1-45

解 (1)

$$\vec{E}_A = \frac{\sigma_e}{2\varepsilon_0}\hat{x}$$

(2)

$$\vec{E}_B = \frac{\sigma_e}{2\varepsilon_0}\hat{x}$$

(3)

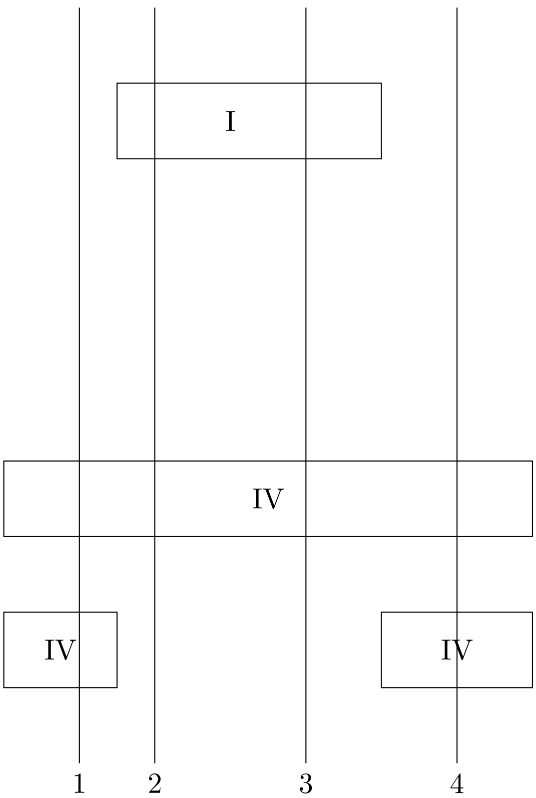
$$\vec{E} = \vec{E}_A + \vec{E}_B = \frac{\sigma_e}{\varepsilon_0}\hat{x}$$

(4) 均匀分布在平板左右两侧

$$E_A' = \frac{\sigma_e}{2\varepsilon_0}\hat{x}$$

1-46

解



(1) 取如 I 所示高斯面,因为导体内部电场为 0,两平行板中间电场与高斯面平行,故该高斯面电通量为 0。故其中没有静电荷即

$$\sigma_2 S + \sigma_3 S = 0 \rightarrow \sigma_1 = -\sigma_2$$

即两平板相向两面的电荷面密度大小相等符号相反。

(2) 取如 II 所示高斯面，由场强叠加原理知该高斯面左右两侧场强大小相等，方向相反。故知 III, IV 两高斯面电通量相等，故两高斯面内静电荷量相等即

$$\sigma_1 S = \sigma_4 S \to \sigma_1 = \sigma_4$$

即两平板相背两面的电荷面密度大小相等符号相反。

(3)

$$\begin{cases} \sigma_1 + \sigma_2 = 3 \\ \sigma_3 + \sigma_4 = 7 \\ \sigma_1 = \sigma_4 \\ \sigma_2 + \sigma_3 = 0 \end{cases}$$

解得

$$\sigma_1 = 5\mu\mathrm{C/m^2}, \sigma_2 = -2\mu\mathrm{C/m^2}, \sigma_3 = 2\mu\mathrm{C/m^2}, \sigma_4 = 5\mu\mathrm{C/m^2}$$

1-52

解 (1)

$$U_2 = \int_{\infty}^{R_3} \frac{q+Q}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r = \frac{q+Q}{4\pi\varepsilon_0 R_3}$$

$$U_1 = U_2 + \int_{R_2}^{R_1} \frac{q}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r = \frac{q+Q}{4\pi\varepsilon_0 R_3} + \frac{q}{4\pi\varepsilon_0}(\frac{1}{R_1} - \frac{1}{R_2})$$

(2)

$$\Delta U = U_1 - U_2 = \frac{q}{4\pi\varepsilon_0}(\frac{1}{R_1} - \frac{1}{R_2})$$

(3)

$$U_1 = U_2 = \frac{q+Q}{4\pi\varepsilon_0 R_3}$$

$$\Delta U = 0$$

(4) 情形 (1):

$$U_2 = 0$$

$$U_1 = U_2 + \int_{R_2}^{R_1} \frac{q}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r = \frac{q}{4\pi\varepsilon_0}(\frac{1}{R_1} - \frac{1}{R_2})$$

$$\Delta U = \int_{R_2}^{R_1} \frac{q}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r = \frac{q}{4\pi\varepsilon_0}(\frac{1}{R_1} - \frac{1}{R_2})$$

情形 (2):

$$U_2 = 0$$

$$U_1 = U_2 = 0$$

$$\Delta U = 0$$

(5) 设平衡后球体所带电荷为 q' 则球壳内表面所带电荷为 $-q'$，外表面所带电荷为 $Q+q'$ 则球壳电势为

$$U_2 = \int_{\infty}^{R_3} \frac{Q+q'}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r = \frac{Q+q'}{4\pi\varepsilon_0 R_3}$$

则球体电势为

$$U_1 = U_2 + \int_{R_2}^{R_1} \frac{q'}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r = \frac{Q+q'}{4\pi\varepsilon_0 R_3} + \frac{q'}{4\pi\varepsilon_0}(\frac{1}{R_1} - \frac{1}{R_2})$$

又因为球体接地，故

$$U_1 = 0$$

解得

$$q' = \frac{Q}{R_3}(\frac{1}{R_2} - \frac{1}{R_1} - \frac{1}{R_3})$$

于是有

$$\begin{aligned}
U_1 &= 0 \\
U_2 &= \frac{1}{4\pi\varepsilon_0}\frac{(R_2-R_1)Q}{R_1R_2+R_2R_3-R_3R_1} \\
\Delta U = U_1 - U_2 &= \frac{1}{4\pi\varepsilon_0}\frac{(R_1-R_2)Q}{R_1R_2+R_2R_3-R_3R_1}
\end{aligned}$$

