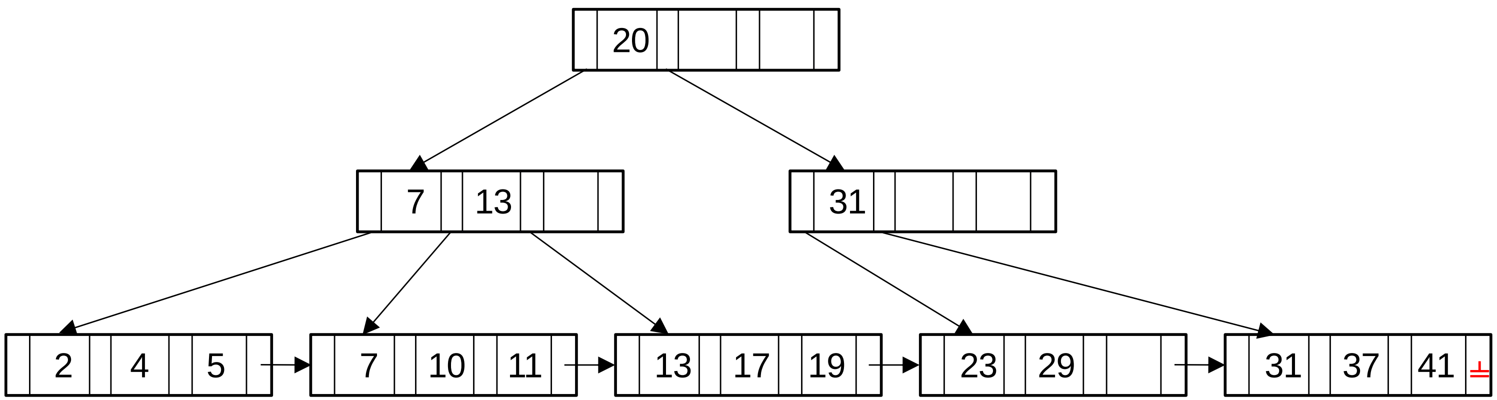
# COMP 3311
# DATABASE MANAGEMENT SYSTEMS

## LECTURE 12 EXERCISES
## INDEXING: B+-TREE

# EXERCISE 1

For the B$^+$-tree below with order 2 and fan out 4, show the tree that would result after *successively* applying the following operations in order.

i. insert 3 ii. insert 8




**Non-leaf nodes:** min $\lceil 4/2 \rceil$ = 2 pointers; min $\lceil 4/2 \rceil$-1 = 1 value

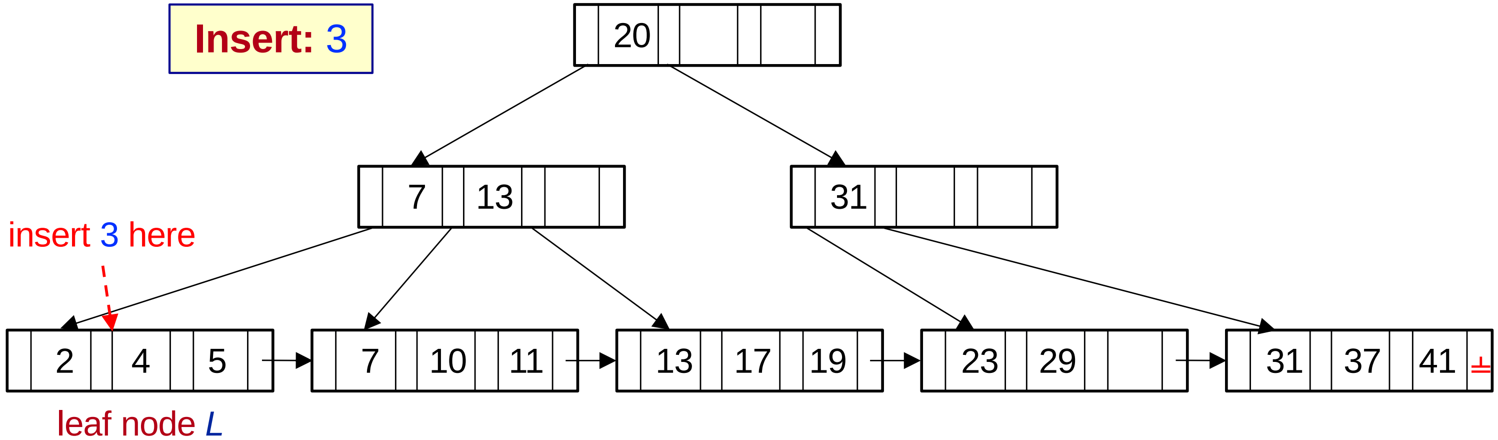
**Leaf nodes:** min $\lceil (4-1)/2 \rceil$ +1 = 3 pointers; min $\lceil (4-1)/2 \rceil$ = 2 values

# EXERCISE 1 (CONT'D)

n = 4
non-leaf nodes: 1 to 3 values
leaf nodes: 2 to 3 values

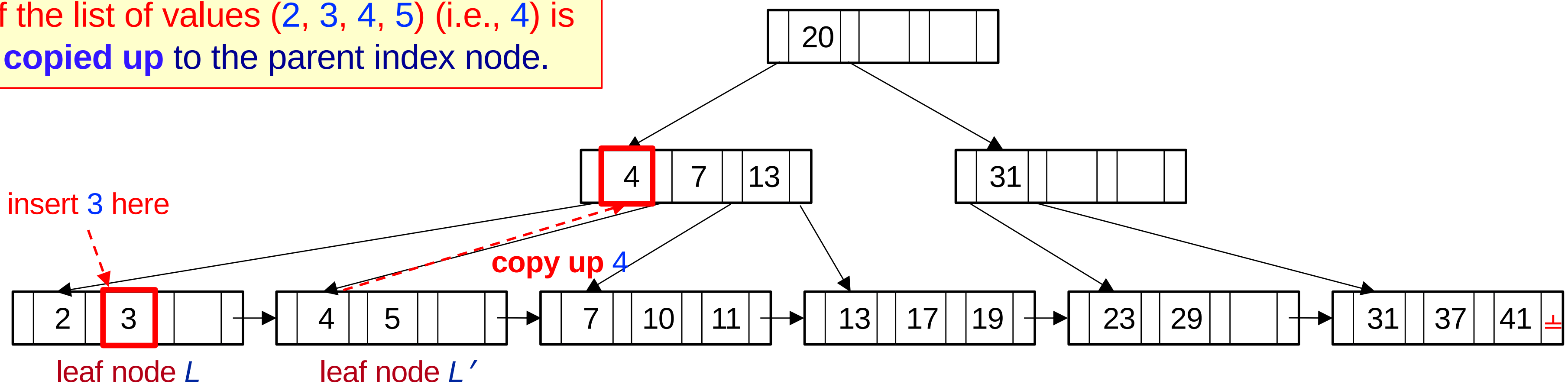
The insertion causes the first leaf node to become overfull.

