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Assignment - 1
Phy 108
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Question-1

a) Given

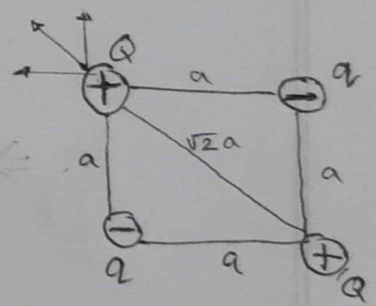
Charge $q_1 = q_4 = Q$ & $q_2 = q_3 = q$

If ~~the~~ Q and q are of same sign which means they are both positively or ~~and~~ negatively signed then we can have net force of zero.

So, we can assume that $Q = +$ charge and $q = -$ charge.

So now the net force of $q_1$ and $q_4$ must be 0.

$$\Sigma F_x = F_{21} - F_{41} \cdot \cos 45^\circ = 0$$




$$= \frac{1}{4\pi\epsilon_0}\frac{qQ}{a^2} - \frac{1}{4\pi\epsilon_0}\frac{QQ}{(\sqrt{2}a)^2}\cos(45)$$

$$= \frac{1}{\cancel{4\pi}\epsilon_0}\frac{q\cancel{Q}}{\cancel{a^2}} = \frac{1}{\cancel{4\pi}\epsilon_0}\frac{\cancel{Q}Q}{2\cancel{a^2}}\cos 45$$

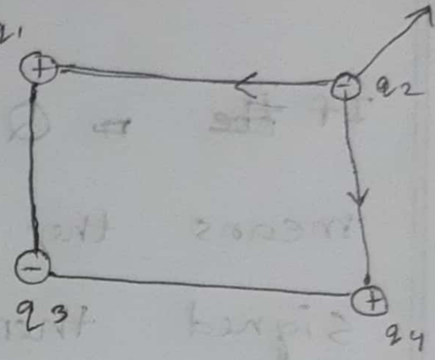
$$\Rightarrow q = \frac{Q}{2} \cdot \frac{\sqrt{2}}{2}$$

$$\Rightarrow \frac{+Q}{-q} = -\frac{4}{\sqrt{2}}$$

$$= \frac{Q}{q} = -2\sqrt{2}$$

(b) All four particles can have net force as 0 only if charge q2 and q3 have same ratio as charge q1 and charge q4.

In charge q2 the net force on y-direction




$$\Sigma F_{2y} = 0$$

$$\Sigma F_{2y} = \frac{1}{4\pi\epsilon_0} \frac{q\,q}{(\sqrt{2}a)^2} \sin 45^\circ - \frac{1}{4\pi\epsilon_0} \frac{qQ}{a^2} = 0$$

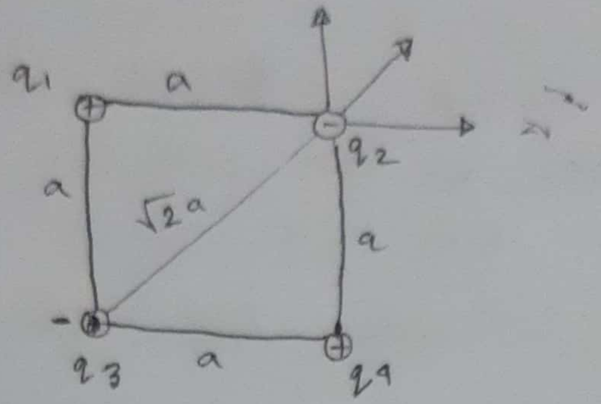
$$\Rightarrow \cancel{\frac{1}{4\pi\epsilon_0}} \frac{q\cancel{q}}{2\cancel{a^2}} \cdot \frac{\sqrt{2}}{2} = \cancel{\frac{1}{4\pi\epsilon_0}} \frac{\cancel{q}Q}{\cancel{a^2}}$$

$$\Rightarrow \frac{q}{2} \cdot \frac{\sqrt{2}}{2} = Q$$

$$\Rightarrow \frac{Q}{q} = \frac{\sqrt{2}}{4}$$

So, the ratios are not equal. So all four charges can't be zero at the same time.

