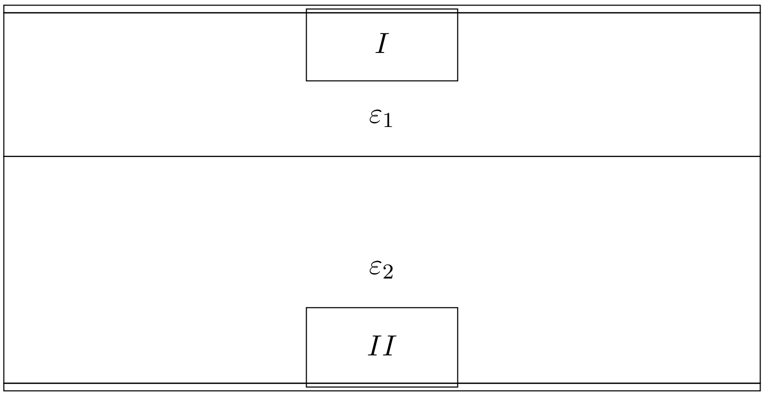
4-3

解



(1) 设上极板所带电荷面密度为 σ，取如图所示二高斯面可知 $D=\sigma$, 则

$$E_1=\frac{D}{\varepsilon_0\varepsilon_1}$$
$$E_2=\frac{D}{\varepsilon_0\varepsilon_2}$$

又

$$E_1d_1+E_2d_2=U$$

解得 $\sigma=4.66\times10^{-5}\mathrm{C/m^2}$ 故

$$P_1=\frac{\varepsilon_1-1}{\varepsilon_1}\sigma=3.7\times10^{-5}\mathrm{C/m^2}$$
$$P_1=\frac{\varepsilon_2-1}{\varepsilon_2}\sigma=1.6\times10^{-5}\mathrm{C/m^2}$$

(2)

$$U=E_2d_2=7.9\times10^{3}\mathrm{V}$$

4-6

解　由对称性知，两平行板之间电场应垂直于导体板，亦即互相平行，故其中间为匀强电场设场强为 E 故有

$$\sigma_1=\varepsilon_0\varepsilon_1E$$

$$\sigma_2=\varepsilon_0\varepsilon_2E$$

又 $Q=\sigma_1S_1+\sigma_2S_2, U=Ed$，故电容为

$$\begin{aligned}C&=\frac{Q}{U}\\&=\frac{(\varepsilon_1S_1+\varepsilon_2S_2)\varepsilon_0}{d}\end{aligned}$$

4-9

解　(1)

$$E = \begin{cases} \dfrac{Q}{4\pi\varepsilon\varepsilon_0 r^2} & R < r < R' \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & r < R' \end{cases}$$

(2)

$$U = \begin{cases} \displaystyle\int_r^{R'} E\,\mathrm{d}r + \int_{R'}^{\infty} E\,\mathrm{d}r = \frac{Q}{4\pi\varepsilon\varepsilon_0}(\frac{1}{r} + \frac{\varepsilon - 1}{R'}) & R < r < R' \\ \displaystyle\int_r^{\infty} E\,\mathrm{d}r = \frac{Q}{4\pi\varepsilon_0 r} & r > R' \end{cases}$$

(3)

$$U = \frac{Q}{4\pi\varepsilon\varepsilon_0}(\frac{1}{R} + \frac{\varepsilon - 1}{R'})$$

4-12

解　$D = \dfrac{Q}{4\pi r^2}$ 故

$$E_1 = \frac{D}{\varepsilon_1\varepsilon_0}$$
$$E_2 = \frac{D}{\varepsilon_2\varepsilon_0}$$

故两极板间电势差为

$$U = \int_{R_1}^{R} E_1\,\mathrm{d}r + \int_{R}^{R_2} E_2\,\mathrm{d}r = \frac{Q}{4\pi\varepsilon_0}\left[(\frac{1}{\varepsilon_1 R_1} - \frac{1}{\varepsilon_1 R}) + (\frac{1}{\varepsilon_2 R} - \frac{1}{\varepsilon_2 R_2})\right]$$

则电容为

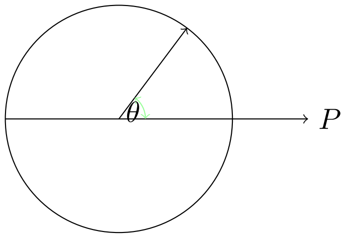
$$C = \frac{Q}{U} = \frac{4\pi\varepsilon_0}{(\frac{1}{\varepsilon_1 R_1} - \frac{1}{\varepsilon_1 R}) + (\frac{1}{\varepsilon_2 R} - \frac{1}{\varepsilon_2 R_2})}$$

(2)

$$\sigma(R_1) = P_1 = \frac{(\varepsilon_1 - 1)Q}{4\pi\varepsilon_1 R_1^2}$$
$$\sigma(R) = \frac{(\varepsilon_2 - 1)Q}{4\pi\varepsilon_2 R^2} - \frac{(\varepsilon_1 - 1)Q}{4\pi\varepsilon_1 R^2} = \frac{(\varepsilon_2 - \varepsilon_1)Q}{4\pi\varepsilon_1\varepsilon_2 R^2}$$
$$\sigma(R_2) = -\frac{(\varepsilon_2 - 1)Q}{4\pi\varepsilon_2 R_2^2}$$

4-19

解



轴线处场强由分界面内部和外部电荷共同作用产生，界面内部极化电荷分布在圆柱表面可看作多个无限长带电直线叠加，极矩 $P=(\varepsilon-1)\varepsilon_0 E_0$, 则极化电荷面密度为 $P\cos\theta$，线密度就为 $\lambda=P\cos\theta r\,\mathrm{d}\theta$, 又因为系统的对称性，故可知何场强方向一定与 P 方向共线则其在轴线处的场强大小为

$$E=\int_0^{2\pi}\frac{\lambda\cos\theta}{2\pi R\varepsilon_0}=\frac{P}{2\pi\varepsilon_0}\int_0^{2\pi}\cos^2\theta\,\mathrm{d}\theta=\frac{P}{2\varepsilon_0}=\frac{\varepsilon-1}{2}E_0$$

又因为该场强与 E_0 方向相反故

$$E=E_0+\frac{\varepsilon-1}{2}E_0=\frac{\varepsilon+1}{2}E_0$$

真挖去后不成立，因为极化不再均匀