



1

解 以自转轴方向为轴向建立球坐标得定解条件

$$\nabla^2 \varphi_1 = 0$$

$$\nabla^2 \varphi_2 = 0$$

$$\frac{1}{R_0} \left( \frac{\partial \varphi_2}{\partial \theta} - \frac{\partial \varphi_1}{\partial \theta} \right) = -\frac{Q\omega \sin \theta}{4\pi R_0} \Big|_{r=R_0}$$

$$\frac{\partial \varphi_1}{\partial r} = \frac{\partial \varphi_2}{\partial r}$$

$$\varphi_1|_{r \rightarrow 0} \text{有限}$$

$$\varphi_2|_{r \rightarrow \infty} = 0$$

通解为

$$\varphi_1 = \sum_{l=0}^{\infty} a_l r^l P_l(\cos \theta)$$

$$\varphi_2 = \sum_{l=0}^{\infty} b_l r^{-l-1} P_l(\cos \theta)$$

代入边界条件得

$$\frac{1}{R_0} \left( \sum_{l=0}^{\infty} a_l R_0^l \frac{l \cos \theta P_l(\cos \theta) - l P_{l-1}(\cos \theta)}{\sin \theta} - \sum_{l=0}^{\infty} b_l R_0^{-l-1} \frac{l \cos \theta P_l(\cos \theta) - l P_{l-1}(\cos \theta)}{\sin \theta} \right) = -\frac{Q\omega \sin \theta}{4\pi R_0}$$

$$\sum_{l=0}^{\infty} l a_l R_0^{l-1} P_l(\cos \theta) = -\sum_{l=0}^{\infty} (l+1) b_l R_0^{-l-2} P_l(\cos \theta)$$

解得

$$a_1 = \frac{-Q\omega}{6\pi R_0}$$

$$b_1 = \frac{Q\omega R_0^2}{12\pi}$$

$$a_l = b_l = 0 (l \neq 1)$$

$$\varphi_1 = \frac{-Q\omega}{6\pi R_0} r \cos \theta$$

$$\varphi_2 = \frac{Q\omega R_0^2}{12\pi r^2} \cos \theta$$

故

$$\vec{B}_1 = -\mu_0 \nabla \varphi_1$$



$$= \frac{Q\vec{\omega}}{6\pi R_0}$$

$$\vec{B}_2 = -\mu_0 \nabla \varphi_2$$

$$= \frac{\mu_0}{4\pi} \left[ \frac{3(\vec{m} \cdot \vec{r})\vec{r}}{r^5} - \frac{\vec{m}}{r^3} \right]$$

2

解 (1)

$$\begin{aligned} \vec{m} &= \frac{1}{2} \iiint \vec{r} \times \vec{j} dV \\ &= \frac{1}{2} \iiint \vec{r} \times \frac{3Q}{4\pi R_0^3} (\vec{\omega} \times \vec{r}) dV \\ &= \frac{1}{2} \frac{3Q}{4\pi R_0^2} \iiint \vec{r} \times (\vec{\omega} \times \vec{r}) r^2 \sin \theta dr d\theta d\phi \\ &= \frac{1}{2} \frac{3Q}{4\pi R_0^2} \iiint (\vec{e}_r \times \vec{e}_\phi) r^4 \sin \theta dr d\theta d\phi \\ &= -\frac{1}{2} \frac{3Q}{4\pi R_0^2 \omega} \iiint \vec{e}_\theta r^4 \sin \theta dr d\theta d\phi \\ &= \frac{3Q\omega}{8\pi R_0^2} \iiint [\sin \theta \vec{e}_z + \cos \theta (-\cos \phi \vec{e}_x - \sin \phi \vec{e}_y)] r^4 \sin \theta dr d\theta d\phi \\ &= \frac{3Q\omega}{8\pi R_0^2} \vec{e}_z \int_0^{2\pi} \int_0^\pi \int_0^{R_0} r^4 \sin^3 \theta dr d\theta d\phi \\ &= \frac{QR_0^2 \vec{\omega}}{5} \end{aligned}$$

(2)

$$\begin{aligned} \vec{L} &= I\vec{\omega} \\ &= \frac{2m_0 R_0^2}{5} \vec{\omega} \\ \frac{m}{L} &= \frac{\frac{QR_0^2 \omega}{5}}{\frac{2m_0 R_0^2}{5} \omega} \\ &= \frac{Q}{2m_0} \end{aligned}$$

2

解 该问题近似于电偶极子在无穷大导体平面边界的问题, 故可使用电像法。令介质平面为  $z=0$  平面。 $\vec{m}$  距其  $d$ , 与  $z$  轴夹角  $\theta$  得到镜像磁矩  $\vec{m}'$  产生的磁标势为

$$\varphi' = \frac{\vec{m}' \cdot \vec{r}}{4\pi r^3}$$



故

$$\begin{aligned}\vec{B}' &= \mu_0(-\nabla\varphi') \\ &= \frac{\mu_0}{4\pi} \left[ \frac{3(\vec{m}' \cdot \vec{r})\vec{r}}{r^5} - \frac{\vec{m}'}{r^3} \right]\end{aligned}$$

则在  $\vec{m}$  处产生的磁感应强度为

$$\vec{B}' = \frac{\mu_0}{4\pi z^3} (3m' \cos\theta \vec{e}_z - \vec{m}')$$

故

$$\begin{aligned}\vec{F} &= -\nabla(-\vec{m} \cdot \vec{B}') \\ &= \nabla \left[ \frac{\mu_0 m^2}{4\pi z^3} (1 + \cos^2 \alpha) \right] \\ &= \vec{e}_z \frac{\partial \frac{\mu_0 m^2}{4\pi z^3} (1 + \cos^2 \theta)}{\partial z} \\ &= \frac{-3\mu_0 m^2}{64\pi d^4} (1 + \cos^2 \theta) \vec{e}_z\end{aligned}$$