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解 连接电源时有

$$R_1 i_1 + L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} + M \frac{\mathrm{d}i_2}{\mathrm{d}t} = \mathcal{E}$$

$$R_2 i_2 + L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} + M \frac{\mathrm{d}i_1}{\mathrm{d}t} = 0$$

换成短接时有

$$R_1 i_1 + L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} + M \frac{\mathrm{d}i_2}{\mathrm{d}t} = 0$$
$$R_2 i_2 + L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} + M \frac{\mathrm{d}i_1}{\mathrm{d}t} = 0$$

由于这两组方程对应的齐次方程相同,故其时间常量相同,下面仅考虑短接时的情况。由短接的第一个方程有

$$\frac{\mathrm{d}i_1}{\mathrm{d}t} = -\frac{M\frac{\mathrm{d}i_2}{\mathrm{d}t} + R_1 i_1}{L_1}$$

代入第二个方程有

$$L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} - \frac{M^2}{L_1} \frac{\mathrm{d}i_2}{\mathrm{d}t} - \frac{MR_1}{L_1} i_1 + R_2 i_2 = 0$$

引入无漏磁条件  $M^2 = L_1L_2$  有

$$-\frac{MR_1}{L_1}i_1 + R_2i_2 = 0$$

$$i_2 = \frac{MR_1}{L_1R_2}i_1$$

$$\frac{di_2}{dt} = \frac{MR_1}{L_1R_2}\frac{di_1}{dt}$$

代入第一个方程有

$$0 = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} + \frac{M^2 R_1}{L_1 R_2} \frac{\mathrm{d}i_1}{\mathrm{d}t} + R_1 i_1$$
$$\frac{\mathrm{d}i_1}{\mathrm{d}t} = -\frac{R_1 R_2}{R_1 L_2 + R_2 L_1} i_1$$

故时间常量为

$$\tau = \frac{1}{\frac{R_1 R_2}{R_1 L_2 + R_2 L_1}} = \frac{R_1 L_2 + R_2 L_1}{R_1 R_2} = \frac{L_1}{R_2} + \frac{L_2}{R_1}$$

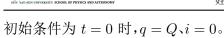
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解 (1)

$$\frac{q}{C} + L\frac{\mathrm{d}i}{\mathrm{d}t} = 0$$

$$\frac{q}{C} + L\frac{\mathrm{d}^2i}{\mathrm{d}t^2} = 0$$

$$q = C_1 \sin(\frac{t}{\sqrt{CL}}) + C_2 \cos(\frac{t}{\sqrt{CL}})$$



故解为

$$q = Q\cos(\frac{t}{\sqrt{LC}})$$

欲令线圈磁场能等于电容中电能,即有

$$\frac{q^2}{2C} = \frac{LI^2}{2}$$
 
$$\frac{q^2}{2C} = \frac{L}{2} (\frac{\mathrm{d}q}{\mathrm{d}t})^2$$
 
$$\frac{Q^2}{2C} \cos^2(\frac{t}{\sqrt{LC}}) = \frac{L}{2} \frac{Q^2}{LC} \cos^2(\frac{t}{\sqrt{LC}})$$
 
$$\tan^2(\frac{t}{\sqrt{LC}}) = 1$$
 
$$t = \frac{\pi}{4} \sqrt{LC}$$

$$q=Q\cos(\frac{\pi}{4})=\frac{\sqrt{2}}{2}Q$$

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解 (1) 并联后总电容为  $C' = 2C = 4\mu F$ 

$$\lambda = \frac{R}{2} \sqrt{\frac{C'}{L}} = 1.58 > 1$$

故不振荡

(2) 并联后总电容为  $C' = \frac{C^2}{2C} = 1\mu F$ 

$$\lambda = \frac{R}{2} \sqrt{\frac{C'}{L}} = 0.79 < 1$$

故振荡