(c) The net electro static force on $q_2$,




$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{a^2}[\cos 0^\circ \hat{i} + \sin 0^\circ \hat{j}]$$

$$= \cancel{\frac{1}{4\pi\epsilon_0}\frac{Q}{}[\hat{i}]} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{a^2}[\cos 0^\circ \hat{i}]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Qq}{a^2}[\hat{i}]$$

$$F_{32} = \frac{1}{4\pi\epsilon_0}\frac{qq}{(\sqrt{2}a)^2}[\cos 45^\circ[\hat{i}] + \sin 45^\circ[\hat{j}]]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{qq}{2a^2}\left[\frac{\sqrt{2}}{2}\hat{i} + \frac{\sqrt{2}}{2}\hat{j}\right]$$

$$F_{42} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{a^2}[\cos 90^\circ \hat{i} + \sin 90^\circ \hat{j}]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Qq}{a^2}[\hat{j}]$$

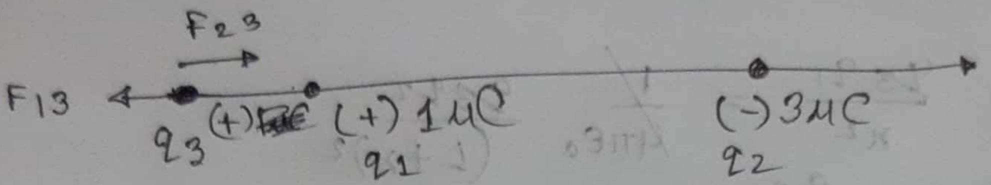
$$F_{net} = \vec{F}_{12} + \vec{F}_{32} + \vec{F}_{42}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Qq}{a^2}[\hat{i}] + \frac{1}{4\pi\epsilon_0}\frac{qq}{2a^2}\left[\frac{\sqrt{2}}{2}\hat{i} + \frac{\sqrt{2}}{2}\hat{j}\right]$$

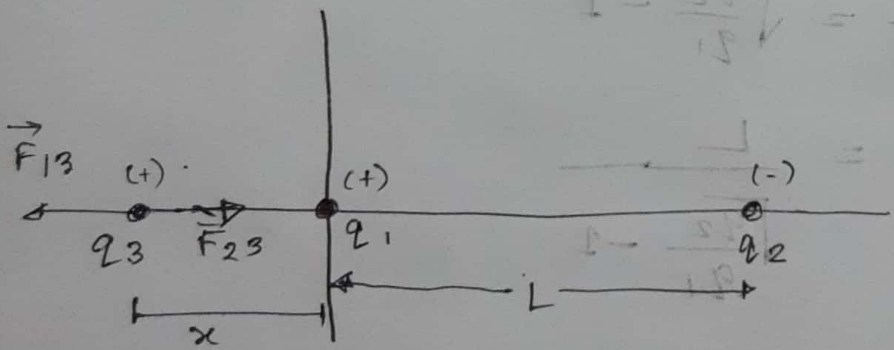
$$+ \frac{1}{4\pi\epsilon_0}\frac{Qq}{a^2}[\hat{j}]$$

Question - 2 (a)

Now, we place $q_3$ on the left.




The direction of forces now are opposite. Now in this case. Now, here considering magnitude charge is greater in $q_2$. So it makes $\vec{F}_{23}$ larger. But $q_3$ is closer to $q_1$ compared to $q_2$. So distance effect makes $\vec{F}_{13}$ larger. So, at some distance on the left ~~the~~ net force can be zero.



For $F_{net} = 0$ we need to have ~~E31~~ $F_{13} = F_{23}$ which is same magnitude.

Now,

$$\vec{F}_{13} = \vec{F}_{23}$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0} \frac{q_3 q_1}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{q_3 q_2}{(L+x)^2}$$

$$\Rightarrow \frac{q_1}{x^2} = \frac{q_2}{(L+x)^2}$$

$$\Rightarrow \frac{(L+x)^2}{x^2} = \frac{q_2}{q_1}$$

$$\Rightarrow \left(\frac{L+x}{x}\right)^2 = \frac{q_2}{q_1}$$

$$\Rightarrow \frac{L}{x} + 1 = \sqrt{\frac{q_2}{q_1}}$$

$$\Rightarrow \frac{L}{x} = \sqrt{\frac{q_2}{q_1}} - 1$$

$$\Rightarrow x = \frac{L}{\sqrt{\frac{q_2}{q_1}} - 1}$$

$$\Rightarrow x = \frac{10\,cm}{\sqrt{\frac{3\times10^{-6}}{1\times10^{-6}}}}$$

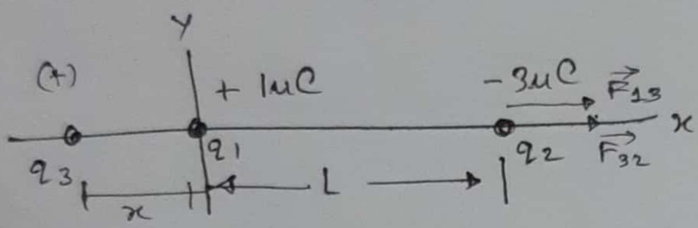
$$\Rightarrow x = 13.7\ cm$$

But we placed $q_3$ on the left side on the axis beside $q_1$.

