1-28

解　设 ne 放在坐标 $(0,0,0)$ 处，$-e$ 放在坐标 $(a,0,0)$ 处

(1) 空间中任一点 (x,y,z) 处电势为

$$
\begin{aligned}
U &= U_1 + U_2 \\
&= \frac{e}{4\pi\varepsilon_0}\left(\frac{n}{\sqrt{x^2+y^2+z^2}} - \frac{1}{\sqrt{(x-a)^2+y^2+z^2}}\right)
\end{aligned}
$$

令 $U=0$，则有

$$\frac{n}{\sqrt{x^2+y^2+z^2}} - \frac{1}{\sqrt{(x-a)^2+y^2+z^2}} = 0$$

$$(x-\frac{n^2a}{n^2-1})^2 + y^2 + z^2 = (\frac{na}{n^2-1})^2$$

(2) 该球面球心坐标为 $(\frac{n^2a}{n^2-1},0,0)$, 符合题意。

(3) 该球面半径为 $\frac{na}{n^2-1}$

1-30

解　基态氢原子核外电子满足

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

故动能 $E_k = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r}$, 所以氢原子机械能 $W = E_k + E_p = \frac{e^2}{8\pi\varepsilon_0 r} + \frac{-e^2}{4\pi\varepsilon_0 r} = \frac{-e^2}{8\pi\varepsilon_0 r}$

因此电离能

$$E = -W = 2.18\times10^{-18}J = 13.625eV$$

1-31

解　(1)

$$U = \frac{e}{4\pi\varepsilon_0 r} = 1.4\times10^6 V$$

$$E_k = e\Delta U = 1.4\times10^6 eV$$

(2)

$$E_k = \frac{3}{2}kT$$

$$T = 1.1\times10^{10}K$$

1-33

解　(1) 以 O 为势能零点

$$
\begin{aligned}
U_P &= \frac{\eta_e}{2\pi\varepsilon_0}\left(\ln\frac{a}{\sqrt{(x-a)^2+y^2}} - \ln\frac{a}{\sqrt{(x+a)^2+y^2}}\right) \\
&= \frac{\eta_e}{4\pi\varepsilon_0}\ln\frac{\sqrt{(x+a)^2+y^2}}{\sqrt{(x-a)^2+y^2}}
\end{aligned}
$$

(2)

$$U=\frac{\eta_e}{4\pi\varepsilon_0}\ln\frac{\sqrt{(x+a)^2+y^2}}{\sqrt{(x-a)^2+y^2}}$$

$$\frac{\sqrt{(x+a)^2+y^2}}{\sqrt{(x-a)^2+y^2}}=\exp(\frac{4\pi\varepsilon_0 U}{\eta_e})=k^2$$

$$x^2+2a\frac{k^2+1}{1-k^2}x+a^2+y^2=0$$

$$(x-\frac{k^2+1}{k^2-1}a)^2+y^2=\frac{4k^2}{(k^2-1)^2}a^2$$

证毕

(3)zOy 平面

1-35

解 (1)

$$\frac{1}{2}mc^2=\Delta Ue$$

$$U=\frac{mc^2}{2e}=2.5\times 10^5 V$$

(2)

$$\frac{1}{2}mc^2=mc^2(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1)$$

$$v=\frac{\sqrt{5}}{3}c=2.2\times 10^8 m/s$$

(3)$U\to\infty$,不可能

1-39

解 (1) 取半径为 R 长为 l 的圆柱形高斯面可得

$$E2\pi Rl=\frac{\int_0^R\frac{\rho_0}{[1+(\frac{r}{a})^2]^2}\mathrm{d}V}{\varepsilon_0}$$

$$\mathrm{d}V=2\pi rldr$$

$$E=\frac{\rho_0 a^2R}{2\varepsilon_0(a^2+R^2)}$$

$$E(r)=\frac{\rho_0 a^2 r}{2\varepsilon_0(a^2+r^2)}$$

(2)

