Assignment 2

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Problem 5

$\forall z \in A_{r,R}(z_0)$, suppose $z = z_0 + le^{i\theta}$. Define $d := \min\{\frac{1}{2}(l-r), \frac{1}{2}(R-l)\}$. We claim that the open neighborhood $B_d(z)$ of z that lies within $A_{r,R}(z_0)$.

Proof of the claim: $\forall w \in B_d(z)$, by the triangle inequality we have

$$r < l - d \leq ||z_0 - z| - |z - w|| \leq |z_0 - w| \leq |z_0 - z| + |z - w| \leq l + d < R$$

Therefore every point in $B_d(z)$ lies in $A_{r,R}(z_0)$.

Therefore $A_{r,R}(z_0)$ is open.

Problem 6

Notice that $\mathbb{Q} \times i\mathbb{Q}$ is a countable dense set in $\mathbb{C}$ due to the following reasons:

1.Q is dense in $\mathbb{R}$, so every complex number can be approximated by a complex number in $\mathbb{Q} \times i\mathbb{Q}$ in the sense that both its real and imaginary parts are approximated to any accuracy.

2. the inequality $|z| \leq |Rez| + |Imz|$ shows the approximation in 1. is also an approximation in the sense of the metric topology on the complex plane.

3.$\mathbb{Q} \times i\mathbb{Q}$ is a countable set, which is due to the diagonal rule of Cantor and the countability of $\mathbb{Q}$ (which is also due to the diagonal rule)