1

解 (1) 易知在 $0 < r < r_1$ 处 $\vec{E} = 0$

在 $r_1 < r < r_2$ 处

$$4\pi r^2 D = \frac{4\pi(r^3 - r_1^3)\rho_f}{3}$$

$$D = \frac{(r^3 - r_1^3)\rho_f}{3r^2}$$

$$\varepsilon E = \frac{(r^3 - r_1^3)\rho_f}{3r^2}$$

$$E = \frac{(r^3 - r_1^3)\rho_f}{3\varepsilon r^2}$$

故 $\vec{E} = \dfrac{(r^3 - r_1^3)\rho_f \vec{r}}{3r^3}$

在 $r > r_2$ 处

$$4\pi r^2 D = \frac{4\pi(r_2^3 - r_1^3)\rho_f}{3}$$

$$D = \frac{(r_2^3 - r_1^3)\rho_f}{3r^2}$$

$$\varepsilon_0 E = \frac{(r_2^3 - r_1^3)\rho_f}{3r^2}$$

$$E = \frac{(r_2^3 - r_1^3)\rho_f}{3\varepsilon_0 r^2}$$

故 $\vec{E} = \dfrac{(r_2^3 - r_1^3)\rho_f \vec{r}}{3r^3}$

(2) 易知只在 $r_1 < r < r_2$ 处有极化体电荷, 有

$$\vec{P} = \vec{D} - \varepsilon_0 \vec{E}$$

$$= \vec{D} - \varepsilon_0 \frac{\vec{D}}{\varepsilon}$$

$$= (1 - \frac{\varepsilon_0}{\varepsilon})\vec{D}$$

故

$$\rho_p = -\nabla \cdot \vec{P}$$

$$= -(1 - \frac{\varepsilon_0}{\varepsilon})\nabla \cdot \vec{D}$$

$$= -(1 - \frac{\varepsilon_0}{\varepsilon})\rho_f$$

考虑内球壳, 此时 $r = r_1$。

$$\sigma_p = -\frac{(\varepsilon - \varepsilon_0)(r^3 - r_1^3)\rho_f}{3\varepsilon r^2}|_{r=r_1} = 0$$

考虑外球壳, 此时 $r = r_2$。

$$\sigma_p = \frac{(\varepsilon - \varepsilon_0)(r^3 - r_1^3)\rho_f}{3\varepsilon r^3}|_{r=r_2} = (1 - \frac{\varepsilon_0}{\varepsilon})\frac{r_2^3 - r_1^3}{3r_2^3}\rho_f$$

2

解　易知在 $0 < r < r_1$ 处 $\vec{B} = 0$

在 $r_1 < r < r_2$ 处

$$2\pi r H = j_f \pi (r^2 - r_1^2)$$

$$H = \frac{j_f(r^2 - r_1^2)}{2r}$$

$$\frac{B}{\mu} = \frac{j_f(r^2 - r_1^2)}{2r}$$

$$B = \frac{\mu j_f(r^2 - r_1^2)}{2r}$$

故 $\vec{B} = \dfrac{\mu(r^2 - r_1^2)\vec{j}_f \times \vec{r}}{2r^2}$

在 $r > r_2$ 处

$$2\pi r H = j_f \pi (r_2^2 - r_1^2)$$

$$H = \frac{j_f(r_2^2 - r_1^2)}{2r}$$

$$\frac{B}{\mu_0} = \frac{j_f(r_2^2 - r_1^2)}{2r}$$

$$B = \frac{\mu_0 j_f(r_2^2 - r_1^2)}{2r}$$

故 $\vec{B} = \dfrac{\mu_0(r_2^2 - r_1^2)\vec{j}_f \times \vec{r}}{2r^2}$

易知只在 $r_1 < r < r_2$ 处有磁化电流, 有

$$\vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H}$$

$$= \frac{\mu\vec{H}}{\mu_0} - \vec{H}$$

$$= (\frac{\mu}{\mu_0} - 1)\vec{H}$$

故

$$\vec{j}_m = \nabla \times \vec{M}$$
$$= (\frac{\mu}{\mu_0} - 1)\nabla \times \vec{H}$$
$$= (\frac{\mu}{\mu_0} - 1)\vec{j}_f$$