$$U(r)=\int_r^0 Edr=\frac{\rho_0 a^2}{4\varepsilon_0}\ln\frac{a^2}{a^2+r^2}$$

1-41

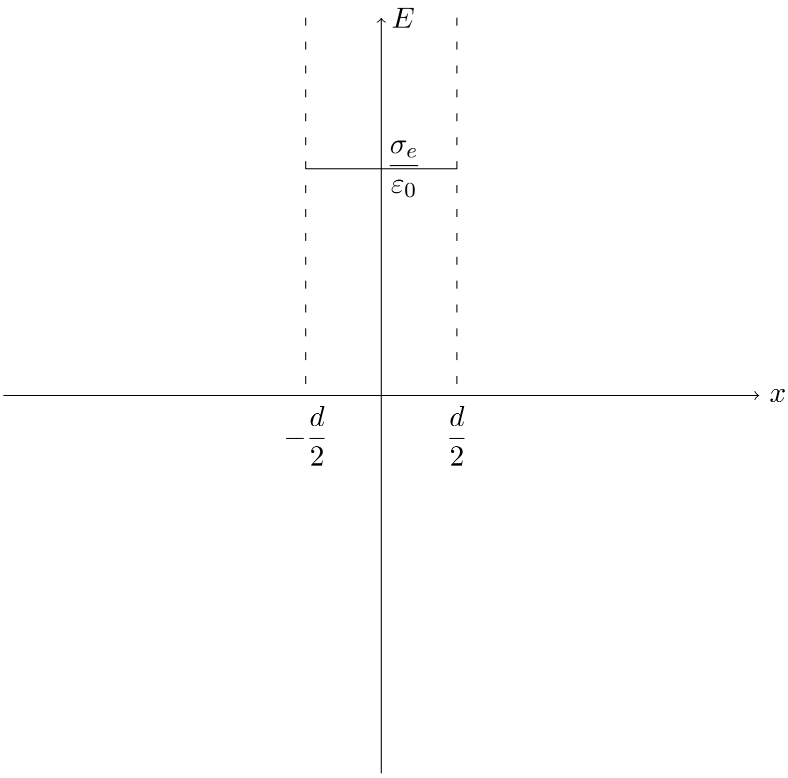
解　由场强叠加原理知

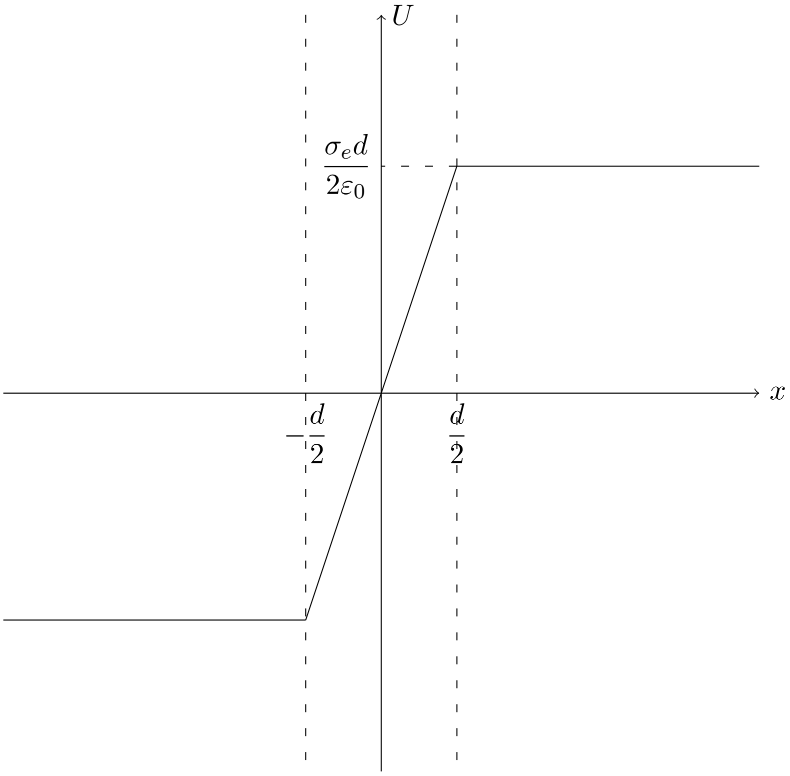
$$E=\begin{cases}0 & (x<-\dfrac{d}{2})\\ \dfrac{\sigma_e}{\varepsilon_0} & (-\dfrac{d}{2}<x<\dfrac{d}{2})\\ 0 & (x>\dfrac{d}{2})\end{cases}$$

