1-4

解　对小球受力分析知

$$\frac{F_q}{mg} = \tan\theta$$

又

$$F_q = \frac{q^2}{4\pi\varepsilon_0(2l\sin\theta)^2}$$

故 $q = \pm\sqrt{16\pi\tan\theta\sin^2\theta l^2\varepsilon_0 mg}$

1-5

解　由油滴受力平衡知

$$Eq = mg = \frac{4}{3}\pi r^3\rho g \to q = \frac{4\pi r^3\rho g}{3E}$$

代入数值得

$$q = -8.03\times 10^{-19}\mathrm{C}$$

1-8

解　在 (r,θ) 处电势为

$$\phi(r,\theta) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{r^2+\frac{l^2}{4}-rl\cos\theta}} - \frac{1}{\sqrt{r^2+\frac{l^2}{4}+rl\cos\theta}}\right)$$

因为 $l \ll r$, 故略去二阶小量 $\frac{l^2}{4}$, 且运用近似 $(1+x)^k = 1+kx(x\ll 1)$ 可得

$$\phi(r,\theta) = \frac{ql\cos\theta}{4\pi\varepsilon_0 r^2}$$

又 $\mathbf{E} = -\nabla\phi$, 且在极坐标中 $\nabla = \frac{\partial}{\partial r}\hat{\mathbf{e}}_r + \frac{\partial}{r\partial\theta}\hat{\mathbf{e}}_\theta$ 故

$$\mathbf{E}(r,\theta) = \frac{ql\cos\theta}{2\pi\varepsilon_0 r^3}\hat{\mathbf{e}}_r + \frac{ql\sin\theta}{4\pi\varepsilon_0 r^3}\hat{\mathbf{e}}_\theta$$

其径向和角向分量为

$$E_r = \frac{ql\cos\theta}{2\pi\varepsilon_0 r^3}, E_\theta = \frac{ql\sin\theta}{4\pi\varepsilon_0 r^3}$$

1-10

解　(1) 其场强大小为

$$E = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(r+l)^2} + \frac{1}{(r-l)^2} - \frac{2}{r^2}\right)$$

泰勒展开并保留二阶余项后得

$$\begin{aligned}E &= \frac{q}{4\pi\varepsilon_0 r^2}(1-\frac{2l}{r}+\frac{3l^2}{r^2}+1+\frac{2l}{r}+\frac{3l^2}{r^2}-2)\\&=\frac{6ql^2}{4\pi\varepsilon_0 r^4}\\&=\frac{3Q}{4\pi\varepsilon_0 r^4}\end{aligned}$$

(2)

$$\begin{aligned}U(r) &= \frac{q}{4\pi\varepsilon_0}(\frac{1}{r+l}+\frac{1}{r-l}-\frac{2}{r})\\&=\frac{q}{4\pi\varepsilon_0 r}\frac{2l^2}{r^2-l^2}\end{aligned}$$

因为 $l \ll r$,故略去二阶小量 l^2 得

$$U(r)=\frac{2ql^2}{4\pi\varepsilon_0 r^3}=\frac{Q}{4\pi\varepsilon_0 r^3}$$

1-11

解 P 点场强大小为

$$\begin{aligned}E &= \frac{ql}{4\pi\varepsilon_0}\left[(x^2+\frac{l^2}{2}-lx)^{-\frac{3}{2}}-(x^2+\frac{l^2}{2}+lx)^{-\frac{3}{2}}\right]\\&=\frac{ql}{4\pi\varepsilon_0}(x^2+\frac{l^2}{2})^{-\frac{3}{2}}\left[(1-\frac{lx}{x^2+\frac{l^2}{2}})^{-\frac{3}{2}}-(1+\frac{lx}{x^2+\frac{l^2}{2}})^{-\frac{3}{2}}\right]\end{aligned}$$

因为 $l \ll x$,故

$$\begin{aligned}(x^2+\frac{l^2}{2})^{-\frac{3}{2}}\left[(1-\frac{lx}{x^2+\frac{l^2}{2}})^{-\frac{3}{2}}-(1+\frac{lx}{x^2+\frac{l^2}{2}})^{-\frac{3}{2}}\right] &= (x^2+\frac{l^2}{2})^{-\frac{3}{2}}\left[1+\frac{3lx}{2(x^2+\frac{l^2}{2})}-1+\frac{3lx}{2(x^2+\frac{l^2}{2})}\right]\\&=x^{-3}\frac{3lx}{x^2}\\&=\frac{3l}{x^4}\end{aligned}$$

故

$$E=\frac{3ql^2}{4\pi\varepsilon_0 x^4}$$

方向竖直向上

1-9

解　(1)

$$\mathrm{F}=\mathrm{F}_1+\mathrm{F}_2$$

$\mathrm{F}_1,\mathrm{F}_2$ 方向相反故

$$F=F_1-F_2$$

因为 $l\ll r$, 故 $F_1-F_2=\dfrac{2Qql}{4\pi\varepsilon_0 r^3}=\dfrac{2Qp}{4\pi\varepsilon_0 r^3}$ 因为 $L_1=L_2=0$, 故 $L=0$ 故

$$\mathbf{F}=\frac{Q}{2\pi\varepsilon_0 r^3}\mathbf{p}$$

$$\mathbf{L}=0$$

(2)

$$F_y=F_{1y}+F_{2y}=\frac{Qql}{4\pi\varepsilon_0 r^3},F_x=F_{1x}-F_{2x}=0$$

$$\mathbf{F}=\frac{Q}{4\pi\varepsilon_0 r^3}\mathbf{p}$$

$$L=F_{1x}\frac{l}{2}+F_{2x}\frac{l}{2}=\frac{Qp}{4\pi\varepsilon_0 r^2}$$

故

$$\mathbf{L}=\frac{Q\mathbf{p}\times\mathbf{r}}{4\pi\varepsilon_0 r^3}$$

1-13

解

$$\begin{aligned}\phi&=\iint\limits_{x^2+y^2+z^2=a^2}\mathbf{E}\cdot\mathbf{n}\mathrm{d}S\\&=\iint\limits_{x^2+y^2\leq a^2}E\cos\theta\frac{\mathrm{d}x\mathrm{d}y}{\cos\theta}\\&=\iint\limits_{x^2+y^2\leq a^2}E\mathrm{d}x\mathrm{d}y\\&=E\pi a^2\end{aligned}$$

1-14

解　取半径为 R 的球面,由于电荷分布是球对称的,故电场强度只有径向分量。该闭合球面所包围的净电荷量为

$$\begin{aligned}q&=\int_0^R-\frac{e}{\pi a_B^3}\mathrm{e}^{-2r/a_B}4\pi r^2\mathrm{d}r+e\\&=\frac{e\mathrm{e}^{\frac{2R}{a_B}}(a_B^2+2a_B+2R^2)}{a_B^2}\end{aligned}$$

由高斯定理

$$4\pi R^2 E = \frac{q}{\varepsilon_0}$$

$$E = \frac{e e^{\frac{2R}{a_B}}(a_B^2 + 2a_B + 2R^2)}{4\pi\varepsilon_0 a_B^2 R^2}$$

方向为由原子中心指向外面

1-15

解 (1) 取底面为面元 dS 的闭合柱面,则该曲面电通量为

$$\phi = (E_1 - E_2)\mathrm{d}S$$

由高斯定理

$$\phi = \frac{q}{\varepsilon_0} = \frac{\rho \mathrm{d}Sh}{\varepsilon_0}$$

则

$$\rho = \frac{(E_1 - E_2)\varepsilon_0}{h} = 4.43 \times 10^{-13} C/m^3$$

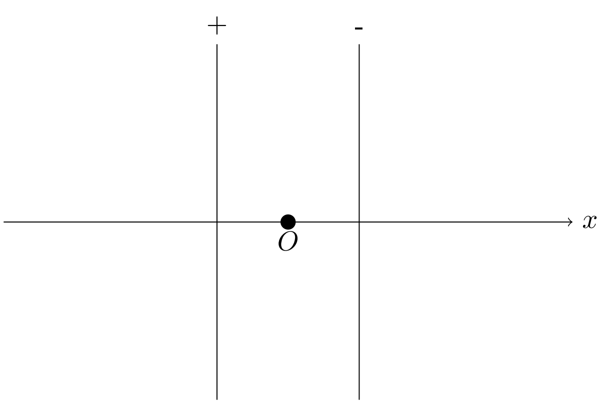
(2)

$$\frac{4\pi R^2 \sigma}{\varepsilon_0} = -4\pi R^2 E_1$$

$$\sigma = -\varepsilon_0 E_1 = -8.85 \times 10^{-10} C/m^2$$

1-12

解 设两条直线如图放置,取两线中点为坐标原点



(1) 设 $\hat{x}$ 为沿 x 轴正方向的单位矢量由场强叠加原理知当 $x < -\frac{a}{2}$ 时场强为

$$\mathbf{E} = \frac{\eta_e}{2\pi\varepsilon_0(-\frac{a}{2} - x)}(-\hat{x}) + \frac{\eta_e}{2\pi\varepsilon_0(\frac{a}{2} - x)}\hat{x} = \frac{-a\eta_e}{2\pi\varepsilon_0(x^2 - \frac{a^2}{4})}\hat{x}$$

当 $-\frac{a}{2} < x < \frac{a}{2}$ 时场强为

$$\mathbf{E} = \frac{\eta_e}{2\pi\varepsilon_0(x + \frac{a}{2})}\hat{x} + \frac{\eta_e}{2\pi\varepsilon_0(\frac{a}{2} - x)}\hat{x} = \frac{-a\eta_e}{2\pi\varepsilon_0(x^2 - \frac{a^2}{4})}\hat{x}$$

当 $\dfrac{a}{2}<x$ 时场强为

$$\mathbf{E}=\frac{\eta_e}{2\pi\varepsilon_0(x+\frac{a}{2})}\hat{x}+\frac{\eta_e}{2\pi\varepsilon_0(x-\frac{a}{2})}(-\hat{x})=\frac{-a\eta_e}{2\pi\varepsilon_0(x^2-\frac{a^2}{4})}\hat{x}$$

综上

$$\mathbf{E}=\frac{\eta_e}{2\pi\varepsilon_0(x+\frac{a}{2})}\hat{x}+\frac{\eta_e}{2\pi\varepsilon_0(x-\frac{a}{2})}(-\hat{x})=\frac{-a\eta_e}{2\pi\varepsilon_0(x^2-\frac{a^2}{4})}\hat{x}$$

(2)

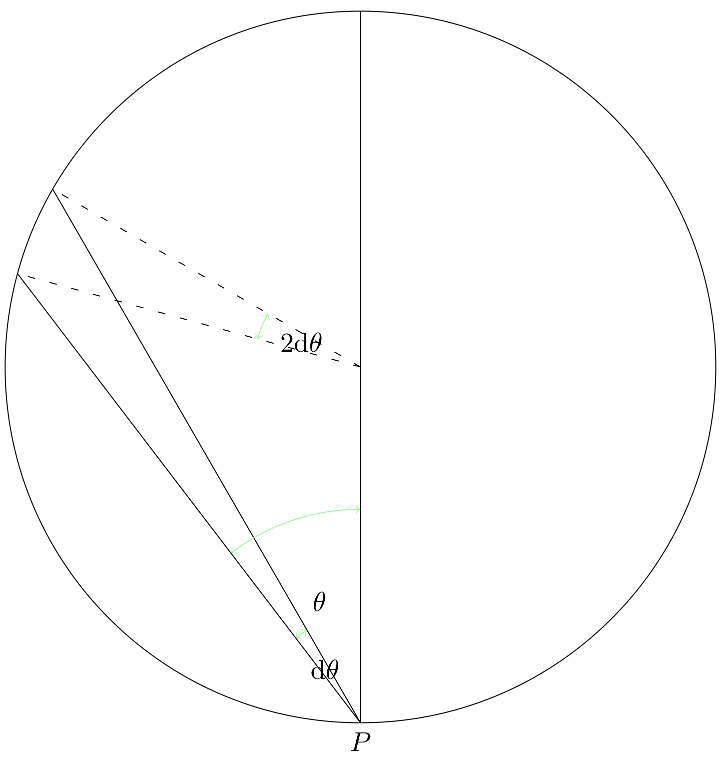
$$F=\eta_e\cdot\frac{\eta_e}{2\pi\varepsilon_0 a}=\frac{\eta_e^2}{2\pi\varepsilon_0 a}$$

1-16

解　由对称性知，场强方向定垂直于轴线，取以轴线为中心线半径为 r 长为 l 的圆柱形高斯面可得

$$2\pi r l E=\frac{q}{\varepsilon_0}$$

又 $r<R$ 时 $q=0$，故 $E=0$。$r>R$ 时 $q=l\lambda$ 故 $E=\dfrac{\lambda}{2\pi\varepsilon_0 r}$。
当 $r=R$ 时高斯定律不适用，可将圆筒沿轴线方向分为无数带电直线