So, $x = -13.7$ cm and $y = 0$.

$\therefore (x, y) = (-13.7\text{ cm}, 0)$

(b) The net force on $q_2$ due to $q_1$ and $q_3$




$$\vec{F}_{32} = \frac{1}{4\pi\epsilon_0} \frac{q_3 q_2}{r^2} \hat{r}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{|q_3|\,|-3\times10^{-6}\text{ C}|}{(23.7)^2} [\cos 0° \hat{i} + \sin 0° \hat{j}]$$

$$= 9\times10^9 \frac{q_3 \times 3\times10^{-6}}{562\text{ cm}^2} [\hat{i}]$$

$$= \frac{2.7\times10^{-6}\, q_3}{5.6\times10^{-2}\text{ m}^2} [\hat{i}] \Rightarrow q_3 \cdot 4.8\times10^{-5}\text{ N/m}^2 [\hat{i}]$$

$$\vec{F}_{13} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

$$= 9\times10^9 \frac{|1\times10^{-6}|\,|-3\times10^{-6}|\,\mu C}{(10\text{ cm})^2} [\cos 0° \hat{i} + \sin 0° \hat{j}]$$

$$= 2.7\text{ N/m}^2$$

$\therefore \vec{F}_{Net} = \vec{F}_{32} + \vec{F}_{13}$

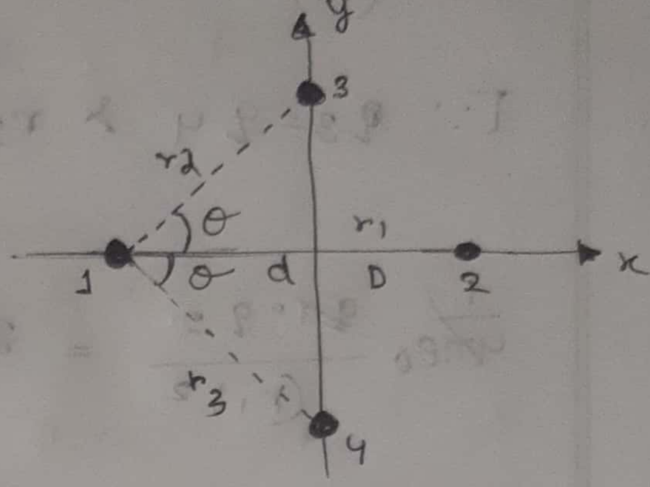
$= 23.4.8 \times 10^{-5}\ N/m^2 [\hat{i}] + 2.7\ N/m^2 [\hat{i}]$

$= 23.2.7\ N/m^2 [\hat{i}]$

Question - 3 (a)

~~$r_{12}$ — $\delta$~~

Given,




$d = 2\,cm$

$\theta = 30°$

$q_2 = +8 \times 10^{-19}\,C$

$q_3 = q_4 = -1.60 \times 10^{-19}\,C$

Now,

$r_1$ to $r_2$ distance $\Rightarrow r_1 = d + D$

Also give that the net force on charge $q_1$ is zero

$\therefore \vec{F}_{net} = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41} = 0$

So, $\vec{\Sigma F_x}$ is also equal to zero.