又因为 O 处电势为 0

$$U(x)=\int_x^0 -E\mathrm{d}x=\frac{\sigma_e}{\varepsilon_0}x$$

其 $E-x$ 与 $U-x$ 图为



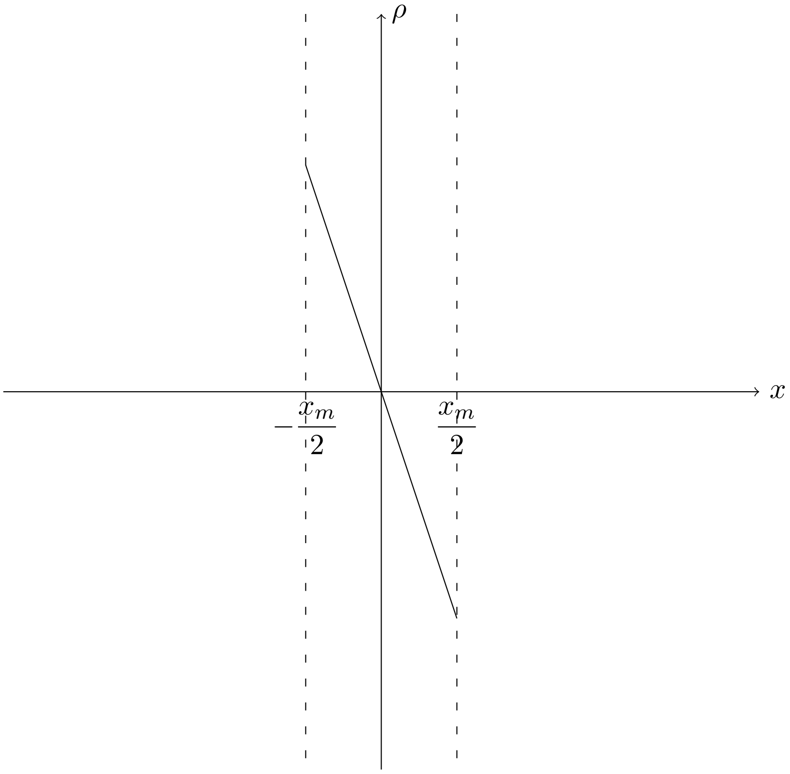


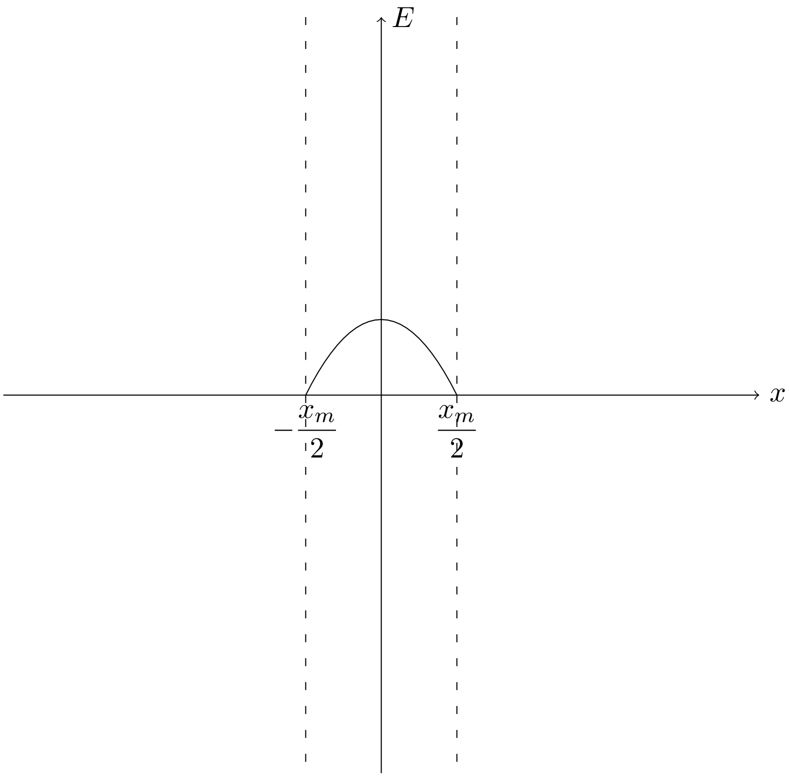
1-43

解　(1) 由高斯定理

$$E\cdot S=\frac{S}{\varepsilon_0}\int_{-x_N}^{X}\rho\mathrm{d}x$$

$$E=\frac{ea}{8\varepsilon_0}(x_m^2-4x^2)$$





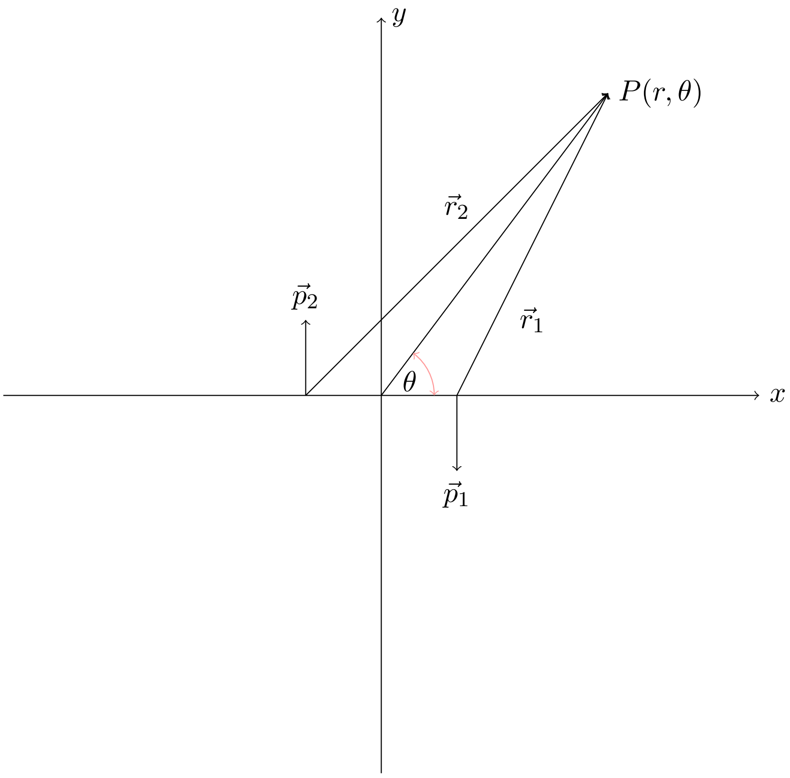
(2) 令 $U=0$, 则 $x=0$,该电势以原点为零点,$\Delta U = U(\frac{x_m}{2}) - U(-\frac{x_m}{2}) = -\frac{eax_m^2}{12\varepsilon_0}$

1-44