**Insert:** 3




B⁺-tree *before* insertion of 3.

The leaf node is split and the search-key value of the entry in position $\lceil n/2 \rceil$+1=3 of the list of values (2, 3, 4, 5) (i.e., 4) is **copied up** to the parent index node.




B⁺-tree *after* insertion of 3.

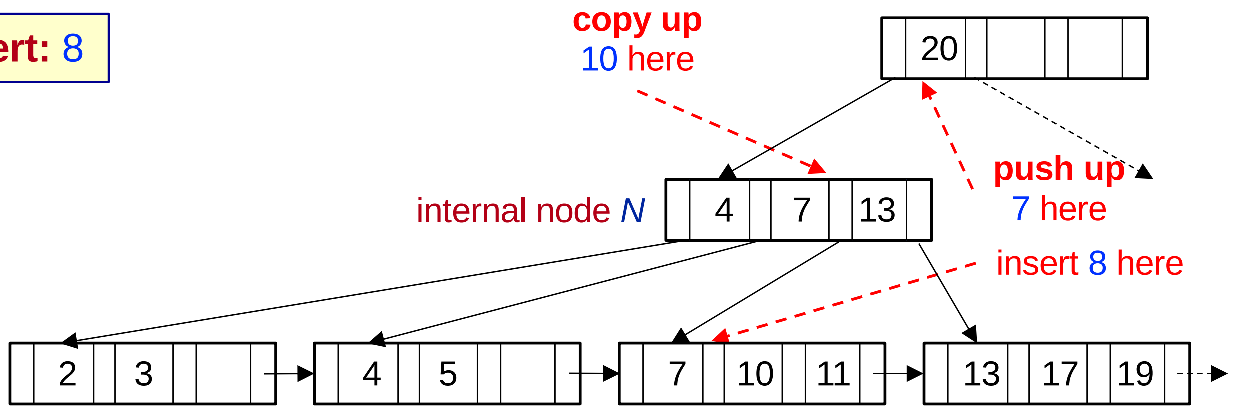
# EXERCISE 1 (cont'd)

n = 4
non-leaf nodes: 1 to 3 values
leaf nodes: 2 to 3 values

The insertion causes the third leaf node to become overfull.

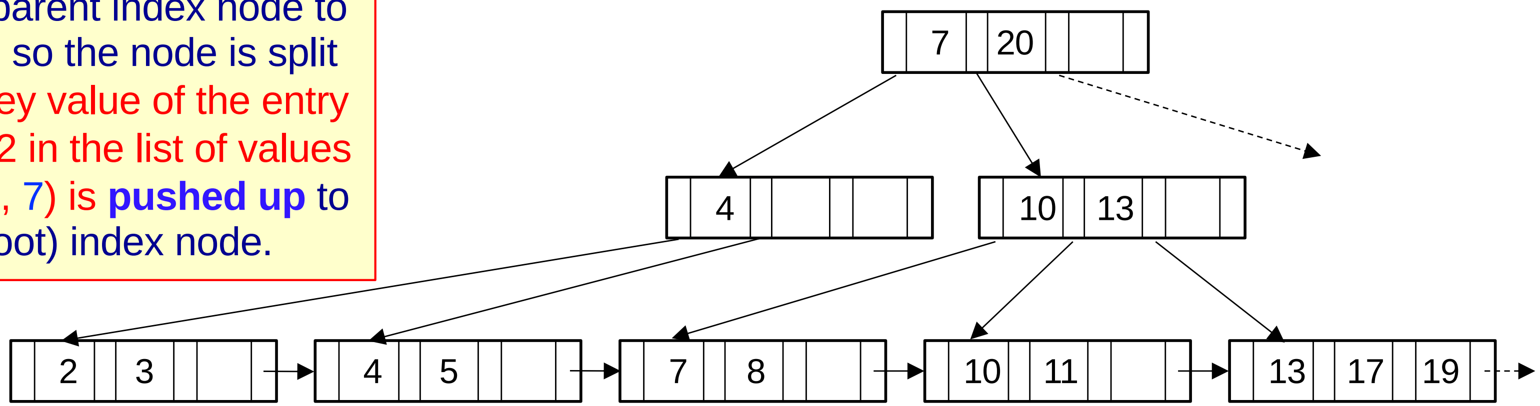
**Insert:** 8

The leaf node is split and the search-key value of the entry in position $\lceil n/2 \rceil+1=3$ of the list of values (7, 8, 10, 11) (i.e., 10) is **copied up** to the parent index node.




B$^+$-tree *before* insertion of 8.

This causes the parent index node to become overfull, so the node is split and the search-key value of the entry in position $\lceil n/2 \rceil=2$ in the list of values (4, 7, 10, 13) (i.e., 7) is **pushed up** to the parent (root) index node.

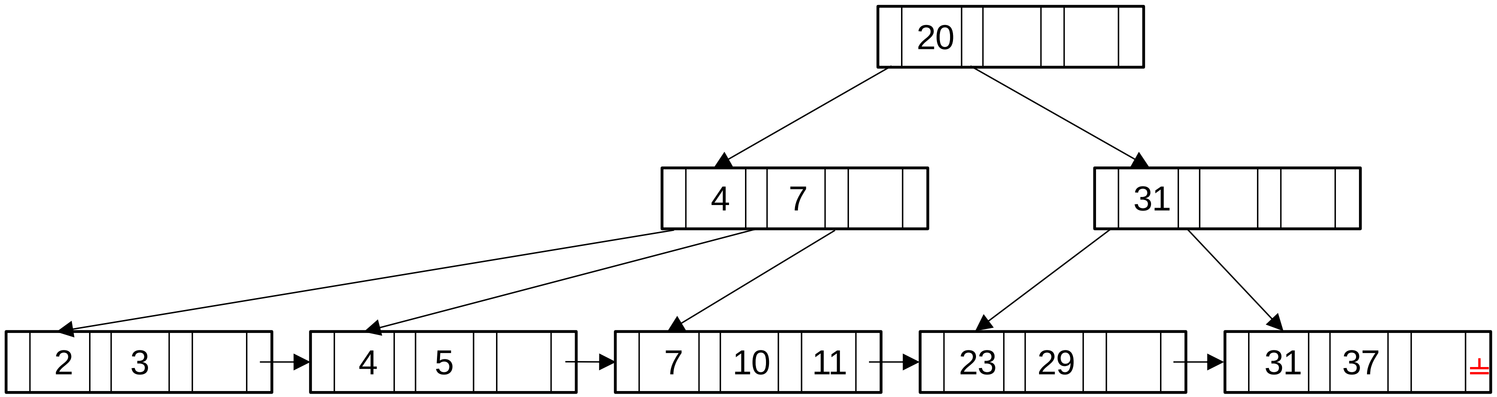



B$^+$-tree *after* insertion of 8.

