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Exercise 1

a

Portfolio	t_0	t_1	t_2	t_3
asset	$50S_0$	$50S_1$	$50S_2$	$50S_3$
ZCB	0	3550	$3550e^{r(t_2-t_1)}+3150$	$3550e^{r(t_3-t_1)}+3150e^{r(t_3-t_2)}$
total	$50S_0$	$50S_1+3550$	$50S_2+3550e^{r(t_2-t_1)}+3150$	$50S_3+3550e^{r(t_3-t_1)}+3150e^{r(t_3-t_2)}$

Table 1: Portfolio A

b

Assume portfolio B consists of a long forward contract underlying 50 assets S and a ZCB of the amount T that start by t_0 and mature at t_3.

The value of portfolio B by t_3 is

$$50S_3 - K + Te^{r(t_3-t_0)}$$

It is expected that portfolio A and B replicate one another, so we let their values at maturity coincide:

$$50S_3 + 3550e^{r(t_3-t_1)} + 3150e^{r(t_3-t_2)} = 50S_3 - K + Te^{r(t_3-t_0)}$$

that is,

$$T = Ke^{r(t_0-t_3)} + 3550e^{r(t_0-t_1)} + 3150e^{r(t_0-t_2)}$$

thus the two portfolios share same value at maturity, and hence they replicate each other.

Portfolio	t_0	t_1	t_2	t_3
forward	0	/	/	$50S_3-K$
ZCB	$Ke^{r(t_0-t_3)}+3550e^{r(t_0-t_1)}+3150e^{r(t_0-t_2)}$	/	/	$K+3550e^{r(t_3-t_1)}+3150e^{r(t_3-t_2)}$
total	$Ke^{r(t_0-t_3)}+3550e^{r(t_0-t_1)}+3150e^{r(t_0-t_2)}$	/	/	$50S_3+3550e^{r(t_3-t_1)}+3150e^{r(t_3-t_2)}$

Table 2: Portfolio B

c

By the one price law, replicating portfolios have identical current value. A forward contract has 0 current value, so we have

$$T = 50S_0$$
$$K = 50S_0 e^{r(t_3-t_0)} - 3550e^{r(t_3-t_1)} - 3150e^{r(t_3-t_2)}$$

Exercise 2

a

Suppose current time $t = 0$.

Tesco may enter a long forward contract underlying 70 tonnes of flour by maturity time $T = 6$ months, at a forward price

$$K = 70S_0 e^{rT} = £70 * 1243 * e^{0.05*1/2} = £89250$$

per tonne.

By doing so however the price of flour will rise 6 months later, Tesco will be able to purchase the flour at a fixed price K.

b

Tesco will receive

$$V_{1/3} = £70 * 1294 - 70 * 1243 * e^{0.05*1/3} = £2107.7$$

c

Waitrose now pays Tesco £1400 and holds the long forward contract. Here is an arbitrage opportunity:

If Waitrose entered a short forward contract agreeing to sell 70 tonnes of flour after 2 months, this contract would have a forward price

$$K' = £70 * S_{1/3} e^{0.05*1/6} = £91338$$

By maturity of the two forward contracts, Waitrose would pay K to fulfill the long forward contract and receive K' from the short forward contract. The received flour would be given away immediately. Thus, Waitrose would receive

$$R = £1400 + K' - K = £688$$

Whatever the price of flour will be by maturity, this amount of money could surely be earned. Hence the strategy shown above is an arbitrage opportunity for Waitrose.