解　同一电场线上任取 AB 两点，过 AB 两点作底面积无限小的柱形高斯面，因为该面中无电荷 $E_AS = E_BS$ 故 $E_A = E_B$，在不同电场线任取 AC 作闭合矩形回路，因为场强环路积分为 0，故 $E_Al = E_Cl$，故 $E_A = E_C$，又因为 ABC 均是任取的，故处处场强相等。

附加题 1

解　该电四极子可视为两个电偶极子叠加，其电偶极距分别为 $\vec{p}_1$,$\vec{p}_2$



$$\begin{aligned} U_1 &= \frac{\vec{p}_1 \cdot \vec{r}_1}{4\pi\varepsilon_0 r_1^3} \\ &= \frac{-rql\sin\theta}{4\pi\varepsilon_0}\frac{1}{(r^2 + \frac{l^2}{4} - rl\cos\theta)^{\frac{3}{2}}} \end{aligned}$$

$$\begin{aligned} U_2 &= \frac{\vec{p}_2 \cdot \vec{r}_2}{4\pi\varepsilon_0 r_2^3} \\ &= \frac{rql\sin\theta}{4\pi\varepsilon_0}\frac{1}{(r^2 + \frac{l^2}{4} + rl\cos\theta)^{\frac{3}{2}}} \end{aligned}$$

