

Optimization Problems, Lecture 1, Segment 1

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Computational Models

- Using computation to help understand the world in which we live
- Experimental devices that help us to understand something that has happened or to predict the future



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- *Optimization models*
- Statistical models
- Simulation models

What Is an Optimization Model?

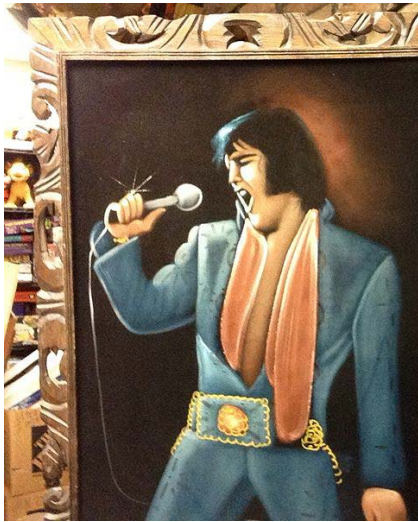
- An objective function that is to be maximized or minimized, e.g.,
 - Minimize time spent traveling from New York to Boston
- A set of constraints (possibly empty) that must be honored, e.g.,
 - Cannot spend more than \$100
 - Must be in Boston before 5:00PM



Takeaways

- Many problems of real importance can be formulated as an optimization problem
- Reducing a seemingly new problem to an instance of a well-known problem allows one to use pre-existing methods for solving them
- Solving optimization problems is computationally challenging
- A greedy algorithm is often a practical approach to finding a pretty good approximate solution to an optimization problem

Knapsack and Bin-packing Problems



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Knapsack Problem

- You have limited strength, so there is a maximum weight knapsack that you can carry
- You would like to take more stuff than you can carry
- How do you choose which stuff to take and which to leave behind?
- Two variants
 - **0/1** knapsack problem
 - **Continuous** or fractional knapsack problem

more difficult
one or not at all



versus



My Least-favorite Knapsack Problem



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1500
Calorie
Capacity

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0/1 Knapsack Problem, Formalized

- Each item is represented by a pair, $\langle \text{value}, \text{weight} \rangle$
- The knapsack can accommodate items with a total weight of no more than w
- A vector, I , of length n , represents the set of available items. Each element of the vector is an item
- A vector, V , of length n , is used to indicate whether or not items are taken. If $V[i] = 1$, item $I[i]$ is taken. If $V[i] = 0$, item $I[i]$ is not taken

0/1 Knapsack Problem, Formalized

Find a V that maximizes

$$\sum_{i=0}^{n-1} V[i] * I[i].value$$

objectives

subject to the constraint that

$$\sum_{i=0}^{n-1} V[i] * I[i].weight \leq w$$

constraints

Brute Force Algorithm

- 1. Enumerate all possible combinations of items. That is to say, generate all subsets of the set of subjects. This is called the power set. *very large*
- 2. Remove all of the combinations whose total units exceeds the allowed weight.
- 3. From the remaining combinations choose any one whose value is the largest.

*may be more than
one optimal solutions
≥*

Often Not Practical

- How big is power set?
- Recall
 - A vector, V , of length n , is used to indicate whether or not items are taken. If $V[i] = 1$, item $I[i]$ is taken. If $V[i] = 0$, item $I[i]$ is not taken
- How many possible different values can V have? 2^n
 - As many different binary numbers as can be represented in n bits
- For example, if there are 100 items to choose from, the power set is of size
126,765,060,022,822,940,149,670,320,5376

Are We Just Being Stupid?

- Alas, no
- 0/1 knapsack problem is inherently exponential
- But don't despair



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