Process Capability

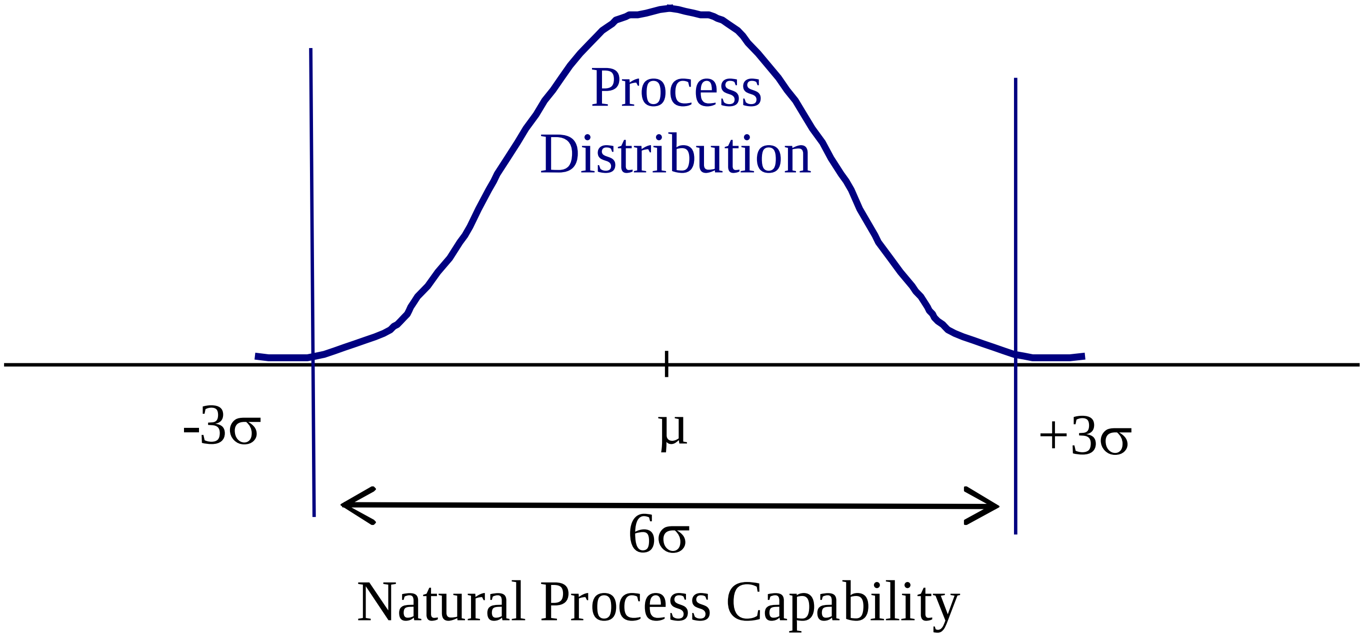
Chapter 11

Need: Calculator
Z table

Karen E. Schmahl Ph.D., P.E.

Process Capability

- The 6σ range inherent process variation



Many everyday decisions are based on process capability.

Design

- What material and processes will enable me to get the form, fit and function of the design that is required?

Production/Process Engineering/Quality Engineering

- Which machines have the capability to process this part?
- How are the tolerances going to stack-up for a given sequence of operations?
- What performance requirements should be specified for new equipment?
- How can variation be reduced in this manufacturing process.
- What sample size and frequency should be used.

