COMP 2012H Honors Object-Oriented Programming and Data Structures

Topic 6: Recursion

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Recursion

- In programming, recursion means that a function calls itself!
- Although it looks strange in the beginning, solving a programming task by recursion renders the program
  - easier to write
  - easier to read (understand)
  - shorter (in codes).

Implement a Recursive Solution

1. Decompose the problem into sub-problems — which are smaller examples of the same problem — plus some additional work that "glues" the solutions of the sub-problems together.
2. The smallest sub-problem has a non-recursive solution.

Example: Factorial Function

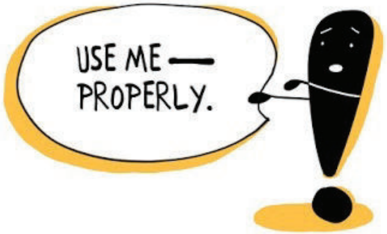
Definition of the factorial function you learn in high school:

$$n! = n \times (n-1) \times (n-2) \times \cdots 2 \times 1$$

Recursive Definition of Factorial Function

- $0! = 1$
- $n! = n \times (n-1)!$ if $n > 0$

- To find the value of $n!$, first find the value of $(n-1)!$ and then multiply the result with n.



Example: Factorial Recursive Function

```
int factorial(int n)     /* factorial.cpp */
{
    if (n < 0)           // Error checking
        return -1;
    else if (n == 0)     // Base case; ending case too!
        return 1;
    else                 // Recursive case
        return n * factorial(n-1);
}
```

Or, equivalently,

```
int factorial(int n)     /* factorial2.cpp */
{
    if (n < 0)           // Error checking
        return -1;
    if (n == 0)          // Base case; ending case too!
        return 1;
    return n * factorial(n-1); // Recursive case
}
```

How the Recursive Factorial Function Works?

factorial(3) :

- 3 < 0 false
- 3 == 0 false
- 3 * factorial(2)

  factorial(2) :

  - 2 < 0 false
  - 2 == 0 false
  - 2 * factorial(1)

    factorial(1) :

    - 1 < 0 false
    - 1 == 0 false
    - 1 * factorial(0)

      factorial(0) :

      - 0 < 0 false
      - 0 == 0 true
      - return 1

    return 1*1 = 1

  return 2*1 = 2

return 3*2 = 6

Factorial Function: Recursive vs. Non-Recursive

```
int factorial(int n)    /* factorial.cpp */
{
    if (n < 0)          // Error checking
        return -1;
    else if (n == 0)    // Base case; ending case too!
        return 1;
    else                // Recursive case
        return n * factorial(n-1);
}
```

```
int factorial(int n)    /* non-recursive-factorial.cpp */
{
    int result = 1;     // Default value for n = 0 or 1
    if (n < 0)          // Error checking
        return -1;

    for (int j = 2; j <= n; j++) // When n >= 2
            result *= j;
    return result;
}
```

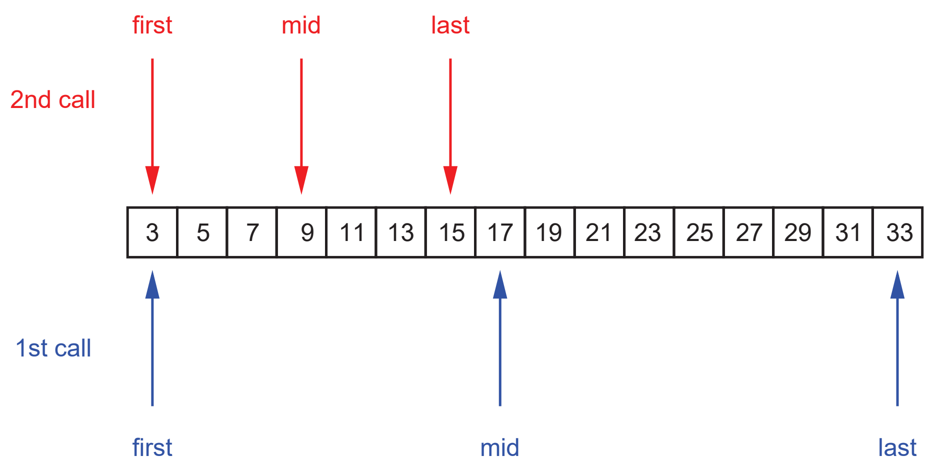

Infinite Recursion!