$F_{21} = F_{31} \cos\theta + F_{41} \cos\theta$

$$\Rightarrow \frac{1}{4\pi\epsilon_0} \frac{q_1 \cdot q_2}{(r_1)^2} = \frac{1}{4\pi\epsilon_0} \frac{q_1 \cdot q_3}{(r_2)^2} \cos\theta + \frac{1}{4\pi\epsilon_0} \frac{q_1 q_4}{(r_3)^2} \cos\theta$$

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$$\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{(r_1)^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1 . q_3}{(r_2)^2}\cos\theta + \frac{1}{4\pi\epsilon_0}\frac{q_1 \cdot q_3}{(r_2)^2}\cos\theta$$

$[\because q_3 = q_4 \ \& \ r_2 = r_3]$

$$\frac{1}{\cancel{4\pi\epsilon_0}}\frac{\cancel{q_1}\cdot q_2}{(r_1)^2} = 2\frac{1}{\cancel{4\pi\epsilon_0}}\frac{\cancel{q_1}\cdot q_3}{(r_2)^2}\cos\theta$$

$$\Rightarrow \frac{q_2}{(r_1)^2} = 2\frac{q_3}{(r_2)^2}\cos\theta$$

$$\frac{q_2}{(d+D)^2} = 2\frac{q_3}{(r_2)^2}\cos\theta$$

Now,

$$r = \frac{d}{\cos\theta}$$

Since $\theta = 30^0$

$$\therefore r_2 = \frac{2}{\frac{\sqrt{3}}{2}}$$

$$r_2 = \frac{4}{\sqrt{3}}$$

Now,

$$\frac{q_2}{(d+D)^2} = 2\frac{q_3}{(r_2)^2}\cos\theta$$

$$\frac{8\times10^{-19}}{(2+D)^2} = 2\cdot\frac{1.6\times10^{-19}}{\left(\frac{4}{\sqrt{3}}\right)^2}\cdot\frac{\sqrt{3}}{2}$$

$$\frac{8\times10^{-19}}{(2+D)^2} = \frac{1.6\times10^{-19}}{\left(\frac{16}{3}\right)}\cdot\sqrt{3}$$

$$\frac{8\times10^{-19}}{(2+D)^2} = \frac{1.6\times10^{-19}}{16}\cdot3\sqrt{3}$$

$$\frac{8\times10^{-19}}{1.6\times10^{-19}}\cdot\frac{16}{3\sqrt{3}} = (2+D)^2$$

$$5\cdot\frac{16}{3\sqrt{3}} = (2+D)^2$$

$$D = \sqrt{5\cdot\frac{16}{3\sqrt{3}}} - 2$$

$$= 1.92\,\text{cm}$$

(b) Here $F_{21} = F_{31} \cos\theta + F_{41} \cos\theta$

$$F_{21} = 2F_{31} \cos\theta$$

So, as the charges $q_3$ and $q_4$ travels toward the origin, $\theta$ becomes 0 and $\cos\theta$ becomes 1

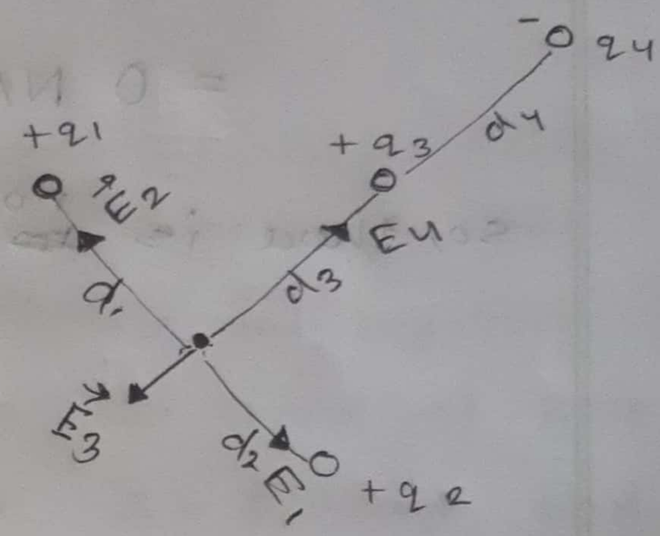
As $\theta$ decreases, the sum of forces $F_{31}\cos\theta$ and $F_{41}\cos\theta$ increases. So, $F_{21}$ also increases to maintain equilibrium.

We know, $F_{21} \propto \frac{1}{(d+D)^2}$. So, if we can increase $F_{21}$ by decreasing distance between $q_1$ and $q_2$ charges. As we move charges $q_3$ and $q_4$ closer to the origin, the value of D is less than 1.92 cm to maintain the equilibrium.

Question - 4

We know positive charge has field radially Outward and negative charge is radially inward.

Now, Given




$q_1 = q_2 = +5e$

$q_3 = +3e$

$q_4 = -12e$

$d = 5\mu m$

The distances are same so, $d_1 = d_2 = d_3 = d_4 = d$

$\vec{E}_1$ and $\vec{E}_2$ are opposite direction and same magnitude so $\vec{E}_1$ and $\vec{E}_2$ cancel out each other.