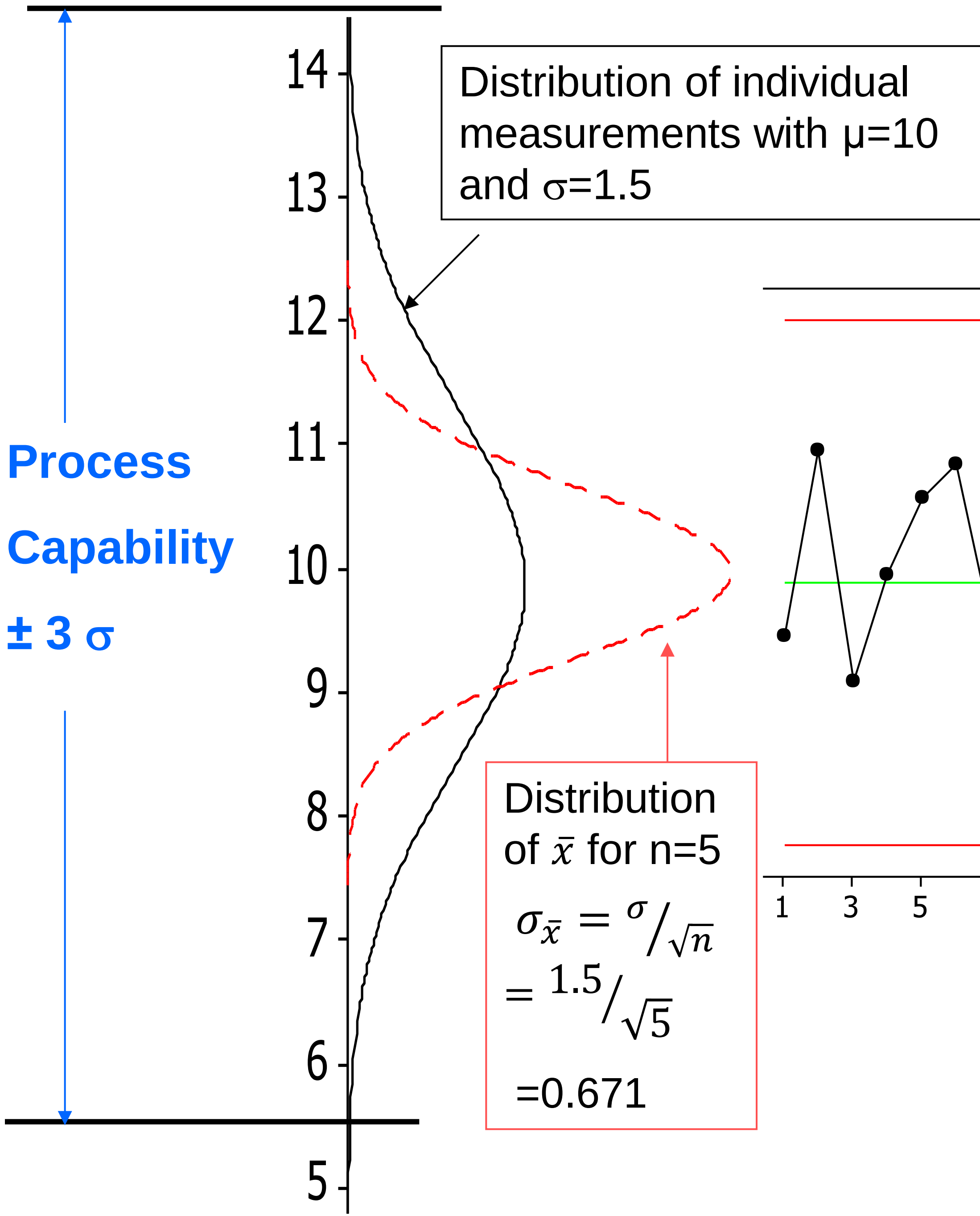
Purchasing/Vendor Quality

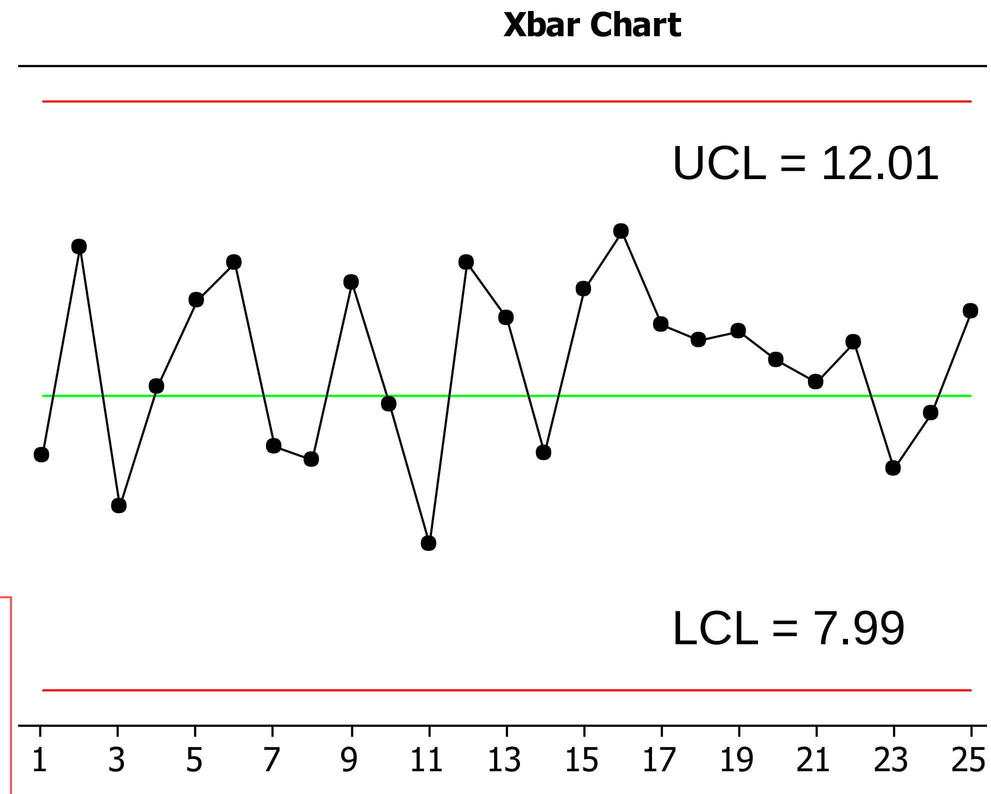
- Which of these vendors has the better product?
- Should I tighten the inspection plan on the next order to this vendor?

Process Capability

- The 6σ range inherent process variation
  - What is it capable of doing
  - σ estimated with $\overline{R}/d_2 \; or \; \overline{s}/c_4$
  - σ_{within} is term often used – variation within samples
- Assumes stable process
- Data is from a normally distributed population

Control Limits Vs Process Capability





Control Charts facilitate keeping process stable.

Process Capability Ratios (Indices)

How well do requirements compare to the inherent variation in process output?

Requirements are given by Specifications

- Are determined by engineers/designers.
- May be referred to as tolerance limits.

Specification Limits

- Communicated to others by engineering documents and drawings.
- A specification will generally have
  - Upper specification limit (USL)
  - Nominal or target value
  - Lower specification limit (LSL)

There is

NO mathematical relationship between

specification limits

and

statistical process control limits!

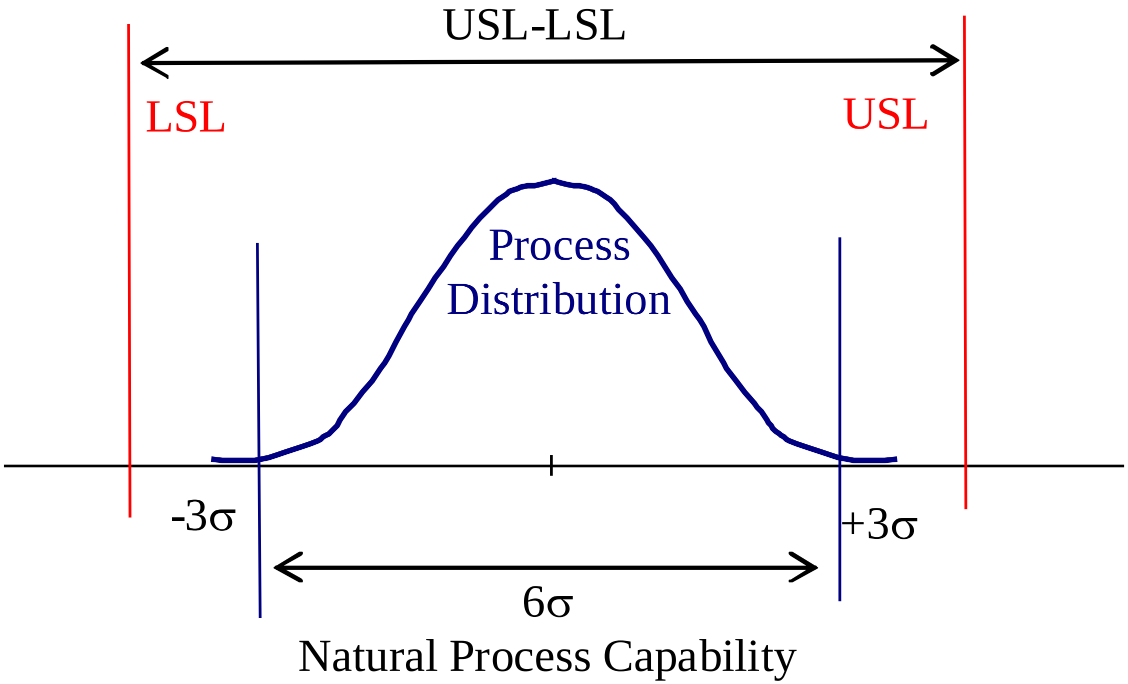
When referring to SPC charts or capability

do not

- refer to control limits as specification limits
- refer to specification limits as control limits!

Process Capability Ratio

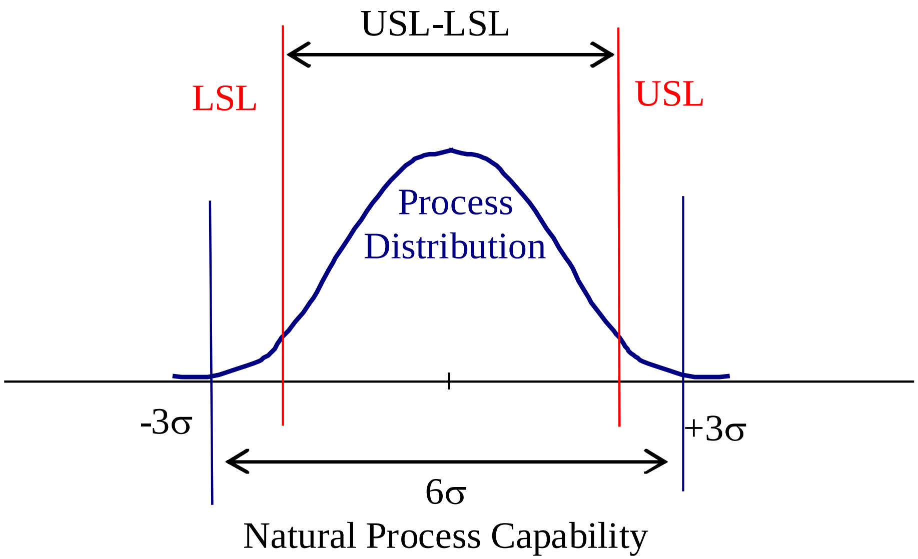
$$C_p = \frac{USL - LSL}{6\sigma}$$



- Where the USL-LSL is greater than 6σ, Cp is greater than 1.
- If the process is centered, and Cp > 1, then a low # of nonconforming items will be produced.

Process Capability Ratio

$$C_p = \frac{USL - LSL}{6\sigma}$$



- Where the USL-LSL is less than 6σ, Cp is less than 1.
- If Cp < 1, then nonconforming items are being produced whether the process is centered or not..

Parts manufactured by an injection molding process are subjected to compressive strength test. Twenty-five samples of five parts each are collected and the compressive strengths shown in the table.

For our problem, we had $\boxed{n = 5}$ $\boxed{m = 25}$

$\boxed{\bar{\bar{x}} = 80.7}$ $\boxed{\bar{R} = 7.8}$ $\boxed{\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{7.8}{2.326} = 3.35}$

Analysis of the control charts provided no indication of potential assignable causes. The process was deemed to be stable.

