1-9

解 (1)

$$\mathrm{F} = \mathrm{F}_1 + \mathrm{F}_2$$

$\mathrm{F}_1, \mathrm{F}_2$ 方向相反故

$$F = F_1 - F_2$$

因为 $l \ll r$, 故 $F_1 - F_2 = \dfrac{2Qql}{4\pi\varepsilon_0 r^3} = \dfrac{2Qp}{4\pi\varepsilon_0 r^3}$ 因为 $L_1 = L_2 = 0$, 故 $L = 0$ 故

$$\mathbf{F} = \frac{Q}{2\pi\varepsilon_0 r^3}\mathbf{p}$$

$$\mathbf{L} = 0$$

(2)

$$F_y = F_{1y} + F_{2y} = \frac{Qql}{4\pi\varepsilon_0 r^3}, F_x = F_{1x} - F_{2x} = 0$$

$$\mathbf{F} = \frac{Q}{4\pi\varepsilon_0 r^3}\mathbf{p}$$

$$L = F_{1x}\frac{l}{2} + F_{2x}\frac{l}{2} = \frac{Qp}{4\pi\varepsilon_0 r^2}$$

故

$$\mathbf{L} = \frac{Q\mathbf{p} \times \mathbf{r}}{4\pi\varepsilon_0 r^3}$$

1-13

解

$$\begin{aligned}
\phi &= \iint\limits_{x^2+y^2+z^2=a^2} \mathbf{E} \cdot \mathbf{n} \mathrm{d}S \\
&= \iint\limits_{x^2+y^2 \le a^2} E\cos\theta \frac{\mathrm{d}x\mathrm{d}y}{\cos\theta} \\
&= \iint\limits_{x^2+y^2 \le a^2} E\mathrm{d}x\mathrm{d}y \\
&= E\pi a^2
\end{aligned}$$

1-14

解 取半径为 R 的球面,由于电荷分布是球对称的,故电场强度只有径向分量。该闭合球面所包围的净电荷量为

$$\begin{aligned}
q &= \int_0^R -\frac{e}{\pi a_B^3}\mathrm{e}^{-2r/a_B} 4\pi r^2 \mathrm{d}r + e \\
&= \frac{e\mathrm{e}^{\frac{2R}{a_B}}(a_B^2 + 2a_B + 2R^2)}{a_B^2}
\end{aligned}$$

由高斯定理

$$4\pi R^2 E = \frac{q}{\varepsilon_0}$$

$$E = \frac{e\mathrm{e}^{\frac{2R}{a_B}}(a_B^2 + 2a_B + 2R^2)}{4\pi\varepsilon_0 a_B^2 R^2}$$

方向为由原子中心指向外面

1-15

解 (1) 取底面为面元 $\mathrm{d}S$ 的闭合柱面,则该曲面电通量为

$$\phi = (E_1 - E_2)\mathrm{d}S$$

由高斯定理

$$\phi = \frac{q}{\varepsilon_0} = \frac{\rho \mathrm{d}Sh}{\varepsilon_0}$$

则

$$\rho = \frac{(E_1 - E_2)\varepsilon_0}{h} = 4.43 \times 10^{-13} C/m^3$$

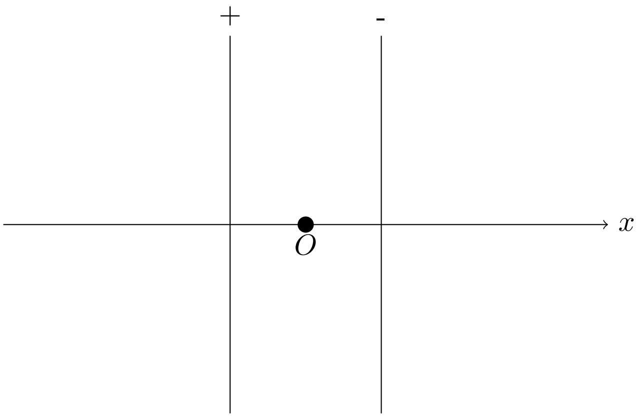
(2)

$$\frac{4\pi R^2 \sigma}{\varepsilon_0} = -4\pi R^2 E_1$$

$$\sigma = -\varepsilon_0 E_1 = -8.85 \times 10^{-10} C/m^2$$

1-12

解 设两条直线如图放置,取两线中点为坐标原点



(1) 设 $\hat{x}$ 为沿 x 轴正方向的单位矢量由场强叠加原理知当 $x < -\frac{a}{2}$ 时场强为

$$\mathbf{E} = \frac{\eta_e}{2\pi\varepsilon_0(-\frac{a}{2} - x)}(-\hat{x}) + \frac{\eta_e}{2\pi\varepsilon_0(\frac{a}{2} - x)}\hat{x} = \frac{-a\eta_e}{2\pi\varepsilon_0(x^2 - \frac{a^2}{4})}\hat{x}$$