每一带电直线的电荷线密度为 $\lambda_0=\dfrac{2\mathrm{d}\theta}{2\pi}\lambda$，于是在 P 点产生的场强就为

$$\mathrm{d}E=\frac{\lambda_0}{2\pi\varepsilon_0 r}$$

又因为 $r = 2R\cos\theta$ 故

$$\mathrm{d}E = \frac{\lambda \mathrm{d}\theta}{4\pi^2\varepsilon_0 R\cos\theta}$$

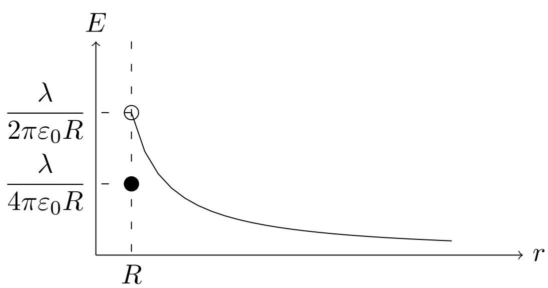
其径向分量为 $\mathrm{d}E\cos\theta = \dfrac{\lambda \mathrm{d}\theta}{4\pi^2\varepsilon_0 R}$ 故

$$E = 2\int_0^{\frac{\pi}{2}} \frac{\lambda \mathrm{d}\theta}{4\pi^2\varepsilon_0 R} = \frac{\lambda}{4\pi\varepsilon_0 R}$$

故

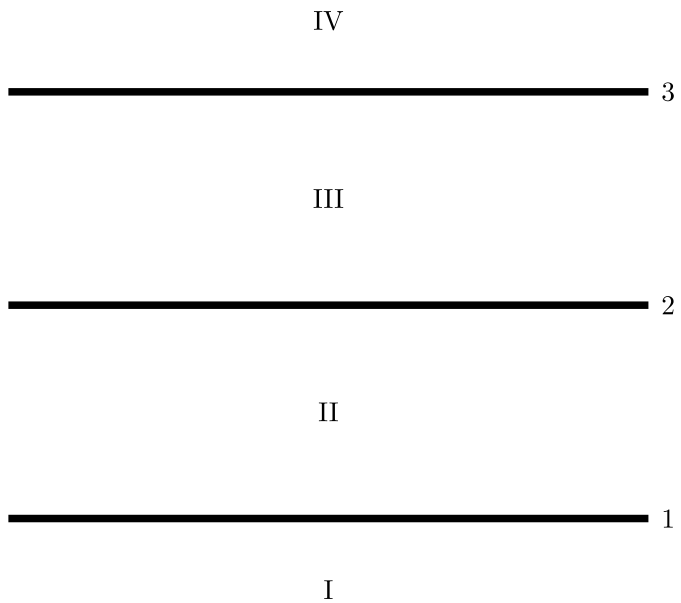
$$E = \begin{cases} 0 & (r < R) \\ \dfrac{\lambda}{4\pi\varepsilon_0 R} & (r = R) \\ \dfrac{\lambda}{2\pi\varepsilon_0 r} & (r > R) \end{cases}$$

$E - r$ 图为



1-19

解　不妨设三个平面如图分布将空间分为 I，II，III，IV 四部分。且令 $\sigma_e > 0$



由场强叠加原理知

(1)$E_i = \dfrac{3\sigma_e}{2\varepsilon_0}, E_{ii} = \dfrac{\sigma_e}{2\varepsilon_0}, E_{iii} = \dfrac{\sigma_e}{2\varepsilon_0}, E_{iv} = \dfrac{3\sigma_e}{2\varepsilon_0}$

方向分别为下下上上

(2)$E_i = \dfrac{\sigma_e}{2\varepsilon_0}, E_{ii} = \dfrac{\sigma_e}{2\varepsilon_0}, E_{iii} = \dfrac{\sigma_e}{2\varepsilon_0}, E_{iv} = \dfrac{\sigma_e}{2\varepsilon_0}$

方向分别为下上下上

(3)$E_i = \dfrac{\sigma_e}{2\varepsilon_0}, E_{ii} = \dfrac{\sigma_e}{2\varepsilon_0}, E_{iii} = \dfrac{\sigma_e}{2\varepsilon_0}, E_{iv} = \dfrac{\sigma_e}{2\varepsilon_0}$

方向分别为上下上下

(4)$E_i = \dfrac{\sigma_e}{2\varepsilon_0}, E_{ii} = \dfrac{3\sigma_e}{2\varepsilon_0}, E_{iii} = \dfrac{\sigma_e}{2\varepsilon_0}, E_{iv} = \dfrac{3\sigma_e}{2\varepsilon_0}$

方向分别为上上上下

1-28

解　设 ne 放在坐标 $(0,0,0)$ 处，$-e$ 放在坐标 $(a,0,0)$ 处

(1) 空间中任一点 (x,y,z) 处电势为

$$\begin{aligned}U &= U_1 + U_2 \\ &= \frac{e}{4\pi\varepsilon_0}\left(\frac{n}{\sqrt{x^2+y^2+z^2}} - \frac{1}{\sqrt{(x-a)^2+y^2+z^2}}\right)\end{aligned}$$

令 $U=0$，则有

$$\frac{n}{\sqrt{x^2+y^2+z^2}} - \frac{1}{\sqrt{(x-a)^2+y^2+z^2}} = 0$$

$$(x-\frac{n^2a}{n^2-1})^2+y^2+z^2=(\frac{na}{n^2-1})^2$$

(2) 该球面球心坐标为 $(\frac{n^2a}{n^2-1},0,0)$, 符合题意。

(3) 该球面半径为 $\frac{na}{n^2-1}$

1-30

解　基态氢原子核外电子满足

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

故动能 $E_k = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r}$, 所以氢原子机械能 $W = E_k + E_p = \frac{e^2}{8\pi\varepsilon_0 r} + \frac{-e^2}{4\pi\varepsilon_0 r} = \frac{-e^2}{8\pi\varepsilon_0 r}$

因此电离能

$$E = -W = 2.18\times10^{-18}J = 13.625eV$$

1-31

解　(1)

$$U = \frac{e}{4\pi\varepsilon_0 r} = 1.4\times10^6 V$$

$$E_k = e\Delta U = 1.4\times10^6 eV$$

(2)

$$E_k = \frac{3}{2}kT$$

$$T = 1.1\times10^{10}K$$

1-33

解　(1) 以 O 为势能零点

$$\begin{aligned}U_P &= \frac{\eta_e}{2\pi\varepsilon_0}\left(\ln\frac{a}{\sqrt{(x-a)^2+y^2}} - \ln\frac{a}{\sqrt{(x+a)^2+y^2}}\right) \\ &= \frac{\eta_e}{4\pi\varepsilon_0}\ln\frac{\sqrt{(x+a)^2+y^2}}{\sqrt{(x-a)^2+y^2}}\end{aligned}$$

(2)

$$U = \frac{\eta_e}{4\pi\varepsilon_0} \ln \frac{\sqrt{(x+a)^2+y^2}}{\sqrt{(x-a)^2+y^2}}$$

$$\frac{\sqrt{(x+a)^2+y^2}}{\sqrt{(x-a)^2+y^2}} = \exp(\frac{4\pi\varepsilon_0 U}{\eta_e}) = k^2$$

$$x^2 + 2a\frac{k^2+1}{1-k^2}x + a^2 + y^2 = 0$$

$$(x - \frac{k^2+1}{k^2-1}a)^2 + y^2 = \frac{4k^2}{(k^2-1)^2}a^2$$

证毕

(3)zOy 平面

1-35

解　(1)

$$\frac{1}{2}mc^2 = \Delta U e$$

$$U = \frac{mc^2}{2e} = 2.5 \times 10^5 V$$

(2)

$$\frac{1}{2}mc^2 = mc^2(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1)$$

$$v = \frac{\sqrt{5}}{3}c = 2.2 \times 10^8 m/s$$

(3)$U \to \infty$,不可能

1-39

解　(1) 取半径为 R 长为 l 的圆柱形高斯面可得

$$E2\pi Rl = \frac{\int_0^R \frac{\rho_0}{[1+(\frac{r}{a})^2]^2}\mathrm{d}V}{\varepsilon_0}$$

$$\mathrm{d}V = 2\pi rldr$$

$$E = \frac{\rho_0 a^2 R}{2\varepsilon_0(a^2+R^2)}$$

$$E(r) = \frac{\rho_0 a^2 r}{2\varepsilon_0(a^2+r^2)}$$

(2)

$$U(r) = \int_r^0 Edr = \frac{\rho_0 a^2}{4\varepsilon_0} \ln \frac{a^2}{a^2+r^2}$$

1-41

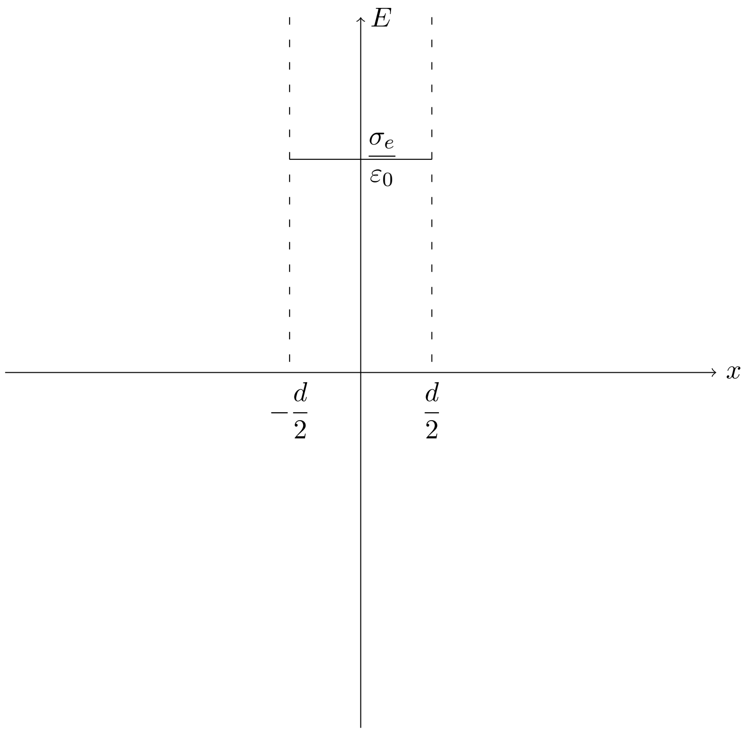
解　由场强叠加原理知

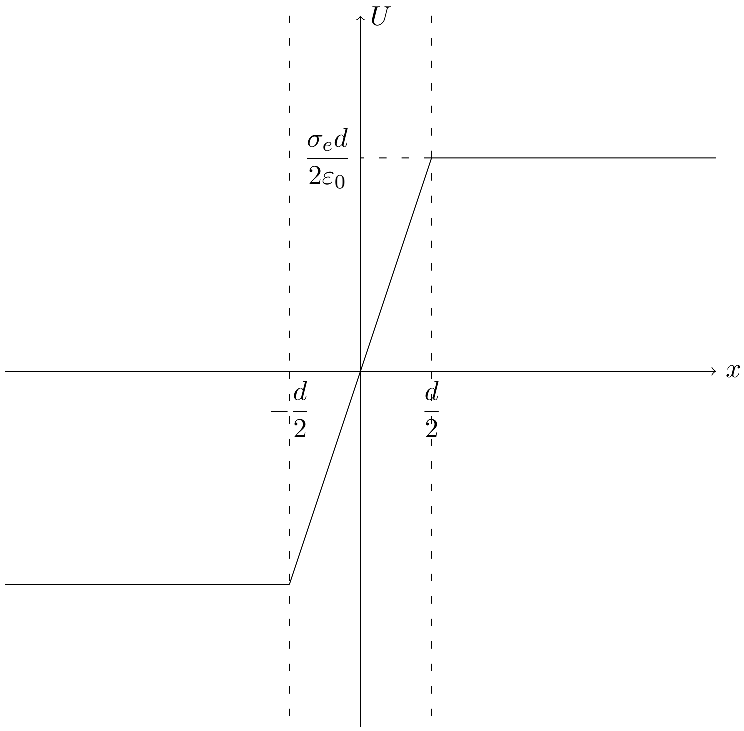
$$E=\begin{cases}0 & (x<-\dfrac{d}{2})\\ \dfrac{\sigma_e}{\varepsilon_0} & (-\dfrac{d}{2}<x<\dfrac{d}{2})\\ 0 & (x>\dfrac{d}{2})\end{cases}$$