# EXERCISE 2

For the B$^+$-tree below with order 2 and fan out 4, show the tree that would result after *successively* applying the following operations in order.

i. delete 5 ii. delete 3 iii. delete 7 iv. delete 11




**Non-leaf nodes:** min $\lceil 4/2 \rceil$ = 2 pointers; min $\lceil 4/2 \rceil$-1 = 1 value

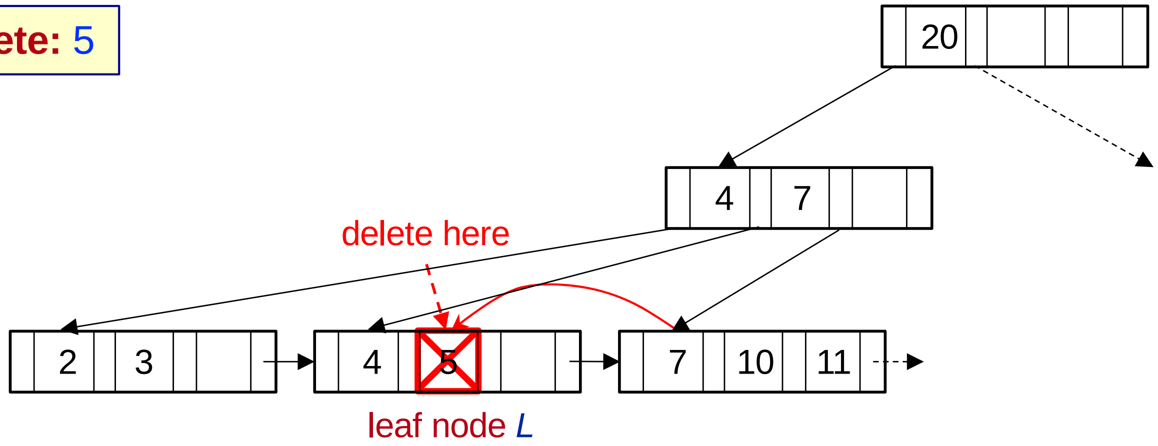
**Leaf nodes:** min $\lceil (4-1)/2 \rceil$ +1 = 3 pointers; min $\lceil (4-1)/2 \rceil$ = 2 values

# EXERCISE 2 (CONT'D)

The deletion causes the second leaf node to become underfull (less than $\lceil (n\text{-}1)/2 \rceil$ = 2 values).

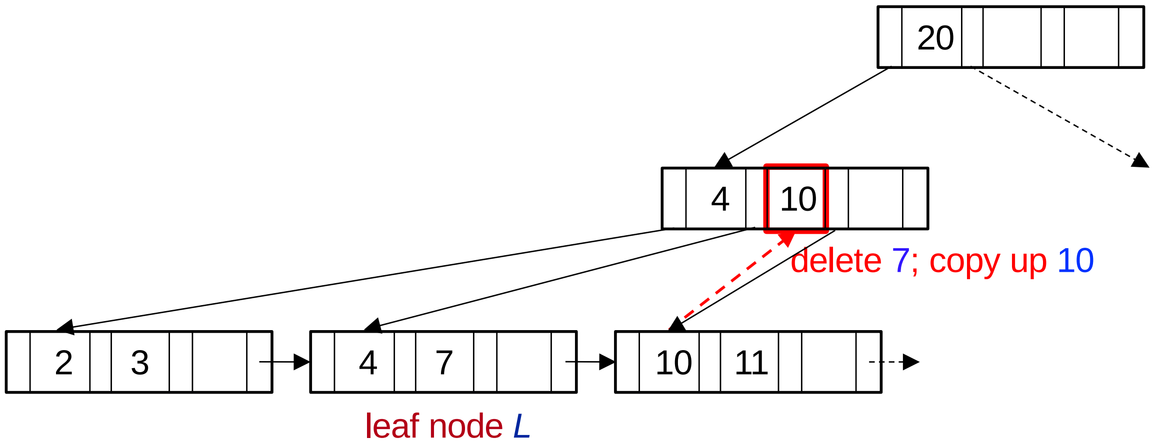
**Delete:** 5

The node can borrow a value (7) from its right sibling.




B+-tree *before* deletion of 5.

The parent index node is adjusted accordingly by deleting 7 and copying up 10.



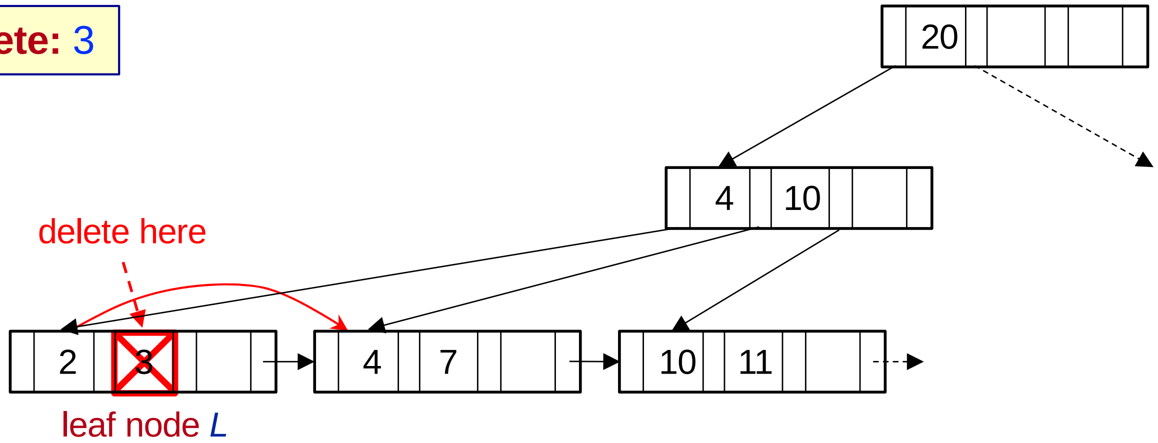

B+-tree *after* deletion of 5.

n = 4
non-leaf nodes: 1 to 3 values
leaf nodes: 2 to 3 values

# EXERCISE 2 (CONT'D)

**Delete:** 3

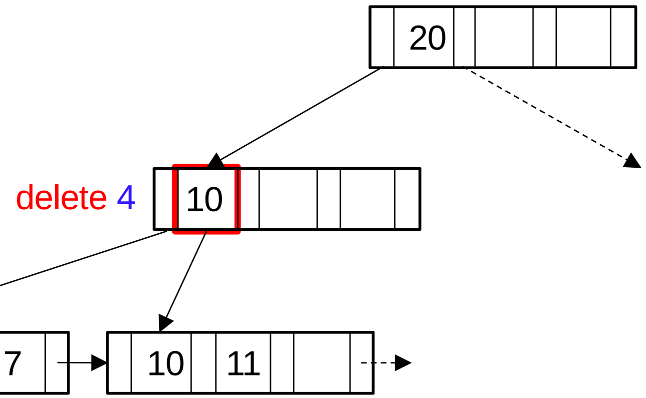
The deletion causes the first leaf node to become underfull (less than $\lceil (n\text{-}1)/2 \rceil$ = 2 values).