For the injection molding problem, we took our samples and found

$$\boxed{\bar{\bar{x}} = 80.7} \qquad \boxed{\bar{R} = 7.8}$$

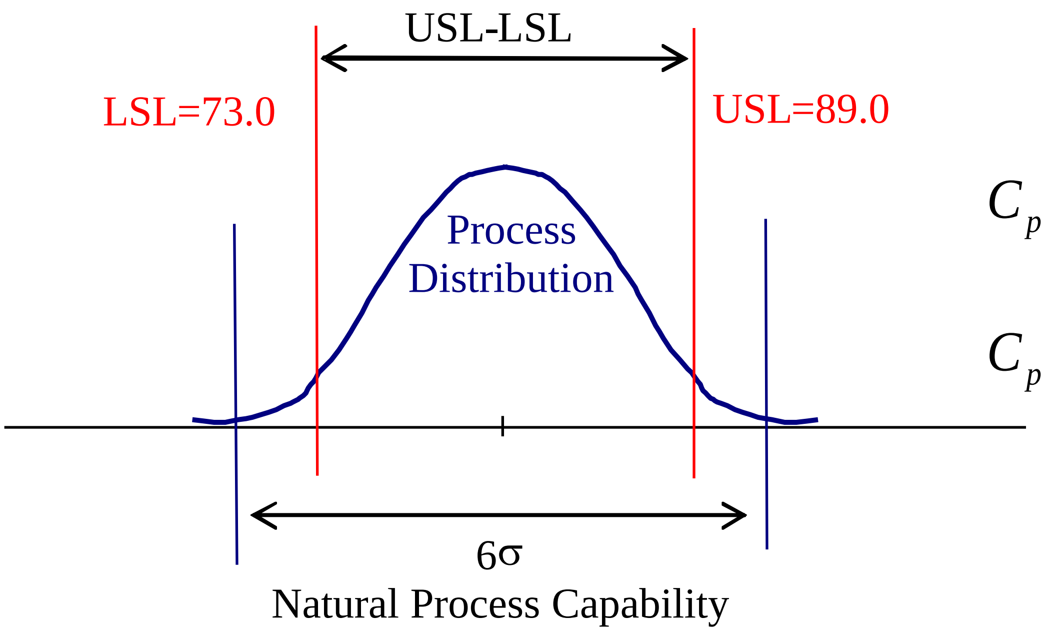
We then estimated σ for the population, where n=5, d_2=2.326

$$\boxed{\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{7.8}{2.326} = 3.35}$$

The specification limits are 81.00± 8.00

$$\boxed{LSL = 73.00} \qquad \boxed{USL = 89.00}$$

What is C_p?



$$C_p = \frac{USL - LSL}{6\sigma}$$

$$C_p = \frac{89.0 - 73.0}{6(3.35)} = \frac{16}{20.1} = 0.796$$

- $C_p < 1$, we would be producing non conforming product even if the process were centered.
- Notice this analysis assumes that the process is centered between the upper and lower spec.

Process Capability Ratio Off-Center Process

- C_p does not take into account where the process mean is *located* relative to the specifications.
- C_{pu} and C_{pl} takes μ and the upper and lower specifications into account.

$$C_{pu} = \frac{USL - \mu}{3\sigma} \qquad C_{pl} = \frac{\mu - LSL}{3\sigma}$$

- A process capability ratio that does take into account centering is C_{pk} defined as

$$C_{pk} = \min(C_{pu}, C_{pl})$$

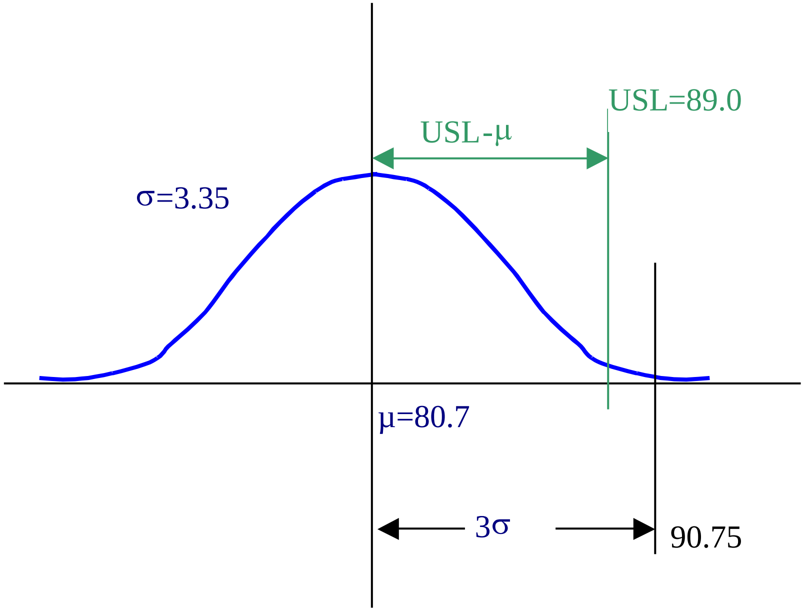
C_{pk} must be greater than 1 for a process to be considered capable

For our injection molded parts problem:

$\boxed{n = 5}$ $\boxed{\widehat{u} = \bar{\bar{x}} = 80.7}$ $\boxed{\bar{R} = 7.8}$ $\boxed{\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{7.8}{2.326} = 3.35}$

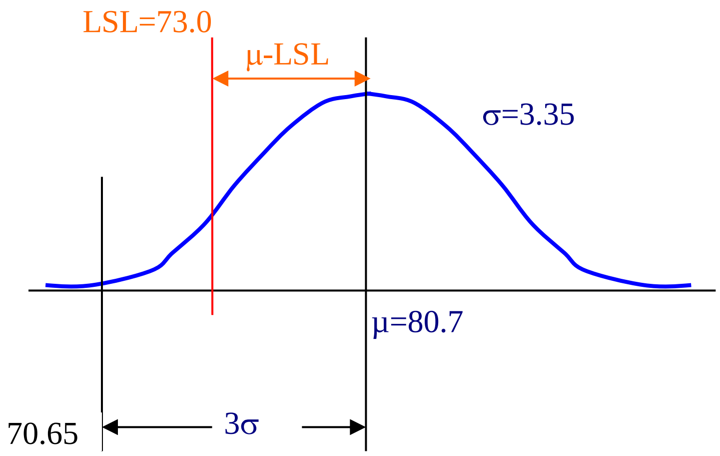
The specification limits are 81.00± 8.00

$\boxed{LSL = 73.00}$ $\boxed{USL = 89.00}$



$$C_{pu} = \frac{USL - \mu}{3\sigma}$$

$$= \frac{89.0 - 80.7}{(3)(3.35)} = .823$$



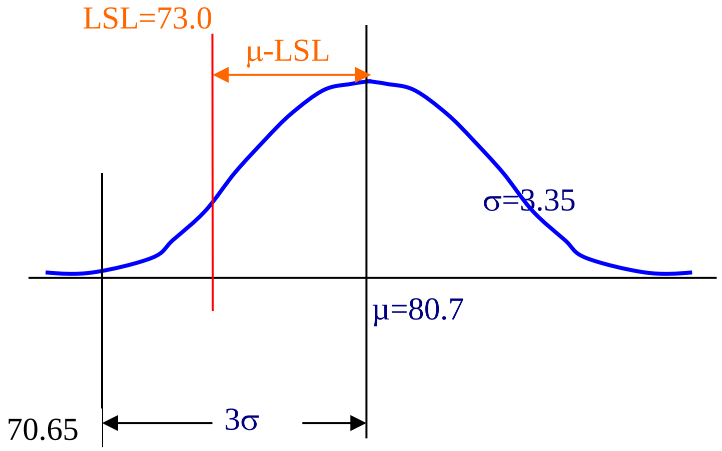
$$C_{pl} = \frac{\mu - LSL}{3\sigma}$$

$$= \frac{80.7 - 73.0}{(3)(3.35)} = 0.77$$

For this problem:

