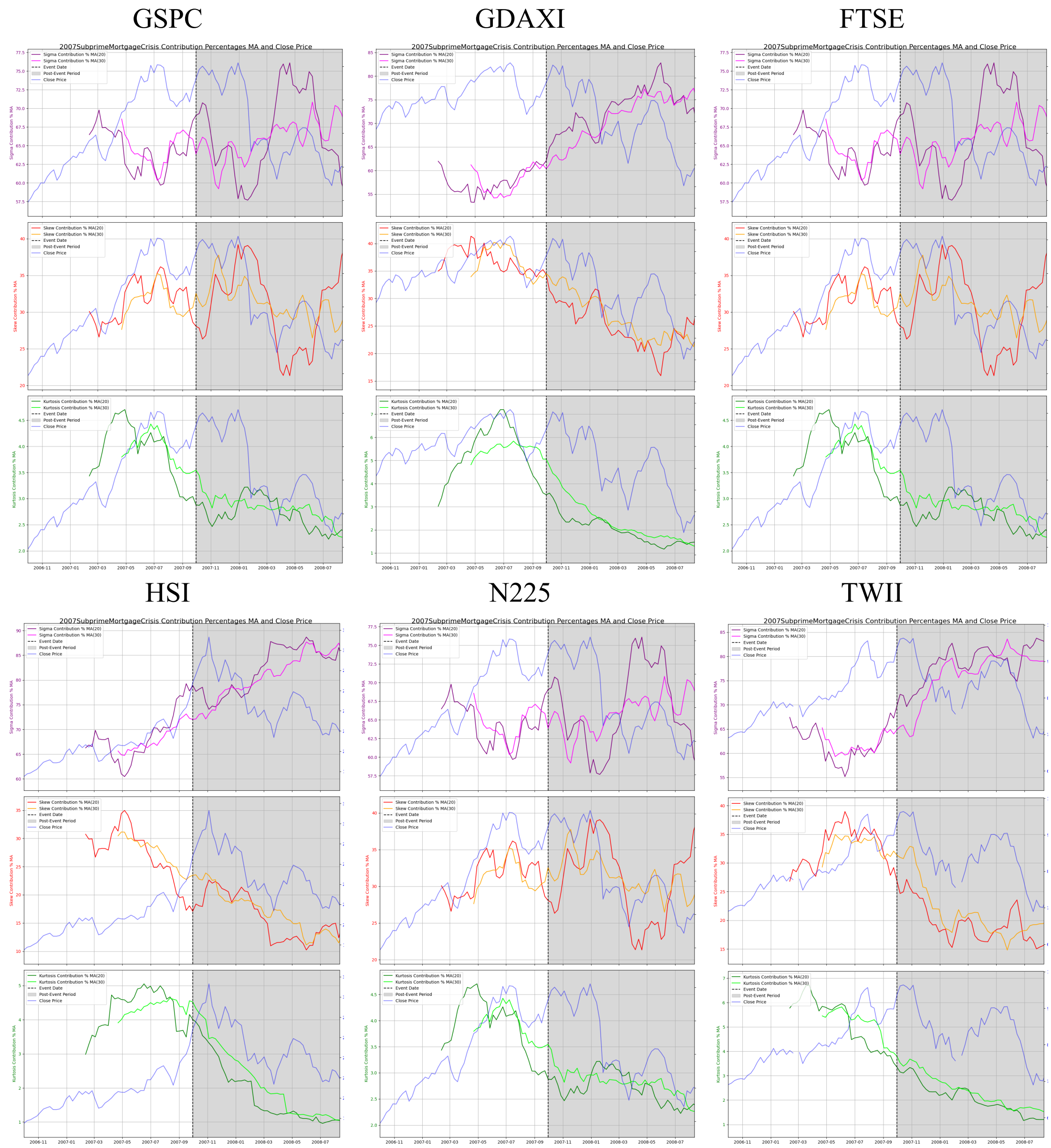

Figure 1. The contribution percentage for each component and the stock price

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