又因为 O 处电势为 0

$$U(x)=\int_x^0 -E\mathrm{d}x=\frac{\sigma_e}{\varepsilon_0}x$$

其 $E-x$ 与 $U-x$ 图为



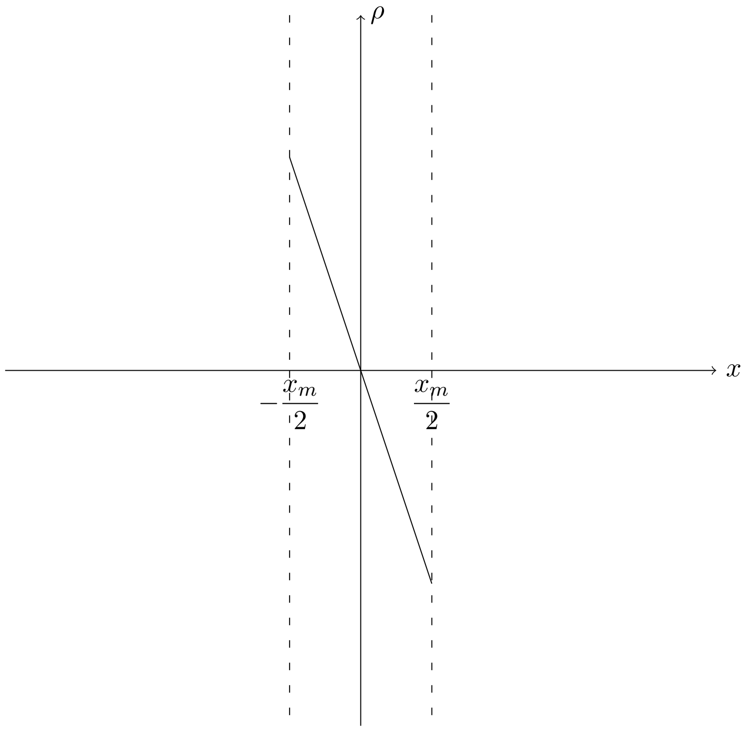


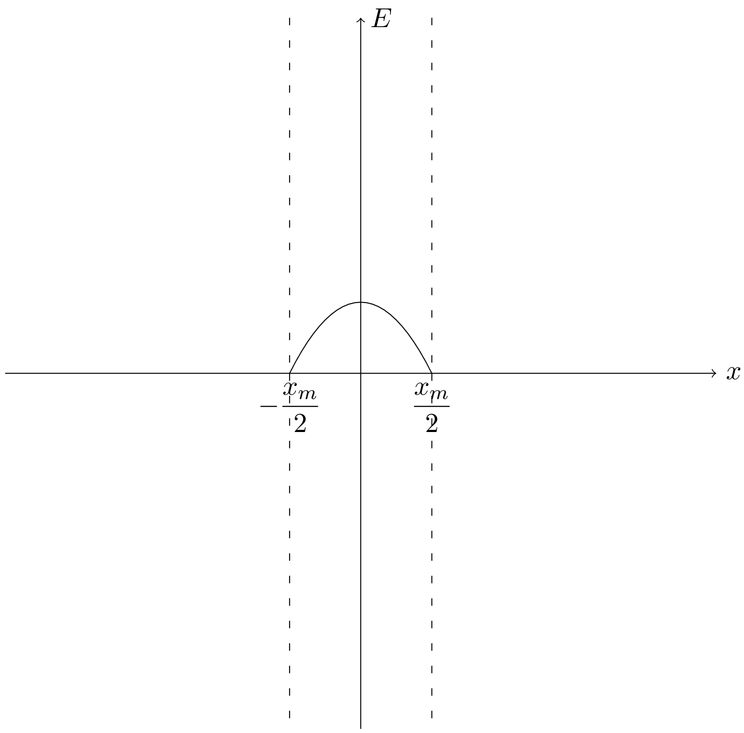
1-43

解　(1) 由高斯定理

$$E\cdot S=\frac{S}{\varepsilon_0}\int_{-x_N}^{X}\rho\mathrm{d}x$$

$$E=\frac{ea}{8\varepsilon_0}(x_m^2-4x^2)$$





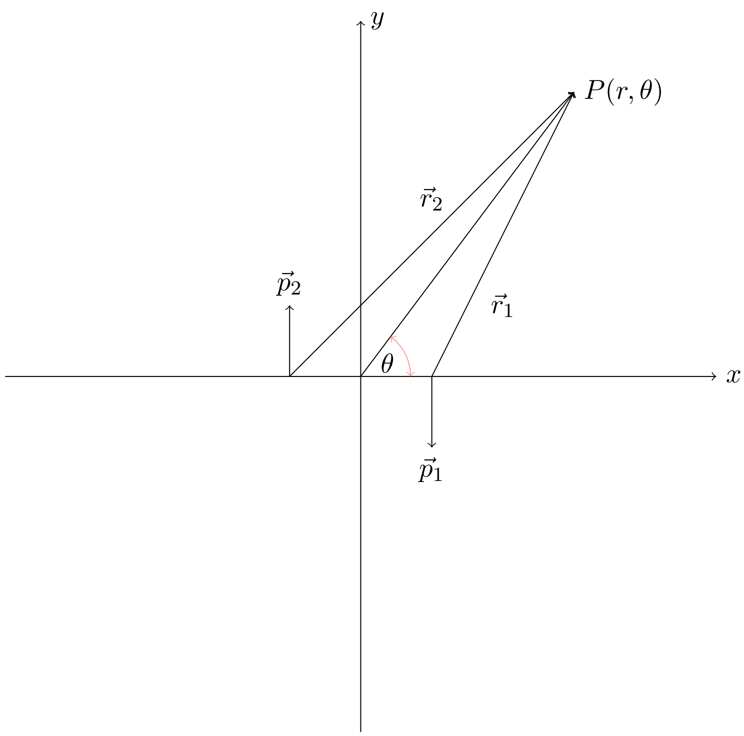
(2) 令 $U=0$, 则 $x=0$,该电势以原点为零点,$\Delta U = U(\frac{x_m}{2}) - U(-\frac{x_m}{2}) = -\frac{eax_m^2}{12\varepsilon_0}$

1-44

解 同一电场线上任取 AB 两点，过 AB 两点作底面积无限小的柱形高斯面，因为该面中无电荷 $E_AS = E_BS$ 故 $E_A = E_B$，在不同电场线任取 AC 作闭合矩形回路，因为场强环路积分为 0，故 $E_Al = E_Cl$，故 $E_A = E_C$，又因为 ABC 均是任取的，故处处场强相等。

附加题 1

解 该电四极子可视为两个电偶极子叠加，其电偶极距分别为 $\vec{p}_1$,$\vec{p}_2$



$$\begin{aligned} U_1 &= \frac{\vec{p}_1 \cdot \vec{r}_1}{4\pi\varepsilon_0 r_1^3} \\ &= \frac{-rql\sin\theta}{4\pi\varepsilon_0}\frac{1}{(r^2+\frac{l^2}{4}-rl\cos\theta)^{\frac{3}{2}}} \end{aligned}$$

$$\begin{aligned} U_2 &= \frac{\vec{p}_2 \cdot \vec{r}_2}{4\pi\varepsilon_0 r_2^3} \\ &= \frac{rql\sin\theta}{4\pi\varepsilon_0}\frac{1}{(r^2+\frac{l^2}{4}+rl\cos\theta)^{\frac{3}{2}}} \end{aligned}$$

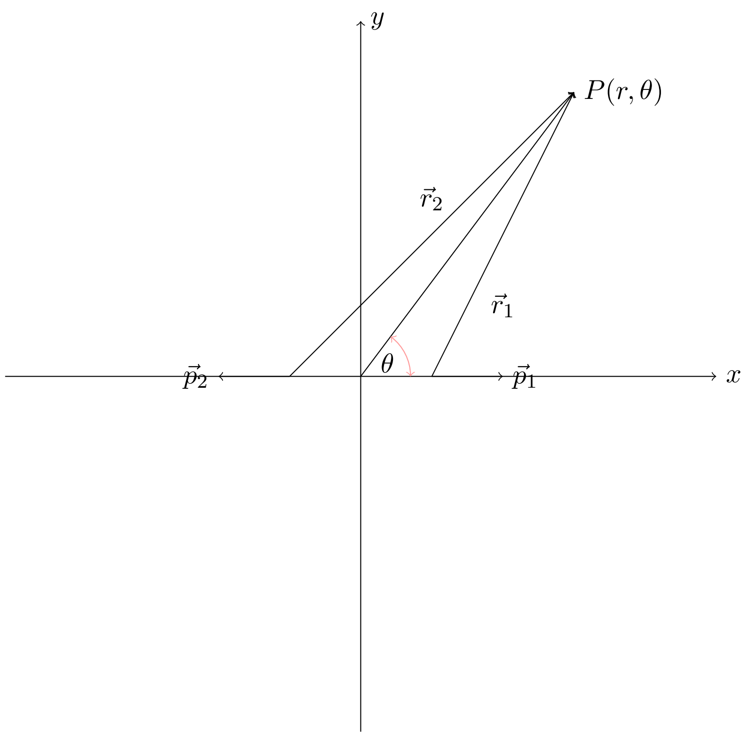
$$\begin{aligned} U_P &= U_1 + U_2 \\ &= \frac{-rql\sin\theta}{4\pi\varepsilon_0}[\frac{1}{(r^2+\frac{l^2}{4}-rl\cos\theta)^{\frac{3}{2}}} - \frac{1}{(r^2+\frac{l^2}{4}+rl\cos\theta)^{\frac{3}{2}}}] \end{aligned}$$

因为 $l \ll r$, 故略去二阶小量 $\frac{l^2}{4}$, 且运用近似 $(1+x)^k = 1+kx(x \ll 1)$ 可得

$$U_P = \frac{-rql\sin\theta}{4\pi\varepsilon_0}\left(\frac{1+\frac{3}{2}\frac{l\cos\theta}{r} - 1 + \frac{3}{2}\frac{l\cos\theta}{r}}{r^3}\right) = \frac{-3ql\sin\theta\cos\theta}{4\pi\varepsilon_0 r^3}$$

附加题 2

解　该电四极子可视为两个电偶极子叠加,其电偶极距分别为 $\vec{p}_1$,$\vec{p}_2$



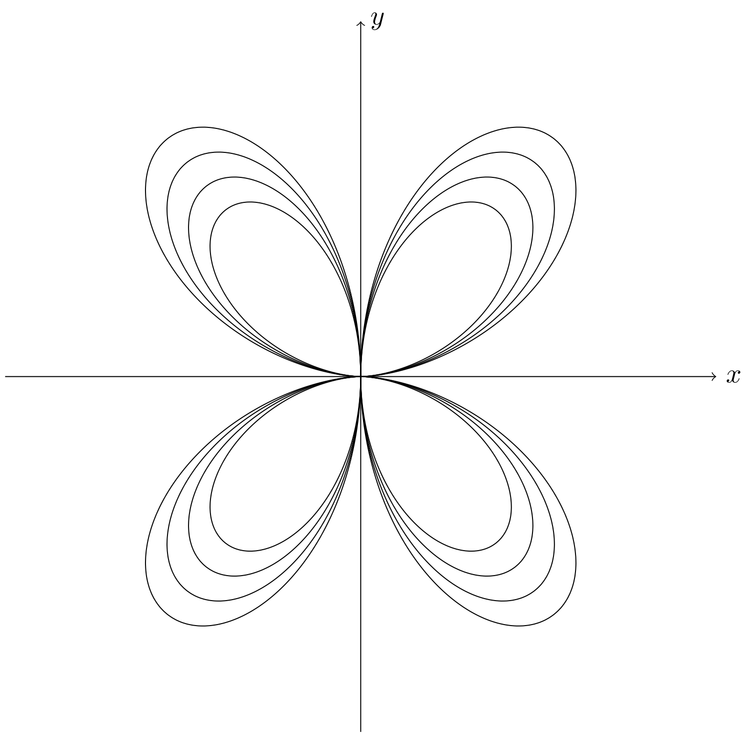
$$\begin{aligned} U_1 &= \frac{\vec{p}_1 \cdot \vec{r}_1}{4\pi\varepsilon_0 r_1^3} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{r\cos\theta ql - \frac{ql^2}{2}}{(r^2 + rl\cos\theta + \frac{l^2}{4})^{\frac{3}{2}}} \end{aligned}$$

$$\begin{aligned} U_2 &= \frac{\vec{p}_2 \cdot \vec{r}_2}{4\pi\varepsilon_0 r_2^3} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{r\cos\theta ql + \frac{ql^2}{2}}{(r^2 + rl\cos\theta - \frac{l^2}{4})^{\frac{3}{2}}} \end{aligned}$$

$$
\begin{aligned}
U_P &= U_1 + U_2 \\
&= \frac{r\cos\theta ql}{4\pi\varepsilon_0 r^3}[(r\cos\theta ql - \frac{ql^2}{2})(1+\frac{3l\cos\theta}{2r}) - (r\cos\theta ql + \frac{ql^2}{2})(1-\frac{3l\cos\theta}{2r})] \\
&= \frac{ql^2(3\cos^2\theta - 1)}{4\pi\varepsilon_0 r^3} = \frac{D(3\cos^2\theta - 1)}{8\pi\varepsilon_0 r^3}
\end{aligned}
$$

$$
\begin{aligned}
\vec{E} &= -\nabla U \\
&= -\frac{\partial U}{\partial r}\hat{e}_r - \frac{\partial U}{r\partial\theta}\hat{e}_\theta \\
&= \frac{3ql^2}{4\pi\varepsilon_0 r^4}[(3\cos^2\theta - 1)\hat{e}_r + 2\sin\theta\cos\theta\hat{e}_\theta]
\end{aligned}
$$