Now,

$$E = E_4 - E_3$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{|q_4|}{(2d)^2} - \frac{1}{4\pi\epsilon_0}\frac{|q_3|}{(d)^2}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q_4}{4d^2} - \frac{1}{4\pi\epsilon_0}\frac{q_3}{d^2}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{12e}{4d^2} - \frac{1}{4\pi\epsilon_0}\frac{3e}{d^2}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{3e}{d^2} - \frac{1}{4\pi\epsilon_0}\frac{3e}{d^2}$$

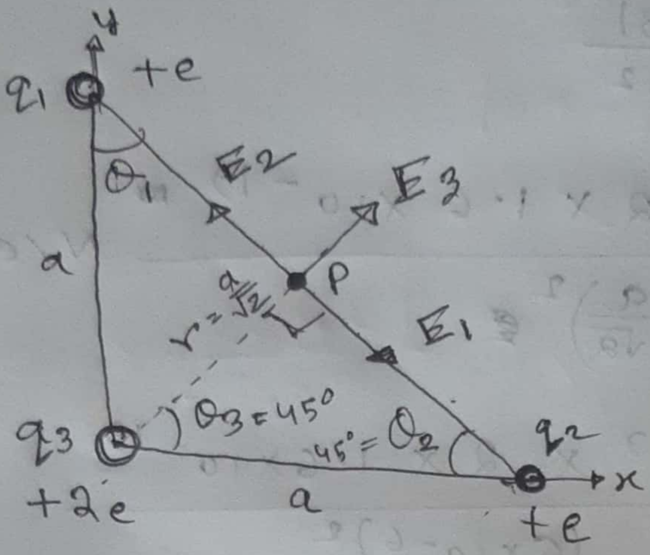
$$E = \frac{1}{4\pi\epsilon_0}\left(\frac{3e}{d^2} - \frac{3e}{d^2}\right)$$

$$= 9\times10^9 \times 0 \ N/C$$

$$= 0 \ N/C$$

so, there is no field at point P.

Question - 5 (a)




Given, $q_1 = q_2 = +e$

$q_3 = +2e$

$a = 6 \mu m = 6 \times 10^{-6}$ m

Now, $r \Rightarrow \sin 45^0 = \frac{r}{a}$

$r = a \cdot \sin 45^0$

$r = \frac{a}{\sqrt{2}}$

Now, since the charges of $q_1$ and $q_2$ are same and also the direction of $E_2$ and $E_1$ are opposite So $E_1$ and $E_2$ cancel out each other.

So the net electric field will be only for $q_3$.

$\therefore E = E_3$

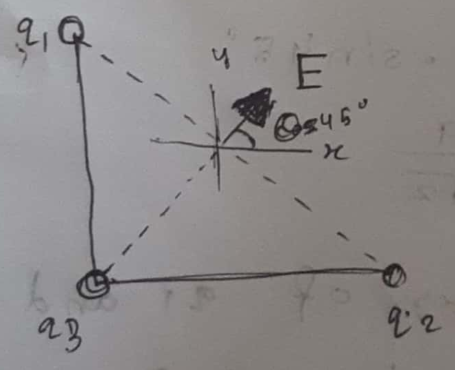
$$E = \frac{1}{4\pi\epsilon_0} \frac{|q_3|}{r^2}$$

$$= \frac{9\times10^9 \times 2 \times 1.6 \times 10^{-19}}{\left(\frac{a}{\sqrt{2}}\right)^2} \text{ N/C}$$

$$= \frac{9\times10^9 \times 2 \times 1.6\times10^{-19}}{\left(\frac{6\times10^{-6}}{2}\right)^2}$$

$$= 1.6\times10^2 \text{ N/C}$$

(b) Direction of the electric field.

