

Assignment 1

Li Yueheng s2306706

2022.1.24

Problem 1

The characteristic function of W_t is $f(s) = e^{-s^2 t/2}$. Since $E[W_t^4] = f^{(4)}(0)$, consider the Taylor expansion $f(s) = 1 - \frac{1}{2}ts^2 + \frac{1}{8}t^2s^4 + \dots$. We find that $\frac{1}{8}t^2 = \frac{f^{(4)}(0)}{4!}$, i.e. $E[W_t^4] = f^{(4)}(0) = 3t^2$.

Since $p(x)$, the probability density function of W_t is an even function while x^5 is an odd function, $E[W_t^5] = \int_{\mathbb{R}} x^5 p(x) dx = 0$.

Problem 2

A forward contract is a contract between two parties agreeing to buy or sell an asset at a certain price by a certain date.

A call option is the right to buy an asset at a certain price within an expiration date.

The fundamental difference between the two is that forward contracts are obligatory, while a call options are not.

Problem 3

$$\begin{aligned} & \int_{\mathbb{R}} \exp\{-ax^2 + bx + c\} dx \\ &= \int_{\mathbb{R}} \exp\{-a(x - \frac{b}{2a})^2 + (c + \frac{b^2}{4a})\} dx \\ &= \sqrt{\frac{\pi}{a}} \exp\{\frac{b^2}{4a} + c\} \end{aligned}$$