

1

解 电场强度可写为

$$E = \frac{q}{4\pi\varepsilon_0} \frac{1}{r^2}$$

其法向导数为

$$\begin{split} \frac{\partial E}{\partial r} &= \frac{q}{4\pi\varepsilon_0} \frac{-2}{r^3} \\ &= \frac{-2E}{r} \end{split}$$

故

$$\frac{1}{E}\frac{\partial E}{\partial n} = -\frac{2}{R}$$

2

解 设该电偶极子在直角坐标下的电偶极矩为 $\vec{p} = (ql, 0, 0)$,则产生的电势为

$$\varphi = \frac{\vec{p} \cdot \vec{r}}{4\pi\varepsilon_0 r^3}$$

$$= \frac{qlx}{4\pi\varepsilon_0 (x^2 + y^2 + z^2)^{\frac{3}{2}}}$$

故电场强度为

$$\begin{split} \vec{E} &= -\nabla \varphi \\ &= [\frac{3lqx^2}{4\pi\varepsilon_0 \left(x^2 + y^2 + z^2\right)^{5/2}} - \frac{lq}{4\pi\varepsilon_0 \left(x^2 + y^2 + z^2\right)^{3/2}}] \vec{e}_x + \\ &\frac{3lqxy}{\left(x^2 + y^2 + z^2\right)^{5/2}} \vec{e}_y + (3lqxz)/(x^2 + y^2 + z^2)^{(5/2)} \vec{e}_z \end{split}$$

故

$$\begin{split} \frac{\partial E_x}{\partial y} &= \frac{3lqy}{4\pi\varepsilon_0 \left(x^2 + y^2 + z^2\right)^{5/2}} - \frac{15lqx^2y}{4\pi\varepsilon_0 \left(x^2 + y^2 + z^2\right)^{7/2}} \\ \frac{\partial E_x}{\partial z} &= \frac{3lqz}{4\pi\varepsilon_0 \left(x^2 + y^2 + z^2\right)^{5/2}} - \frac{15lqx^2z}{4\pi\varepsilon_0 \left(x^2 + y^2 + z^2\right)^{7/2}} \\ \frac{\partial E_y}{\partial x} &= \frac{3lqy}{4\pi\varepsilon_0 \left(x^2 + y^2 + z^2\right)^{5/2}} - \frac{15lqx^2y}{4\pi\varepsilon_0 \left(x^2 + y^2 + z^2\right)^{7/2}} \\ \frac{\partial E_y}{\partial z} &= -\frac{15lqxyz}{4\pi\varepsilon_0 \left(x^2 + y^2 + z^2\right)^{7/2}} \\ \frac{\partial E_z}{\partial x} &= \frac{3lqy}{4\pi\varepsilon_0 \left(x^2 + y^2 + z^2\right)^{5/2}} - \frac{15lqx^2y}{4\pi\varepsilon_0 \left(x^2 + y^2 + z^2\right)^{7/2}} \end{split}$$



$$\frac{\partial E_z}{\partial y} = -\frac{15lqxyz}{4\pi\varepsilon_0 \left(x^2 + y^2 + z^2\right)^{7/2}}$$

因为

$$\frac{\partial E_x}{\partial y} = \frac{\partial E_y}{\partial x}$$
$$\frac{\partial E_x}{\partial z} = \frac{\partial E_z}{\partial x}$$
$$\frac{\partial E_y}{\partial z} = \frac{\partial E_z}{\partial y}$$

故

$$\nabla \times \vec{E} = 0$$

3

解 设该电荷位于坐标原点,则其产生的电势为

$$\varphi = \frac{q}{4\pi\varepsilon_0 r}$$

$$= \frac{q}{4\pi\varepsilon_0} \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

考察其穿过平面 z = a(-a < x < a, -a < y < a) 的电通量,只需计算电场的 z 方向分量

$$E_z = \frac{\partial \varphi}{\partial z}$$

$$= -\frac{q}{4\pi\varepsilon_0} \frac{z}{(x^2 + y^2 + z^2)^{3/2}}$$

故其其穿过平面 z = a(-a < x < a, -a < y < a) 的电通量为

$$\begin{split} I &= \iint \frac{q}{4\pi\varepsilon_0} \frac{a}{(x^2 + y^2 + a^2)^{3/2}} \, \mathrm{d}x \, \mathrm{d}y \\ &= \frac{aq}{4\pi\varepsilon_0} \int_{-a}^a \mathrm{d}y \int_{-a}^a \frac{1}{(x^2 + y^2 + a^2)^{3/2}} \, \mathrm{d}x \\ &= \frac{aq}{4\pi\varepsilon_0} \int_{-a}^a \frac{2a}{(a^2 + y^2)\sqrt{2a^2 + y^2}} \, \mathrm{d}y \\ &= \frac{aq}{4\pi\varepsilon_0} \frac{2\pi}{3a} \\ &= \frac{q}{6\varepsilon_0} \end{split}$$



解 由对称性知,该点电场方向必为径向,故只需计算球上各点在该点产生电场的径向 分量,取圆环微元该微元带电量为

$$q = 2\pi R^2 \sigma \sin\theta \,\mathrm{d}\theta$$

其在考察点产生的电场强度为

$$\begin{split} &= \frac{\mathrm{d}q}{4\pi\varepsilon_0} \frac{R - R\cos\theta}{[R^2\sin^2\theta + (R - R\cos\theta)^2]^{\frac{3}{2}}} \\ &= \frac{2\pi R^2\sigma\sin\theta\,\mathrm{d}\theta}{4\pi\varepsilon_0} \frac{R - R\cos\theta}{[R^2\sin^2\theta + (R - R\cos\theta)^2]^{\frac{3}{2}}} \end{split}$$

故

$$E = \int dE$$

$$= \int_{\theta_0}^{\pi} \frac{2\pi R^2 \sigma \sin \theta \, d\theta}{4\pi \varepsilon_0} \frac{R - R \cos \theta}{\left[R^2 \sin^2 \theta + (R - R \cos \theta)^2\right]^{\frac{3}{2}}}$$

$$= \frac{\sigma}{2\varepsilon_0} \int_{\theta_0}^{\pi} \frac{\sin \theta (1 - \cos \theta) \, d\theta}{\left[\sin^2 \theta + (1 - \cos \theta)^2\right]^{\frac{3}{2}}}$$

$$= \frac{\sigma}{2\varepsilon_0} (1 - \sin \frac{\theta_0}{2})$$

故该点电场强度为

$$E = \frac{\sigma}{2\varepsilon_0} (1 - \sin\frac{\theta_0}{2})$$

方向沿径向