B$^+$-tree *before* deletion of 3.

The node cannot borrow a value from its right sibling, so it must be merged with it.

The parent index node is adjusted accordingly by deleting 4.



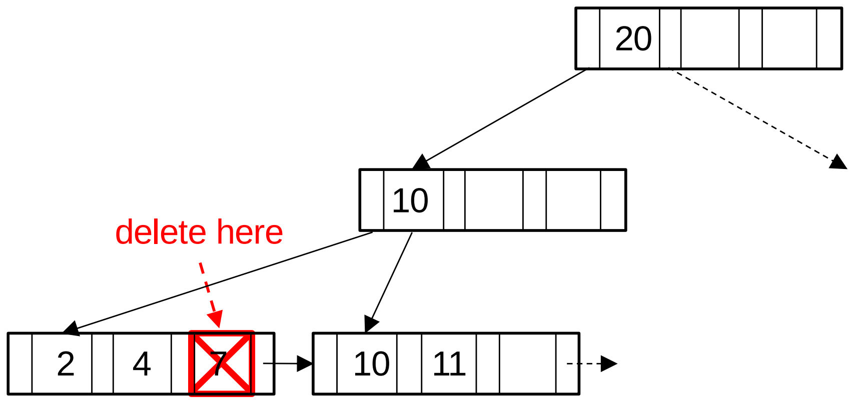

B$^+$-tree *after* deletion of 3.

# EXERCISE 2 (CONT'D)

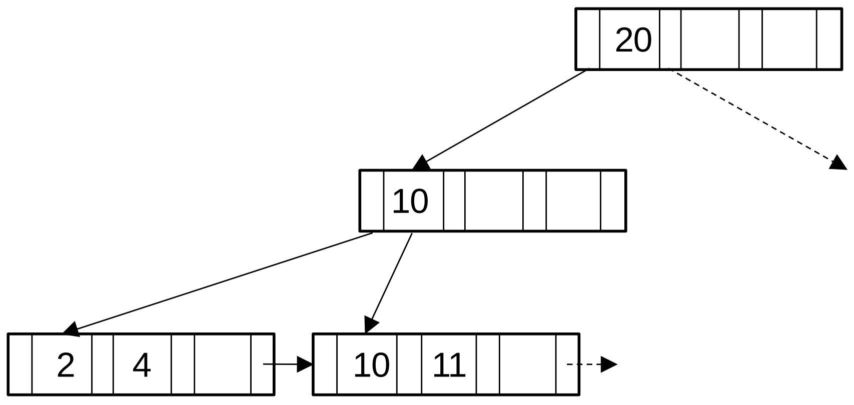
n = 4
non-leaf nodes: 1 to 3 values
leaf nodes: 2 to 3 values

The value is deleted in the first leaf node.

**Delete:** 7




B$^+$-tree *before* deletion of 7.



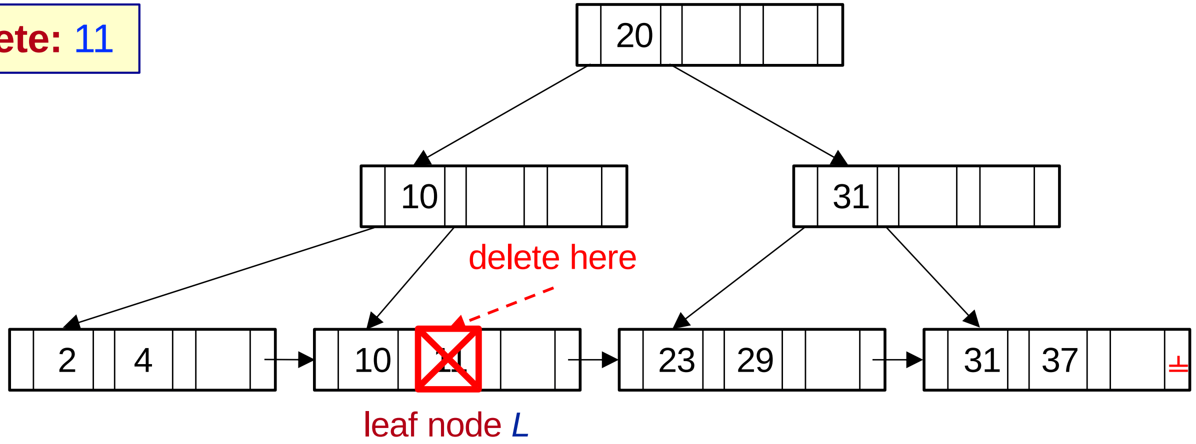

B$^+$-tree *after* deletion of 7.

n = 4
non-leaf nodes: 1 to 3 values
leaf nodes: 2 to 3 values

# EXERCISE 2 (CONT'D)

The deletion causes the second leaf node to become underfull (less than $\lceil (n\text{-}1)/2 \rceil$ = 2 values).

**Delete:** 11

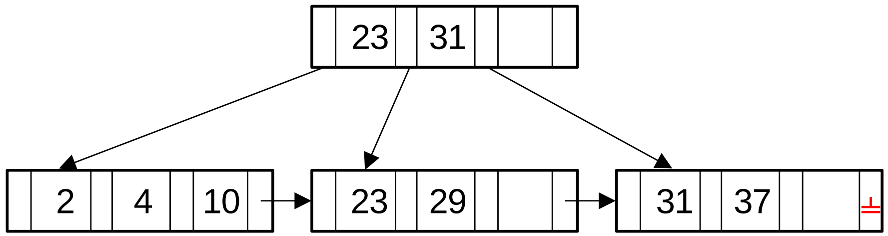



B$^+$-tree *before* deletion of 11.

The node cannot borrow a value from either of its siblings, so it must be merged (pick left sibling).

This causes the parent index node to now have only 1 pointer, but it needs 2. Therefore, it must be merged with its sibling and the index values adjusted.

This merge causes the root index node to now have only 1 pointer, so it can be deleted and the tree shrinks one level.