在内表面,此时 $r = r_1$。

$$\alpha_m = \vec{n} \times M|_{r=r_1} = 0$$

在外表面,此时 $r = r_2$。

$$\alpha_m = -\vec{n} \times M|_{r=r_2} = -(\frac{\mu}{\mu_0} - 1)\frac{r_2^2 - r_1^2}{2r^2}\vec{j}_f$$

3

解 (1)

$$E_1 l_1 + E_2 l_2 = \mathscr{E}$$
$$D_1 = D_2$$
$$D_1 = \varepsilon_1 E_1$$
$$D_2 = \varepsilon_2 E_2$$

解得

$$D = \frac{\varepsilon_1 \varepsilon_2 E}{l_1 \varepsilon_2 + l2 \varepsilon_1}$$

故

$$\omega_{f2} = -\omega_{f1} = D = \frac{\varepsilon_1 \varepsilon_2 E}{l_1 \varepsilon_2 + l2 \varepsilon_1}$$

(2)

$$\omega_{f3} = 0$$

若介质漏电,设漏电电流为 $\vec{j}_1, \vec{j}_2$

(1)

$$\vec{n} \cdot (\vec{j}_2 - \vec{j}_1) = 0$$
$$l_1 \frac{\vec{j}_1}{\sigma_1} + l_2 \frac{\vec{j}_2}{\sigma_2} = \mathscr{E}$$

解得

$$E_1 = \frac{\sigma_2 \mathscr{E}}{l_1\sigma_2 + l_2\sigma_1}$$

$$E_2 = \frac{\sigma_1 \mathscr{E}}{l_1\sigma_2 + l_2\sigma_1}$$

故

$$\omega_{f1} = D_1 = \frac{\varepsilon_1\sigma_2 \mathscr{E}}{l_1\sigma_2 + l_2\sigma_1}$$

$$\omega_{f2} = D_2 = \frac{\varepsilon_2\sigma_1 \mathscr{E}}{l_1\sigma_2 + l_2\sigma_1}$$

$$\omega_{f3} = D_2 - D_1 = \frac{\varepsilon_2\sigma_1 \mathscr{E} - \varepsilon_1\sigma_2 \mathscr{E}}{l_1\sigma_2 + l_2\sigma_1}$$

4

解　(1) 由 $\vec{D}$ 的法向连续条件知

$$D_1 \cos\theta_1 = D_2 \cos\theta_2$$

$$E_1\varepsilon_1 \cos\theta_1 = E_2\varepsilon_2 \cos\theta_2$$

又由 $\vec{E}$ 的切向连续条件知

$$E_1 \sin\theta_1 = E_2 \sin\theta_2$$

故有

$$\frac{\tan\theta_2}{\tan\theta_1} = \frac{\varepsilon_2}{\varepsilon_1}$$

(2) 由 $\vec{j}$ 的法向连续条件知

$$j_1 \cos\theta_1 = j_2 \cos\theta_2$$

$$E_1\sigma_1 \cos\theta_1 = E_2\sigma_2 \cos\theta_2$$

又由 $\vec{E}$ 的切向连续条件知

$$E_1 \sin\theta_1 = E_2 \sin\theta_2$$

故有

$$\frac{\tan\theta_2}{\tan\theta_1} = \frac{\sigma_2}{\sigma_1}$$

5

解   不妨设电场方程为

$$\vec{E} = \vec{E}_0 \sin(\omega t - kx)$$

则有

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$

$$= -\vec{E}_0 \times \nabla(\sin(\omega t - kx))$$

$$= -\vec{E}_0 \times (-k\vec{e}_x \cos(\omega t - kx))$$

$$= k\cos(\omega t - kx)\vec{E}_0 \times \vec{e}_x$$

故

$$\vec{B} = \int \cos(\omega t - kx)\,\mathrm{d}t\, k\vec{E}_0 \times \vec{e}_x$$

