Assignment 3

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Question 5

Notice that

$$(1+z^2)^{1/2} = ((z+i)(z-i))^{1/2} = \exp\{\frac{1}{2}(\log(z+i)+\log(z-i))\}$$
$$=: \exp\{\frac{1}{2}(g(z)+g(z))\}$$

Since g and h are compositions of the logarithmic multifunction and translations, among them translations are obviously holomorphic on $D_1(0)$, it suffices to choose appropriate branches of logarithmic multifunction so that their brunch cuts do not intersect the images of $D_1(0)$ under translations. In doing so, by the composition law of holomorphic functions, the original multifunction would be holomorphic on $D_1(0)$.

Notice that $i + D_1(0) = D_1(i)$ and $D_1(0) - i = D_1(-i)$. For g to be holomorphic, we may define $g(z) = Log_{1,0}(z+i)$. While for h, define $h(z) = Log_{1,0}(z-i)$.

Question 6

Notice that

$$(1+z^2)^{1/2} = (z^2(1+z^{-2}))^{1/2} = (z^2)^{1/2}(1+z^{-2})^{1/2}$$

on $\mathbb{C} - \overline{D_1(0)}$.

Simply choose $(z^2)^{1/2} = z$, which is holomorphic on the whole complex plane.

As for $(1+z^{-2})^{1/2} = \exp\{\frac{1}{2}\log(1+z^{-2})\}$, choose $(1+z^{-2})^{1/2} = \exp\{\frac{1}{2}Log_{3,0}(1+z^{-2})\}$.

Since $(1+z^{-2})(\mathbb{C} - \overline{D_1(0)}) \subset D_2(0)$, while the branch cut of $Log_{3,0}$ lies outside $\underline{D_3(0)}$, they will not intersect. Thus the branch we chose is holomorphic on $\mathbb{C} - \overline{D_1(0)}$.