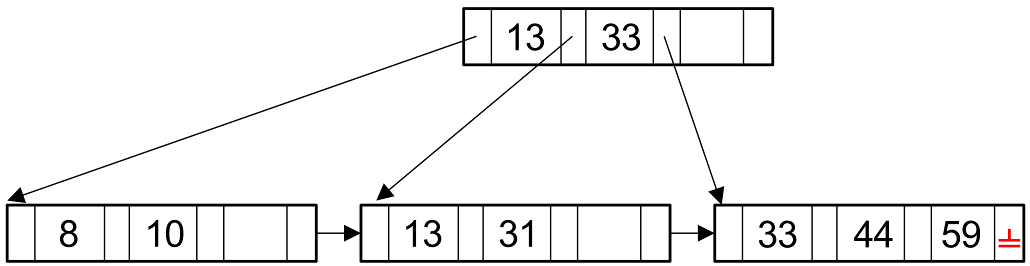



B$^+$-tree *after* deletion of 11.

# EXERCISE 3

For the B+-tree below with order 2 and fan out 4, show the tree that would result after *successively* applying the following operations in order. Add nodes to or cross out nodes in the empty B+-tree below as necessary.

i. insert 45 ii. insert 35 iii. insert 40 iv. delete 59

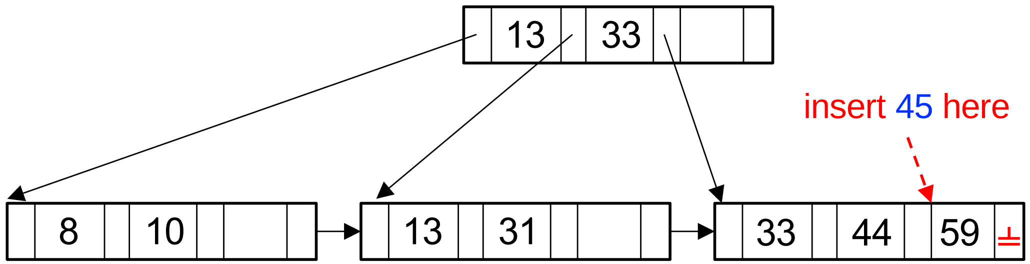



**Non-leaf nodes:** min $\lceil 4/2 \rceil$ = 2 pointers; min $\lceil 4/2 \rceil$-1 = 1 value

**Leaf nodes:** min $\lceil (4-1)/2 \rceil$ +1 = 3 pointers; min $\lceil (4-1)/2 \rceil$ = 2 values

n = 4
non-leaf nodes: 1 to 3 values
leaf nodes: 2 to 3 values

# EXERCISE 3 (CONT'D)

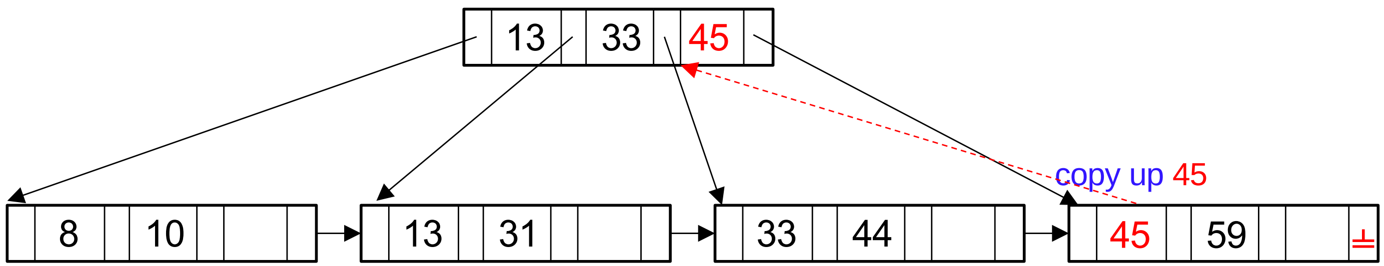



**Insert: 45**

The value is inserted in the right-most leaf node.

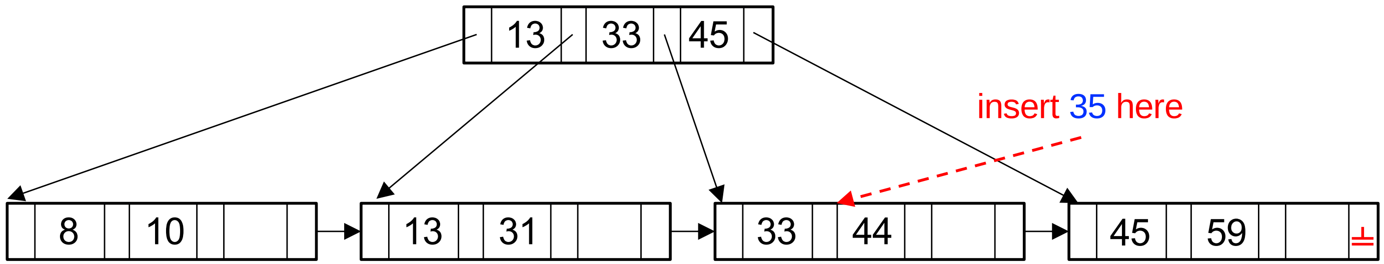
This causes the node to become overfull and split.

The search-key value at position $\lceil n/2 \rceil+1=3$ (i.e., 45) in the list of values (33, 44, 45, 59) is **copied up** to the parent index node.



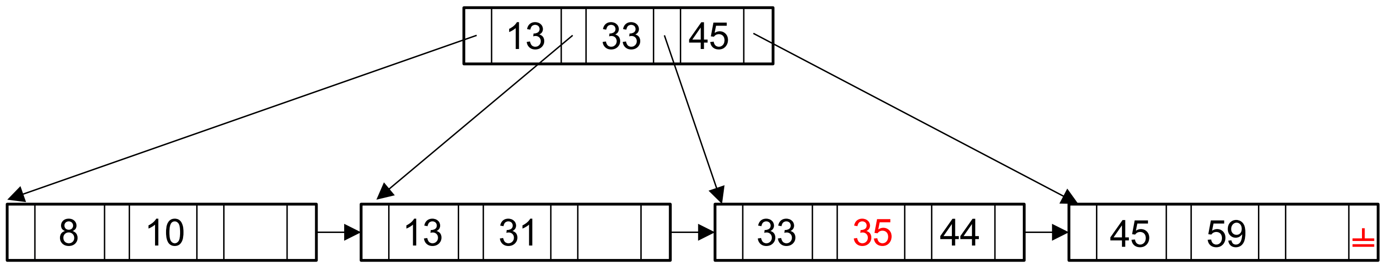
n = 4
non-leaf nodes: 1 to 3 values
leaf nodes: 2 to 3 values

