

1-66

解 (1)

$$j = \frac{I}{S} = \sigma E$$

故

$$E_1 = \frac{I}{\sigma_1 S}$$

$$E_2 = \frac{I}{\sigma_2 S}$$

(2)

$$U_{AB} = E_1 d_1 = \frac{I d_1}{\sigma_1 S}$$

$$U_{AB} = E_2 d_2 = \frac{I d_2}{\sigma_2 S}$$

2-3(思考题)

解

$$\begin{aligned}
 \vec{F}_{12} &= \frac{\mu_0}{4\pi} \oint_{(L_1)} \oint_{(L_2)} \frac{I_1 I_2 d\vec{l}_1 \times (d\vec{l}_2 \times \hat{r}_{12})}{r_{12}^2} \\
 &= \frac{\mu_0}{4\pi} \oint_{(L_1)} \oint_{(L_2)} \frac{I_1 I_2 [(d\vec{l}_1 \cdot \hat{r}_{12}) d\vec{l}_2 - (d\vec{l}_1 \cdot d\vec{l}_2) \hat{r}_{12}]}{r_{12}^2} \\
 &= -\frac{\mu_0}{4\pi} \oint_{(L_1)} \oint_{(L_2)} \frac{I_1 I_2 (d\vec{l}_1 \cdot d\vec{l}_2) \hat{r}_{12}}{r_{12}^2}
 \end{aligned}$$

又因为被积函数连续, 故积分可交换顺序, 即

$$\vec{F}_{12} = -\frac{\mu_0}{4\pi} \oint_{(L_2)} \oint_{(L_1)} \frac{I_1 I_2 (d\vec{l}_1 \cdot d\vec{l}_2) \hat{r}_{12}}{r_{12}^2}$$

同理

$$\vec{F}_{21} = -\frac{\mu_0}{4\pi} \oint_{(L_2)} \oint_{(L_1)} \frac{I_1 I_2 (d\vec{l}_1 \cdot d\vec{l}_2) \hat{r}_{21}}{r_{21}^2}$$

又因为 $\hat{r}_{21} = -\hat{r}_{12}$, 故

$$\vec{F}_{12} = -\vec{F}_{21}$$

2-4

解

$$B = \frac{\mu_0}{2\pi d} \sqrt{I_1^2 + I_2^2} = 7.2 \times 10^{-5} \text{T}$$

由两个分量组成, 其中

$$B_1 = \frac{\mu_0 I_1}{2\pi d} = 4.0 \times 10^{-5} \text{T}$$

方向垂直纸面向里

$$B_2 = \frac{\mu_0 I_2}{2\pi d} = 6.0 \times 10^{-5} \text{T}$$

方向平行纸面向右

2-5

解 (1) 由系统对称性知, 磁感应强度一定沿轴线方向, 且三条边贡献相等不妨设其中一条边为线段 $(0, 0, 0) \rightarrow (2a, 0, 0)$ 且其余两边都在 xOy 平面上且均在第一象限则轴线上一点坐标为 $(a, \sqrt{3}a, r_0)$, 则位于点 $(x, 0, 0)$ 处的电流元在该点产生的磁感应强度为

$$\begin{aligned} B &= \frac{\mu_0 I d\mathbf{x} \times \hat{\mathbf{r}}}{4\pi r^2} \\ &= \frac{\mu_0 I}{4\pi} \frac{-r_0 dx \vec{j} + \frac{\sqrt{3}}{2} a dx \vec{k}}{[(a-x)^2 + \frac{3}{4}a^2 + r_0^2]^{\frac{3}{2}}} \end{aligned}$$

我们只取沿轴线即 z 轴方向分量则

$$\begin{aligned} \vec{B} &= 3 \int_0^{2a} \frac{\mu_0 I}{4\pi} \frac{\frac{\sqrt{3}}{2} a \vec{k}}{[(a-x)^2 + \frac{3}{4}a^2 + r_0^2]^{\frac{3}{2}}} dx \\ &= \frac{9\mu_0 I a^2}{2\pi(3r_0^2 + a^2)\sqrt{3r_0^2 + 4a^2}} \vec{k} \end{aligned}$$

(2) 当 $r_0 \gg a$ 时 $B = \frac{\sqrt{3}\mu_0 I a^2}{2\pi r_0^3}$, 而 $S = \sqrt{3}a^2$ 故

$$B = \frac{\mu_0 m}{2\pi r_0^3}$$

2-6

解 (1) 将载流板分割为无数无限细无限长的载流导线, 叠加得

$$B = \int dB \cos \theta = \int_{-a}^a \frac{\mu_0}{4\pi} \frac{2(\frac{I}{2a}) dl}{\sqrt{x^2 + l^2}} \frac{x}{\sqrt{x^2 + l^2}} = \frac{\mu_0 I}{2\pi a} \arctan \frac{a}{x}$$

(2)

$$B = \frac{\mu_0 I}{2}$$