当 $-\frac{a}{2} < x < \frac{a}{2}$ 时场强为

$$\mathbf{E} = \frac{\eta_e}{2\pi\varepsilon_0(x + \frac{a}{2})}\hat{x} + \frac{\eta_e}{2\pi\varepsilon_0(\frac{a}{2} - x)}\hat{x} = \frac{-a\eta_e}{2\pi\varepsilon_0(x^2 - \frac{a^2}{4})}\hat{x}$$

当 $\dfrac{a}{2} < x$ 时场强为

$$\mathbf{E} = \frac{\eta_e}{2\pi\varepsilon_0(x+\frac{a}{2})}\hat{x} + \frac{\eta_e}{2\pi\varepsilon_0(x-\frac{a}{2})}(-\hat{x}) = \frac{-a\eta_e}{2\pi\varepsilon_0(x^2-\frac{a^2}{4})}\hat{x}$$

综上

$$\mathbf{E} = \frac{\eta_e}{2\pi\varepsilon_0(x+\frac{a}{2})}\hat{x} + \frac{\eta_e}{2\pi\varepsilon_0(x-\frac{a}{2})}(-\hat{x}) = \frac{-a\eta_e}{2\pi\varepsilon_0(x^2-\frac{a^2}{4})}\hat{x}$$

(2)

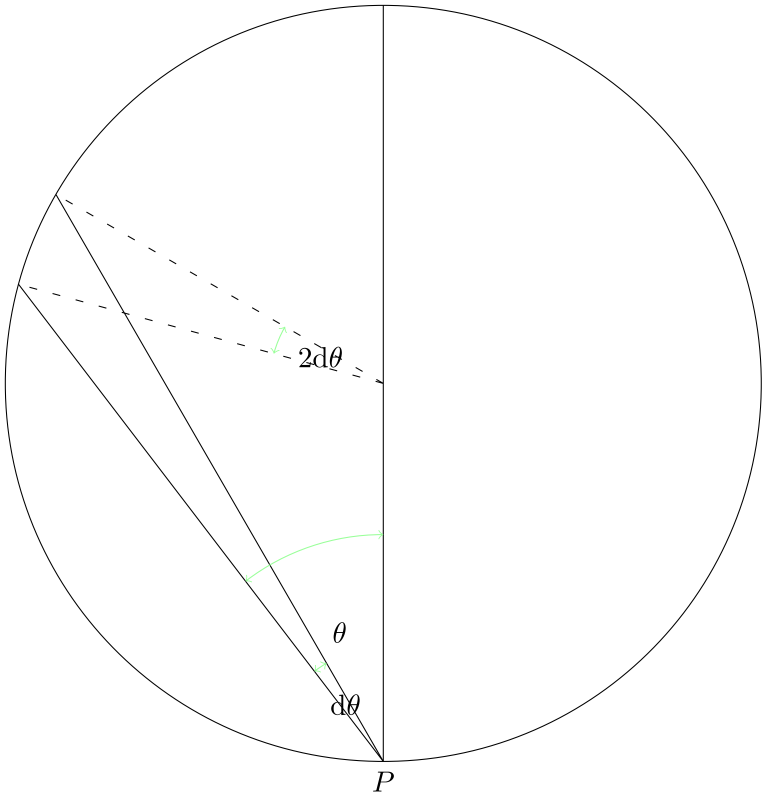
$$F = \eta_e \cdot \frac{\eta_e}{2\pi\varepsilon_0 a} = \frac{\eta_e^2}{2\pi\varepsilon_0 a}$$

1-16

解 由对称性知,场强方向定垂直于轴线,取以轴线为中心线半径为 r 长为 l 的圆柱形高斯面可得

$$2\pi r l E = \frac{q}{\varepsilon_0}$$

又 $r < R$ 时 $q = 0$,故 $E = 0$。$r > R$ 时 $q = l\lambda$ 故 $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$。
当 $r = R$ 时高斯定律不适用,可将圆筒沿轴线方向分为无数带电直线



每一带电直线的电荷线密度为 $\lambda_0 = \dfrac{2\mathrm{d}\theta}{2\pi}\lambda$,于是在 P 点产生的场强就为

$$\mathrm{d}E = \frac{\lambda_0}{2\pi\varepsilon_0 r}$$

又因为 $r = 2R\cos\theta$ 故

$$\mathrm{d}E = \frac{\lambda \mathrm{d}\theta}{4\pi^2\varepsilon_0 R\cos\theta}$$