# EXERCISE 3 (CONT'D)



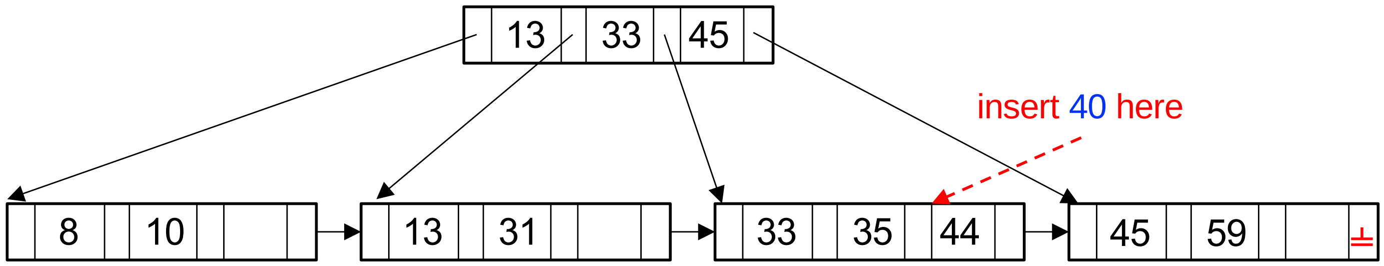

**Insert:** 35

The value is inserted in the third leaf node from the left in order.



n = 4
non-leaf nodes: 1 to 3 values
leaf nodes: 2 to 3 values

# EXERCISE 3 (CONT'D)

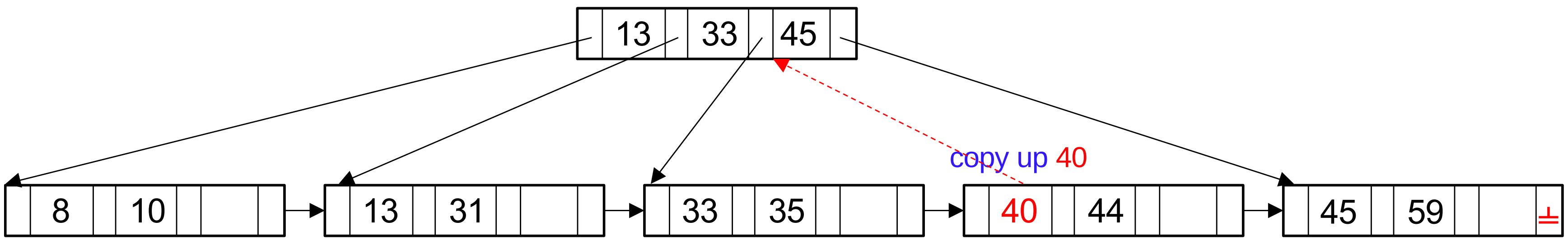



**Insert: 40**

The value is inserted in the third leaf node from the left.

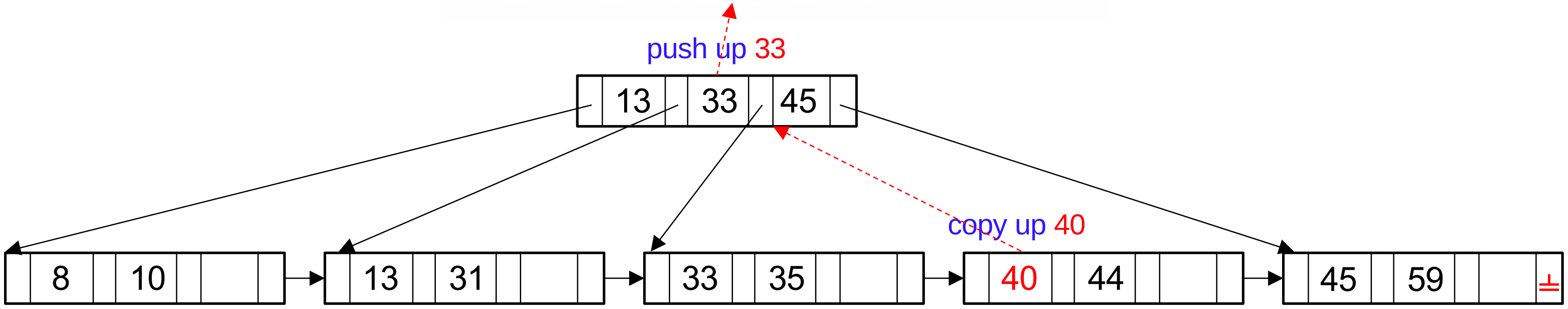
This causes the node to become overfull and split.

The search-key value at position $\lceil n/2 \rceil+1=3$ (i.e., 40) in in the list of values (33, 35, 40, 44) is **copied up** to the parent index node.



n = 4
non-leaf nodes: 1 to 3 values
leaf nodes: 2 to 3 values

# EXERCISE 3 (CONT'D)

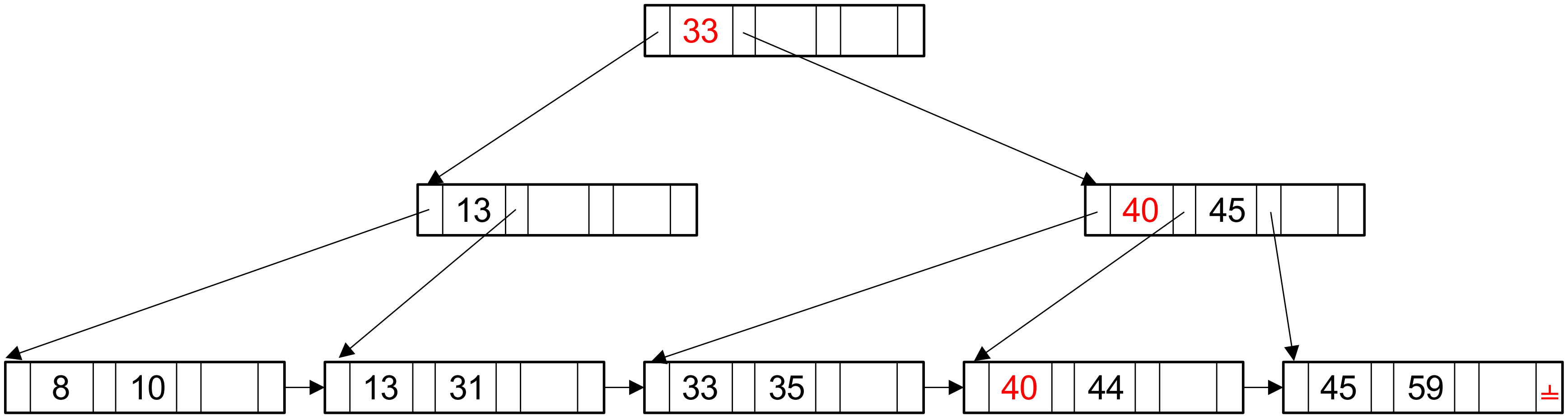



**Insert:** 40

This causes the parent node to become overfull and split.

