1

解　(1)

$$\vec{A}_1 = \vec{A}_0 \cos\left(k_1 z - \omega_1 t\right)$$
$$\vec{A}_2 = \vec{A}_0 \cdot \cos\left(k_2 z - \omega_2 t\right)$$

故

$$\begin{aligned}\vec{A}_1 + \vec{A}_2 &= \vec{A}_0 \cdot 2\left(\cos\left(k_1 z - \omega_1 t\right) + \cos\left(k_2 z - \omega_2 t\right)\right) \\ &= \vec{A}_0 \cdot 2\cos\frac{\left(k_1 - k_2\right)z - \left(\omega_1 - \omega_2\right)t}{2}\cos\frac{\left(k_1 + k_2\right)z - \left(\omega_1 + \omega_2\right)t}{2}\end{aligned}$$

又

$$k_1 = k + \mathrm{d}k$$
$$k2 = k - \mathrm{d}k$$
$$\omega_1 = \omega + \mathrm{d}\omega$$
$$\omega_2 = \omega + \mathrm{d}\omega$$

故

$$\begin{aligned}\vec{A} &= \vec{A}_0 \cdot 2\cos\left(\mathrm{d}k \cdot z - \mathrm{d}\omega \cdot t\right)\cos\left(k \cdot z - \omega \cdot t\right) \\ &= \vec{A}_0 \cdot 2\cos\left(\mathrm{d}k \cdot z - \mathrm{d}\omega \cdot t\right) e^{\mathrm{i}(kz - \omega t)}\end{aligned}$$

(2) 相速度:

$$kx - \omega t = 0$$
$$v_p = \frac{\omega}{k}$$

群速度:

$$kx - \mathrm{d}\omega t = 0$$
$$v_g = \frac{\mathrm{d}\omega}{\mathrm{d}k}$$

2

解　(1)

$$\nabla \cdot \vec{B} = 0$$
$$\mathrm{i}\vec{k} \cdot \vec{B} = 0$$
$$\vec{k} \cdot \vec{B} = 0$$

$$\nabla\cdot\vec{D}=0$$
$$\mathrm{i}\vec{k}\cdot\vec{D}=0$$
$$\vec{k}\cdot\vec{D}=0$$
$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$
$$\mathrm{i}\vec{k}\times\vec{E}=\mathrm{i}\omega\vec{B}$$
$$\vec{B}=\frac{\vec{k}\times\vec{E}}{\omega}$$
$$\vec{B}\cdot\vec{E}=0$$
$$\nabla\times\vec{H}=\frac{\partial\vec{D}}{\partial t}$$
$$\mathrm{i}\vec{k}\times\vec{H}=-\mathrm{i}\omega\vec{D}$$
$$\vec{D}=\frac{-\vec{k}\times\vec{B}}{\omega\mu}$$

故

$$\vec{k}\cdot\vec{B}=\vec{k}\cdot\vec{B}=\vec{B}\cdot\vec{D}=\vec{B}\cdot\vec{D}=\vec{B}\cdot\vec{E}=0$$

$$\nabla\cdot\vec{E}=\frac{\rho}{\varepsilon_0}$$
$$\mathrm{i}\vec{k}\cdot\vec{E}=\frac{\rho}{\varepsilon_0}$$
$$\vec{k}\cdot\vec{E}=\frac{\rho}{\mathrm{i}\varepsilon_0}$$

而介质中由于极化电荷的存在 ρ 一般不为 0。故一般 $\vec{k}\cdot\vec{E}\neq 0$。

(2)

$$\begin{aligned}\vec{D}&=\frac{-\vec{k}\times\vec{B}}{\omega\mu}\\&=\frac{-\vec{k}\times\frac{\vec{k}\times\vec{E}}{\omega}}{\omega\mu}\\&=\frac{-\vec{k}\times(\vec{k}\times\vec{E})}{\omega^2\mu}\\&=\frac{k^2\vec{E}-(\vec{k}\cdot\vec{E})\vec{k}}{\omega^2\mu}\end{aligned}$$

(3)

$$\vec{S}=\vec{E}\times\vec{H}$$

$$
\begin{aligned}
&= \vec{E} \times \frac{\vec{k} \times \vec{E}}{\mu\omega} \\
&= \frac{E^2\vec{k} - (\vec{k}\vec{E})\vec{E}}{\mu\omega}
\end{aligned}
$$

若要令 $\vec{k}$ 与 $\vec{S}$ 在同一方向那么就要使 $(\vec{k}\vec{E})\vec{E} = 0$,一般不满足。

3

解

$$
\begin{aligned}
A_x &= A_0 \cos(kz - \omega t) \\
A_y &= A_0 \cos\left(kz - \omega t + \frac{\pi}{2}\right)
\end{aligned}
$$

故

$$
A_x^2 + A_y^2 = A_0^2
$$

即圆偏振

4

解

$$
\begin{aligned}
\vec{A}_1 &= \left(a\cos(kz - \omega t), a\cos\left(kz - \omega t + \frac{\pi}{2}\right)\right) \\
\vec{A}_2 &= \left(b\cos(kz - \omega t), -b\cos\left(kz - \omega t + \frac{\pi}{2}\right)\right)
\end{aligned}
$$

故

$$
\begin{aligned}
\vec{A} &= \left(a\cos(kz - \omega t) + b\cos(kz - \omega t), a\cos\left(kz - \omega t + \frac{\pi}{2}\right) - b\cos\left(kz - \omega t + \frac{\pi}{2}\right)\right) \\
&= \left((a+b)\cos(kz - \omega t), (a-b)\cos\left(kz - \omega t + \frac{\pi}{2}\right)\right)
\end{aligned}
$$

若 $\frac{a}{b} = \pm 1$,则线偏振。

若 $\frac{a}{b} \neq \pm 1$,则椭圆偏振。