电场线如图



1-45

解　(1)

$$\vec{E}_A=\frac{\sigma_e}{2\varepsilon_0}\hat{x}$$

(2)

$$\vec{E}_B=\frac{\sigma_e}{2\varepsilon_0}\hat{x}$$

(3)

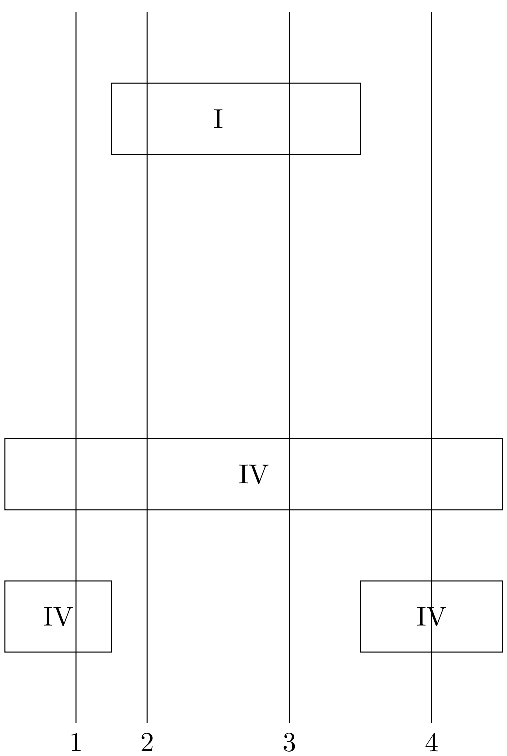
$$\vec{E}=\vec{E}_A+\vec{E}_B=\frac{\sigma_e}{\varepsilon_0}\hat{x}$$

(4) 均匀分布在平板左右两侧

$$E'_A=\frac{\sigma_e}{2\varepsilon_0}\hat{x}$$

1-46

解



(1) 取如 I 所示高斯面,因为导体内部电场为 0,两平行板中间电场与高斯面平行,故该高斯面电通量为 0。故其中没有静电荷即

$$\sigma_2 S+\sigma_3 S=0\rightarrow\sigma_1=-\sigma_2$$

即两平板相向两面的电荷面密度大小相等符号相反。

(2) 取如 II 所示高斯面，由场强叠加原理知该高斯面左右两侧场强大小相等，方向相反。故知 III, IV 两高斯面电通量相等，故两高斯面内静电荷量相等即

$$\sigma_1 S = \sigma_4 S \to \sigma_1 = \sigma_4$$

即两平板相背两面的电荷面密度大小相等符号相反。

(3)

$$\begin{cases} \sigma_1 + \sigma_2 = 3 \\ \sigma_3 + \sigma_4 = 7 \\ \sigma_1 = \sigma_4 \\ \sigma_2 + \sigma_3 = 0 \end{cases}$$

解得

$$\sigma_1 = 5\mu\mathrm{C/m^2}, \sigma_2 = -2\mu\mathrm{C/m^2}, \sigma_3 = 2\mu\mathrm{C/m^2}, \sigma_4 = 5\mu\mathrm{C/m^2}$$

1-52

解 (1)

$$U_2 = \int_{\infty}^{R_3} \frac{q+Q}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r = \frac{q+Q}{4\pi\varepsilon_0 R_3}$$

$$U_1 = U_2 + \int_{R_2}^{R_1} \frac{q}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r = \frac{q+Q}{4\pi\varepsilon_0 R_3} + \frac{q}{4\pi\varepsilon_0}(\frac{1}{R_1} - \frac{1}{R_2})$$

(2)

$$\Delta U = U_1 - U_2 = \frac{q}{4\pi\varepsilon_0}(\frac{1}{R_1} - \frac{1}{R_2})$$

(3)

$$U_1 = U_2 = \frac{q+Q}{4\pi\varepsilon_0 R_3}$$

$$\Delta U = 0$$

(4) 情形 (1):

$$U_2 = 0$$

$$U_1 = U_2 + \int_{R_2}^{R_1} \frac{q}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r = \frac{q}{4\pi\varepsilon_0}(\frac{1}{R_1} - \frac{1}{R_2})$$

$$\Delta U = \int_{R_2}^{R_1} \frac{q}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r = \frac{q}{4\pi\varepsilon_0}(\frac{1}{R_1} - \frac{1}{R_2})$$

情形 (2):

$$U_2 = 0$$

$$U_1 = U_2 = 0$$

$$\Delta U = 0$$

(5) 设平衡后球体所带电荷为 q' 则球壳内表面所带电荷为 $-q'$，外表面所带电荷为 $Q+q'$ 则球壳电势为

$$U_2 = \int_{\infty}^{R_3} \frac{Q+q'}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r = \frac{Q+q'}{4\pi\varepsilon_0 R_3}$$

则球体电势为

$$U_1 = U_2 + \int_{R_2}^{R_1} \frac{q'}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r = \frac{Q+q'}{4\pi\varepsilon_0 R_3} + \frac{q'}{4\pi\varepsilon_0}(\frac{1}{R_1} - \frac{1}{R_2})$$

又因为球体接地，故

$$U_1 = 0$$

解得

$$q' = \frac{Q}{R_3}(\frac{1}{R_2} - \frac{1}{R_1} - \frac{1}{R_3})$$

于是有

$$\begin{aligned} U_1 &= 0 \\ U_2 &= \frac{1}{4\pi\varepsilon_0}\frac{(R_2-R_1)Q}{R_1R_2+R_2R_3-R_3R_1} \\ \Delta U = U_1 - U_2 &= \frac{1}{4\pi\varepsilon_0}\frac{(R_1-R_2)Q}{R_1R_2+R_2R_3-R_3R_1} \end{aligned}$$

1-57

解　(1) 设上极板带电 Q, 则中间导体上表面带电 $-Q$, 下表面带电 Q, 下极板带电 $-Q$, 则电容器中间除导体内部的区域的场强为

$$E = \frac{Q}{\varepsilon_0 S}$$

则两极板电势差为

$$U = E(d-t)$$

故

$$C = \frac{Q}{U} = \frac{\varepsilon_0 S}{d-t}$$

(2) 上面讨论与极板位置无关，故远近无影响。

1-62

解　(1)

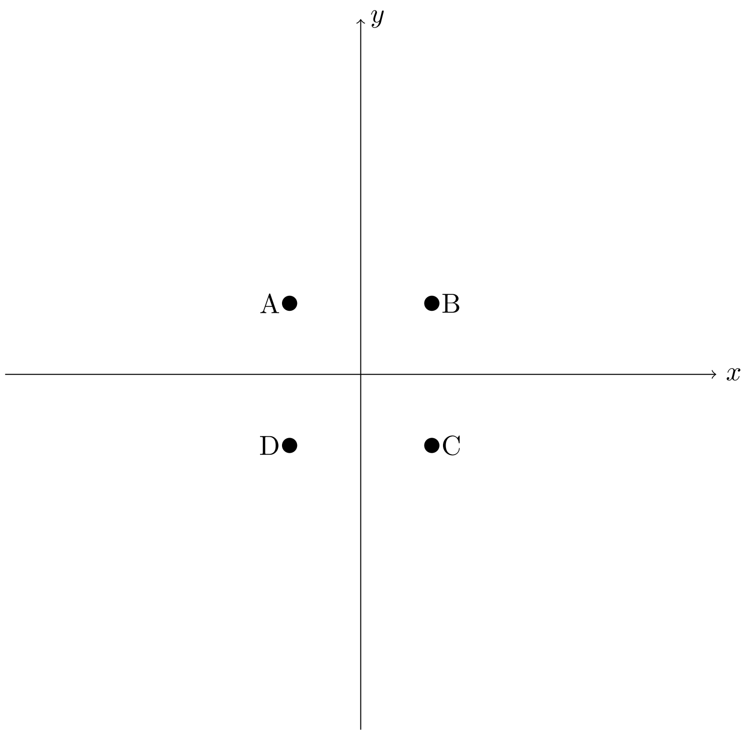
$$\begin{aligned} U &= \int_{R_1}^{R_2} \frac{Q}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r + \int_{R_3}^{R_4} \frac{Q}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r \\ &= \frac{Q}{4\pi\varepsilon_0}(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3} - \frac{1}{R_4}) \end{aligned}$$

(2)

$$C = \frac{Q}{U} = \frac{4\pi\varepsilon_0}{\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3} - \frac{1}{R_4}}$$

附加题

解　因为导线无限长由对称性可知，电荷在导线上均匀分布，设线密度大小为 λ 又电



场方向向右，故 AD 带 $-\lambda$，BC 带 λ。以 A 为电势零点则 B 电势为

$$U_B = -E_0 a + \frac{-\lambda}{2\pi\varepsilon_0}(\ln r - \ln a) + \frac{-\lambda}{2\pi\varepsilon_0}(\ln a - \ln\sqrt{2}a) + \frac{\lambda}{2\pi\varepsilon_0}(\ln\sqrt{2}a - \ln a) + \frac{\lambda}{2\pi\varepsilon_0}(\ln a - \ln r)$$
$$= -E_0 a + \frac{\lambda}{\pi\varepsilon_0}(\ln\sqrt{2}a - \ln r)$$

又因为 $U_B = U_A = 0$ 解得

$$\lambda = \frac{\pi\varepsilon_0 E_0 a}{\ln\sqrt{2}a - \ln r}$$

故 x 轴上场强分布为

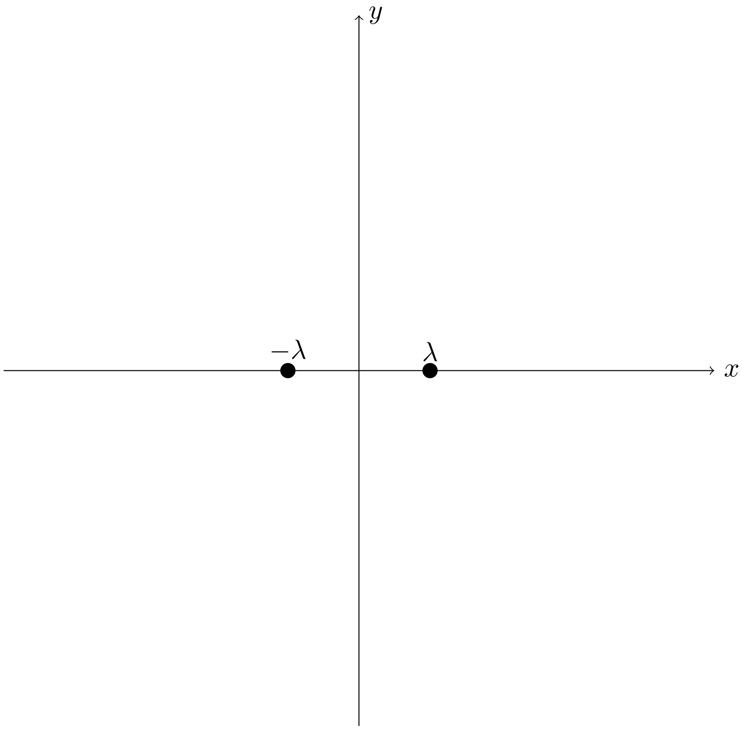
$$E(x) = E_0 + 2\frac{\lambda}{2\pi\varepsilon_0\sqrt{\frac{a^2}{4} + (x - \frac{a}{2})^2}}\frac{x - \frac{a}{2}}{\sqrt{\frac{a^2}{4} + (x - \frac{a}{2})^2}} - 2\frac{\lambda}{2\pi\varepsilon_0\sqrt{\frac{a^2}{4} + (x - \frac{a}{2})^2}}\frac{x + \frac{a}{2}}{\sqrt{\frac{a^2}{4} + (x + \frac{a}{2})^2}}$$
$$= E_0\left\{1 + \frac{a(x - \frac{a}{2})}{\ln\frac{\sqrt{2}a}{r}[\frac{a^2}{4} + (x - \frac{a}{2})^2]} - \frac{a(x + \frac{a}{2})}{\ln\frac{\sqrt{2}a}{r}[\frac{a^2}{4} + (x + \frac{a}{2})^2]}\right\}$$

