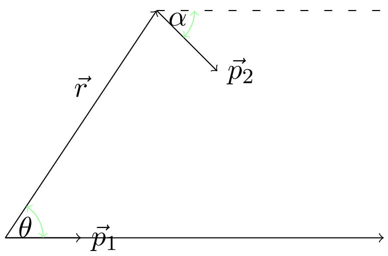
附加题 1

解



$\vec{p}_1$ 在 $\vec{p}_2$ 处产生的电势为

$$U = \frac{\vec{p}_1 \cdot \vec{r}}{4\pi\varepsilon_0 r^3} = \frac{p_1}{4\pi\varepsilon_0}\frac{x}{(x^2+y^2)^{\frac{3}{2}}}$$

故场强为

$$\vec{E} = -\nabla U = -\frac{\partial U}{\partial x}\hat{x} - \frac{\partial U}{\partial y}\hat{y} = \frac{p_1}{4\pi\varepsilon_0}\frac{2x^2-y^2}{(x^2+y^2)^{\frac{5}{2}}}\hat{x} + \frac{p_1}{4\pi\varepsilon_0}\frac{3xy}{(x^2+y^2)^{\frac{5}{2}}}\hat{y}$$

又 $\vec{p}_2 = p_2\cos\alpha\hat{x} - p_2\sin\alpha\hat{y}$ 故相互作用能

$$W = -\vec{p}_2 \cdot \vec{E} = -[\frac{p_1p_2\cos\alpha}{4\pi\varepsilon_0}\frac{2x^2-y^2}{(x^2+y^2)^{\frac{5}{2}}} - \frac{p_1p_2\sin\alpha}{4\pi\varepsilon_0}\frac{3xy}{(x^2+y^2)^{\frac{5}{2}}}]$$

代入 $x = r\cos\theta, y = r\sin\theta$ 得

$$W = \frac{p_1p_2}{4\pi\varepsilon_0 r^3}[\sin\theta\sin(\theta+\alpha) - 2\cos\theta\cos(\theta+\alpha)]$$

故

$$\vec{F} = -\nabla W = -\frac{\partial W}{\partial r}\hat{e}_r - \frac{\partial W}{r\partial\theta}\hat{e}_\theta = \frac{p_1p_2}{4\pi\varepsilon_0}\frac{3[\sin\theta\sin(\alpha+\theta) - 2\cos\theta\cos(\alpha+\theta)]}{r^4}\hat{e}_r - \frac{p_1p_2}{4\pi\varepsilon_0}\frac{3\sin\theta\cos(\alpha+\theta) + 3\cos\theta\sin(\alpha+\theta)}{r^4}\hat{e}_\theta$$

附加题 2

解　设电子经典半径为 a, 因为电荷在其中均匀分布,故其电荷体密度 $\rho=\dfrac{3\mathrm{e}}{4\pi a^3}$ 取半径为 r 的球形高斯面,当 $r<a$ 时可得

$$4\pi r^2E=\frac{\rho\frac{4\pi r^3}{3}}{\varepsilon_0}\to E=\frac{\rho r}{3\varepsilon_0}$$

当 $r>a$ 时可得

$$4\pi r^2E=\frac{\mathrm{e}}{\varepsilon_0}\to E=\frac{\mathrm{e}}{4\pi r^2\varepsilon_0}$$

则其自能为

$$\begin{aligned}W&=\frac{\varepsilon_0}{2}\iiint E^2\,\mathrm{d}V\\&=\int_0^a\frac{\varepsilon_0}{2}(\frac{\rho r}{3\varepsilon_0})^2 4\pi r^2\,\mathrm{d}r+\int_a^\infty\frac{\varepsilon_0}{2}(\frac{\mathrm{e}}{4\pi\varepsilon_0 r^2})\,\mathrm{d}r\\&=\frac{3\mathrm{e}^2}{20\pi\varepsilon_0 a}\end{aligned}$$

则

$$\begin{aligned}W&=m_\mathrm{e}\mathrm{c}^2\\\frac{3\mathrm{e}^2}{20\pi\varepsilon_0 a}&=m_\mathrm{e}\mathrm{c}^2\\a&=\frac{3\mathrm{e}^2}{20\pi\varepsilon_0 m_\mathrm{e}\mathrm{c}^2}=1.69\times10^{-15}\mathrm{m}\end{aligned}$$