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解 (1) 易知在 $0 < r < r_1$ 处 $\vec{E} = 0$

在 $r_1 < r < r_2$ 处

$$4\pi r^2 D = \frac{4\pi(r^3 - r_1^3)\rho_f}{3}$$

$$D = \frac{(r^3 - r_1^3)\rho_f}{3r^2}$$

$$\varepsilon E = \frac{(r^3 - r_1^3)\rho_f}{3r^2}$$

$$E = \frac{(r^3 - r_1^3)\rho_f}{3\varepsilon r^2}$$

$$\text{故 } \vec{E} = \frac{(r^3 - r_1^3)\rho_f \vec{r}}{3r^3}$$

在 $r > r_2$ 处

$$4\pi r^2 D = \frac{4\pi(r_2^3 - r_1^3)\rho_f}{3}$$

$$D = \frac{(r_2^3 - r_1^3)\rho_f}{3r^2}$$

$$\varepsilon_0 E = \frac{(r_2^3 - r_1^3)\rho_f}{3r^2}$$

$$E = \frac{(r_2^3 - r_1^3)\rho_f}{3\varepsilon_0 r^2}$$

$$\text{故 } \vec{E} = \frac{(r_2^3 - r_1^3)\rho_f \vec{r}}{3r^3}$$

(2) 易知只在 $r_1 < r < r_2$ 处有极化体电荷, 有

$$\begin{aligned} \vec{P} &= \vec{D} - \varepsilon_0 \vec{E} \\ &= \vec{D} - \varepsilon_0 \frac{\vec{D}}{\varepsilon} \\ &= \left(1 - \frac{\varepsilon_0}{\varepsilon}\right) \vec{D} \end{aligned}$$

故

$$\begin{aligned} \rho_p &= \nabla \cdot \vec{P} \\ &= -\left(1 - \frac{\varepsilon_0}{\varepsilon}\right) \nabla \cdot \vec{D} \\ &= -\left(1 - \frac{\varepsilon_0}{\varepsilon}\right) \rho_f \end{aligned}$$



考虑内球壳, 此时 $r = r_1$ 。

$$\sigma_p = -\frac{(\varepsilon - \varepsilon_0)(r^3 - r_1^3)\rho_f}{3\varepsilon r^2}|_{r=r_1} = 0$$

考虑外球壳, 此时 $r = r_2$ 。

$$\sigma_p = \frac{(\varepsilon - \varepsilon_0)(r^3 - r_1^3)\rho_f}{3\varepsilon r^3}|_{r=r_2} = (1 - \frac{\varepsilon_0}{\varepsilon})\frac{r_2^3 - r_1^3}{3r_2^3}\rho_f$$

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解 易知在 $0 < r < r_1$ 处 $\vec{B} = 0$

在 $r_1 < r < r_2$ 处

$$2\pi r H = j_f \pi (r^2 - r_1^2)$$

$$H = \frac{j_f (r^2 - r_1^2)}{2r}$$

$$\frac{B}{\mu} = \frac{j_f (r^2 - r_1^2)}{2r}$$

$$B = \frac{\mu j_f (r^2 - r_1^2)}{2r}$$

$$\text{故 } \vec{B} = \frac{\mu(r^2 - r_1^2)\vec{j}_f \times \vec{r}}{2r^2}$$

在 $r > r_2$ 处

$$2\pi r H = j_f \pi (r_2^2 - r_1^2)$$

$$H = \frac{j_f (r_2^2 - r_1^2)}{2r}$$

$$\frac{B}{\mu_0} = \frac{j_f (r_2^2 - r_1^2)}{2r}$$

$$B = \frac{\mu_0 j_f (r_2^2 - r_1^2)}{2r}$$

$$\text{故 } \vec{B} = \frac{\mu_0(r_2^2 - r_1^2)\vec{j}_f \times \vec{r}}{2r^2}$$

易知只在 $r_1 < r < r_2$ 处有磁化电流, 有

$$\begin{aligned} \vec{M} &= \frac{\vec{B}}{\mu_0} - \vec{H} \\ &= \frac{\mu \vec{H}}{\mu_0} - \vec{H} \\ &= \left(\frac{\mu}{\mu_0} - 1\right) \vec{H} \end{aligned}$$



故

$$\begin{aligned}\vec{j}_m &= \nabla \times \vec{M} \\ &= \left(\frac{\mu}{\mu_0} - 1\right) \nabla \times \vec{H} \\ &= \left(\frac{\mu}{\mu_0} - 1\right) \vec{j}_f\end{aligned}$$