代入数值得

$$E(x) = 1 + \frac{x - 0.005}{\ln(100\sqrt{2})[0.25 + (x - 0.005)^2]} - \frac{x + 0.005}{\ln(100\sqrt{2})[0.25 + (x + 0.005)^2]}\mathrm{V/m}$$

1-65

解　由电像法可将电场分布视为两无限长导线产生的电场，以地面为 $x=0$ 平面建立如图坐标系 设地面电势为 0，导线离地面距离为 a，导线上电荷线密度为 λ 则



$$U(x,y)=-\int_a^{\sqrt{(x-a)^2+y^2}}\frac{\lambda}{2\pi r\varepsilon_0}\,\mathrm{d}r+(-\int_a^{\sqrt{(x+a)^2+y^2}}\frac{-\lambda}{2\pi r\varepsilon_0}\,\mathrm{d}r)=\frac{\lambda}{4\pi\varepsilon_0}\ln\frac{(x+a)^2+y^2}{(x-a)^2+y^2}$$

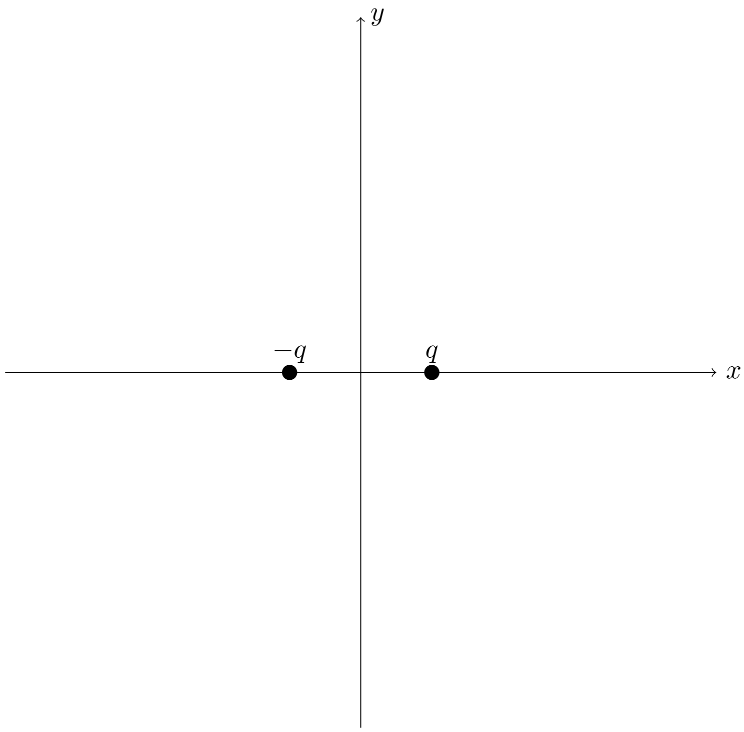
$$\begin{aligned}\vec{E}&=-\nabla U\\&=-\frac{\partial U}{\partial x}\hat{x}-\frac{\partial U}{\partial y}\hat{y}\\&=\frac{\lambda}{2\pi\varepsilon_0}[\frac{x-a}{(x-a)^2+y^2}-\frac{x+a}{(x+a)^2+y^2}]\hat{x}+\frac{\lambda}{\pi\varepsilon_0}\frac{2axy}{[(x+a)^2+y^2][(x-a)^2+y^2]}\hat{y}\end{aligned}$$

令 $x=0$ 则 $E=\dfrac{-\lambda a}{\pi\varepsilon_0(y^2+a^2)}$ 那么地面上的电荷密度为

$$\sigma=E\varepsilon_0=\frac{-\lambda a}{\pi(y^2+a^2)}$$

附加题 1

解　由电像法可将电场分布视为两点电荷产生的电场，以导体平面为 $x=0$ 平面建立如图坐标系



则平面上场强分布为

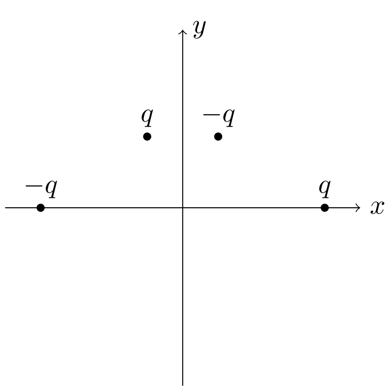
$$E = 2\frac{\sqrt{2}}{2}\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2+y^2}\frac{a}{\sqrt{a^2+y^2}} = \frac{\sqrt{2}aq}{4\pi\varepsilon_0(a^2+y^2)^{\frac{3}{2}}}$$

则电荷分布为

$$\sigma = \varepsilon_0 E = \frac{\sqrt{2}aq}{4\pi(a^2+y^2)^{\frac{3}{2}}}$$

附加题 2

解　由电像法可将电场分布视为四个点电荷产生的电场, 以导体平面为 $x=0$ 平面建立如图坐标系

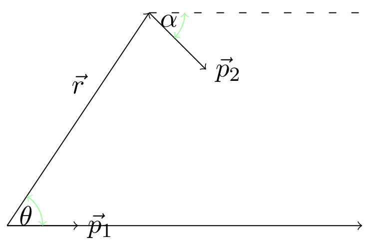


则电势分布为

$$U = \frac{q}{4\pi\varepsilon_0}[\frac{1}{\sqrt{(x-d_2)^2+y^2}}+\frac{1}{\sqrt{(x+d_1)^2+(y-d_0)^2}}-\frac{1}{\sqrt{(x+d_2)^2+y^2}}-\frac{1}{\sqrt{(x-d_1)^2+(y-d_0)^2}}]$$

附加题 1

解



$\vec{p}_1$ 在 $\vec{p}_2$ 处产生的电势为

$$\begin{aligned} U &= \frac{\vec{p}_1 \cdot \vec{r}}{4\pi\varepsilon_0 r^3} \\ &= \frac{p_1}{4\pi\varepsilon_0} \frac{x}{(x^2+y^2)^{\frac{3}{2}}} \end{aligned}$$

故场强为

$$\begin{aligned} \vec{E} &= -\nabla U \\ &= -\frac{\partial U}{\partial x}\hat{x} - \frac{\partial U}{\partial y}\hat{y} \\ &= \frac{p_1}{4\pi\varepsilon_0} \frac{2x^2-y^2}{(x^2+y^2)^{\frac{5}{2}}}\hat{x} + \frac{p_1}{4\pi\varepsilon_0} \frac{3xy}{(x^2+y^2)^{\frac{5}{2}}}\hat{y} \end{aligned}$$

又 $\vec{p}_2 = p_2\cos\alpha\hat{x} - p_2\sin\alpha\hat{y}$ 故相互作用能

$$\begin{aligned} W &= -\vec{p}_2 \cdot \vec{E} \\ &= -[\frac{p_1p_2\cos\alpha}{4\pi\varepsilon_0} \frac{2x^2-y^2}{(x^2+y^2)^{\frac{5}{2}}} - \frac{p_1p_2\sin\alpha}{4\pi\varepsilon_0} \frac{3xy}{(x^2+y^2)^{\frac{5}{2}}}] \end{aligned}$$

代入 $x = r\cos\theta, y = r\sin\theta$ 得

$$W = \frac{p_1p_2}{4\pi\varepsilon_0 r^3}\left[\sin\theta\sin(\theta+\alpha) - 2\cos\theta\cos(\theta+\alpha)\right]$$

故

$$\begin{aligned} \vec{F} &= -\nabla W \\ &= -\frac{\partial W}{\partial r}\hat{e}_r - \frac{\partial W}{r\partial\theta}\hat{e}_\theta \\ &= \frac{p_1p_2}{4\pi\varepsilon_0} \frac{3[\sin\theta\sin(\alpha+\theta) - 2\cos\theta\cos(\alpha+\theta)]}{r^4}\hat{e}_r - \frac{p_1p_2}{4\pi\varepsilon_0} \frac{3\sin\theta\cos(\alpha+\theta) + 3\cos\theta\sin(\alpha+\theta)}{r^4}\hat{e}_\theta \end{aligned}$$

附加题 2

解 设电子经典半径为 a, 因为电荷在其中均匀分布，故其电荷体密度 $\rho = \frac{3\mathrm{e}}{4\pi a^3}$ 取半径为 r 的球形高斯面，当 $r < a$ 时可得

$$4\pi r^2 E = \frac{\rho \frac{4\pi r^3}{3}}{\varepsilon_0} \rightarrow E = \frac{\rho r}{3\varepsilon_0}$$

当 $r > a$ 时可得

$$4\pi r^2 E = \frac{\mathrm{e}}{\varepsilon_0} \rightarrow E = \frac{\mathrm{e}}{4\pi r^2 \varepsilon_0}$$

则其自能为

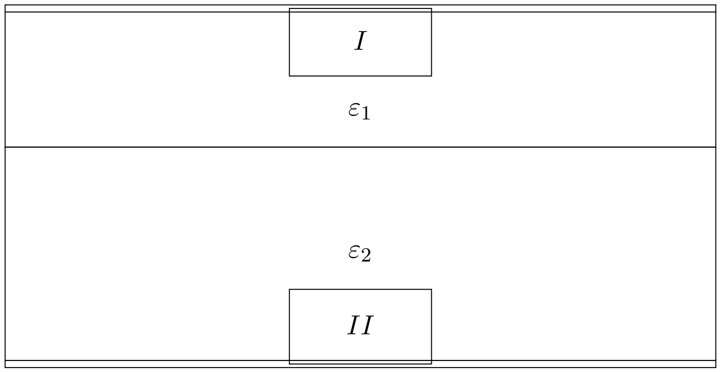
$$\begin{aligned} W &= \frac{\varepsilon_0}{2} \iiint E^2 \,\mathrm{d}V \\ &= \int_0^a \frac{\varepsilon_0}{2} (\frac{\rho r}{3\varepsilon_0})^2 4\pi r^2 \,\mathrm{d}r + \int_a^\infty \frac{\varepsilon_0}{2} (\frac{\mathrm{e}}{4\pi\varepsilon_0 r^2}) \,\mathrm{d}r \\ &= \frac{3\mathrm{e}^2}{20\pi\varepsilon_0 a} \end{aligned}$$

则

$$\begin{aligned} W &= m_{\mathrm{e}}\mathrm{c}^2 \\ \frac{3\mathrm{e}^2}{20\pi\varepsilon_0 a} &= m_{\mathrm{e}}\mathrm{c}^2 \\ a &= \frac{3\mathrm{e}^2}{20\pi\varepsilon_0 m_{\mathrm{e}}\mathrm{c}^2} = 1.69 \times 10^{-15}\mathrm{m} \end{aligned}$$

4-3

解



(1) 设上极板所带电荷面密度为 σ，取如图所示二高斯面可知 $D=\sigma$, 则

$$E_1=\frac{D}{\varepsilon_0\varepsilon_1}$$
$$E_2=\frac{D}{\varepsilon_0\varepsilon_2}$$

又

$$E_1d_1+E_2d_2=U$$

解得 $\sigma=4.66\times10^{-5}\mathrm{C/m^2}$ 故

$$P_1=\frac{\varepsilon_1-1}{\varepsilon_1}\sigma=3.7\times10^{-5}\mathrm{C/m^2}$$
$$P_1=\frac{\varepsilon_2-1}{\varepsilon_2}\sigma=1.6\times10^{-5}\mathrm{C/m^2}$$

(2)

$$U=E_2d_2=7.9\times10^{3}\mathrm{V}$$

4-6

解　由对称性知，两平行板之间电场应垂直于导体板，亦即互相平行，故其中间为匀强电场设场强为 E 故有

$$\sigma_1=\varepsilon_0\varepsilon_1E$$

$$\sigma_2=\varepsilon_0\varepsilon_2E$$

又 $Q=\sigma_1S_1+\sigma_2S_2, U=Ed$，故电容为

$$\begin{aligned}C&=\frac{Q}{U}\\&=\frac{(\varepsilon_1S_1+\varepsilon_2S_2)\varepsilon_0}{d}\end{aligned}$$

4-9

解 (1)

$$E = \begin{cases} \dfrac{Q}{4\pi\varepsilon\varepsilon_0 r^2} & R < r < R' \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & r < R' \end{cases}$$

