



1

解 电场强度可写为

$$E = \frac{q}{4\pi\epsilon_0} \frac{1}{r^2}$$

其法向导数为

$$\begin{aligned} \frac{\partial E}{\partial r} &= \frac{q}{4\pi\epsilon_0} \frac{-2}{r^3} \\ &= \frac{-2E}{r} \end{aligned}$$

故

$$\frac{1}{E} \frac{\partial E}{\partial n} = -\frac{2}{R}$$

2

解 设该电偶极子在直角坐标下的电偶极矩为 $\vec{p} = (ql, 0, 0)$, 则产生的电势为

$$\begin{aligned} \varphi &= \frac{\vec{p} \cdot \vec{r}}{4\pi\epsilon_0 r^3} \\ &= \frac{qlx}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{\frac{3}{2}}} \end{aligned}$$

故电场强度为

$$\begin{aligned} \vec{E} &= -\nabla\varphi \\ &= \left[\frac{3lqx^2}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{5/2}} - \frac{lq}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{3/2}} \right] \vec{e}_x + \\ &\quad \frac{3lqxy}{(x^2 + y^2 + z^2)^{5/2}} \vec{e}_y + \frac{3lqxz}{(x^2 + y^2 + z^2)^{5/2}} \vec{e}_z \end{aligned}$$

故

$$\begin{aligned} \frac{\partial E_x}{\partial y} &= \frac{3lqy}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{5/2}} - \frac{15lqx^2y}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{7/2}} \\ \frac{\partial E_x}{\partial z} &= \frac{3lqz}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{5/2}} - \frac{15lqx^2z}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{7/2}} \\ \frac{\partial E_y}{\partial x} &= \frac{3lqy}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{5/2}} - \frac{15lqx^2y}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{7/2}} \\ \frac{\partial E_y}{\partial z} &= -\frac{15lqxyz}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{7/2}} \\ \frac{\partial E_z}{\partial x} &= \frac{3lqy}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{5/2}} - \frac{15lqx^2y}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{7/2}} \end{aligned}$$



$$\frac{\partial E_z}{\partial y} = -\frac{15lqxyz}{4\pi\epsilon_0(x^2 + y^2 + z^2)^{7/2}}$$

因为

$$\begin{aligned}\frac{\partial E_x}{\partial y} &= \frac{\partial E_y}{\partial x} \\ \frac{\partial E_x}{\partial z} &= \frac{\partial E_z}{\partial x} \\ \frac{\partial E_y}{\partial z} &= \frac{\partial E_z}{\partial y}\end{aligned}$$

故

$$\nabla \times \vec{E} = 0$$

3

解 设该电荷位于坐标原点, 则其产生的电势为

$$\begin{aligned}\varphi &= \frac{q}{4\pi\epsilon_0 r} \\ &= \frac{q}{4\pi\epsilon_0} \frac{1}{\sqrt{x^2 + y^2 + z^2}}\end{aligned}$$

考察其穿过平面 $z = a$ ($-a < x < a, -a < y < a$) 的电通量, 只需计算电场的 z 方向分量

$$\begin{aligned}E_z &= \frac{\partial \varphi}{\partial z} \\ &= -\frac{q}{4\pi\epsilon_0} \frac{z}{(x^2 + y^2 + z^2)^{3/2}}\end{aligned}$$

故其穿过平面 $z = a$ ($-a < x < a, -a < y < a$) 的电通量为

$$\begin{aligned}I &= \iint \frac{q}{4\pi\epsilon_0} \frac{a}{(x^2 + y^2 + a^2)^{3/2}} dx dy \\ &= \frac{aq}{4\pi\epsilon_0} \int_{-a}^a dy \int_{-a}^a \frac{1}{(x^2 + y^2 + a^2)^{3/2}} dx \\ &= \frac{aq}{4\pi\epsilon_0} \int_{-a}^a \frac{2a}{(a^2 + y^2) \sqrt{2a^2 + y^2}} dy \\ &= \frac{aq}{4\pi\epsilon_0} \frac{2\pi}{3a} \\ &= \frac{q}{6\epsilon_0}\end{aligned}$$

4



解 由对称性知, 该点电场方向必为径向, 故只需计算球上各点在该点产生电场的径向分量, 取圆环微元该微元带电量为

$$q = 2\pi R^2 \sigma \sin \theta d\theta$$

其在考察点产生的电场强度为

$$\begin{aligned} &= \frac{dq}{4\pi\epsilon_0} \frac{R - R \cos \theta}{[R^2 \sin^2 \theta + (R - R \cos \theta)^2]^{\frac{3}{2}}} \\ &= \frac{2\pi R^2 \sigma \sin \theta d\theta}{4\pi\epsilon_0} \frac{R - R \cos \theta}{[R^2 \sin^2 \theta + (R - R \cos \theta)^2]^{\frac{3}{2}}} \end{aligned}$$

故

$$\begin{aligned} E &= \int dE \\ &= \int_{\theta_0}^{\pi} \frac{2\pi R^2 \sigma \sin \theta d\theta}{4\pi\epsilon_0} \frac{R - R \cos \theta}{[R^2 \sin^2 \theta + (R - R \cos \theta)^2]^{\frac{3}{2}}} \\ &= \frac{\sigma}{2\epsilon_0} \int_{\theta_0}^{\pi} \frac{\sin \theta (1 - \cos \theta) d\theta}{[\sin^2 \theta + (1 - \cos \theta)^2]^{\frac{3}{2}}} \\ &= \frac{\sigma}{2\epsilon_0} (1 - \sin \frac{\theta_0}{2}) \end{aligned}$$

故该点电场强度为

$$E = \frac{\sigma}{2\epsilon_0} (1 - \sin \frac{\theta_0}{2})$$

方向沿径向