略去不属于电磁波部分的常数后可得

$$\vec{B} = \frac{k}{\omega} \sin(\omega t - kx)\vec{E}_0 \times \vec{e}_x$$

则能量密度为

$$w = \frac{1}{2}(\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B})$$

$$= \varepsilon_0 E_0^2 \sin^2(\omega t - kx)$$

平均值为

$$\overline{w} = \frac{1}{T}\int_0^T \varepsilon_0 E_0^2 \sin^2(\omega t - kx)\,\mathrm{d}t$$

$$= \frac{1}{2}\varepsilon_0 E_0^2$$

坡印廷矢量的瞬时值为

$$\vec{S} = \Re E \times \Re H$$

$$= \vec{E}_0 \cos(\omega t - kx - \frac{\pi}{2}) \times (\frac{k}{\omega\mu_0}\cos(\omega t - kx - \frac{\pi}{2})\vec{E}_0 \times \vec{e}_x)$$

$$= \frac{k\vec{e}_x}{\omega\mu_0}E_0^2 \cos^2(\omega t - kx - \frac{\pi}{2})$$

则均值为

$$\overline{S} = \frac{1}{T}\int_0^T \vec{S}\,\mathrm{d}t$$

$$= \frac{E_0^2 k \vec{e}_x}{2\omega\mu_0}$$

6

解　(1) 静电条件下，导体内部电场为 0。由 $\vec{E}$ 的切向连续可知

$$\vec{n} \times \vec{E}_{外} = 0$$

故 $\vec{E}_{外}$ 垂直于导体表面。

(2) 稳恒电流条件下导体表面 $\sigma_f = 0$。故由 $\vec{D}$ 的切向连续可知

$$\vec{n} \cdot (\vec{D}_{内} - \vec{D}_{外}) = 0$$

又因为 $\vec{D}_{外} = 0$，故有

$$\vec{n} \cdot \vec{D}_{内} = 0$$

$$\vec{n} \cdot \vec{E}_{内} = 0$$

即电场方向平行于导体平面

7

解　(1) 由高斯定理知

$$\nabla \cdot \vec{D} = \rho_f$$

代入电荷守恒方程可得

$$\nabla \cdot \vec{j} + \frac{\partial \nabla \cdot \vec{D}}{\partial t} = 0$$

$$\nabla \cdot (\vec{j} + \frac{\partial \vec{D}}{\partial t}) = 0$$

$$\vec{j} + \frac{\partial \vec{D}}{\partial t} = 0$$

(2) 由高斯定理知

$$\vec{D} = \frac{\lambda_f}{2\pi r}\vec{e}_r$$

$$\vec{E} = \frac{\lambda_f}{2\pi\varepsilon r}\vec{e}_r$$

又

$$\vec{j} + \frac{\partial \vec{D}}{\partial t} = 0$$

$$\sigma\vec{E} + \varepsilon\frac{\partial \vec{E}}{\partial t} = 0$$

$$\vec{E} = \vec{E}_0 e^{-\frac{\sigma}{\varepsilon}t}$$

故

$$\frac{\lambda_f}{2\pi\varepsilon r}\vec{e}_r = \frac{\lambda_{f0}}{2\pi\varepsilon r}e^{-\frac{\sigma}{\varepsilon}t}\vec{e}_r$$

$$\lambda_f = \lambda_{f0}e^{-\frac{\sigma}{\varepsilon}t}$$

(3)

$$w = \vec{j}^2\rho$$

$$= \sigma^2 E^2 \frac{1}{\sigma}$$

$$= (\frac{\lambda_f}{2\pi\varepsilon r})^2\sigma$$

(4)

$$P = \int_a^b (\frac{\lambda_f}{2\pi\varepsilon r})^2 \sigma 2\pi r l \, \mathrm{d}r$$

$$= \frac{\lambda_f^2 \sigma l}{2\pi\varepsilon^2}\ln\frac{b}{a}$$

静电能

$$W = \int \frac{\vec{E}\cdot\vec{D}}{2}\,\mathrm{d}V$$

$$= \int_a^b \frac{\lambda_f^2 l}{4\pi\varepsilon r}\,\mathrm{d}r$$

$$= \frac{\lambda_f^2 l}{4\pi\varepsilon}\ln\frac{b}{a}$$

$$-\frac{\mathrm{d}W}{\mathrm{d}t} = -\frac{\lambda_f l}{2\pi\varepsilon}\ln\frac{b}{a}\frac{\mathrm{d}\lambda_f}{\mathrm{d}t}$$

$$= \frac{\lambda_f^2 \sigma l}{2\pi\varepsilon^2}\ln\frac{b}{a}$$

8
解 (1)