在内表面, 此时 $r = r_1$ 。

$$\alpha_m = \vec{n} \times M|_{r=r_1} = 0$$

在外表面, 此时 $r = r_2$ 。

$$\alpha_m = -\vec{n} \times M|_{r=r_2} = -\left(\frac{\mu}{\mu_0} - 1\right) \frac{r_2^2 - r_1^2}{2r^2} \vec{j}_f$$

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解 (1)

$$E_1 l_1 + E_2 l_2 = \mathcal{E}$$

$$D_1 = D_2$$

$$D_1 = \varepsilon_1 E_1$$

$$D_2 = \varepsilon_2 E_2$$

解得

$$D = \frac{\varepsilon_1 \varepsilon_2 E}{l_1 \varepsilon_2 + l_2 \varepsilon_1}$$

故

$$\omega_{f2} = -\omega_{f1} = D = \frac{\varepsilon_1 \varepsilon_2 E}{l_1 \varepsilon_2 + l_2 \varepsilon_1}$$

(2)

$$\omega_{f3} = 0$$

若介质漏电, 设漏电电流为 \vec{j}_1, \vec{j}_2

(1)

$$\vec{n} \cdot (\vec{j}_2 - \vec{j}_1) = 0$$

$$l_1 \frac{\vec{j}_1}{\sigma_1} + l_2 \frac{\vec{j}_2}{\sigma_2} = \mathcal{E}$$



解得

$$E_1 = \frac{\sigma_2 \mathcal{E}}{l_1 \sigma_2 + l_2 \sigma_1}$$

$$E_2 = \frac{\sigma_1 \mathcal{E}}{l_1 \sigma_2 + l_2 \sigma_1}$$

故

$$\omega_{f1} = D_1 = \frac{\varepsilon_1 \sigma_2 \mathcal{E}}{l_1 \sigma_2 + l_2 \sigma_1}$$

$$\omega_{f2} = D_2 = \frac{\varepsilon_2 \sigma_1 \mathcal{E}}{l_1 \sigma_2 + l_2 \sigma_1}$$

$$\omega_{f3} = D_2 - D_1 = \frac{\varepsilon_2 \sigma_1 \mathcal{E} - \varepsilon_1 \sigma_2 \mathcal{E}}{l_1 \sigma_2 + l_2 \sigma_1}$$

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解 (1) 由 \vec{D} 的法向连续条件知

$$D_1 \cos \theta_1 = D_2 \cos \theta_2$$

$$E_1 \varepsilon_1 \cos \theta_1 = E_2 \varepsilon_2 \cos \theta_2$$

又由 \vec{E} 的切向连续条件知

$$E_1 \sin \theta_1 = E_2 \sin \theta_2$$

故有

$$\frac{\tan \theta_2}{\tan \theta_1} = \frac{\varepsilon_2}{\varepsilon_1}$$

(2) 由 \vec{j} 的法向连续条件知

$$j_1 \cos \theta_1 = j_2 \cos \theta_2$$

$$E_1 \sigma_1 \cos \theta_1 = E_2 \sigma_2 \cos \theta_2$$

又由 \vec{E} 的切向连续条件知

$$E_1 \sin \theta_1 = E_2 \sin \theta_2$$

故有

$$\frac{\tan \theta_2}{\tan \theta_1} = \frac{\sigma_2}{\sigma_1}$$

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解 不妨设电场方程为

$$\vec{E} = \vec{E}_0 \sin(\omega t - kx)$$

则有

$$\begin{aligned} \frac{\partial \vec{B}}{\partial t} &= -\nabla \times \vec{E} \\ &= -\vec{E}_0 \times \nabla(\sin(\omega t - kx)) \\ &= -\vec{E}_0 \times (-k\vec{e}_x \cos(\omega t - kx)) \\ &= k \cos(\omega t - kx) \vec{E}_0 \times \vec{e}_x \end{aligned}$$

故

$$\vec{B} = \int \cos(\omega t - kx) dt k \vec{E}_0 \times \vec{e}_x$$

略去不属于电磁波部分的常数后可得

$$\vec{B} = \frac{k}{\omega} \sin(\omega t - kx) \vec{E}_0 \times \vec{e}_x$$

则能量密度为

$$\begin{aligned} w &= \frac{1}{2}(\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B}) \\ &= \varepsilon_0 E_0^2 \sin^2(\omega t - kx) \end{aligned}$$

