EIN 5226

Binomial Distribution

Applications to Quality

Note: Need Calculator

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The Binomial Distribution

Binomial distribution assumptions:

1. All trials are identical.
2. Each outcome is either a "success" or "failure".
3. **The probability of a success is constant.**
4. All trials are independent

Understanding the assumptions

For each of these situations/questions, indicate if binomial distribution is applicable

Yes	/ No	A coin is flipped 6 times. What is the probability of exactly 2 heads out of the 6 flips?
Yes	/ No	A box contains 30 shapes: 6 balls, 15 blocks, and 9 pyramids. Drawing 4 shapes out, without replacement. What is the probability of exactly 2 blocks out of the 4?
Yes	/ No	A process produces shapes in the following proportions: 20% balls, 40% blocks, and 40% pyramids. Drawing 4 shapes from the output of the process, what is the probability of getting exactly 3 blocks out of the 4?

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Understanding the assumptions

For each of these situations/questions, indicate if binomial distribution is applicable

Yes / No	A process has a historical 1st pass yield rate of 92%. If a sample of 100 parts is taken from the process, what is the probability that exactly 1 part is defective?

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Binomial Distribution in Quality

Examples – questions answered based on binomial distribution

- How many items must I sample in my receiving inspection process to ensure adequate vendor quality is received?
- How much improvement in yield was realized after overhaul of the equipment?
- Did our proportion of orders that have complaints significantly decline after implementation of the new process.
- Can I really say that one vendor is better than the other based on the percent defective in the samples that they provided.

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The Binomial Distribution

- A number of trials, n, are made with outcomes observed.
- One outcome is labeled “success,” and the other outcome is labeled “failure.”
- The probability of a success is denoted by *p*. The probability of a failure (often denoted by q) is then 1 – *p.*

A process has a historical 1st pass yield rate of 92%. A sample of 100 parts is taken from the process.

n = # of trials = 100 parts

Success: defective part >>>>> P(defective part) = p = .08
Failure: good part >>>>> P (good part) = q = 0.92

A process has a historical 1st pass yield rate of 92%. A sample of 100 parts is taken from the process.

What is the probability that the first 3 parts taken for the sample are defective and the rest are good?

A. 0.920 B. 0.08 C. $(0.08)^3(0.92)^{97}$ D. $(0.92)^3(0.08)^{97}$

What is the probability that the last 3 parts taken for the sample are defective and the rest are good?

A. 0.920 B. 0.08 C. $(0.08)^{97}(0.92)^3$ D. $(0.92)^{97}(0.08)^3$

How many different outcomes (combinations) can you have that have 3 defective parts and 97 good parts?

A. 147,440 B. 161,700 C. 204,118 D. 198,240

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A process has a historical 1st pass yield rate of 92%. A sample of 100 parts is taken from the process.

What is the probability that the 5th, 23rd and 42nd parts taken for the sample are defective and the rest are good?

A. $(0.08)^5(0.08)^{23}(0.08)^{42}$ B. $(0.08)^3(0.92)^{97}$ C. $(0.92)^3(0.08)^{97}$

True / False Since each different combination of three is mutually exclusive, to find the probability of getting exactly 3 in the sample, I could add up the probabilities of each of the individual outcomes with 3 defectives in it.

What is the probability of getting exactly 3 defective parts in the sample?

A. 0.0543 B. 0.0800 C. 0.0157 D. 0.0254

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Binomial Distribution

The probability of exactly x occurrences in n trials in an event that has a constant probability of occurrence

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

where p = probability of success
n = the number of trials

x = the number successes

and $\binom{n}{x} = \frac{n!}{x!(n-x)!} = nCx$

A process has a historical 1st pass yield rate of 92%. What is the probability of getting exactly 3 defectives in a sample of size 100?

$$p(3) = \binom{3}{100} 0.08^3 (1-0.08)^{100-3}$$
$$= (161{,}700) 0.08^3 (0.92)^{97}$$
$$= 0.0254$$

where p = probability of defective = 0.08
n = number in sample = 100

x = defective in sample = 3

$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$

A package distribution company has a complaint rate of 10% on orders shipped. 30 orders are shipped.

What is the probability that exactly 2 orders result in complaints?

The number of combinations possible which result in 2 order complaints out of the 30 shipped is

A. 324 B. 435 C. 524 D. 456

The probability of any one of those combinations is

A. $0.10^2 * 0.90^{28}$ B. $0.10^{28} * 0.90^2$ C. $0.10^2 * 0.90^{30}$

The probability that exactly 2 orders result in complaints is

A. 0.2277 B. 0.0800 C. 0.0157 D. 0.0254

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$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$

A package distribution company has a complaint rate of 10% on orders shipped. 30 orders are shipped.

What is the probability that none of the orders result in complaints?

A. 0.0424 B. 0.0800 C. 0.0157 D. 0.0254

What is the probability that there will be less than 3 complaints?

A. 0.0543 B. 0.589 C. 0.411 D. 0.0254

What is the probability that there will at least 1 complaint?

A. 0.0543 B. 0.0800 C. 0.9576 D. 0.0254

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The Binomial Distribution

Binomial distribution assumptions:

1. All trials are identical.
2. Each outcome is either a “success” or “failure”.
3. **The probability of a success is constant.**
4. All trials are independent

A lot contains 1000 components, 5% of which are known to be defective. Ten components are sampled from the lot. What is the probability that exactly 1 is defective?