$C_p = 0.80$ $\quad$ $C_{pu} = 0.82$ $\quad$ $C_{pl} = 0.77$

$C_{pk} = \min(C_{pu}, C_{pl}) = \min(0.82, 0.77) = 0.77$



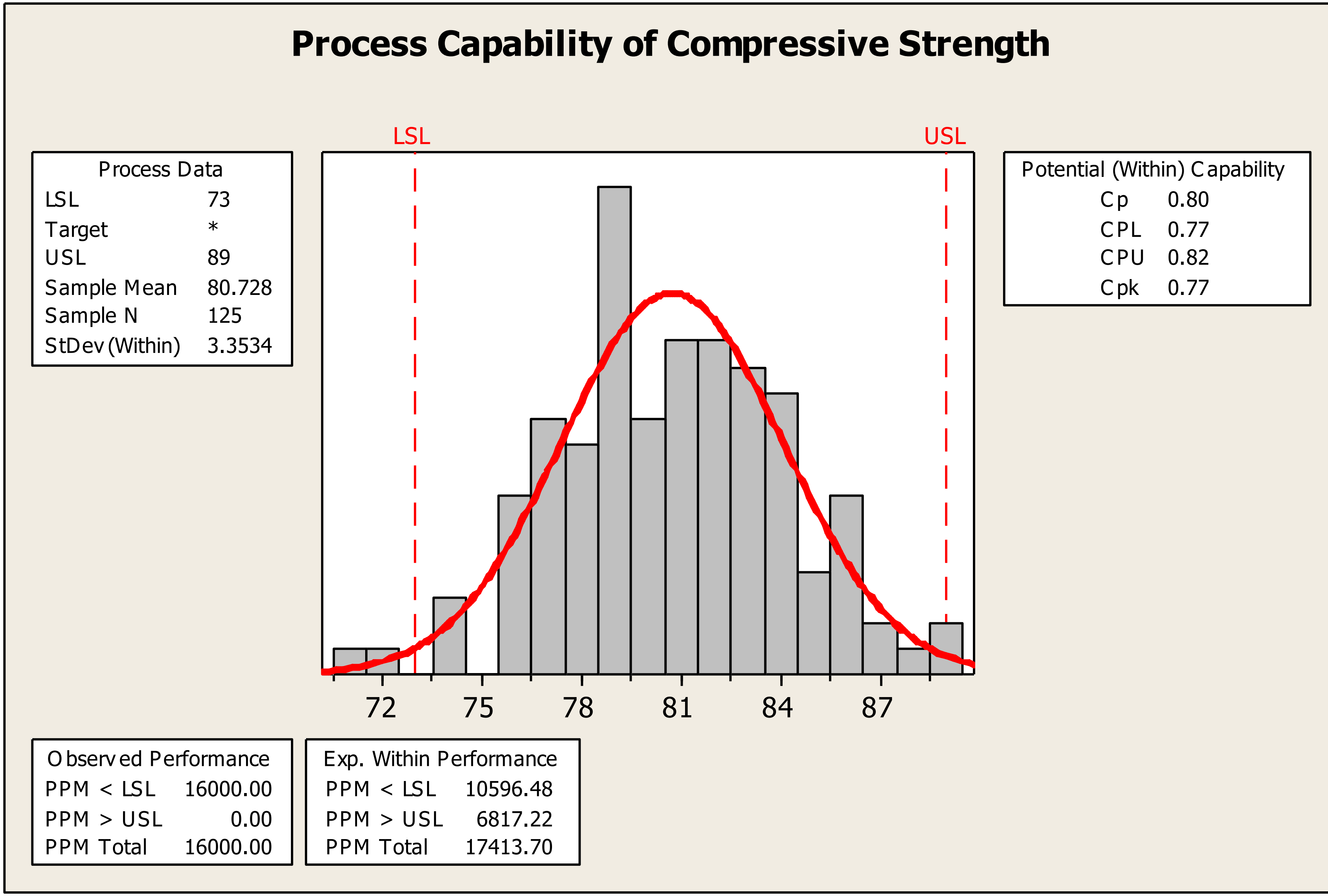
$$C_{pl} = \frac{\mu - LSL}{3\sigma}$$
$$= \frac{80.7 - 73.0}{(3)(3.35)} = 0.77$$

For this problem:

$C_p = 0.80$ $\quad$ $C_{pu} = 0.82$ $\quad$ $C_{pl} = 0.77$

$C_{pk} = \min(C_{pu}, C_{pl}) = \min(0.82, 0.77) = 0.77$

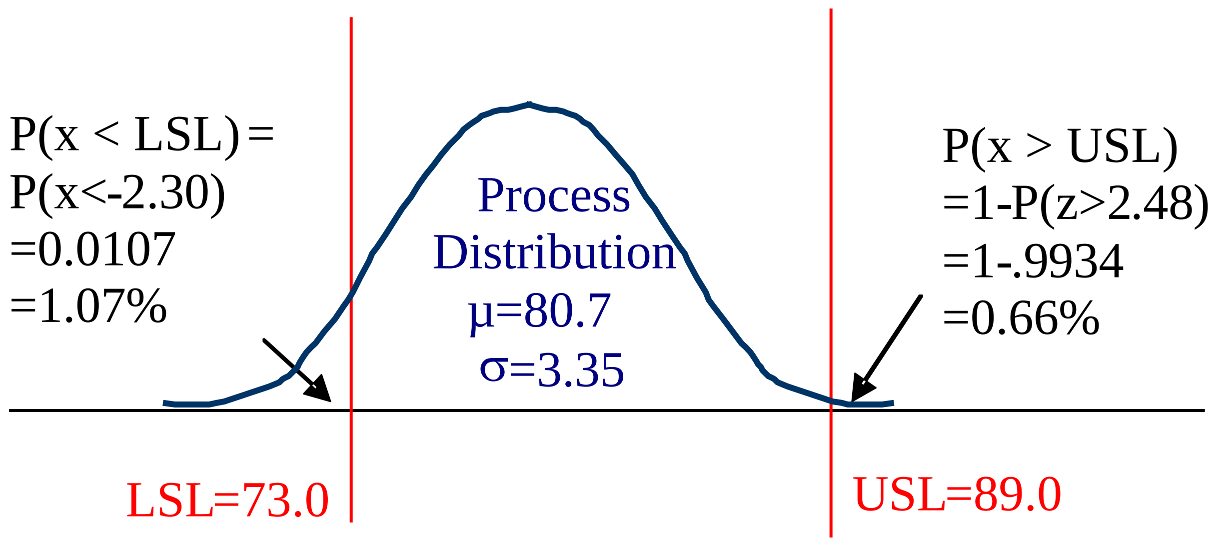
T / F From the capability ratios for this process, I conclude the process is not stable..


Process Capability of Compressive Strength
LSL
USL
Process Data
LSL 73
Target *
USL 89
Sample Mean 80.728
Sample N 125
StDev(Within) 3.3534
Potential (Within) Capability
Cp 0.80
CPL 0.77
CPU 0.82
Cpk 0.77
72
75
78
81
84
87
Observed Performance
PPM < LSL 16000.00
PPM > USL 0.00
PPM Total 16000.00
Exp. Within Performance
PPM < LSL 10596.48
PPM > USL 6817.22
PPM Total 17413.70

Non-Conforming Product

- Observed performance
  - How many values were actually outside of the specification limits?
    - 2/125 = 1.6% = .016 * 1,000,000 = 16000 ppm
- Within performance
  - Based on estimated inherent capability and location of the data, how many would be expected to be out of the specification limits?

What is the % nonconforming?



$$Z = \frac{73-80.7}{3.35} = -2.3$$

$$Z = \frac{89-80.7}{3.35} = +2.48$$

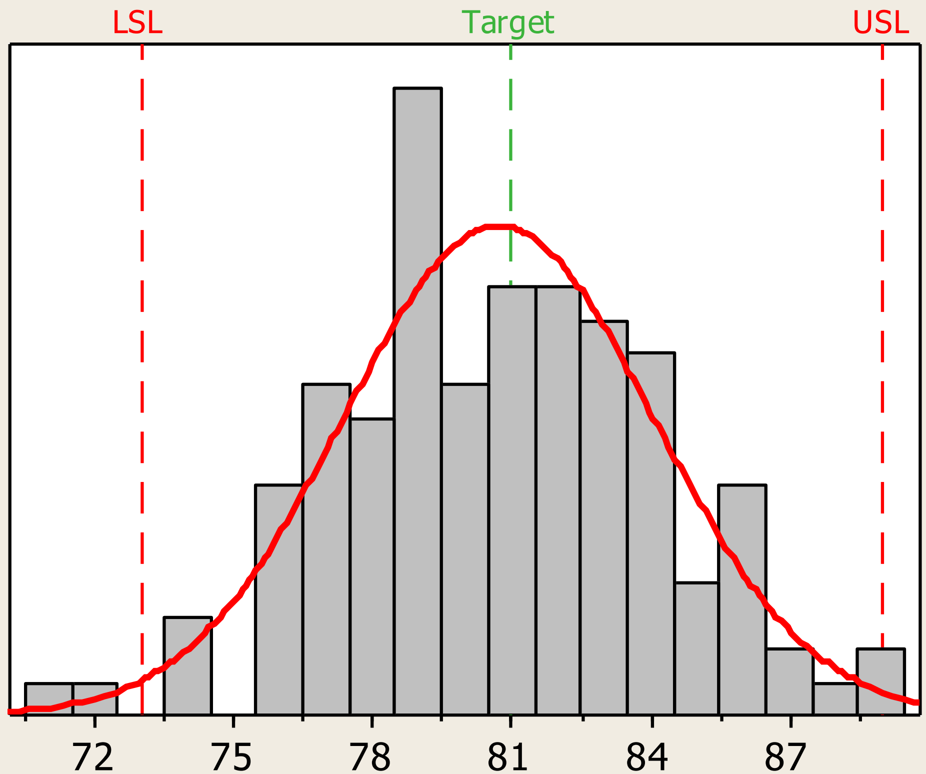
Total % nonconforming = 1.07 + 0.66 = 1.73%