平均值为

$$\begin{aligned} \bar{w} &= \frac{1}{T} \int_0^T \varepsilon_0 E_0^2 \sin^2(\omega t - kx) dt \\ &= \frac{1}{2} \varepsilon_0 E_0^2 \end{aligned}$$

坡印廷矢量的瞬时值为

$$\begin{aligned} \vec{S} &= \Re E \times \Re H \\ &= \vec{E}_0 \cos(\omega t - kx - \frac{\pi}{2}) \times (\frac{k}{\omega \mu_0} \cos(\omega t - kx - \frac{\pi}{2}) \vec{E}_0 \times \vec{e}_x) \\ &= \frac{k \vec{e}_x}{\omega \mu_0} E_0^2 \cos^2(\omega t - kx - \frac{\pi}{2}) \end{aligned}$$

则均值为

$$\bar{S} = \frac{1}{T} \int_0^T \vec{S} dt$$



$$= \frac{E_0^2 k \vec{e}_x}{2\omega\mu_0}$$

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解 (1) 静电条件下, 导体内部电场为 0。由 \vec{E} 的切向连续可知

$$\vec{n} \times \vec{E}_{\text{外}} = 0$$

故 $\vec{E}_{\text{外}}$ 垂直于导体表面。

(2) 稳恒电流条件下导体表面 $\sigma_f = 0$ 。故由 \vec{D} 的切向连续可知

$$\vec{n} \cdot (\vec{D}_{\text{内}} - \vec{D}_{\text{外}}) = 0$$

又因为 $\vec{D}_{\text{外}} = 0$, 故有

$$\vec{n} \cdot \vec{D}_{\text{内}} = 0$$

$$\vec{n} \cdot \vec{E}_{\text{内}} = 0$$

即电场方向平行于导体平面

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解 (1) 由高斯定理知

$$\nabla \cdot \vec{D} = \rho_f$$

代入电荷守恒方程可得

$$\nabla \cdot \vec{j} + \frac{\partial \nabla \cdot \vec{D}}{\partial t} = 0$$

$$\nabla \cdot (\vec{j} + \frac{\partial \vec{D}}{\partial t}) = 0$$

$$\vec{j} + \frac{\partial \vec{D}}{\partial t} = 0$$

(2) 由高斯定理知

$$\vec{D} = \frac{\lambda_f}{2\pi r} \vec{e}_r$$

$$\vec{E} = \frac{\lambda_f}{2\pi\epsilon r} \vec{e}_r$$

又

$$\vec{j} + \frac{\partial \vec{D}}{\partial t} = 0$$

$$\sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} = 0$$



$$\vec{E} = \vec{E}_0 e^{-\frac{\sigma}{\epsilon} t}$$

故

$$\begin{aligned} \frac{\lambda_f}{2\pi\epsilon r} \vec{e}_r &= \frac{\lambda_{f0}}{2\pi\epsilon r} e^{-\frac{\sigma}{\epsilon} t} \vec{e}_r \\ \lambda_f &= \lambda_{f0} e^{-\frac{\sigma}{\epsilon} t} \end{aligned}$$

(3)

$$\begin{aligned} w &= j^2 \rho \\ &= \sigma^2 E^2 \frac{1}{\sigma} \\ &= \left(\frac{\lambda_f}{2\pi\epsilon r} \right)^2 \sigma \end{aligned}$$

(4)

$$\begin{aligned} P &= \int_a^b \left(\frac{\lambda_f}{2\pi\epsilon r} \right)^2 \sigma 2\pi r l \, dr \\ &= \frac{\lambda_f^2 \sigma l}{2\pi\epsilon^2} \ln \frac{b}{a} \end{aligned}$$

静电能

$$\begin{aligned} W &= \int \frac{\vec{E} \cdot \vec{D}}{2} \, dV \\ &= \int_a^b \frac{\lambda_f^2 l}{4\pi\epsilon r} \, dr \\ &= \frac{\lambda_f^2 l}{4\pi\epsilon} \ln \frac{b}{a} \end{aligned}$$

$$\begin{aligned} -\frac{dW}{dt} &= -\frac{\lambda_f l}{2\pi\epsilon} \ln \frac{b}{a} \frac{d\lambda_f}{dt} \\ &= \frac{\lambda_f^2 \sigma l}{2\pi\epsilon^2} \ln \frac{b}{a} \end{aligned}$$