Does this problem meet the above assumptions?

Yes No

A lot contains 1000 components, 5% of which are defective. Ten components are sampled from the lot. What is the probability that exactly 1 is defective?

N=1000
n=10

$$p(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

Define success as defect
r= successes in population = 5% times 1000 = 50
x= successes in sample = 1

$\binom{r}{x}$	$\binom{N-r}{n-x}$	$\binom{N}{n}$	P(1)
50	1.67E+21	2.63E+23	0.317

Large Finite Populations

A lot contains 1000 components, 5% of which are defective. Ten components are sampled from the lot. What is the probability that exactly 1 is defective?

If the sample size is no more than 10% of a large population, the binomial distribution may be used to model the number of successes.

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Large Finite Populations

A lot contains 1000 components, 5% of which are defective. Ten components are sampled from the lot. What is the probability that exactly 1 is defective?

Trials not truly independent, p not constant

5% defective: 50 out of 1000 are defective.

P(1st component defective) = 50/1000 = .05

P(2nd component defective/1st component defective)

= 49/999 = .049

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Large Finite Populations

A lot contains 1000 components, 5% of which are defective. Ten components are sampled from the lot. What is the probability that exactly 1 is defective?

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

n=10
p=.05
x=1

$$p(1) = \binom{10}{1} .05^1 (1-.05)^{10-1} = 0.3151$$

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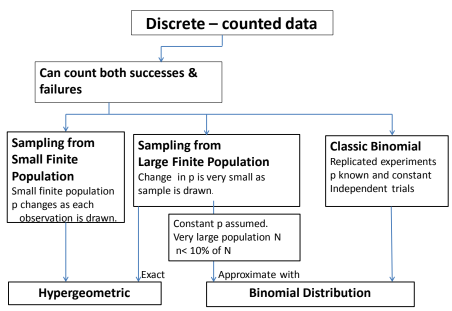
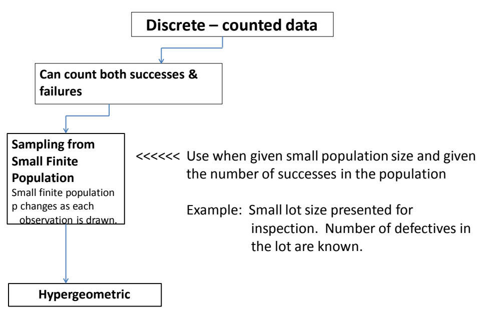
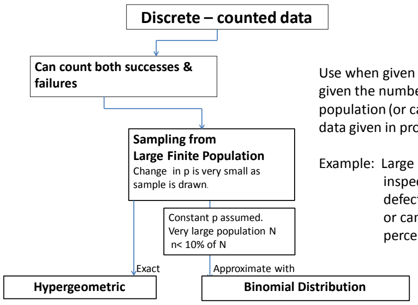
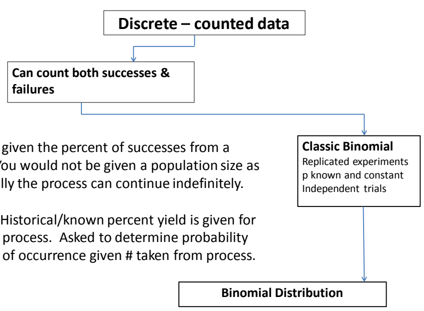






Use when given large population size and given the number of successes in the population (or can be determined with data given in problem)

Example: Large lot size presented for inspection. Number of defectives in the lot are known or can be determined if give percent of the lot defective.



Use when given the percent of successes from a process. You would not be given a population size as theoretically the process can continue indefinitely.

Example: Historical/known percent yield is given for process. Asked to determine probability of occurrence given # taken from process.

$p(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$ $\binom{n}{x}p^{x}(1-p)^{n-x}$ $\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$

Practice Problem

A new medicine was developed to ease the symptoms of the common cold. Studies showed that 60% of the people that take the medicine obtain significant relief. 400 people are prescribed the medicine. A sample of 15 of these people was taken. What is the probability that 7 people in the sample obtained significant relief?

True / False For this problem I can use the binomial approximation to the hypergeometric.

P(x=7) = ?

A. 0.118 B. 0.205 C. 0.175 D. 0.301

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$p(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$ $\binom{n}{x}p^{x}(1-p)^{n-x}$ $\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$

Practice Problem

At a career fair, 65 companies had a booth? 25 of these had a candy dish at the booth. You only have time to visit 12 booths. What is the probability that you will visit exactly 8 booths with a candy dish?

True / False For this problem I can use the binomial approximation to the hypergeometric.

P(x=8) = ?
A. 0.2013 B. 0.0515 C. 0.0245 D. 0.0345

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$p(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$ $\binom{n}{x}p^{x}(1-p)^{n-x}$ $\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$

Practice Problem

A machining center has consistently produced 12% of its parts below the specification limit? 10 parts are taken at random from the process. What is the probability that more than 1 of them will be out of the spec limit?

True / False For this problem I can use the binomial approximation to the hypergeometric.

P(x>1) = ?
A. 0.1658 B. 0.342 C. 0.175 D. 0.301

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End of lecture