1-57

解 (1) 设上极板带电 Q, 则中间导体上表面带电 $-Q$, 下表面带电 Q, 下极板带电 $-Q$, 则电容器中间除导体内部的区域的场强为

$$E = \frac{Q}{\varepsilon_0 S}$$

则两极板电势差为

$$U = E(d-t)$$

故

$$C = \frac{Q}{U} = \frac{\varepsilon_0 S}{d-t}$$

(2) 上面讨论与极板位置无关，故远近无影响。

1-62

解 (1)

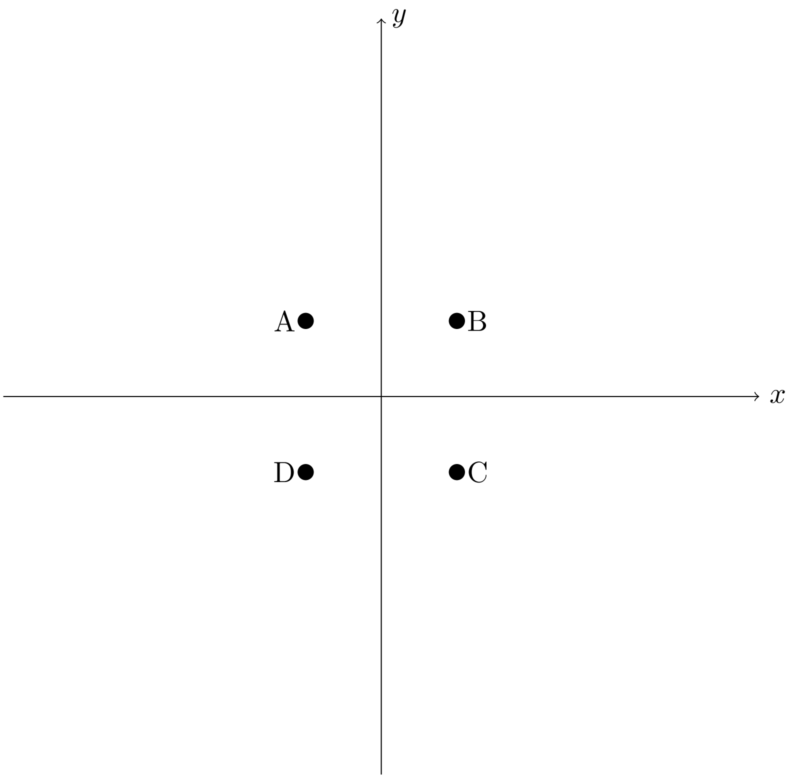
$$\begin{aligned}
U &= \int_{R_1}^{R_2} \frac{Q}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r + \int_{R_3}^{R_4} \frac{Q}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r \\
&= \frac{Q}{4\pi\varepsilon_0}(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3} - \frac{1}{R_4})
\end{aligned}$$

(2)

$$C = \frac{Q}{U} = \frac{4\pi\varepsilon_0}{\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3} - \frac{1}{R_4}}$$

附加题

解 因为导线无限长由对称性可知，电荷在导线上均匀分布，设线密度大小为 λ 又电



场方向向右，故 AD 带 $-\lambda$，BC 带 λ。以 A 为电势零点则 B 电势为

$$\begin{aligned} U_B &= -E_0 a + \frac{-\lambda}{2\pi\varepsilon_0}(\ln r - \ln a) + \frac{-\lambda}{2\pi\varepsilon_0}(\ln a - \ln\sqrt{2}a) + \frac{\lambda}{2\pi\varepsilon_0}(\ln\sqrt{2}a - \ln a) + \frac{\lambda}{2\pi\varepsilon_0}(\ln a - \ln r) \\ &= -E_0 a + \frac{\lambda}{\pi\varepsilon_0}(\ln\sqrt{2}a - \ln r) \end{aligned}$$

又因为 $U_B = U_A = 0$ 解得

$$\lambda = \frac{\pi\varepsilon_0 E_0 a}{\ln\sqrt{2}a - \ln r}$$

故 x 轴上场强分布为

$$\begin{aligned} E(x) &= E_0 + 2\frac{\lambda}{2\pi\varepsilon_0\sqrt{\frac{a^2}{4} + (x - \frac{a}{2})^2}} \frac{x - \frac{a}{2}}{\sqrt{\frac{a^2}{4} + (x - \frac{a}{2})^2}} - 2\frac{\lambda}{2\pi\varepsilon_0\sqrt{\frac{a^2}{4} + (x - \frac{a}{2})^2}} \frac{x + \frac{a}{2}}{\sqrt{\frac{a^2}{4} + (x + \frac{a}{2})^2}} \\ &= E_0\left\{1 + \frac{a(x - \frac{a}{2})}{\ln\frac{\sqrt{2}a}{r}[\frac{a^2}{4} + (x - \frac{a}{2})^2]} - \frac{a(x + \frac{a}{2})}{\ln\frac{\sqrt{2}a}{r}[\frac{a^2}{4} + (x + \frac{a}{2})^2]}\right\} \end{aligned}$$

代入数值得

$$E(x) = 1 + \frac{x - 0.005}{\ln(100\sqrt{2})[0.25 + (x - 0.005)^2]} - \frac{x + 0.005}{\ln(100\sqrt{2})[0.25 + (x + 0.005)^2]} \mathrm{V/m}$$