Parts per million nonconforming: 10700 + 6600 = 17300 ppm

Process Capability of Compressive Strength

Use of Z Value

Process Data	
LSL	73
Target	81
USL	89
Sample Mean	80.728
Sample N	125
StDev(Within)	3.3534



Potential (Within) Capability	
Z.Bench	2.11
Z.LSL	2.30
Z.USL	2.47
Cpk	0.77

Based on number of sample observations – one time snap shot

Observed Performance	
PPM < LSL	16000.00
PPM > USL	0.00
PPM Total	16000.00

Exp. Within Performance	
PPM < LSL	10596.48
PPM > USL	6817.22
PPM Total	17413.70

Area under the curve outside of spec limits – could use to project future performance

- A process must be in statistical control for capability analysis to be meaningful.
  - Use control chart to determine if stable
- A process may be in statistical control but not capable.
  - Calculate capability indices to determine if requirements are being met.

Practice Problem (1a)

The quality control plan for a certain production process is to be developed taking samples of size 4. 25 samples are taken with the results.

$\bar{\bar{X}}$ and $\bar{R}$ were determined to be 34.0 and 2.4, respectively.

What is the estimate of the population standard deviation?

A) 6.4 B) 1.032 C) 1.166 D) 2.059

Practice (1b)

$$C_p = \frac{USL - LSL}{6\sigma} \qquad C_{pl} = \frac{\mu - LSL}{3\sigma} \qquad C_{pu} = \frac{USL - \mu}{3\sigma}$$

The quality control plan for a certain production process is to be developed taking samples of size 4. 25 samples are taken with the results.

$\bar{\bar{X}}$ and $\bar{R}$ were determined to be 34.0 and 2.4, respectively.

The specification limits for the part are 35.0 $\pm$ 4

What is Cp for the process?

A) 0.571 B) 1.144 C) 2.287 D) 1.835

T / F From the Cp for this process, I conclude the process is capable.

Practice (1c)

$$C_p = \frac{USL - LSL}{6\sigma} \qquad C_{pl} = \frac{\mu - LSL}{3\sigma} \qquad C_{pu} = \frac{USL - \mu}{3\sigma}$$

The quality control plan for a certain production process is to be developed taking samples of size 4. 25 samples are taken with the results.

$\bar{\bar{X}}$ and $\bar{R}$ were determined to be 34.0 and 2.4, respectively.

The specification limits for the part are 35.0 $\pm$ 4

What is Cpl for the process?

A) 1.144 B) 1.166 C) 1.429 D) 0.858

Practice (1d)

$$C_p = \frac{USL - LSL}{6\sigma} \qquad C_{pl} = \frac{\mu - LSL}{3\sigma} \qquad C_{pu} = \frac{USL - \mu}{3\sigma}$$

The quality control plan for a certain production process is to be developed taking samples of size 4. 25 samples are taken with the results.

$\bar{\bar{X}}$ and $\bar{R}$ were determined to be 34.0 and 2.4, respectively.

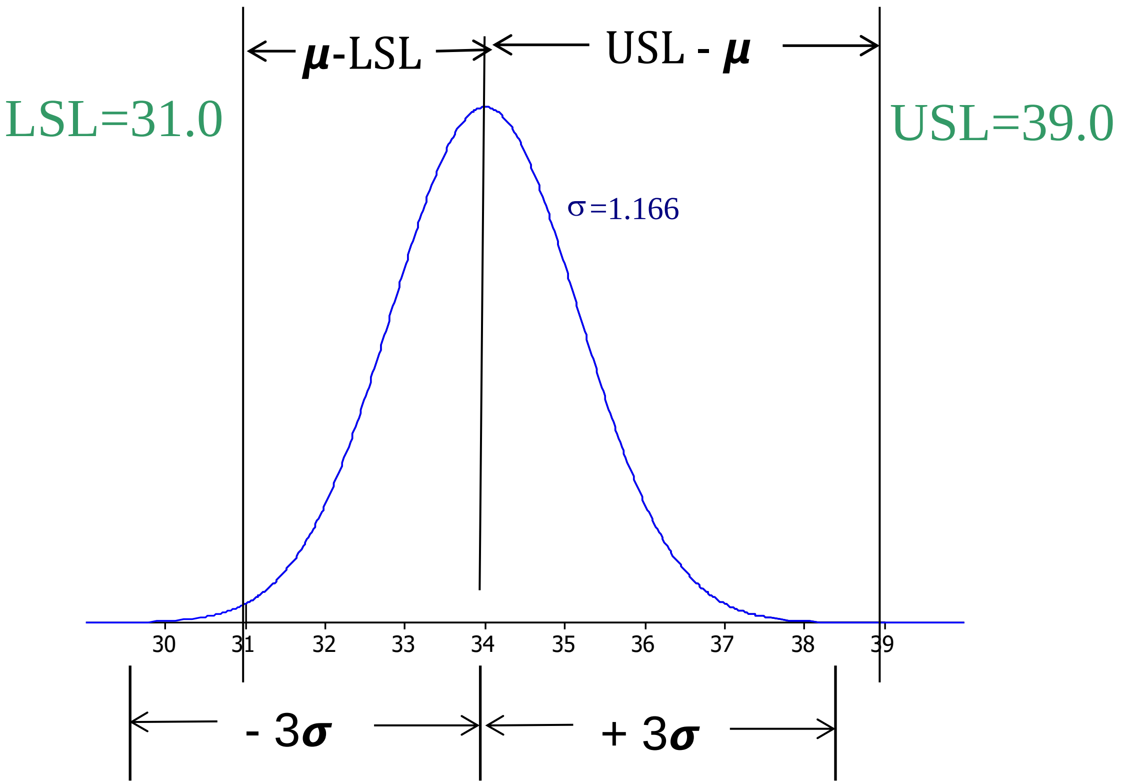
The specification limits for the part are 35.0 $\pm$ 4

What is Cpu for the process?

A) 1.144 B) 0.715 C) 1.429 D) 0.858

What is Cpk for the process?