- We have to be careful that a recursion will eventually end up with a non-recursive base case.
- Otherwise, we will get infinite recursion!

```
int factorial(int n)
{
    // Forget the base case, which is the ending case too!
    return n * factorial(n-1);
}
```

```
int factorial(int n)
{
    // Forget checking if n < 0
    if (n == 0)
        return 1;

    // Infinite recursion for negative n
    return n * factorial(n-1);
}
```


Binary Search



binary search for the value 9

Example: Recursive Solution of Binary Search

```
const int NOT_FOUND = -1;       /* File: bsearch.cpp */
int bsearch(const int data[ ], // sorted in ascending order
            int first,          // lower bound index
            int last,           // upper bound index
            int value)          // value to search
{
    if (last < first)           // Base case #1
        return NOT_FOUND;

    int mid = (first + last)/2;

    if (data[mid] == value)
        return mid;              // Base case #2

    else if (data[mid] > value) // Search the lower half
        return bsearch(data, first, mid-1, value);

    else                         // Search the upper half
        return bsearch(data, mid+1, last, value);
}
```


Example: Non-Recursive Solution of Binary Search

```
const int NOT_FOUND = -1;       /* File: non-recursive-bsearch.cpp */
int bsearch(const int data[ ], // sorted in ascending order
            int size,           // number of data in the array
            int value)          // value to search
{
    int first = 0;
    int last = size - 1;

    while (first <= last)
    {
        int mid = (first + last)/2;
        if (data[mid] == value)
            return mid;         // Value found!
        else if (data[mid] > value)
            last = mid - 1;     // Set up for searching the lower half
        else
            first = mid + 1;    // Set up for searching the upper half
    }

    return NOT_FOUND;
}
```


Disadvantages of Recursion

- The greater programming productivity is achieved at the expenses of the more computing resources. To run recursion, it usually requires
  - more memory
  - more computational time
- The reason is that whenever a function is called, the computer
  - has to memorize its current state, and passes control from the caller to the callee.
  - sets up a new data structure (you may think of it as a scratch paper for rough work) called activation record which contains information such as
    - where the caller stops
    - what actual parameters are passed to the callee
    - create new local variables required by the callee function
    - the return value of the function at the end
  - removes the activation record of the callee when it finishes.
  - passes control back to the caller.

That's all!

Any questions?

Further Reading

Example: Fibonacci Numbers

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, ...

Fibonacci (1202) investigated how fast rabbits could breed:

- A newly-born pair of rabbits, one male, one female, are put in a field.
- Rabbits mate at the age of one month so that at the end of its 2nd month, a female can produce another pair of rabbits.
- Suppose that our rabbits never die.
- Suppose the female always produces one new pair (one male, one female) every month from the 2nd month on.
- How many pairs will there be in one year?

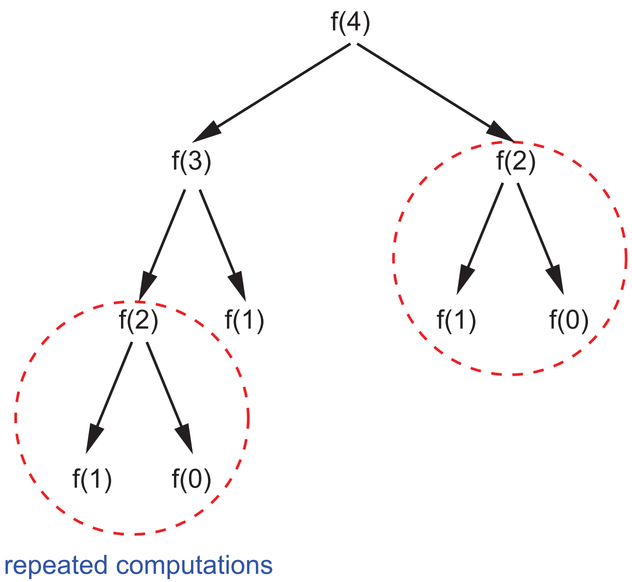
Question : What is special with the above numbers?

Answer : Except for the first 2 numbers, each number is the sum of the last 2 numbers in the sequence.

Example: Fibonacci Function as a Recursion

```
int fibonacci(int n)     /* File: fibonacci.cpp */
{
    if (n == 0)          // Base case #1
        return 0;

    if (n == 1)          // Base case #2
        return 1;

    return fibonacci(n-1) + fibonacci(n-2);
}
```

