5-31

解　连接电源时有

$$R_1 i_1 + L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} + M\frac{\mathrm{d}i_2}{\mathrm{d}t} = \mathscr{E}$$

$$R_2 i_2 + L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} + M\frac{\mathrm{d}i_1}{\mathrm{d}t} = 0$$

换成短接时有

$$R_1 i_1 + L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} + M\frac{\mathrm{d}i_2}{\mathrm{d}t} = 0$$

$$R_2 i_2 + L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} + M\frac{\mathrm{d}i_1}{\mathrm{d}t} = 0$$

由于这两组方程对应的齐次方程相同,故其时间常量相同,下面仅考虑短接时的情况。由短接的第一个方程有

$$\frac{\mathrm{d}i_1}{\mathrm{d}t} = -\frac{M\frac{\mathrm{d}i_2}{\mathrm{d}t} + R_1 i_1}{L_1}$$

代入第二个方程有

$$L_2\frac{\mathrm{d}i_2}{\mathrm{d}t} - \frac{M^2}{L_1}\frac{\mathrm{d}i_2}{\mathrm{d}t} - \frac{MR_1}{L_1}i_1 + R_2 i_2 = 0$$

引入无漏磁条件 $M^2 = L_1L_2$ 有

$$-\frac{MR_1}{L_1}i_1 + R_2 i_2 = 0$$

$$i_2 = \frac{MR_1}{L_1R_2}i_1$$

$$\frac{\mathrm{d}i_2}{\mathrm{d}t} = \frac{MR_1}{L_1R_2}\frac{\mathrm{d}i_1}{\mathrm{d}t}$$

代入第一个方程有

$$0 = L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} + \frac{M^2R_1}{L_1R_2}\frac{\mathrm{d}i_1}{\mathrm{d}t} + R_1 i_1$$

$$\frac{\mathrm{d}i_1}{\mathrm{d}t} = -\frac{R_1R_2}{R_1L_2 + R_2L_1}i_1$$

故时间常量为

$$\tau = \frac{1}{\frac{R_1R_2}{R_1L_2+R_2L_1}} = \frac{R_1L_2 + R_2L_1}{R_1R_2} = \frac{L_1}{R_2} + \frac{L_2}{R_1}$$

5-32

解　(1)

$$\frac{q}{C} + L\frac{\mathrm{d}i}{\mathrm{d}t} = 0$$

$$\frac{q}{C} + L\frac{\mathrm{d}^2 i}{\mathrm{d}t^2} = 0$$

$$q = C_1\sin(\frac{t}{\sqrt{CL}}) + C_2\cos(\frac{t}{\sqrt{CL}})$$

初始条件为 $t=0$ 时，$q=Q$、$i=0$。

故解为

$$q = Q\cos(\frac{t}{\sqrt{LC}})$$

欲令线圈磁场能等于电容中电能，即有

$$\begin{aligned}
\frac{q^2}{2C} &= \frac{LI^2}{2} \\
\frac{q^2}{2C} &= \frac{L}{2}(\frac{\mathrm{d}q}{\mathrm{d}t})^2 \\
\frac{Q^2}{2C}\cos^2(\frac{t}{\sqrt{LC}}) &= \frac{L}{2}\frac{Q^2}{LC}\cos^2(\frac{t}{\sqrt{LC}}) \\
\tan^2(\frac{t}{\sqrt{LC}}) &= 1 \\
t &= \frac{\pi}{4}\sqrt{LC}
\end{aligned}$$

(2)

$$q = Q\cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}Q$$

5-33

解　(1) 并联后总电容为 $C' = 2C = 4\mu\mathrm{F}$

$$\lambda = \frac{R}{2}\sqrt{\frac{C'}{L}} = 1.58 > 1$$

故不振荡

(2) 并联后总电容为 $C' = \frac{C^2}{2C} = 1\mu\mathrm{F}$

$$\lambda = \frac{R}{2}\sqrt{\frac{C'}{L}} = 0.79 < 1$$

故振荡