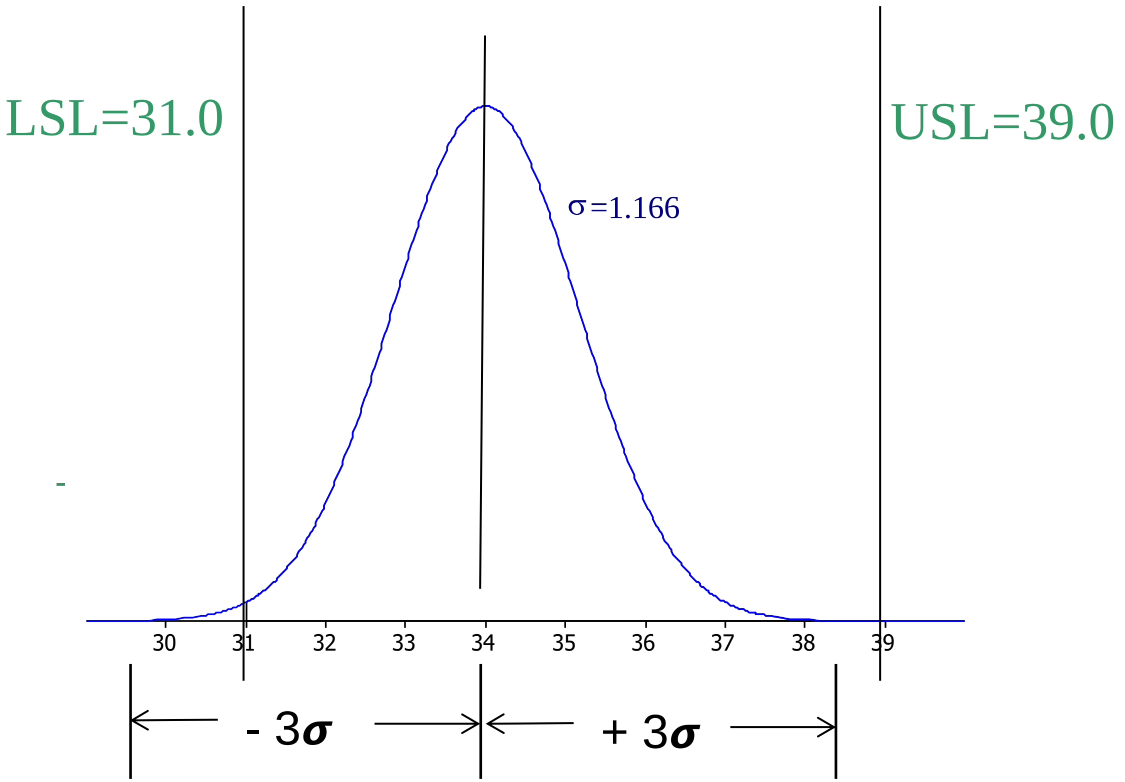
A) 1.144 B) 0.715 C) 1.429 D) 0.858



$$C_{pl} = \frac{\mu - LSL}{3\sigma} \qquad C_{pu} = \frac{USL - \mu}{3\sigma}$$

$$C_{pl} = \frac{34 - 31}{3(1.166)} = .858 \qquad C_{pu} = \frac{39 - 34}{3(1.166)} = 1.429$$

Practice (1e)



The percent defective for this process is

A) 1.01% B) 0.50% C) 3.21% D) 2.33%

The PPM defective for this process is

A) 23300 B) 101000 C) 5000 D) 500,000

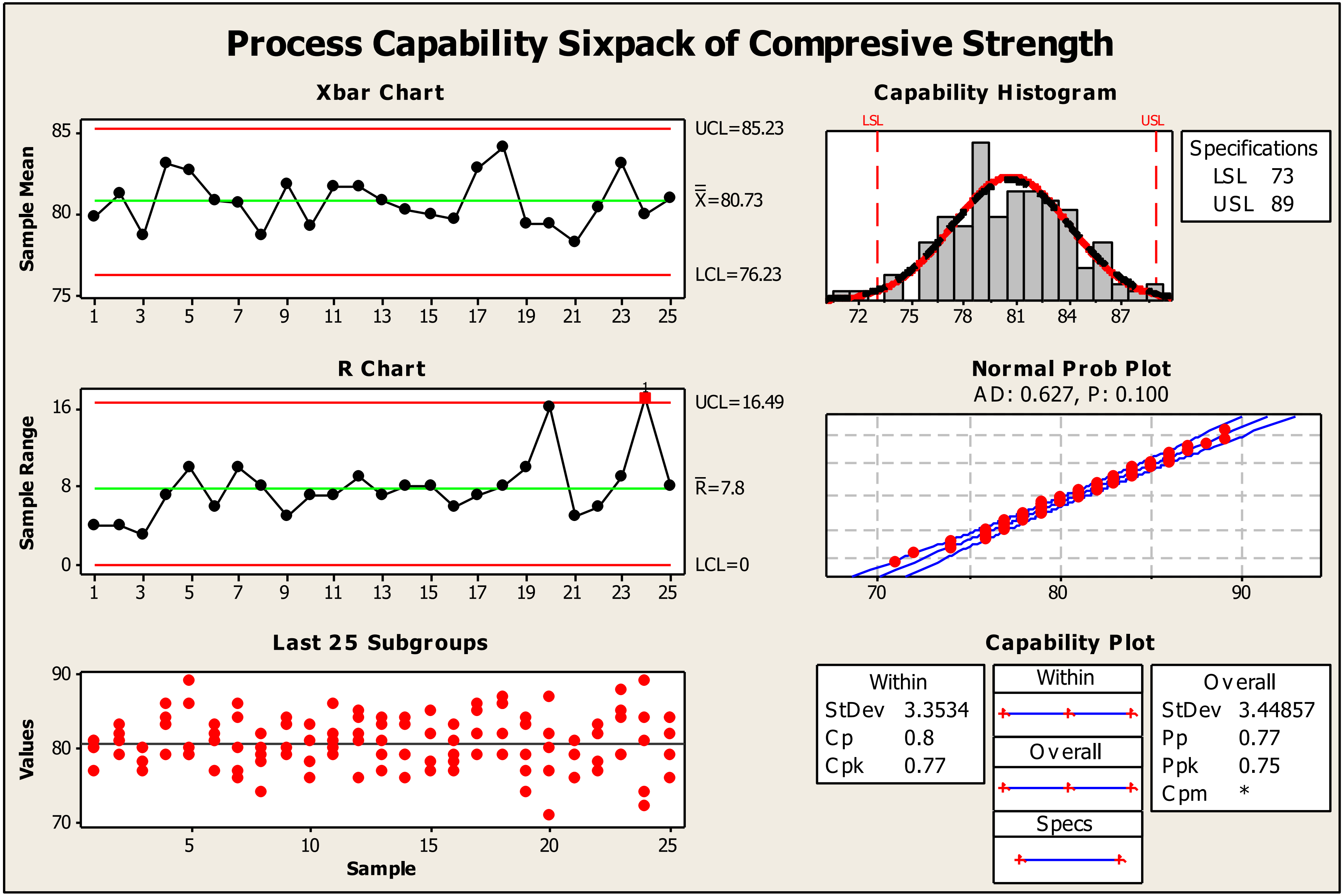
Steps for Process Capability
Control Chart Approach

- Data Collection
- Verify Process is stable
  - Stability over time
- Verify population normality
- Estimate process standard deviation
  - Within subgroup variation

$$\hat{\sigma} = \frac{\bar{R}}{d_2} \qquad \hat{\sigma} = \frac{\bar{s}}{c_4}$$

- Compare to specifications / Calculate indices
  - C_p, C_{pk}

- Control Chart Method
  - Control chart
- Histogram
- Normality Plot



Process Capability Sixpack of Compresive Strength
Xbar Chart
Sample Mean
UCL=85.23
X̿=80.73
LCL=76.23
Capability Histogram
LSL
USL
Specifications
LSL 73
USL 89
R Chart
Sample Range
UCL=16.49
R̄=7.8
LCL=0
Normal Prob Plot
AD: 0.627, P: 0.100
Last 25 Subgroups
Values
Sample
Capability Plot
Within
StDev 3.3534
Cp 0.8
Cpk 0.77
Within
Overall
Specs
Overall
StDev 3.44857
Pp 0.77
Ppk 0.75
Cpm *

If Process is NOT capable:

