1-4

解　对小球受力分析知

$$\frac{F_q}{mg} = \tan\theta$$

又

$$F_q = \frac{q^2}{4\pi\varepsilon_0(2l\sin\theta)^2}$$

故 $q = \pm\sqrt{16\pi\tan\theta\sin^2\theta l^2\varepsilon_0 mg}$

1-5

解　由油滴受力平衡知

$$Eq = mg = \frac{4}{3}\pi r^3\rho g \to q = \frac{4\pi r^3\rho g}{3E}$$

代入数值得

$$q = -8.03\times10^{-19}\mathrm{C}$$

1-8

解　在 (r,θ) 处电势为

$$\phi(r,\theta) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{r^2+\frac{l^2}{4}-rl\cos\theta}} - \frac{1}{\sqrt{r^2+\frac{l^2}{4}+rl\cos\theta}}\right)$$

因为 $l \ll r$, 故略去二阶小量 $\frac{l^2}{4}$, 且运用近似 $(1+x)^k = 1+kx(x\ll 1)$ 可得

$$\phi(r,\theta) = \frac{ql\cos\theta}{4\pi\varepsilon_0 r^2}$$

又 $\mathbf{E} = -\nabla\phi$, 且在极坐标中 $\nabla = \frac{\partial}{\partial r}\hat{\mathbf{e}}_r + \frac{\partial}{r\partial\theta}\hat{\mathbf{e}}_\theta$ 故

$$\mathbf{E}(r,\theta) = \frac{ql\cos\theta}{2\pi\varepsilon_0 r^3}\hat{\mathbf{e}}_r + \frac{ql\sin\theta}{4\pi\varepsilon_0 r^3}\hat{\mathbf{e}}_\theta$$

其径向和角向分量为

$$E_r = \frac{ql\cos\theta}{2\pi\varepsilon_0 r^3}, E_\theta = \frac{ql\sin\theta}{4\pi\varepsilon_0 r^3}$$

1-10

解　(1) 其场强大小为

$$E = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(r+l)^2} + \frac{1}{(r-l)^2} - \frac{2}{r^2}\right)$$

泰勒展开并保留二阶余项后得

$$\begin{aligned}E &= \frac{q}{4\pi\varepsilon_0 r^2}(1-\frac{2l}{r}+\frac{3l^2}{r^2}+1+\frac{2l}{r}+\frac{3l^2}{r^2}-2)\\ &= \frac{6ql^2}{4\pi\varepsilon_0 r^4}\\ &= \frac{3Q}{4\pi\varepsilon_0 r^4}\end{aligned}$$

(2)

$$\begin{aligned}U(r) &= \frac{q}{4\pi\varepsilon_0}(\frac{1}{r+l}+\frac{1}{r-l}-\frac{2}{r})\\ &= \frac{q}{4\pi\varepsilon_0 r}\frac{2l^2}{r^2-l^2}\end{aligned}$$

因为 $l \ll r$，故略去二阶小量 l^2 得

$$U(r) = \frac{2ql^2}{4\pi\varepsilon_0 r^3} = \frac{Q}{4\pi\varepsilon_0 r^3}$$

1-11

解　P 点场强大小为

$$\begin{aligned}E &= \frac{ql}{4\pi\varepsilon_0}\left[(x^2+\frac{l^2}{2}-lx)^{-\frac{3}{2}}-(x^2+\frac{l^2}{2}+lx)^{-\frac{3}{2}}\right]\\ &= \frac{ql}{4\pi\varepsilon_0}(x^2+\frac{l^2}{2})^{-\frac{3}{2}}\left[(1-\frac{lx}{x^2+\frac{l^2}{2}})^{-\frac{3}{2}}-(1+\frac{lx}{x^2+\frac{l^2}{2}})^{-\frac{3}{2}}\right]\end{aligned}$$

因为 $l \ll x$，故

$$\begin{aligned}(x^2+\frac{l^2}{2})^{-\frac{3}{2}}\left[(1-\frac{lx}{x^2+\frac{l^2}{2}})^{-\frac{3}{2}}-(1+\frac{lx}{x^2+\frac{l^2}{2}})^{-\frac{3}{2}}\right] &= (x^2+\frac{l^2}{2})^{-\frac{3}{2}}\left[1+\frac{3lx}{2(x^2+\frac{l^2}{2})}-1+\frac{3lx}{2(x^2+\frac{l^2}{2})}\right]\\ &= x^{-3}\frac{3lx}{x^2}\\ &= \frac{3l}{x^4}\end{aligned}$$

故

$$E = \frac{3ql^2}{4\pi\varepsilon_0 x^4}$$

方向竖直向上

解　(1)

$$\begin{aligned}C &= \frac{\varepsilon_0 S}{d}\\ C' &= \frac{\varepsilon_r\varepsilon_0 S}{d}\\ \Delta Q = U(C'-C) &= \frac{(\varepsilon_r-1U\varepsilon_0 S)}{d}\end{aligned}$$

(2)

$$D_1 = D_2$$

$$\varepsilon_r \varepsilon_0 E_1 = \varepsilon_0 E_2$$

$$E_1 d + E_2 d = U$$

解得

$$D = \frac{\varepsilon_r \varepsilon_0 U}{d(\varepsilon_r + 1)}$$

又因为 $D = \sigma$,故

$$Q = \sigma S = \frac{\varepsilon_r \varepsilon_0 U S}{d(\varepsilon_r + 1)}$$

$$\begin{aligned} W &= U(Q - C'U) \\ &= \frac{-\varepsilon_r^2 \varepsilon_0 U^2 S}{(\varepsilon_r + 1)d} \end{aligned}$$

(3)

$$\begin{aligned} C'' &= \frac{Q}{U} \\ &= \frac{\varepsilon_r \varepsilon_0 S}{d(\varepsilon_r + 1)} \end{aligned}$$

$$C''' = \frac{\varepsilon_0 S}{2d}$$

$$\begin{aligned} A &= \frac{Q^2}{2C'''} - \frac{Q^2}{2C''} \\ &= \frac{(C'' - C''')Q^2}{2C''C'''} \\ &= \frac{(\frac{\varepsilon_r \varepsilon_0 S}{d(\varepsilon_r+1)} - \frac{\varepsilon_0 S}{2d})(\frac{\varepsilon_r \varepsilon_0 U S}{d(\varepsilon_r+1)})^2}{2\frac{\varepsilon_r \varepsilon_0 S}{d(\varepsilon_r+1)}\frac{\varepsilon_0 S}{2d}} \end{aligned}$$