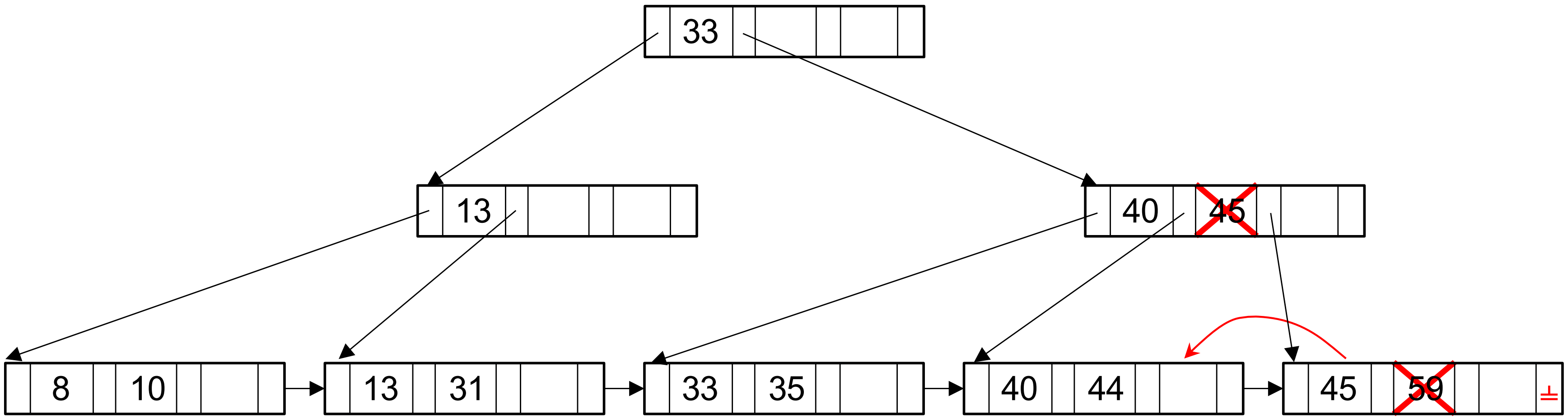
The first ⌈n/2⌉-1=1 value is placed in the existing node. The search-key value at position ⌈n/2⌉=2 (i.e., 33) in the list of values (13, 33, 40, 45) is **pushed up** into the new root node.

The remaining 2 values are placed in a new internal node.



n = 4
non-leaf nodes: 1 to 3 values
leaf nodes: 2 to 3 values

# EXERCISE 3 (CONT'D)




**Delete:** 59

Deleting 59 causes the right-most leaf node to become underfull (i.e., it has less than 2 values).

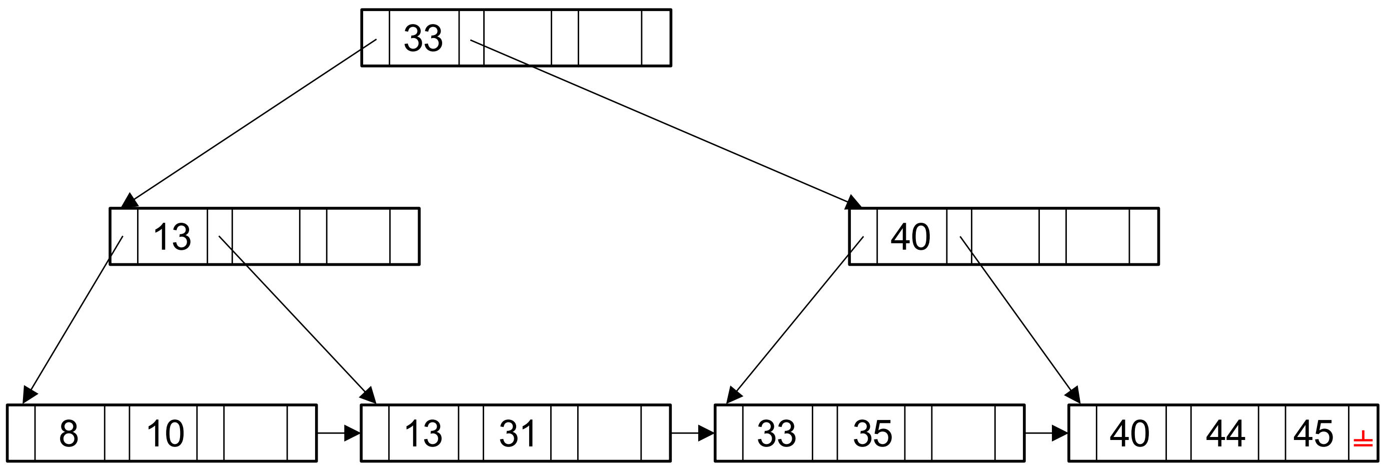
Since a value cannot be borrowed from its right sibling, the left-most leaf node is merged with its right sibling.

The parent node is adjusted by deleting 45.

n = 4
non-leaf nodes: 1 to 3 values
leaf nodes: 2 to 3 values

# EXERCISE 3 (CONT'D)

## Final $B^+$-tree



# EXERCISE 4

Construct a B$^+$-tree for the following set of search-key values using bulk loading, which creates leaf nodes from left to right. Assume each node can hold 4 pointers (i.e., 3 values) and that each leaf node is loaded with the minimum number of values.

2 3 5 7 11 17 19 23 29 31

Since n=4, the minimum number of values in a leaf node is $\lceil n/2 \rceil = 2$.

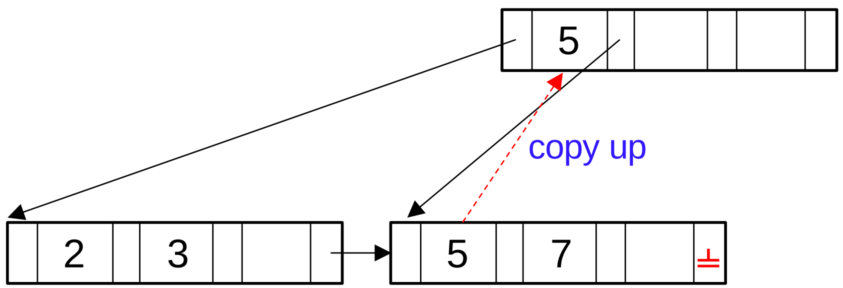
- Load the first two records into the root node.

| | | | | | | |
|---|---|---|---|---|---|---|
| | 2 | | 3 | | | ⊥ |

# EXERCISE 4 (CONT'D)

~~2 3~~ $\boxed{5 \quad 7}$ 11 17 19 23 29 31

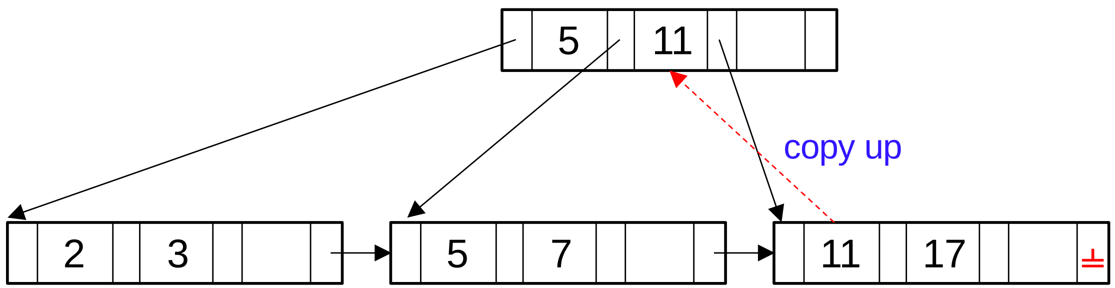
- Load the next two records into a new leaf node.
- Create a new root node.
- Copy up the minimum value in the new leaf node to the root node.