(2)

$$U = \begin{cases} \displaystyle\int_r^{R'} E\,\mathrm{d}r + \int_{R'}^{\infty} E\,\mathrm{d}r = \frac{Q}{4\pi\varepsilon\varepsilon_0}(\frac{1}{r} + \frac{\varepsilon - 1}{R'}) & R < r < R' \\ \displaystyle\int_r^{\infty} E\,\mathrm{d}r = \frac{Q}{4\pi\varepsilon_0 r} & r > R' \end{cases}$$

(3)

$$U = \frac{Q}{4\pi\varepsilon\varepsilon_0}(\frac{1}{R} + \frac{\varepsilon - 1}{R'})$$

4-12

解 $D = \dfrac{Q}{4\pi r^2}$ 故

$$E_1 = \frac{D}{\varepsilon_1\varepsilon_0}$$

$$E_2 = \frac{D}{\varepsilon_2\varepsilon_0}$$

故两极板间电势差为

$$U = \int_{R_1}^{R} E_1\,\mathrm{d}r + \int_{R}^{R_2} E_2\,\mathrm{d}r = \frac{Q}{4\pi\varepsilon_0}\left[(\frac{1}{\varepsilon_1 R_1} - \frac{1}{\varepsilon_1 R}) + (\frac{1}{\varepsilon_2 R} - \frac{1}{\varepsilon_2 R_2})\right]$$

则电容为

$$C = \frac{Q}{U} = \frac{4\pi\varepsilon_0}{(\frac{1}{\varepsilon_1 R_1} - \frac{1}{\varepsilon_1 R}) + (\frac{1}{\varepsilon_2 R} - \frac{1}{\varepsilon_2 R_2})}$$

(2)

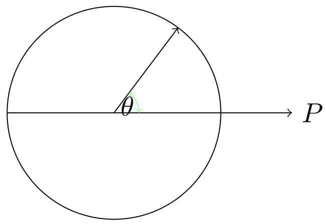
$$\sigma(R_1) = P_1 = \frac{(\varepsilon_1 - 1)Q}{4\pi\varepsilon_1 R_1^2}$$

$$\sigma(R) = \frac{(\varepsilon_2 - 1)Q}{4\pi\varepsilon_2 R^2} - \frac{(\varepsilon_1 - 1)Q}{4\pi\varepsilon_1 R^2} = \frac{(\varepsilon_2 - \varepsilon_1)Q}{4\pi\varepsilon_1\varepsilon_2 R^2}$$

$$\sigma(R_2) = -\frac{(\varepsilon_2 - 1)Q}{4\pi\varepsilon_2 R_2^2}$$

4-19

解



轴线处场强由分界面内部和外部电荷共同作用产生，界面内部极化电荷分布在圆柱表面可看作多个无限长带电直线叠加，极矩 $P = (\varepsilon - 1)\varepsilon_0 E_0$, 则极化电荷面密度为 $P\cos\theta$, 线密度就为 $\lambda = P\cos\theta r\,\mathrm{d}\theta$, 又因为系统的对称性，故可知何场强方向一定与 P 方向共线则其在轴线处的场强大小为

$$E = \int_0^{2\pi} \frac{\lambda\cos\theta}{2\pi R\varepsilon_0} = \frac{P}{2\pi\varepsilon_0}\int_0^{2\pi} \cos^2\theta\,\mathrm{d}\theta = \frac{P}{2\varepsilon_0} = \frac{\varepsilon - 1}{2}E_0$$

又因为该场强与 E_0 方向相反故

$$E = E_0 + \frac{\varepsilon - 1}{2}E_0 = \frac{\varepsilon + 1}{2}E_0$$

真挖去后不成立，因为极化不再均匀

4-59

解　(1) 插入前:

$$E_0 = \frac{Q^2}{2C_0} = \frac{Q^2 d}{2\varepsilon_0 S}$$

插入后:

$$E = \frac{Q^2}{2C} = \frac{Q^2 d}{2\varepsilon\varepsilon_0 S}$$

故

$$\Delta E = E - E_0 = \frac{Q^2 d}{2\varepsilon_0 S}(\frac{1}{\varepsilon} - 1)$$

(2) 设介质板面积为 $S = ab$ 插入深度为 x 则

$$C = C_1 + C_2 = \frac{\varepsilon\varepsilon_0 bx}{d} + \frac{\varepsilon_0 b(a-x)}{d} = \frac{\varepsilon_0 b}{d}[a + (\varepsilon - 1)x]$$

$$W = \frac{Q^2}{2C} = \frac{Q^2 d}{2b\varepsilon_0[a + (\varepsilon - 1)x]}$$

$$F = -\frac{\mathrm{d}W}{\mathrm{d}x} = \frac{Q^2 d(\varepsilon - 1)}{2b\varepsilon_0[a + (\varepsilon - 1)x]^2}$$

故做功为

$$A = \int_0^a F\,\mathrm{d}x = \frac{Q^2 d}{2\varepsilon_0 S}(1 - \frac{1}{\varepsilon})$$

4-60

解　(1) 插入前:

$$E_0 = \frac{C_0 U^2}{2} = \frac{\varepsilon_0 S U^2}{2d}$$

插入后:

$$E = \frac{C_0 U^2}{2} = \frac{\varepsilon\varepsilon_0 S U^2}{2d}$$

故

$$\Delta E = E - E_0 = \frac{\varepsilon_0 S U^2}{2d}(\varepsilon - 1)$$

(2)

$$\Delta Q = CU - C_0 U = \frac{\varepsilon_0 S U}{\varepsilon - 1}$$

故电源移动电荷做功为

$$W = \Delta Q U = \frac{\varepsilon_0 S U^2}{\varepsilon - 1}$$

(3) 与上题分析类似可得

$$A = \frac{\varepsilon_0 S U^2(\varepsilon - 1)}{2d}$$

4-61

解

$$C = C_1 + C_2 = \frac{\varepsilon\varepsilon_0 ax}{d} + \frac{\varepsilon_0 a(a-x)}{d} = \frac{\varepsilon_0 a}{d}[a + (\varepsilon - 1)x]$$

$$W = \frac{Q^2}{2C} = \frac{Q^2 d}{2a\varepsilon_0[a + (\varepsilon - 1)x]}$$

$$F = -\frac{\mathrm{d}W}{\mathrm{d}x} = \frac{Q^2 d(\varepsilon - 1)}{2a\varepsilon_0[a + (\varepsilon - 1)x]^2}$$

令 $x = \dfrac{a}{2}$ 得 $F = \dfrac{2(\varepsilon - 1)Q^2 d}{\varepsilon_0(\varepsilon + 1)^2 a^3}$

4-62

解　并联总电容 $C = C_1 + C_2 = \dfrac{\varepsilon_0 S}{d} + \dfrac{\varepsilon\varepsilon_0 S}{d} = \dfrac{\varepsilon_0 S}{d}(\varepsilon + 1)$ 则总能量为

$$W = \frac{CU^2}{2} = 5.4 \times 10^{-5}\mathrm{J}$$

中间是空气的电容器两端电荷为 $Q_1 = C_1 U = \dfrac{\varepsilon_0 SU}{d}$, 中间插入酒精的极板两端电荷为 $Q_2 = C_2 U = \dfrac{\varepsilon\varepsilon_0 SU}{d}$ 则用导线连接后总电荷为 $Q = Q_2 - Q_1$, 总能量为

$$E = \frac{Q^2}{2C} = 4.6 \times 10^{-5}\mathrm{J}$$

$$\Delta E = W - E = 7.8 \times 10^{-6}\mathrm{J}$$

损失的能量部分转换为导线产生的焦耳热，部分转换为电磁波辐射到了外界

$$\mathbb{V}, \mathbf{V}, \mathrm{V}, V, \mathcal{V}, \nu, \mathsf{V}, \mathrm{V}, V$$

1-66

解　(1)

$$j = \frac{I}{S} = \sigma E$$

故

$$E_1 = \frac{I}{\sigma_1 S}$$

$$E_2 = \frac{I}{\sigma_2 S}$$

(2)

$$U_{AB} = E_1 d_1 = \frac{I d_1}{\sigma_1 S}$$

$$U_{AB} = E_2 d_2 = \frac{I d_2}{\sigma_2 S}$$

2-3(思考题)

解

$$\begin{aligned}\vec{F}_{12} &= \frac{\mu_0}{4\pi} \oint\limits_{(L_1)} \oint\limits_{(L_2)} \frac{I_1 I_2 \,\mathrm{d}\vec{l}_1 \times (\mathrm{d}\vec{l}_2 \times \hat{\mathbf{r}}_{12})}{r_{12}^2} \\ &= \frac{\mu_0}{4\pi} \oint\limits_{(L_1)} \oint\limits_{(L_2)} \frac{I_1 I_2 [(\mathrm{d}\vec{l}_1 \cdot \hat{\mathbf{r}}_{12}) \,\mathrm{d}\vec{l}_2 - (\mathrm{d}\vec{l}_1 \cdot \mathrm{d}\vec{l}_2) \hat{\mathbf{r}}_{12}]}{r_{12}^2} \\ &= -\frac{\mu_0}{4\pi} \oint\limits_{(L_1)} \oint\limits_{(L_2)} \frac{I_1 I_2 (\mathrm{d}\vec{l}_1 \cdot \mathrm{d}\vec{l}_2) \hat{\mathbf{r}}_{12}}{r_{12}^2}\end{aligned}$$

又因为被积函数连续，故积分可交换顺序，即

$$\vec{F}_{12} = -\frac{\mu_0}{4\pi} \oint\limits_{(L_2)} \oint\limits_{(L_1)} \frac{I_1 I_2 (\mathrm{d}\vec{l}_1 \cdot \mathrm{d}\vec{l}_2) \hat{\mathbf{r}}_{12}}{r_{12}^2}$$

同理

$$\vec{F}_{21} = -\frac{\mu_0}{4\pi} \oint\limits_{(L_2)} \oint\limits_{(L_1)} \frac{I_1 I_2 (\mathrm{d}\vec{l}_1 \cdot \mathrm{d}\vec{l}_2) \hat{\mathbf{r}}_{21}}{r_{21}^2}$$

又因为 $\hat{\mathbf{r}}_{21} = -\hat{\mathbf{r}}_{12}$, 故

$$\vec{F}_{12} = -\vec{F}_{21}$$

2-4

解

$$B = \frac{\mu_0}{2\pi d} \sqrt{I_1^2 + I_2^2} = 7.2 \times 10^{-5} \mathrm{T}$$

由两个分量组成，其中

$$B_1 = \frac{\mu_0 I_1}{2\pi d} = 4.0 \times 10^{-5}\mathrm{T}$$

方向垂直纸面向里

$$B_2 = \frac{\mu_0 I_2}{2\pi d} = 6.0 \times 10^{-5}\mathrm{T}$$

方向平行纸面向右

2-5

解 (1) 由系统对称性知，磁感应强度一定沿轴线方向，且三条边贡献相等不妨设其中一条边为线段 $(0,0,0) \to (2a,0,0)$ 且其余两边都在 xOy 平面上且均在第一象限则轴线上一点坐标为 $(a, \sqrt{3}a, r_0)$, 则位于点 $(x,0,0)$ 处的电流元在该点产生的磁感应强度为

$$\begin{aligned} B &= \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}\mathbf{x} \times \hat{\mathbf{r}}}{r^2} \\ &= \frac{\mu_0 I}{4\pi}\frac{-r_0\,\mathrm{d}x\vec{j} + \frac{\sqrt{3}}{2}a\,\mathrm{d}x\vec{k}}{\left[(a-x)^2 + \frac{3}{4}a^2 + r_0^2\right]^{\frac{3}{2}}} \end{aligned}$$

我们只取沿轴线即 z 轴方向分量则

$$\begin{aligned} \vec{B} &= 3\int_0^{2a} \frac{\mu_0 I}{4\pi}\frac{\frac{\sqrt{3}}{2}a\vec{k}}{\left[(a-x)^2 + \frac{3}{4}a^2 + r_0^2\right]^{\frac{3}{2}}}\,\mathrm{d}x \\ &= \frac{9\mu_0 I a^2}{2\pi(3r_0^2 + a^2)\sqrt{3r_0^2 + 4a^2}}\vec{k} \end{aligned}$$

(2) 当 $r_0 \gg a$ 时 $B = \dfrac{\sqrt{3}\mu_0 I a^2}{2\pi r_0^3}$, 而 $S = \sqrt{3}a^2$ 故

$$B = \frac{\mu_0 m}{2\pi r_0^3}$$

2-6

解 (1) 将载流板分割为无数无限细无限长的载流导线，叠加得

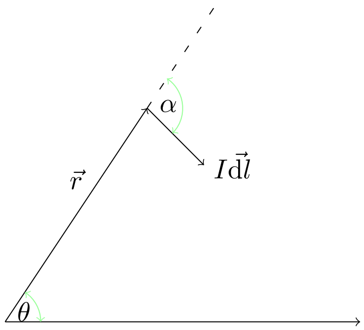
$$B = \int \mathrm{d}B\cos\theta = \int_{-a}^{a} \frac{\mu_0}{4\pi}\frac{2(\frac{I}{2a})\,\mathrm{d}l}{\sqrt{x^2+l^2}}\frac{x}{\sqrt{x^2+l^2}} = \frac{\mu_0 I}{2\pi a}\arctan\frac{a}{x}$$