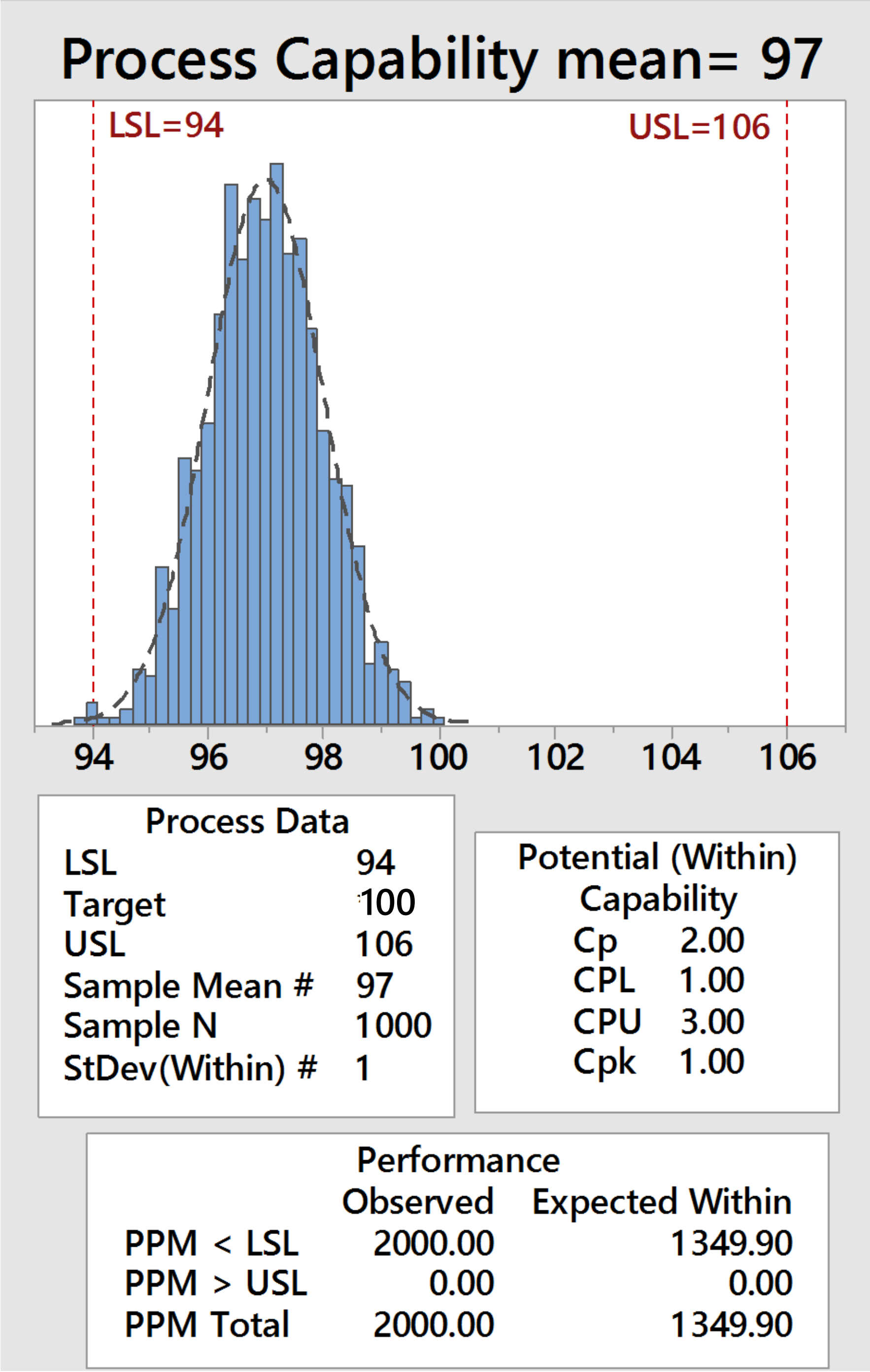
- Change the process.
  - Reduce common cause variation
  - Center the process
  - Use a different process/machine
  - Subcontract the work

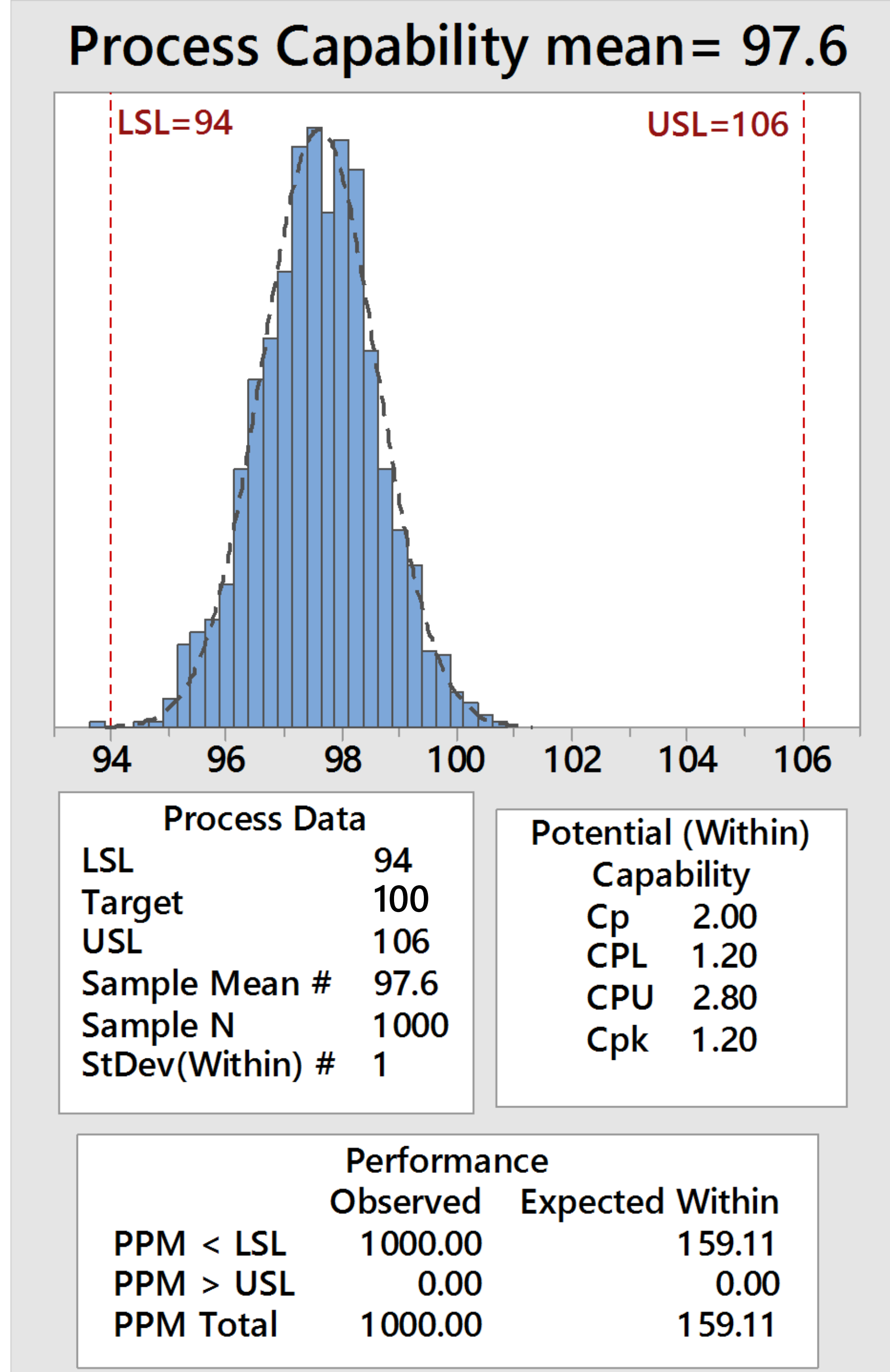
If Process is NOT capable:

- Change the process.
- Change the requirements
  - Change the specification limits.
  - Redesign the product
- Obtain a deviation from the customer
- Manage the non-conforming material
  - 100% inspection.
- Discontinue making the product.

If Process is barely capable ($1<C_{pk}<1.2$)

- Must consider the risk of shifting
- Should be closely monitored to detect a shift.
  - Monitor on a continuing basis with a control chart
  - May need to use larger sample size to detect shift more quickly and/or take samples more frequently
- Efforts should be made to further improve the process.
- Company targets are typically $C_{pk}>1.3$


Process Capability mean= 97
LSL=94
USL=106
94
96
98
100
102
104
106
Process Data
LSL 94
Target 100
USL 106
Sample Mean # 97
Sample N 1000
StDev(Within) # 1
Potential (Within) Capability
Cp 2.00
CPL 1.00
CPU 3.00
Cpk 1.00
Performance
Observed Expected Within
PPM < LSL 2000.00 1349.90
PPM > USL 0.00 0.00
PPM Total 2000.00 1349.90


Process Capability mean= 97.6
LSL=94
USL=106
94
96
98
100
102
104
106
Process Data
LSL 94
Target 100
USL 106
Sample Mean # 97.6
Sample N 1000
StDev(Within) # 1
Potential (Within) Capability
Cp 2.00
CPL 1.20
CPU 2.80
Cpk 1.20
Performance
Observed Expected Within
PPM < LSL 1000.00 159.11
PPM > USL 0.00 0.00
PPM Total 1000.00 159.11

Steps for Process Capability
Historical data

- Data Collection
  - history on file/ i.e similar part
- Verify Process is stable
  - history on file
  - Stability over time
- Verify population normality
  - history on file
- Estimate process variation
  - Use historical standard deviation for process
- Compare to specifications
  - C_p, C_{pk}

Practice (2a)

$$C_p = \frac{USL - LSL}{6\sigma} \qquad C_{pl} = \frac{\mu - LSL}{3\sigma} \qquad C_{pu} = \frac{USL - \mu}{3\sigma}$$

A customer has learned about process capability ratios and requests analysis of the prior years data on a critical part characteristic with specification of 2.5 +/- .05

Inspection data for the past year indicates the process mean for the characteristic is 2.510 with a standard deviation of 0.015.