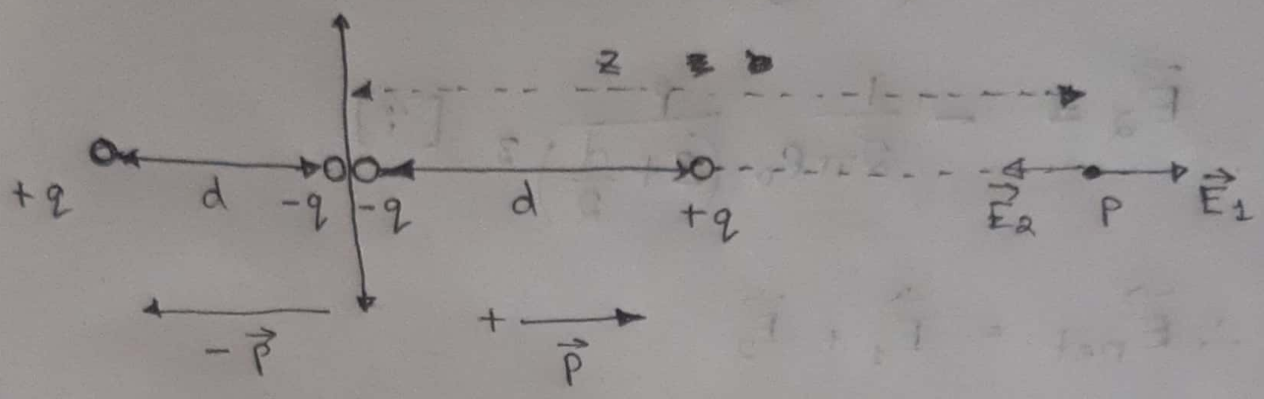



The direction is,

$$\hat{r} = [\cos 45° \hat{i} + \sin 45° \hat{j}]$$

$$= \left[\frac{\sqrt{2}}{2}\hat{i} + \frac{\sqrt{2}}{2}\hat{j}\right]$$

Question - 6 (a)




The electric field ~~due~~ to a dipole is -

$$E = \frac{1}{2\pi\epsilon_0} \frac{p}{r^3}$$

p is the dipole moment of the dipole and r is the distance between the center of the dipole and point P.

$\therefore$ for the dipole 1,

$$r = \left(z - \frac{d}{2}\right)$$

$$\therefore \vec{E_1} = \frac{1}{2\pi\epsilon_0} \frac{p}{\left(z - \frac{d}{2}\right)^3} [\hat{i}]$$

For dipole-2

$$r = \left(z + \frac{d}{2}\right)$$

$$\vec{E}_2 = -\frac{1}{2\pi\epsilon_0} \frac{p}{\left(z+\frac{d}{2}\right)^3} [\hat{i}]$$

$$\therefore \vec{E}_{net} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{1}{2\pi\epsilon_0} \frac{p}{\left(z-\frac{d}{2}\right)^3} \hat{i} - \frac{1}{2\pi\epsilon_0} \frac{p}{\left(z+\frac{d}{2}\right)^2} \hat{i}$$

$$= \frac{1}{2\pi\epsilon_0} \frac{p}{z^3} \left( \frac{1}{\left(1-\frac{d}{2z}\right)^3} - \frac{1}{\left(1+\frac{d}{2z}\right)^3} \right)$$

$$\vec{E}_{net} = \frac{1}{2\pi\epsilon_0} \frac{p}{z^3} \left\{ \left(1-\frac{d}{2z}\right)^{-3} - \left(1+\frac{d}{2z}\right)^{-3} \right\}$$

(b) From (a) we got,

$$\vec{E}_{net} = \frac{1}{2\pi\epsilon_0} \frac{p}{z^3} \left\{ \left(1-\frac{d}{2z}\right)^{-3} - \left(1+\frac{d}{2z}\right)^{-3} \right\}$$

This is similar to Binomial theorem,

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \dots$$

Considering, $x = -\frac{d}{2z}$ for the first term and $x = \frac{d}{2z}$ for the second, where $n = -3$ we get,

$$\vec{E}_{net} = \frac{1}{2\pi\epsilon_0}\frac{P}{z^3}\left\{\left(1 + \frac{3d}{2z}\right) - \left(1 - \frac{3d}{2z}\right)\right\}$$

$$\vec{E}_{net} = \frac{1}{2\pi\epsilon_0}\frac{P}{z^3}\left(\frac{3d}{z}\right)$$

$$\vec{E}_{net} = \frac{6Pd}{4\pi\epsilon_0 z^4}$$

$$\vec{E}_{net} = \frac{6qd^2}{4\pi\epsilon_0 z^4} \quad [P = qd]$$

$$\vec{E}_{net} = \frac{3Q}{4\pi\epsilon_0 z^4} \quad [qd^2 = Q]$$

$$\therefore \vec{E}_{net} = \frac{3Q}{4\pi\epsilon_0 z^4}$$

<u>showed</u>