# EXERCISE 4 (CONT'D)

~~2 3 5 7~~ $\boxed{11 \quad 17}$ 19 23 29 31

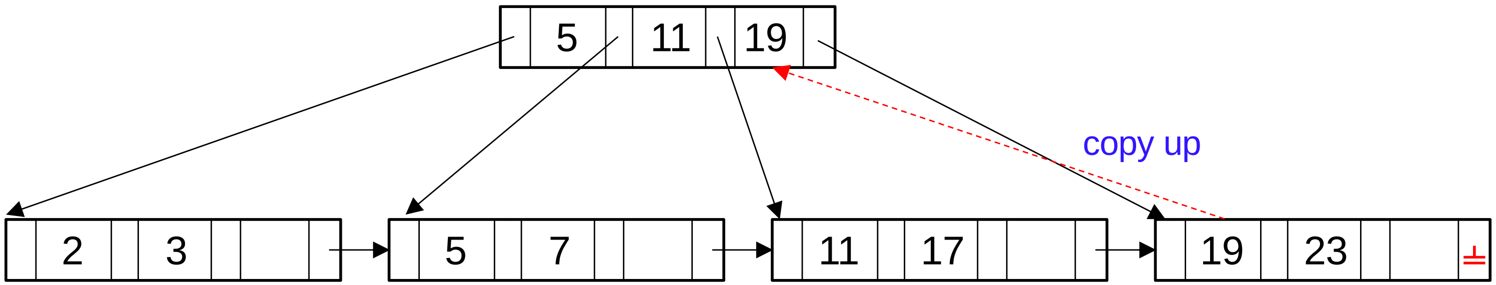
- Load the next two records into a new leaf node.
- Copy up the minimum value in the new leaf node to the parent node.



# EXERCISE 4 (CONT'D)

~~2 3 5 7 11 17~~ 19 23 29 31

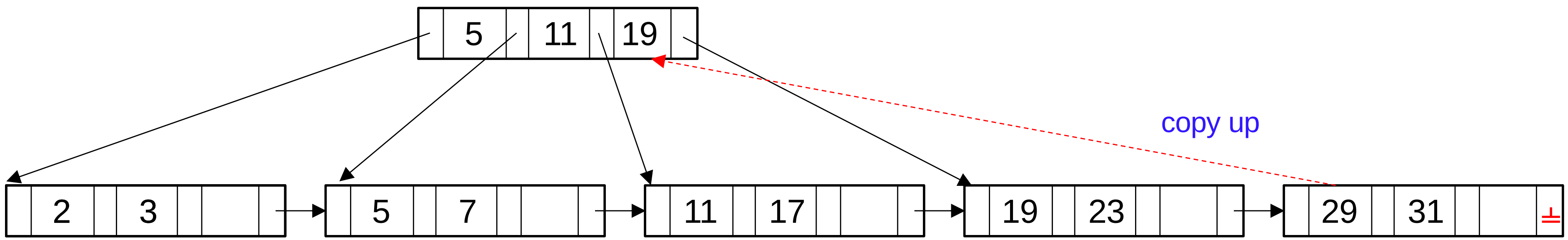
- Load the next two records into a new leaf node.
- Copy up the minimum value in the new leaf node to the parent node.



# EXERCISE 4 (CONT'D)

~~2 3 5 7 11 17 19 23~~ 29 31

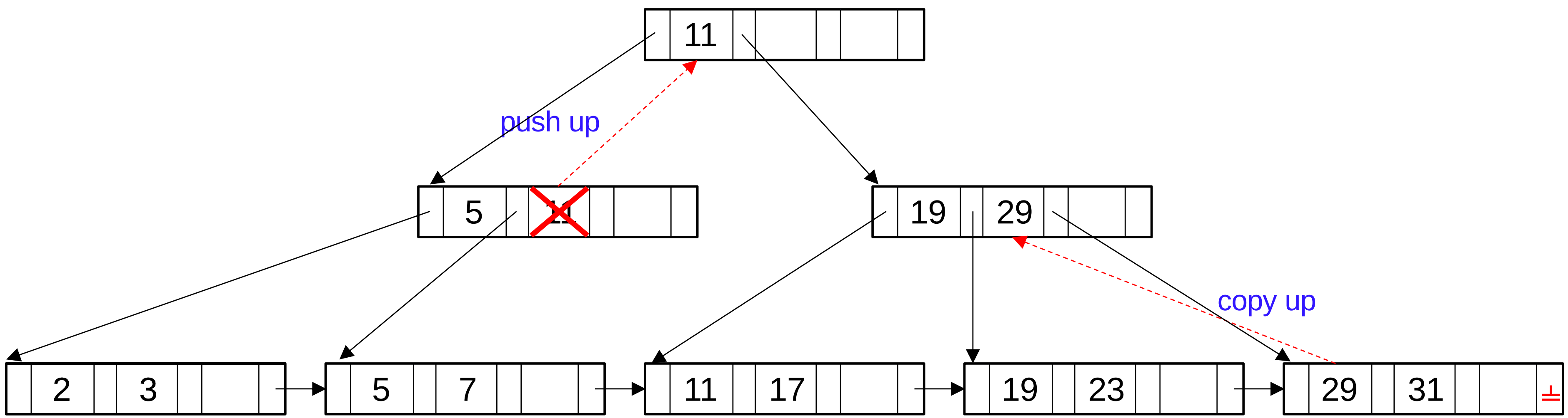
- Load the final two records into a new leaf node.
- Copy up the minimum value in the new leaf node to the parent node.
- This requires the parent (root) node to be split.



# EXERCISE 4 (CONT'D)

~~2 3 5 7 11 17 19 23~~ $\boxed{29 \quad 31}$

- The value at position $\lceil n/2 \rceil = 2$ (i.e., 11) is pushed up into a new root node.
- Pointers are adjusted.



# EXERCISE 4 (cont'd)

~~2 3 5 7 11 17 19 23 29 31~~

- Done.