$$\rho_p = -\nabla\cdot\vec{P}$$

$$= -[(\nabla\frac{K}{r^2})\cdot\vec{r} + \frac{K}{r^2}(\nabla\cdot\vec{r})]$$

$$= -(\frac{K}{r^2} + 3\frac{K}{r^2})$$

$$= -\frac{K}{r^2}$$

$$\sigma_p = -\vec{n} \cdot (\vec{P}_2 - \vec{P}_1)|_{r=R}$$

$$= \vec{n} \cdot \vec{P}_1|_{r=R}$$

$$= \vec{n} \cdot K \frac{\vec{r}}{r^2}|_{r=R}$$

$$= \frac{K}{R}$$

(2) 由

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{D} = \varepsilon \vec{E}$$

可得

$$\vec{D} = \frac{\varepsilon \vec{P}}{\varepsilon - \varepsilon_0}$$

故

$$\rho_f = \nabla \cdot \vec{D}$$

$$= \frac{\varepsilon}{\varepsilon - \varepsilon_0} \nabla \cdot \vec{P}$$

$$= \frac{\varepsilon}{\varepsilon - \varepsilon_0} \frac{K}{r^2}$$

$$= \frac{\varepsilon K}{(\varepsilon - \varepsilon_0)r^2}$$

(3) 在球内部

$$\vec{E} = \frac{\vec{P}}{\varepsilon - \varepsilon_0}$$

$$= \frac{K\vec{r}}{(\varepsilon - \varepsilon_0)r^2}$$

球外部

$$4\pi r^2 E = \int_0^R 4\pi r^2 \frac{\rho_f}{\varepsilon_0} \, \mathrm{d}r$$

$$4\pi r^2 E = \int_0^R 4\pi r^2 \frac{\varepsilon K}{\varepsilon_0(\varepsilon - \varepsilon_0)r^2} \, \mathrm{d}r$$

$$4\pi r^2 E = \frac{4\pi \varepsilon K R}{\varepsilon_0(\varepsilon - \varepsilon_0)}$$

$$E = \frac{\varepsilon KR}{\varepsilon_0(\varepsilon - \varepsilon_0)r^2}$$

故

$$\vec{E} = \frac{\varepsilon KR}{\varepsilon_0(\varepsilon - \varepsilon_0)r^3}\vec{r}$$

故

$$\varphi = \int_{\infty}^{r} \frac{\varepsilon KR}{\varepsilon_0(\varepsilon - \varepsilon_0)r^3}\vec{r} \cdot \mathrm{d}\vec{r}$$

$$= \frac{\varepsilon KR}{(\varepsilon - \varepsilon_0)\varepsilon_0 r}$$

在球内部

$$\vec{E} = \frac{\vec{P}}{\varepsilon - \varepsilon_0}$$

$$= \frac{K\vec{r}}{(\varepsilon - \varepsilon_0)r^2}$$

故

$$\varphi = \varphi(R) + \int_{R}^{r} \frac{K\vec{r}}{(\varepsilon - \varepsilon_0)r^2} \cdot \mathrm{d}\vec{r}$$

$$= \frac{K}{\varepsilon - \varepsilon_0}(\ln\frac{R}{r} + \frac{\varepsilon}{\varepsilon_0})$$

(4)

$$W = \int_{0}^{R} \frac{\varepsilon E^2}{2}4\pi r^2 \,\mathrm{d}r + \int_{R}^{\infty} \frac{\varepsilon_0 E^2}{2}4\pi r^2 \,\mathrm{d}r$$

$$= \int_{0}^{R} \frac{2\pi\varepsilon K^2}{(\varepsilon - \varepsilon_0)} \,\mathrm{d}r + \int_{R}^{\infty} \frac{2\pi\varepsilon^2 K^2 R}{\varepsilon_0(\varepsilon - \varepsilon_0)^2 r^2} \,\mathrm{d}r$$

$$= 2\pi\varepsilon R(\frac{K}{\varepsilon - \varepsilon_0})^2 + \frac{2\pi\varepsilon^2 RK^2}{\varepsilon_0(\varepsilon - \varepsilon_0)^2}$$

$$= 2\pi\varepsilon R(1 + \frac{\varepsilon}{\varepsilon_0})(\frac{K}{\varepsilon - \varepsilon_0})^2$$