(2)

$$B = \frac{\mu_0 \iota}{2}$$

附加题 1

解



以中心为极点建立极坐标系设线圈边界方程为 $r = r_0 + \Delta r(\theta)$ 由毕奥-萨伐尔定律知，在 θ 附近的电流元在中心产生的磁感应强度为

$$\mathrm{d}\vec{B} = \frac{\mu_0 I}{4\pi}\frac{\vec{r}\times \mathrm{d}\vec{l}}{r^3}$$

同一闭合回路的电流元在中心产生的磁感应强度方向相同故中心磁感应强度的大小为

$$B = \int \frac{\mu_0 I}{4\pi}\frac{r\sin\alpha}{r_0^3(1+\frac{\Delta r}{r_0})^3}\,\mathrm{d}l$$

又 $r\sin\alpha\,\mathrm{d}l = 2\,\mathrm{d}S$，且 $\dfrac{\Delta r}{r_0} \ll 1$，故

$$B = \frac{\mu_0 I}{4\pi r_0^3}\int\frac{2\,\mathrm{d}S}{1} = \frac{\mu_0 IS}{2\pi r_0^3} = \frac{\mu_0 m}{2\pi r_0^3}$$

2-21

解　(1) 取半径为 $r(\frac{D_2}{2}<r<\frac{D_1}{2})$ 的环形回路，由对称性知该环路上的磁感应强度均沿切向，则由安培环路定理知

$$2\pi r B=\mu_0 N I$$

则

$$B=\frac{\mu_0 N I}{2\pi r}$$

(2)

$$\begin{aligned}\Phi_B&=\int_{\frac{D_2}{2}}^{\frac{D_1}{2}}Bh\,\mathrm{d}r\\&=\int_{\frac{D_2}{2}}^{\frac{D_1}{2}}\frac{\mu_0 N I}{2\pi r}h\,\mathrm{d}r\\&=\frac{\mu_0 N I h}{2\pi}\ln\frac{D_1}{D_2}\end{aligned}$$

2-22

解　由对称性知磁感应强度与平面平行且与电流方向垂直，取一穿过载流板的矩形回路则由安培环路定理知

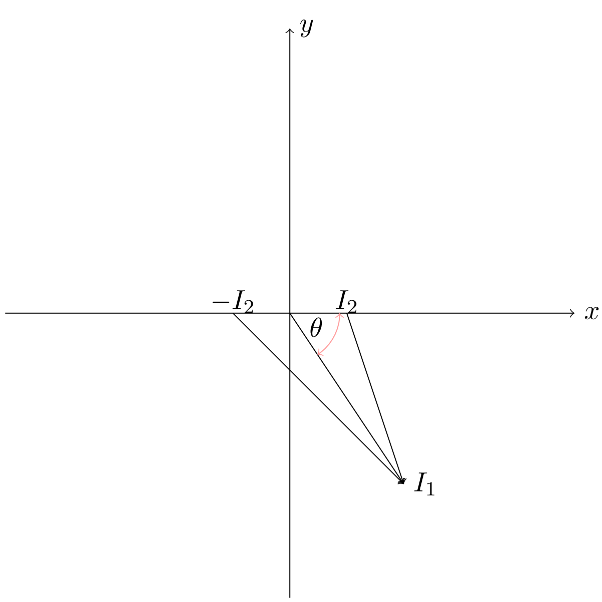
$$B\cdot 2l=\mu_0\iota l$$

则

$$B=\frac{\mu_0\iota}{2}$$

2-32

解　(1)



$$
\begin{aligned}
\vec{F}_1 &= \frac{\mu_0 I_1 I_2 a}{\pi\sqrt{a^2+b^2-2ab\cos\theta}}\frac{(b\cos\theta-a)\hat{x}-b\sin\theta\hat{y}}{\sqrt{(b\cos\theta-a)^2+b^2\sin^2\theta}} \\
&= \frac{\mu_0 I_1 I_2 a(b\cos\theta-a)}{\pi(a^2+b^2-2ab\cos\theta)}\hat{x}-\frac{\mu_0 I_1 I_2 ab\sin\theta}{\pi(a^2+b^2-2ab\cos\theta)}\hat{y}
\end{aligned}
$$

同理

$$
\begin{aligned}
\vec{F}_2 &= \frac{-\mu_0 I_1 I_2 a}{\pi\sqrt{a^2+b^2+2ab\cos\theta}}\frac{(b\cos\theta+a)\hat{x}-b\sin\theta\hat{y}}{\sqrt{(b\cos\theta+a)^2+b^2\sin^2\theta}} \\
&= \frac{-\mu_0 I_1 I_2 a(b\cos\theta+a)}{\pi(a^2+b^2+2ab\cos\theta)}\hat{x}+\frac{\mu_0 I_1 I_2 ab\sin\theta}{\pi(a^2+b^2+2ab\cos\theta)}\hat{y}
\end{aligned}
$$

故合力为

$$
\begin{aligned}
\vec{F} &= \vec{F}_1+\vec{F}_2 \\
&= \frac{\mu_0 I_1 I_2 a}{\pi}\left(\frac{b\cos\theta-a}{a^2+b^2-2ab\cos\theta}-\frac{b\cos\theta+a}{a^2+b^2+2ab\cos\theta}\right)\hat{x} \\
&+ \frac{\mu_0 I_1 I_2 ab\sin\theta}{\pi}\left(\frac{1}{a^2+b^2+2ab\cos\theta}-\frac{1}{a^2+b^2-2ab\cos\theta}\right)\hat{y}
\end{aligned}
$$

故合力矩为

$$
\begin{aligned}
\vec{L} &= \vec{r}_1\times\vec{F}_1+\vec{r}_2\times\vec{F}_2 \\
&= a\hat{x}\times\vec{F}_1-a\hat{x}\times\vec{F}_2 \\
&= a\hat{x}\times\left(\vec{F}_1-\vec{F}_2\right) \\
&= a(F_{1y}-F_{2y})\hat{z} \\
&= \frac{-\mu_0 I_1 I_2 a^2 b\sin\theta}{\pi}\left(\frac{1}{a^2+b^2+2ab\cos\theta}+\frac{1}{a^2+b^2-2ab\cos\theta}\right)\hat{z}
\end{aligned}
$$

(2) 欲使线圈平衡则

$$L=0$$

即 $\sin\theta=0$ 则

$$\theta=\begin{cases}0\\ \pi\end{cases}$$

(3)

$$
\begin{aligned}
W &= \int_0^{\frac{\pi}{2}} L\,\mathrm{d}\theta \\
&= -\frac{\mu_0 I_1 I_2 a}{\pi}\ln\frac{b-a}{b+a}
\end{aligned}
$$

2-33

解　由对称性知线圈受力一定垂直于导线方向

$$\begin{aligned} F &= \int \mathrm{d}F\cos\theta \\ &= 2\int_0^{\pi} \frac{\mu_0 I_1 I_2 \cos\theta\,\mathrm{d}\theta}{2\pi(l - r\cos\theta)} \\ &= \mu_0 I_1 I_2 \left(1 - \frac{l}{\sqrt{l^2 - r^2}}\right) \end{aligned}$$

2-35

解　(1)

$$\begin{aligned} L_{\text{磁}} &= NIBS \\ &= NIabB \\ &= 1.0 \times 10^{-6}\mathrm{N\cdot m} \end{aligned}$$

(2)

$$\begin{aligned} D &= \frac{L_{\text{磁}}}{\varphi} \\ &= 1.9 \times 10^{-6}\mathrm{N\cdot m} \end{aligned}$$

2-43

解

$$\frac{1}{2}mv^2 = Uq \to v = \sqrt{\frac{2Uq}{m}}$$

$$qvB = \frac{mv^2}{r}$$

$$qB = \frac{m\sqrt{\frac{2Uq}{m}}}{\frac{x}{2}}$$

$$m = \frac{qB^2}{8U}x^2$$

2-45

解　(1)

$$E = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E}{m}}$$

$$B = \frac{mv}{qR} = \frac{\sqrt{2mE}}{qR} = 0.48\mathrm{T}$$

(2)

$$n = \frac{E}{Uq} = 200$$

$$F = ma$$

$$\frac{U}{d}q = ma$$

$$a = \frac{Uq}{md}$$

又因为圆周运动满足 $R = \frac{mv}{Bq}$,故运动周期为

$$T = \frac{2\pi R}{v} = \frac{2\pi m}{Bq}$$

故

$$t = \frac{v}{a} + n\frac{T}{2} = \frac{\sqrt{\frac{2E}{m}}}{\frac{Uq}{md}} + \frac{200\pi m}{Bq} = 1.38 \times 10^{-5}\mathrm{s}$$

2-50

解　(1)N 型

(2)

$$\frac{U}{b}e = Bev \to v = \frac{U}{Bb}$$

$$I = neSv$$

$$n = \frac{I}{nev} = \frac{BI}{eaU} = 2.9 \times 10^{20}/\mathrm{m}^3$$

2-50

解 (1)N 型

(2)

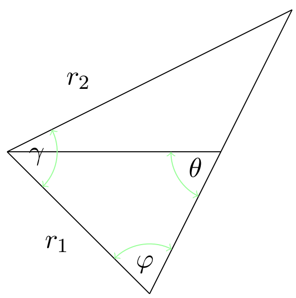
$$\frac{U}{b}e = Bev \to v = \frac{U}{Bb}$$

$$I = neSv$$

$$n = \frac{I}{nev} = \frac{BI}{eaU} = 2.9 \times 10^{20}/\mathrm{m}^3$$

3-5

解



$$\begin{aligned}\Phi &= \int \mathbf{B} \cdot \mathrm{d}\mathbf{S} \\ &= \int_0^{\gamma} 2a \frac{r\,\mathrm{d}\alpha}{\sin\beta} \frac{\mu_0 I}{2\pi r} \cos\beta \\ &= \frac{a\mu_0 I}{\pi} \int_0^{\gamma} \frac{\cos\beta\,\mathrm{d}\alpha}{\sin\beta}\end{aligned}$$

又因为 $\beta = \varphi + \alpha$ 故

$$\begin{aligned}\Phi &= \frac{a\mu_0 I}{\pi} \int_{\varphi}^{\gamma+\varphi} \frac{\cos\beta\,\mathrm{d}\beta}{\sin\beta} \\ &= \frac{a\mu_0 I}{\pi} \ln \frac{\sin(\varphi+\alpha)}{\sin(\phi)} \\ &= \frac{a\mu_0 I}{2\pi} \ln \frac{r_1}{r_2} \\ &= \frac{a\mu_0 I}{2\pi} \left(\ln(a^2+b^2-2ab\cos\theta) - \ln(a^2+b^2+2ab\cos\theta)\right)\end{aligned}$$

故电动势为

$$\begin{aligned}\mathscr{E} &= \frac{\mathrm{d}\Phi}{\mathrm{d}t} \\ &= \frac{\mathrm{d}\Phi}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t} \\ &= \frac{-\mu_0 I a^2 b\omega\sin(\omega t)}{\pi}\left(\frac{1}{a^2+b^2+2ab\cos(\omega t)}+\frac{1}{a^2+b^2-2ab\cos(\omega t)}\right)\end{aligned}$$

3-8

解

$$\begin{aligned}Q &= I\,\mathrm{d}t \\ Q &= \frac{\mathrm{d}\Phi}{R\,\mathrm{d}t}\,\mathrm{d}t \\ \Delta Q &= \frac{\Delta\Phi}{R} \\ \Delta Q &= \frac{N\pi d^2 B}{2R} \\ B &= \frac{2R\Delta Q}{N\pi d^2} = 1.3\times10^{-4}\mathrm{T}\end{aligned}$$

3-11

解

$$\begin{aligned}\mathscr{E} &= \int_a^b \frac{v\mu_0 I \,\mathrm{d}r}{2\pi r} \\ &= \frac{v\mu_0 I}{2\pi}\ln\frac{b}{a} \\ &= 3.7\times 10^{-5}\mathrm{V}\end{aligned}$$

a 端电势高

3-13

解　(1)

$$\begin{aligned}\mathscr{E} &= \int_0^R \omega r B \,\mathrm{d}r \\ &= \frac{\omega B R^2}{2}\end{aligned}$$