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解 (1)

$$\begin{aligned} \rho_p &= -\nabla \cdot \vec{P} \\ &= -\left[\left(\nabla \frac{K}{r^2} \right) \cdot \vec{r} + \frac{K}{r^2} (\nabla \cdot \vec{r}) \right] \\ &= -\left(\frac{K}{r^2} + 3 \frac{K}{r^2} \right) \end{aligned}$$



$$= -\frac{K}{r^2}$$

$$\begin{aligned}\sigma_p &= -\vec{n} \cdot (\vec{P}_2 - \vec{P}_1)|_{r=R} \\ &= \vec{n} \cdot \vec{P}_1|_{r=R} \\ &= \vec{n} \cdot K \frac{\vec{r}}{r^2}|_{r=R} \\ &= \frac{K}{R}\end{aligned}$$

(2) 由

$$\begin{aligned}\vec{D} &= \epsilon_0 \vec{E} + \vec{P} \\ \vec{D} &= \epsilon \vec{E}\end{aligned}$$

可得

$$\vec{D} = \frac{\epsilon \vec{P}}{\epsilon - \epsilon_0}$$

故

$$\begin{aligned}\rho_f &= \nabla \cdot \vec{D} \\ &= \frac{\epsilon}{\epsilon - \epsilon_0} \nabla \cdot \vec{P} \\ &= \frac{\epsilon}{\epsilon - \epsilon_0} \frac{K}{r^2} \\ &= \frac{\epsilon K}{(\epsilon - \epsilon_0)r^2}\end{aligned}$$

(3) 在球内部

$$\begin{aligned}\vec{E} &= \frac{\vec{P}}{\epsilon - \epsilon_0} \\ &= \frac{K \vec{r}}{(\epsilon - \epsilon_0)r^2}\end{aligned}$$

球外部

$$\begin{aligned}4\pi r^2 E &= \int_0^R 4\pi r^2 \frac{\rho_f}{\epsilon_0} dr \\ 4\pi r^2 E &= \int_0^R 4\pi r^2 \frac{\epsilon K}{\epsilon_0(\epsilon - \epsilon_0)r^2} dr \\ 4\pi r^2 E &= \frac{4\pi \epsilon K R}{\epsilon_0(\epsilon - \epsilon_0)}\end{aligned}$$



$$E = \frac{\varepsilon K R}{\varepsilon_0(\varepsilon - \varepsilon_0)r^2}$$

故

$$\vec{E} = \frac{\varepsilon K R}{\varepsilon_0(\varepsilon - \varepsilon_0)r^3} \vec{r}$$

故

$$\begin{aligned} \varphi &= \int_{\infty}^r \frac{\varepsilon K R}{\varepsilon_0(\varepsilon - \varepsilon_0)r^3} \vec{r} \cdot d\vec{r} \\ &= \frac{\varepsilon K R}{(\varepsilon - \varepsilon_0)\varepsilon_0 r} \end{aligned}$$

在球内部

$$\begin{aligned} \vec{E} &= \frac{\vec{P}}{\varepsilon - \varepsilon_0} \\ &= \frac{K \vec{r}}{(\varepsilon - \varepsilon_0)r^2} \end{aligned}$$

故

$$\begin{aligned} \varphi &= \varphi(R) + \int_R^r \frac{K \vec{r}}{(\varepsilon - \varepsilon_0)r^2} \cdot d\vec{r} \\ &= \frac{K}{\varepsilon - \varepsilon_0} \left(\ln \frac{R}{r} + \frac{\varepsilon}{\varepsilon_0} \right) \end{aligned}$$

(4)

$$\begin{aligned} W &= \int_0^R \frac{\varepsilon E^2}{2} 4\pi r^2 dr + \int_R^{\infty} \frac{\varepsilon_0 E^2}{2} 4\pi r^2 dr \\ &= \int_0^R \frac{2\pi \varepsilon K^2}{(\varepsilon - \varepsilon_0)} dr + \int_R^{\infty} \frac{2\pi \varepsilon^2 K^2 R}{\varepsilon_0(\varepsilon - \varepsilon_0)^2 r^2} dr \\ &= 2\pi \varepsilon R \left(\frac{K}{\varepsilon - \varepsilon_0} \right)^2 + \frac{2\pi \varepsilon^2 R K^2}{\varepsilon_0(\varepsilon - \varepsilon_0)^2} \\ &= 2\pi \varepsilon R \left(1 + \frac{\varepsilon}{\varepsilon_0} \right) \left(\frac{K}{\varepsilon - \varepsilon_0} \right)^2 \end{aligned}$$