Workshop 1

Li Yueheng

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Problem 6

Note that the equation would lead to a contradiction

$$1 = |z| = |z + 2(1 + i)| \geq ||z| - |2(1 + i)|| = 2\sqrt{2} - 1 > 1$$

Therefore such z must not exist.

Problems 7&8

let $z = a + bi$. Note that

$$\begin{aligned} |z| &= \sqrt{a^2 + b^2} \\ |Re(z)| &= |a| \\ |Im(z)| &= |b| \end{aligned}$$

Since $2|ab| \leq a^2 + b^2$, we have

$$|z|^2 = a^2 + b^2 \leq a^2 + b^2 + 2|ab| = (|a| + |b|)^2 \leq 2(a^2 + b^2) = (\sqrt{2}|z|)^2$$

Since all the terms in the desired inequalities are nonnegative, the above inequalities give the results we need.

The first inequality becomes an equality iff $ab = 0$, i.e. z is real or imaginary. While the second becomes an equality iff $|a| = |b|$. These directly follows from the inequalities above.

Problem 9

Suppose there is such relation, consider i. Without loss of generality, assume $i > 0$, then it leads to $-1 = i^2 > 0$. So an order on $\mathbb{C}$ cannot be consistent with the order on $\mathbb{R}$.