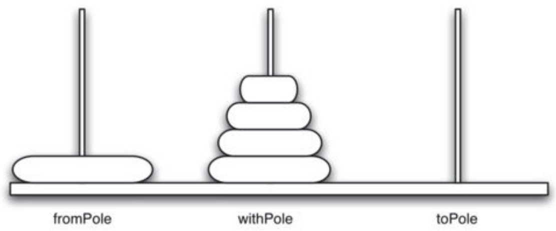

Inefficiency of Recursive Fibonacci Function



Example: Non-Recursive Fibonacci Function

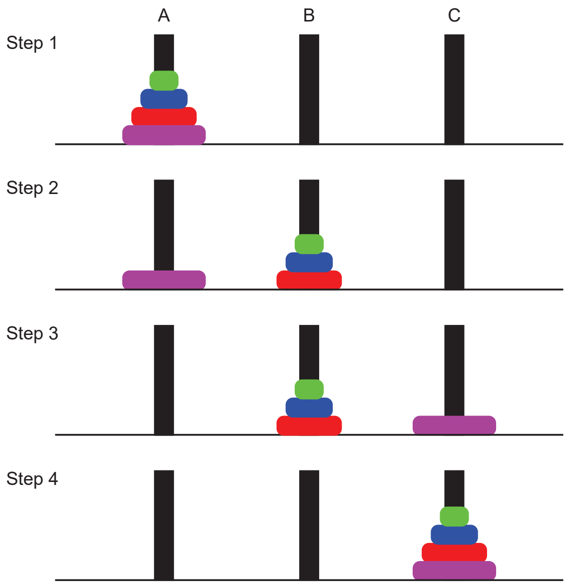
```
int fibonacci(int n)      /* non-recursive-fibonacci.cpp */
{
    int fn;               // keep track of f(n)
    int fn_1 = 1;         // keep track of f(n-1)
    int fn_2 = 0;         // keep track of f(n-2)

    if (n == 0) return 0; // Base case #1
    if (n == 1) return 1; // Base case #2

    for (int j = 2; j <= n; j++)
    {
        fn = fn_1 + fn_2; // f(n) = f(n-1) + f(n-2)

        // Prepare for the calculation of the next fibonacci number
        fn_2 = fn_1;     // f(n-2) = f(n-1)
        fn_1 = fn;       // f(n-1) = f(n)
    }

    return fn;
}
```


Example: Tower of Hanoi Game



- It consists of 3 pegs, and a stack of discs of different sizes.
- It starts with all discs stacked up on one peg with smaller discs sitting on top of bigger discs.
- The goal is to move the entire stack of discs to another peg, making use of the remaining peg.
- Rules:
  - only one disc may be moved at a time
  - no disc may be placed on top of a smaller disc

Recursive Solution of Tower of Hanoi



Example: Recursive Solution of Tower of Hanoi

```
#include <iostream>     /* File: toh.cpp */
using namespace std;

void tower_of_hanoi(int num_discs, char pegA, char pegB, char pegC)
{
    if (num_discs == 0) // Base case
        return;

    tower_of_hanoi(num_discs-1, pegA, pegC, pegB);

    cout << "move disc " << num_discs
         << " from peg " << pegA << " to peg " << pegC << endl;

    tower_of_hanoi(num_discs-1, pegB, pegA, pegC);
}
```

Example: Counting Zeros in an Integer

- Example: for the integer 120809, there are 2 zeros.
- Basic idea:
  - Break down the number into quotient and remainder.
  - Count the number of zeros in quotient and remainder.

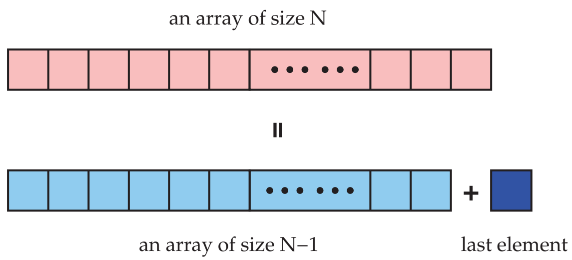
```
int num_zeros(int n)    /* File: num-zeros.cpp */
{
    if (n == 0)              // Base case #1
        return 1;

    else if (n < 10 && n > 0) // Base case #2
        return 0;

    else
        return num_zeros(n/10) + num_zeros(n%10);
}
```


Example: Factoring

- Goal: find how many times factor m appears in the integer n.
- Example: if $n = 48$ and $m = 4$, since $48 = 4 \times 4 \times 3$, the answer is 2.
- Basic idea:
  - Divide n by m until the remainder is non-zero.
  - Increment the count by 1 for every successful division.

```
int num_factors(int n, int m) /* File: factor.cpp */
{
    if (n % m != 0)     // Base case
        return 0;
    else
        return 1 + num_factors(n/m, m);
}
```


Array and Recursion



- Array is a recursive data structure in nature.
- For many problems, one may define a recursion on an array of size N which
  - will call *itself* with only $N - 1$ elements (either the top $N - 1$ or the last $N - 1$ elements),
  - with some extra codes to deal with the remaining element (last or first element).

Example: Sum Up Array Elements

```
#include <iostream> /* File: array-sum.cpp */
using namespace std;

// Summing up x[0] + x[1] + ... + x[num_elements-1]
int array_sum(const int x[], int num_elements)
{
    if (num_elements <= 0) return 0; // Base case
    return array_sum(x, num_elements-1) + x[num_elements-1];
}

int main()
{
    int a[] = { 1, 2, 3, 4, 5, 6 };
    int n;               // #elements in an array to sum
    while (cin >> n)
        cout << array_sum(a, n) << endl;
    return 0;
}
```

Question: What happens if you pass a value bigger than the size of the array size to n?