COT 5407 Introduction to Algorithms
Homework 3

Due *in class* on Monday, November 20, 2017

This homework covers Ch 11, 12, 22, 23, 24.

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1. **[10 points]**

   Demonstrate what happens when we insert the following keys 5, 28, 37, 15, 20, 33, 12, 17, 30 into a hash table with collisions resolved by chaining. Let the table have 9 slots, and let the hash function be $h(k) = kmod9$.

2. **[10 points]**

   Suppose that you use a hash function h to hash n distinct keys into an array T that has length m. Assuming simple uniform hashing, what is the expected number of collisions? More formally, what is the expected cardinality of $\{\{k, l\} : k \neq l$ and $h(k) = h(l)\}$?

3. **[5 points]** Solve exercise 11.3-3 from Cormen.

4. **[5 points]** Solve exercise 11.4-1 from Cormen.

5. **[10 points]**

   For the set of $\{1, 4, 5, 11, 16, 19, 21\}$ of keys, draw binary search trees of heights 2, 3, 4, 5, and 6.

6. **[10 points]**

   Briefly mention what is the difference between the binary-search-tree property and the min-heap property. Is it possible to use the min-heap property print out the keys of an tree with n nodes in $O(n)$? Justify your answer.

7. **[10 points]** Solve exercise 12.2-1 from Cormen.

8. **[10 points]** Solve exercise 12.3-3 from Cormen.

9. **[10 points]** Solve exercise 12.3-3 from Cormen. 12.3-4

10. **[15 points]** Consider an undirected connected graph $G = (V, E)$ with weights $w : E \to \Re$, assume that all edge weights are distinct. Consider a cycle $< v_1, v_2, \ldots, v_k, v_{k+1} >$ in G, where $v_{k+1} = v_1$, and let (v_i, v_{i+1}) be the edge in the cycle with the largest edge weight. Show that (v_i, v_{i+1}) is not part of the minimum spanning tree T of G.

11. **[15 points]** Consider two graphs with positive weights. $G = (V, E, w)$ and $G' = (V, E, w')$ with the same vertices V and edges E such that for any edge $e \in E$, we have $w(e) = w(e)^2$. Prove or disprove the following statement: For any two vertices $u, v \in V$, any shortest path between u and v in G is also a shortest path in G'.

12. **[10 points]** Solve exercise 22.1-5 in Cormen

13. **[10 points]** Solve exercise 22.3-2 in Cormen

14. **[10 points]** Solve exercise 22.4-1 in Cormen

15. **[10 points]** Solve exercise 22.5-2 in Cormen

16. **[10 points]** Solve exercise 23.5-2 in Cormen 23.2-2

17. **[10 points]** 24.2-1 in Cormen

18. **[10 points]** 24.3-1 in Cormen

19. **[10 points]** (a) Using Kruskal's algorithm find a Minimum Spanning Tree of the graph below. Enumerate the edges in the order they are selected, draw the minimal spanning tree you obtain, and give the total weight of the minimum spanning tree.

20. **[10 points]** b) Using Prims's algorithm find a Minimum Spanning Tree of the graph below. Enumerate the edges in the order they are selected, draw the minimal spanning tree you obtain, and give the total weight of the minimal spanning tree.