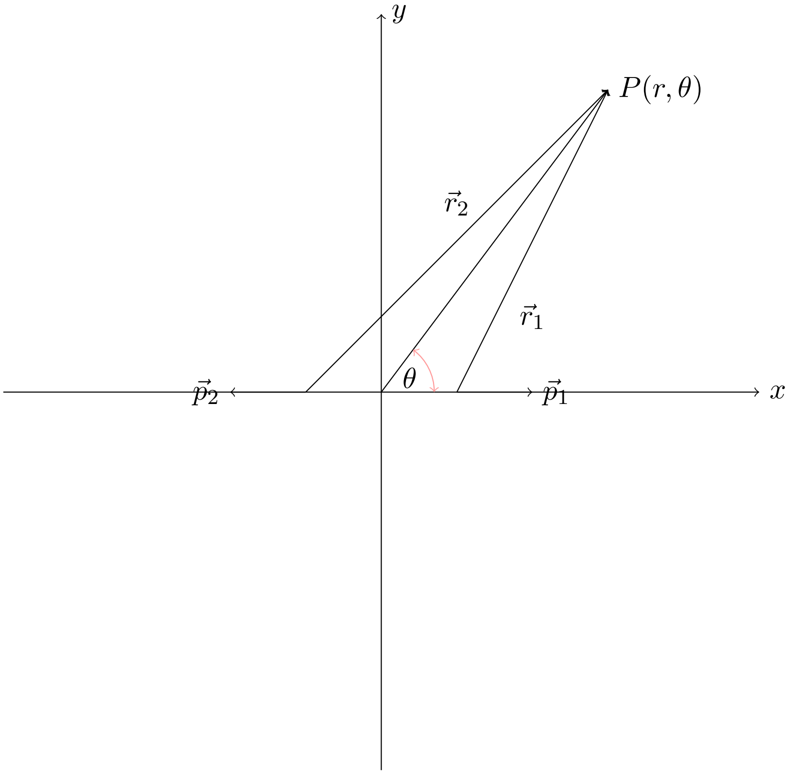
$$\begin{aligned} U_P &= U_1 + U_2 \\ &= \frac{-rql\sin\theta}{4\pi\varepsilon_0}[\frac{1}{(r^2 + \frac{l^2}{4} - rl\cos\theta)^{\frac{3}{2}}} - \frac{1}{(r^2 + \frac{l^2}{4} + rl\cos\theta)^{\frac{3}{2}}}] \end{aligned}$$

因为 $l \ll r$, 故略去二阶小量 $\dfrac{l^2}{4}$, 且运用近似 $(1+x)^k = 1 + kx(x \ll 1)$ 可得

$$U_P = \frac{-rql\sin\theta}{4\pi\varepsilon_0}\left(\frac{1+\frac{3}{2}\frac{l\cos\theta}{r} - 1 + \frac{3}{2}\frac{l\cos\theta}{r}}{r^3}\right) = \frac{-3ql\sin\theta\cos\theta}{4\pi\varepsilon_0 r^3}$$

附加题 2

解 该电四极子可视为两个电偶极子叠加,其电偶极距分别为 $\vec{p}_1$,$\vec{p}_2$



$$\begin{aligned} U_1 &= \frac{\vec{p}_1 \cdot \vec{r}_1}{4\pi\varepsilon_0 r_1^3} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{r\cos\theta ql - \frac{ql^2}{2}}{(r^2 + rl\cos\theta + \frac{l^2}{4})^{\frac{3}{2}}} \end{aligned}$$

$$\begin{aligned} U_2 &= \frac{\vec{p}_2 \cdot \vec{r}_2}{4\pi\varepsilon_0 r_2^3} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{r\cos\theta ql + \frac{ql^2}{2}}{(r^2 + rl\cos\theta - \frac{l^2}{4})^{\frac{3}{2}}} \end{aligned}$$

$$
\begin{aligned}
U_P &= U_1 + U_2 \\
&= \frac{r\cos\theta ql}{4\pi\varepsilon_0 r^3}[(r\cos\theta ql - \frac{ql^2}{2})(1+\frac{3l\cos\theta}{2r}) - (r\cos\theta ql + \frac{ql^2}{2})(1-\frac{3l\cos\theta}{2r})] \\
&= \frac{ql^2(3\cos^2\theta - 1)}{4\pi\varepsilon_0 r^3} = \frac{D(3\cos^2\theta-1)}{8\pi\varepsilon_0 r^3}
\end{aligned}
$$

$$
\begin{aligned}
\vec{E} &= -\nabla U \\
&= -\frac{\partial U}{\partial r}\hat{e}_r - \frac{\partial U}{r\partial\theta}\hat{e}_\theta \\
&= \frac{3ql^2}{4\pi\varepsilon_0 r^4}[(3\cos^2\theta - 1)\hat{e}_r + 2\sin\theta\cos\theta\hat{e}_\theta]
\end{aligned}
$$

电场线如图