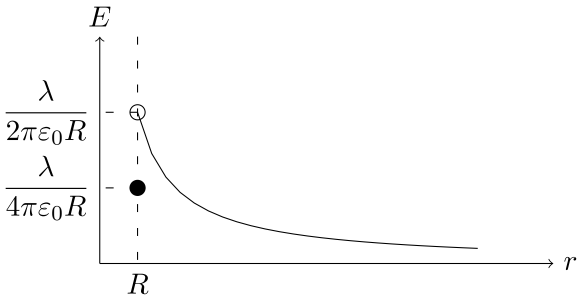
其径向分量为 $\mathrm{d}E\cos\theta = \dfrac{\lambda \mathrm{d}\theta}{4\pi^2\varepsilon_0 R}$ 故

$$E = 2\int_0^{\frac{\pi}{2}} \frac{\lambda \mathrm{d}\theta}{4\pi^2\varepsilon_0 R} = \frac{\lambda}{4\pi\varepsilon_0 R}$$

故

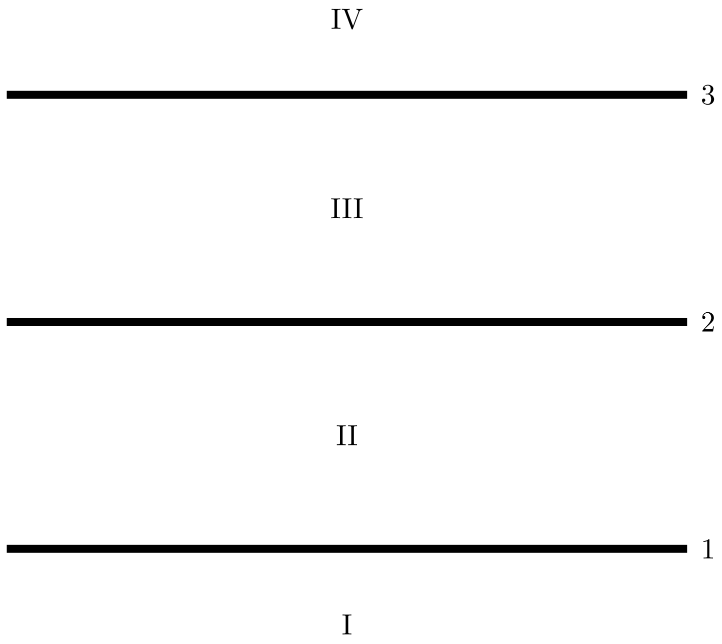
$$E = \begin{cases} 0 & (r < R) \\ \dfrac{\lambda}{4\pi\varepsilon_0 R} & (r = R) \\ \dfrac{\lambda}{2\pi\varepsilon_0 r} & (r > R) \end{cases}$$

$E - r$ 图为



1-19

解　不妨设三个平面如图分布将空间分为 I,II,III,IV 四部分。且令 $\sigma_e > 0$



由场强叠加原理知

(1)$E_i = \dfrac{3\sigma_e}{2\varepsilon_0}, E_{ii} = \dfrac{\sigma_e}{2\varepsilon_0}, E_{iii} = \dfrac{\sigma_e}{2\varepsilon_0}, E_{iv} = \dfrac{3\sigma_e}{2\varepsilon_0}$

方向分别为下下上上

(2)$E_i = \dfrac{\sigma_e}{2\varepsilon_0}, E_{ii} = \dfrac{\sigma_e}{2\varepsilon_0}, E_{iii} = \dfrac{\sigma_e}{2\varepsilon_0}, E_{iv} = \dfrac{\sigma_e}{2\varepsilon_0}$

方向分别为下上下上

(3)$E_i = \dfrac{\sigma_e}{2\varepsilon_0}, E_{ii} = \dfrac{\sigma_e}{2\varepsilon_0}, E_{iii} = \dfrac{\sigma_e}{2\varepsilon_0}, E_{iv} = \dfrac{\sigma_e}{2\varepsilon_0}$

方向分别为上下上下

(4)$E_i = \dfrac{\sigma_e}{2\varepsilon_0}, E_{ii} = \dfrac{3\sigma_e}{2\varepsilon_0}, E_{iii} = \dfrac{\sigma_e}{2\varepsilon_0}, E_{iv} = \dfrac{3\sigma_e}{2\varepsilon_0}$

方向分别为上上上下