Problem Set 4
Non-Code Part

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Exercise 1

1.1

Note that the following is the Euler equation of Brock and Mirman's Neoclassical growth model;

$$\frac{1}{C_t} = \beta \frac{\alpha e^{z_t}(K_{t+1})^{\alpha-1}}{C_{t+1}}$$

We are solving this problem with Guess-Verify approach.
Using the resource constraint, $C_t + K_{t+1} = Y_t = e^{z_t}K_t^{\alpha}$, we assume that

$$\begin{aligned} C_t =& \phi Y_t \\ K_{t+1} =& (1-\phi)Y_t \end{aligned}$$

Then, using this rule, the Euler equation becomes ;

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$$\begin{aligned}\frac{1}{\phi Y_t} &= \beta \mathbb{E} \frac{\alpha e^{Z_{t+1}} K_{t+1}^{\alpha-1}}{\phi Y_{t+1}} \\ \leftrightarrow \frac{1}{\phi e^{Z_t} K_t} &= \beta \mathbb{E} \frac{\alpha e^{Z_{t+1}} K_{t+1}^{\alpha-1}}{\phi e^{Z_{t+1}} K_{t+1}^{\alpha-1}} \\ \leftrightarrow \frac{1}{\phi e^{Z_t} K_t} &= \beta \alpha / K_{t+1}\end{aligned}$$

Thus, $K_{t+1} = \beta \alpha e^{Z_t} K_t^{\alpha}$, and $C_t = (1-\beta\alpha) e^{Z_t} K_t^{\alpha}$

1.2

The intra-temporal condition of this economy is;

$$\frac{-a}{1-l_t} + \frac{w_t(1-\tau)}{c_t} = 0$$

The inter-temporal condition of this economy is;

$$\frac{1}{c_t} = \beta \mathbb{E}_t \{ \frac{(r_{t+1}-\delta)(1-\tau)+1}{c_t} \}$$

with $w_t = (1-\alpha) e^{z_t} K_t^{\alpha} L_t^{-\alpha}$ and $r_t = \alpha e^{z_t} K_t^{\alpha-1} L_t^{1-\alpha}$
The banlanced budget equation and the exogenous law of motion follows the same form in the section 3.

1.3

The intra-temporal condition of this economy is;

$$\frac{-a}{1-l_t} + \frac{w_t(1-\tau)}{c_t^{\gamma}} = 0$$

The inter-temporal condition of this economy is;

$$\frac{1}{c_t^{\gamma}} = \beta \mathbb{E}_t \{ \frac{(r_{t+1} - \delta)(1-\tau) + 1}{c_{t+1}^{\gamma}} \}$$

with $w_t = (1-\alpha)e^{z_t} K_t^{\alpha} L_t^{-\alpha}$ and $r_t = \alpha e^{z_t} K_t^{\alpha-1} L_t^{1-\alpha}$
The banlanced budget equation, the exogenous law of motion follows the same form in the section 3.

1.4

The intra-temporal condition of this economy is;

$$\frac{-a}{(1-l_t)^{\xi}} + \frac{w_t(1-\tau)}{c_t^{\gamma}} = 0$$

The inter-temporal condition of this economy is;

$$\frac{1}{c_t^{\gamma}} = \beta \mathbb{E}_t \{ \frac{(r_{t+1} - \delta)(1-\tau) + 1}{c_{t+1}^{\gamma}} \}$$

with $w_t = (1-\alpha) L_t^{\eta-1} \{ \alpha K_t^{\eta} + (1-\alpha) L_t^{\eta} \}^{1/\eta - 1}$ and $r_t = \alpha K_t^{\eta-1} \{ \alpha K_t^{\eta} + (1-\alpha) L_t^{\eta} \}^{1/\eta-1}$
The banlanced budget equation, the exogenous law of motion follows the same form in the section 3.

1.5(part)

The inter-temporal condition of this economy is;

$$\frac{1}{c_t^{\gamma}} = \beta \mathbb{E}_t \{ \frac{(r_{t+1} - \delta)(1-\tau) + 1}{c_{t+1}^{\gamma}} \}$$

with $w_t = (1-\alpha)e^{(1-\alpha)z_t} K_t^{\alpha} L_t^{-\alpha}$ and $r_t = \alpha e^{(1-\alpha)z_t} K_t^{\alpha-1} L_t^{1-\alpha}$
The banlanced budget equation, the exogenous law of motion follows the same form in the section 3.

In the steady-state, the Euler equation above becomes the following;

$$\frac{1}{c_{ss}^{\gamma}} = \beta\{\frac{(r_{ss}-\delta)(1-\tau)+1}{c_{ss}^{\gamma}}\}$$

Thus,

$$r_{ss} = \alpha K_{ss}^{\alpha-1} = \frac{1/\beta - 1}{1-\tau} + \delta$$

$$K_{ss} = ((\frac{1/\beta - 1}{1-\tau} + \delta)/\alpha)^{\frac{1}{\alpha-1}}$$

With the paremeters $\beta = 0.98$, $\alpha = 0.4$, $\delta = 0.1$, $\bar{z} = 0$, $\tau = 0.05$, the steady-state capital is $K_{ss} = 7.2875$, $I_{ss} = \delta K_{ss} = 0.72875$, $Y_{ss} = K_{ss}^{\alpha} = 2.213$

1.6(part)

The characterizing equation of this economy is the same with Exercise 4. The steady-state version of this is just all the equations with ss scripts. With python, $C_{ss} = 1.159$,$K_{ss} = 2.2574$,$L_{ss} = 0.3097$. Thus,

$$Y_{ss} = K_{ss}^{\alpha} L_{ss}^{1-\alpha} = 0.6855$$

$$I_{ss} = \delta K_{ss} = 0.2257$$

Exercise 2

2.3(part)

$$\begin{aligned}
&\mathbb{E}_t\{F\tilde{X}_{t+1} + G\tilde{X}_t + H\tilde{X}_{t-1} + L\tilde{Z}_{t+1} + M\tilde{Z}_t\} = 0 \\
&\mathbb{E}_t\{F(P\tilde{X}_t + Q\tilde{Z}_{t+1}) + G(P\tilde{X}_{t-1} + Q\tilde{Z}_t) + H\tilde{X}_{t-1} + L(N\tilde{Z}_t + \varepsilon_{t+1}) + M\tilde{Z}_t\} = 0
\end{aligned}$$

This leads to $[(FP+G)P+H]\tilde{X}_{t-1} + Q[(FP+G)Q+(FQ+L)N+M]Z_t = 0$