Workshop 1

Li Yueheng s2306706

2022.1.24

Problem 4(c)

Consider the Fourier series of $f(x) = (x - \frac{1}{2})^2$:

$$
\begin{aligned}
c_n =& \int_{\mathbb{T}} f(x) e^{-i2\pi n} dx \\
=& \frac{2}{(2\pi n)^2}, \forall n \neq 0 \\
c_0 =& \frac{1}{12}
\end{aligned}
$$

therefore

$$f(x) \sim \frac{1}{12} + \sum_{n \in \mathbb{Z}} c_n e^{i2\pi nx} = \frac{1}{12} + \sum_{n \geq 1} \frac{con(2\pi nx)}{(\pi n)^2}$$

Since right hand side series converges absolutely and each term is a continuous function, it converges to a continuous function. While f is continuous, the function and its Fourier series coincide.

Problem 4(d)

take $x = 1$ in the equality above to find $\sum_{n \geq 1} \frac{1}{n^2} = \frac{\pi^2}{6}$