Probabilistic Deep Learning:
Theoretical Part Assignment 1

Deadline: 08.02.2024, 12:00 (noon)

## Exercise 1 [0.5 points]

Exercise 2.3 from the book 'Deep Learning: Foundations and Concepts'.

**2.3** ($\star$) Consider a variable $\mathbf{y}$ given by the sum of two independent random variables $\mathbf{y} = \mathbf{u} + \mathbf{v}$ where $\mathbf{u} \sim p_{\mathbf{u}}(\mathbf{u})$ and $\mathbf{v} \sim p_{\mathbf{v}}(\mathbf{v})$. Show that the distribution $p_{\mathbf{y}}(\mathbf{y})$ is given by

$$p(\mathbf{y}) = \int p_{\mathbf{u}}(\mathbf{u}) p_{\mathbf{v}}(\mathbf{y} - \mathbf{u}) \, \mathrm{d}\mathbf{u}. \tag{2.119}$$

This is known as the *convolution* of $p_{\mathbf{u}}(\mathbf{u})$ and $p_{\mathbf{v}}(\mathbf{v})$.

## Exercise 2 [1 point]

Exercise 2.10 from the book 'Deep Learning: Foundations and Concepts'.

**2.10** ($\star$) Suppose that the two variables $x$ and $z$ are statistically independent. Show that the mean and variance of their sum satisfies

$$\mathbb{E}[x + z] = \mathbb{E}[x] + \mathbb{E}[z] \tag{2.120}$$

$$\mathrm{var}[x + z] = \mathrm{var}[x] + \mathrm{var}[z]. \tag{2.121}$$

## Exercise 3 [1.5 points]

Exercise 2.11 part 1 from the book 'Deep Learning: Foundations and Concepts'.

Consider the variables x and y with joint distribution $p(x, y)$. Prove the following result:

$$E[x] = E_y[E_x[x|y]]$$

Where $E_x[x|y]$ denotes the expectation of x under the conditional distribution $p(x|y)$.

## Exercise 4 [2 points]

Exercise 2.15 from the book 'Deep Learning: Foundations and Concepts'.

$$f(\mu, \sigma^2) = \ln(p(x|, \sigma^2) = -\frac{1}{2\sigma^2}\sum_{n=1}^{N}(x_n - \mu)^2 - \frac{N}{2}\ln(\sigma^2) - \frac{N}{2}\ln(2\pi)$$

Set the derivative of $f(\mu, \sigma^2)$ with respect to $\mu$ to zero and with respect to $\sigma^2$ to zero and show:

$$\mu_{ML} = \frac{1}{N}\sum_{n=1}^{N} x_n$$

$$\sigma^2_{ML} = \frac{1}{N}\sum_{n=1}^{N}(x_n - \mu)^2$$