(2) 从 b 到 a

(3)

$$\begin{aligned}L &= \int_0^R rBIdr \\ &= \frac{BIR^2}{2}\end{aligned}$$

方向垂直纸面向里

(4) 会

(5) 相当于多个电阻并联,感应电动势不变

3-30

解　(1) 取半径为 $r(\frac{D_2}{2} < r < \frac{D_1}{2})$ 的环形回路,由对称性知该环路上的磁感应强度均沿切向,则由安培环路定理知

$$2\pi r B = \mu_0 N I$$

则

$$B = \frac{\mu_0 N I}{2\pi r}$$

$$\begin{aligned}\Phi_B &= \int_{\frac{D_2}{2}}^{\frac{D_1}{2}} Bh \,\mathrm{d}r \\ &= \int_{\frac{D_2}{2}}^{\frac{D_1}{2}} \frac{\mu_0 N I}{2\pi r} h \,\mathrm{d}r \\ &= \frac{\mu_0 N I h}{2\pi}\ln\frac{D_1}{D_2}\end{aligned}$$

$$\begin{aligned}\Psi &= N\Phi_B \\ &= \frac{\mu_0 N^2 I h}{2\pi}\ln\frac{D_1}{D_2}\end{aligned}$$

故自感系数为

$$\begin{aligned}L &= \frac{\Psi}{I} \\ &= \frac{\mu_0 N^2 h}{2\pi}\ln\frac{D_1}{D_2}\end{aligned}$$

(2)

$$L = \frac{4\pi\times10^{-7}\times1000\times1000\times0.01}{2\pi}\ln\frac{0.2}{0.1}\mathrm{H} = 1.4\times10^{-3}\mathrm{H}$$

3-34

解

$$\begin{cases} L_1 + L_2 + 2M = 1\mathrm{H} \\ L_1 + L_2 - 2M = 0.4\mathrm{H} \end{cases}$$

解得 $M = 0.15\mathrm{H}$

3-35

解　(1)

$$B = \frac{\mu_0 N I}{2\pi r} + \frac{\mu_0 I}{2\pi(d-r)}$$

$$\begin{aligned}\Phi &= \int_a^{d-a} B\,\mathrm{d}r \\ &= \frac{\mu_0 I}{\pi}\ln\frac{d-a}{a}\end{aligned}$$

自感系数

$$L = \frac{\Phi}{I} = \frac{\mu_0}{\pi}\ln\frac{d-a}{a}$$

因为 $a \ll d$ 故

$$L \approx \frac{\mu_0}{\pi}\ln\frac{d}{a}$$

故

$$L = \frac{4\pi\times10^{-7}}{\pi}\ln\frac{200}{1}\mathrm{H} = 2.1\times10^{-6}\mathrm{H}$$

(2)

$$\begin{aligned}A &= \int F \,\mathrm{d}r \\ &= \int_d^{2d} \frac{\mu_0 I^2}{2\pi r}\,\mathrm{d}r \\ &= \frac{\mu_0 I^2}{2\pi}\ln 2 \\ &= 5.5\times 10^{-5}\mathrm{J}\end{aligned}$$

(3)

$$\begin{aligned}\Delta W &= W_2 - W_1 \\ &= \frac{L_2 I^2}{2} - \frac{L_1 I^2}{2} \\ &\frac{1}{2}(\frac{\mu_0}{\pi}\ln\frac{2d}{a} - \frac{\mu_0}{\pi}\ln\frac{d}{a}) \\ &= \frac{\mu_0 I^2}{\pi}\ln 2 \\ &= 5.5\times 10^{-5}\mathrm{J}\end{aligned}$$

能量增加,来自电源

5-31

解　连接电源时有

$$R_1 i_1 + L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} + M\frac{\mathrm{d}i_2}{\mathrm{d}t} = \mathscr{E}$$

$$R_2 i_2 + L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} + M\frac{\mathrm{d}i_1}{\mathrm{d}t} = 0$$

换成短接时有

$$R_1 i_1 + L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} + M\frac{\mathrm{d}i_2}{\mathrm{d}t} = 0$$

$$R_2 i_2 + L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} + M\frac{\mathrm{d}i_1}{\mathrm{d}t} = 0$$

由于这两组方程对应的齐次方程相同，故其时间常量相同，下面仅考虑短接时的情况。由短接的第一个方程有

$$\frac{\mathrm{d}i_1}{\mathrm{d}t} = -\frac{M\frac{\mathrm{d}i_2}{\mathrm{d}t} + R_1 i_1}{L_1}$$

代入第二个方程有

$$L_2\frac{\mathrm{d}i_2}{\mathrm{d}t} - \frac{M^2}{L_1}\frac{\mathrm{d}i_2}{\mathrm{d}t} - \frac{MR_1}{L_1}i_1 + R_2 i_2 = 0$$

引入无漏磁条件 $M^2 = L_1 L_2$ 有

$$-\frac{MR_1}{L_1}i_1 + R_2 i_2 = 0$$

$$i_2 = \frac{MR_1}{L_1R_2}i_1$$

$$\frac{\mathrm{d}i_2}{\mathrm{d}t} = \frac{MR_1}{L_1R_2}\frac{\mathrm{d}i_1}{\mathrm{d}t}$$

代入第一个方程有

$$0 = L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} + \frac{M^2R_1}{L_1R_2}\frac{\mathrm{d}i_1}{\mathrm{d}t} + R_1 i_1$$

$$\frac{\mathrm{d}i_1}{\mathrm{d}t} = -\frac{R_1R_2}{R_1L_2 + R_2L_1}i_1$$

故时间常量为

$$\tau = \frac{1}{\frac{R_1R_2}{R_1L_2+R_2L_1}} = \frac{R_1L_2 + R_2L_1}{R_1R_2} = \frac{L_1}{R_2} + \frac{L_2}{R_1}$$

5-32

解　(1)

$$\frac{q}{C} + L\frac{\mathrm{d}i}{\mathrm{d}t} = 0$$

$$\frac{q}{C} + L\frac{\mathrm{d}^2 i}{\mathrm{d}t^2} = 0$$

$$q = C_1 \sin\left(\frac{t}{\sqrt{CL}}\right) + C_2\cos\left(\frac{t}{\sqrt{CL}}\right)$$

初始条件为 $t=0$ 时，$q=Q$、$i=0$。

故解为

$$q = Q\cos(\frac{t}{\sqrt{LC}})$$

欲令线圈磁场能等于电容中电能，即有

$$\frac{q^2}{2C} = \frac{LI^2}{2}$$

$$\frac{q^2}{2C} = \frac{L}{2}(\frac{\mathrm{d}q}{\mathrm{d}t})^2$$

$$\frac{Q^2}{2C}\cos^2(\frac{t}{\sqrt{LC}}) = \frac{L}{2}\frac{Q^2}{LC}\cos^2(\frac{t}{\sqrt{LC}})$$

$$\tan^2(\frac{t}{\sqrt{LC}}) = 1$$

$$t = \frac{\pi}{4}\sqrt{LC}$$

(2)

$$q = Q\cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}Q$$

5-33

解　(1) 并联后总电容为 $C' = 2C = 4\mu\mathrm{F}$

$$\lambda = \frac{R}{2}\sqrt{\frac{C'}{L}} = 1.58 > 1$$

故不振荡

(2) 并联后总电容为 $C' = \frac{C^2}{2C} = 1\mu\mathrm{F}$

$$\lambda = \frac{R}{2}\sqrt{\frac{C'}{L}} = 0.79 < 1$$

故振荡

4-25

解

$$M=\frac{m}{V}=\frac{4M}{\pi d^2 l}$$

$$i'=M=\frac{4\times 12000}{\pi(0.025)^2\times 0.075}=3.3\times 10^8\mathrm{A/m}$$

4-66

解

$$\mathbf{D}=\varepsilon_0\mathbf{E}$$

电场能量密度为

$$\rho_E=\frac{\mathbf{D}\cdot\mathbf{E}}{2}=\frac{\varepsilon_0 E^2}{2}$$

因为该电场在该空间处处均匀,故该空间电场能为

$$W_e=\frac{\varepsilon_0 E^2 V}{2}=4.43\times 10^{-5}\mathrm{J}$$

$$\mathbf{H}=\frac{B}{\mu_0}$$

磁场能量密度为

$$\rho_B=\frac{\mathbf{B}\cdot\mathbf{H}}{2}=\frac{B^2}{2\mu_0}$$

因为该磁场在该空间处处均匀,故该空间电场能为

$$W_B=\frac{B^2 V}{2\mu_0}=397.89\mathrm{J}$$

4-68

解　由安培环路定理可知该同轴线产生的磁场分布为

$$B=\begin{cases}\dfrac{\mu_0 rI}{2\pi a^2} & 0<r<a\\[2ex] \dfrac{\mu_0 I}{2\pi r} & a<r<b\\[2ex] \dfrac{\mu_0 I(c^2-r^2)}{2\pi r(c^2-b^2)} & b<r<c\\[2ex] 0 & r>c\end{cases}$$

(1)

①导线内,即 $0<r<a$ 处

$$B=\frac{\mu_0 rI}{2\pi a^2}$$

$$H=\frac{B}{\mu_0}=\frac{rI}{2\pi a^2}$$

故该处单位长度能量为

$$\begin{aligned}W &= \iint \frac{\mathbf{B}\cdot\mathbf{H}}{2}\,\mathrm{d}S \\ &= \int_0^a \frac{\mu_0 I^2 r^2}{8\pi^2 a^4} 2\pi r\,\mathrm{d}r \\ &= \frac{\mu_0 I^2}{16\pi}\end{aligned}$$

②导线和圆筒之间,即 $a < r < b$ 处

$$\begin{aligned}B &= \frac{\mu_0 I}{2\pi r} \\ H &= \frac{B}{\mu_0} = \frac{I}{2\pi r}\end{aligned}$$

故该处单位长度能量为

$$\begin{aligned}W &= \iint \frac{\mathbf{B}\cdot\mathbf{H}}{2}\,\mathrm{d}S \\ &= \int_a^b \frac{\mu_0 I^2}{8\pi^2 r^2} 2\pi r\,\mathrm{d}r \\ &= \frac{\mu_0 I^2}{4\pi}\ln\frac{b}{a}\end{aligned}$$

③圆筒内,即 $b < r < c$ 处

$$\begin{aligned}B &= \frac{\mu_0 I(c^2-r^2)}{2\pi r(c^2-b^2)} \\ H &= \frac{I(c^2-r^2)}{2\pi r(c^2-b^2)}\end{aligned}$$

故该处单位长度能量为

$$\begin{aligned}W &= \iint \frac{\mathbf{B}\cdot\mathbf{H}}{2}\,\mathrm{d}S \\ &= \int_b^c \frac{\mu_0 I^2 (c^2-r^2)^2}{8\pi^2 r^2 (c^2-b^2)^2} 2\pi r\,\mathrm{d}r \\ &= \frac{\mu_0 I^2}{16\pi(c^2-b^2)^2}(4c^4\ln\frac{c}{b} - 3c^4 + 4b^2c^2 - b^4)\end{aligned}$$

④圆筒外,即 $r > c$ 处

$$B = 0$$

$$H = 0$$

故该处单位长度能量为

$$W = \iint \frac{\mathbf{B}\cdot\mathbf{H}}{2}\,\mathrm{d}S$$

$$= 0$$

(2) 代入数值有

$$W_1 = 2.5 \times 10^{-6}\mathrm{J}$$

$$W_2 = 1.4 \times 10^{-5}\mathrm{J}$$

$$W_3 = 6.8 \times 10^{-7}\mathrm{J}$$

$$W_4 = 0$$

6-4

解

$$S = \frac{1}{2}E_0H_0$$

可知 $E_0 = \sqrt{2S\sqrt{\frac{\mu\mu_0}{\varepsilon\varepsilon_0}}} = 1.01 \times 10^3\mathrm{V/m}$

故

$$\sqrt{\overline{E^2}} = \frac{\sqrt{2}}{2}E_0 = 7.3 \times 10^2\mathrm{V/m}$$

同理

$$\sqrt{\overline{H^2}} = \frac{\sqrt{2}}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0 = 1.9\mathrm{A/m}$$

6-9

解　(1)$\vec{E}$ 竖直向下,$\vec{H}$ 与侧面相切,故 $\vec{S}$ 垂直于侧面

(2)

$$\begin{aligned} P &= S \cdot 2\pi Rl \\ &= EH \cdot 2\pi Rl \\ &= \frac{q}{\varepsilon_0 A}\frac{I}{2\pi R} \cdot 2\pi Rl \\ &= \frac{q}{C}\frac{\mathrm{d}q}{\mathrm{d}t} \\ &= \frac{\mathrm{d}}{\mathrm{d}t}(\frac{q^2}{2C}) \end{aligned}$$