T / F In order to calculate the ratios, the assumption of normality can be verified by a normal probability plot.

T / F The second assumption of stability over time should be verified if possible.

What is Cp for the process?

A) 0.450 B) 0.900 C) 2.222 D) 1.111

Practice (2b)

$$C_p = \frac{USL - LSL}{6\sigma} \qquad C_{pl} = \frac{\mu - LSL}{3\sigma} \qquad C_{pu} = \frac{USL - \mu}{3\sigma}$$

A customer has learned about process capability ratios and requests analysis of the prior years data on a critical part characteristic with specification of 2.5 +/- .05

Inspection data for the past year indicates the process mean for the characteristic is 2.510 with a standard deviation of 0.015.

What is Cpu for the process?

A) 0.889 B) 1.333 C) 0.645 D) 1.214

What is Cpl for the process?

A) 0.889 B) 1.333 C) 1.481 D) 1.214

What is Cpk for the process?

A) 0.889 B) 1.333 C) 0.645 D) 1.111

Practice (2c)

A customer has learned about process capability ratios and requests analysis of the prior years data on a critical part characteristic with specification of 2.5 +/- .05
Inspection data for the past year indicates the process mean for the characteristic is 2.510 with a standard deviation of 0.015.

The percent defective for this process is

A) 0.38% under spec   B) 0.68% over spec

C) 3.2% under spec   D) 0.38 % over spec

The parts per million defective for this process is

A) 3800   B) 6800   C) 380   D) 3200

Practice (2d)

A customer has learned about process capability ratios and requests analysis of the prior years data on a critical part characteristic with specification of 2.5 +/- .05 Inspection data for the past year indicates the process mean for the characteristic is 2.510 with a standard deviation of 0.015.

The customer uses 110,000 parts per year. They are purchasing a new automated assembly machine. If the part is over spec, a jam will occur in the machine that cost approximately $1100 to clear. If the customer continues to purchase your parts, how much would they have to spend each year on fixing jams?

A) $126K B) $230K C) $460K D) $510K

Practice (2f)

$$C_p = \frac{USL - LSL}{6\sigma} \qquad C_{pl} = \frac{\mu - LSL}{3\sigma} \qquad C_{pu} = \frac{USL - \mu}{3\sigma}$$

A customer has learned about process capability ratios and requests analysis of the prior years data on a critical part characteristic with specification of 2.5 +/- .05 Inspection data for the past year indicates the process mean for the characteristic is 2.510 with a standard deviation of 0.015.

Based on the ratios, which of the following actions might you want to take regarding the process?

A) No actions needed as the process is capable.

B) Adjust the process to move the mean closer to the center of the specification.

C) Brainstorm causes of variation and take actions where possible to reduce the variation.

D) Both B and C.

Steps for Process Capability
Large Sample

- Data Collection
  - i.e. sample measurements from vendor's parts
- Verify Process is stable
  - Stability over time
  - information likely not available
- Verify population normality
  - Histogram, Normality Plot
- Estimate process variation
  - Overall data variation

$$\hat{\sigma} = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{(n-1)}}$$

- Compare to specifications
  - C_p, C_{pk}

C_p vs. P_p

Process Capability Analysis – C_p, C_{pk}

Uses sample ranges or sample standard deviation to estimate of process variation

Called “within” or “short term” variation

Considered to estimate capability of process.

Performance Capability Analysis – P_p, P_{pk}

Uses standard deviation of all data to estimate process variation

Called “overall” or “long term” variation

Considered to reflect actual performance of process over time.

Process Performance Indices – Pp, Ppk

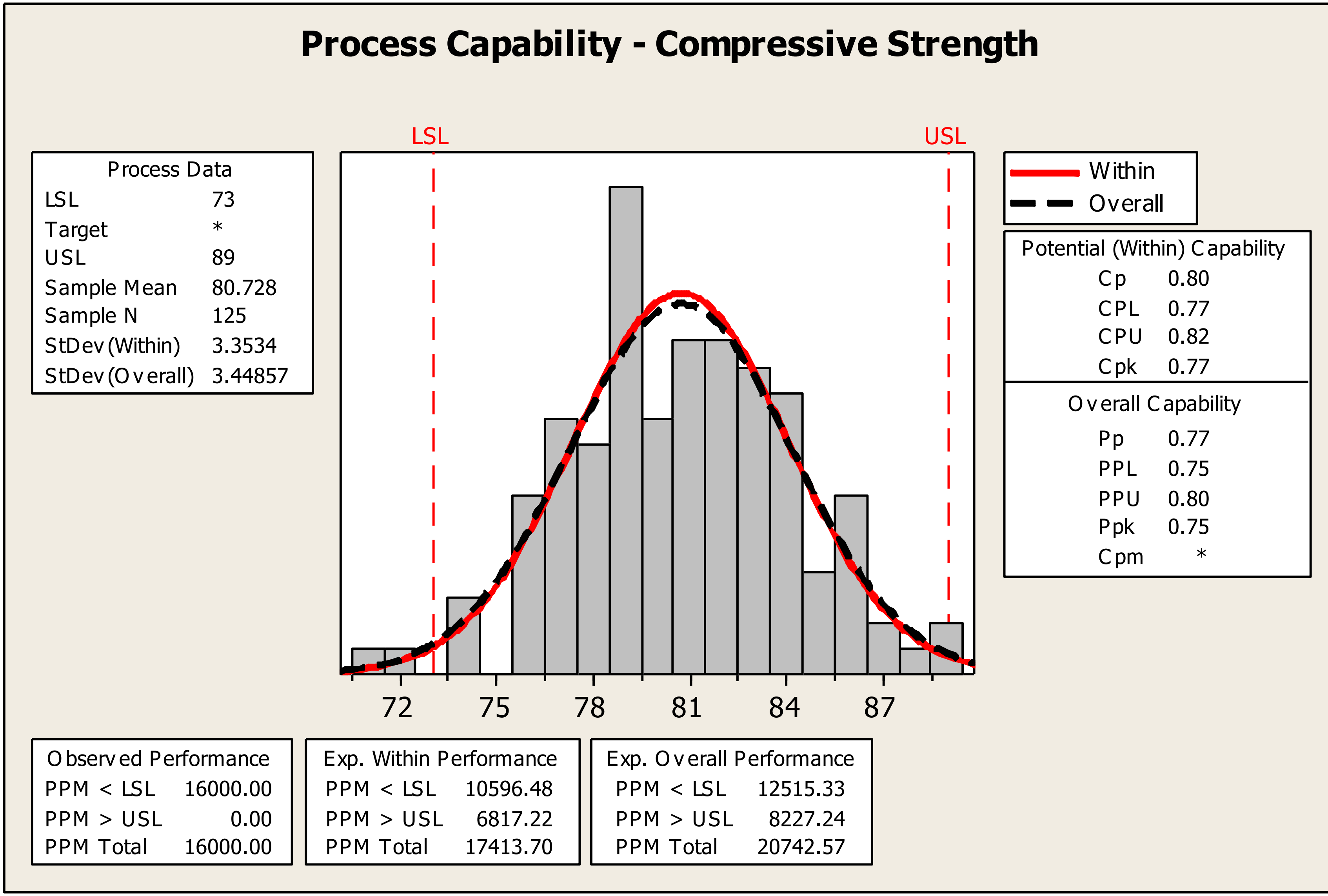
$$\hat{P}_p = \frac{USL - LSL}{6S}$$

$$P_{pk} = \min(P_{pu}, P_{pl})$$

where

$$P_{pu} = \frac{USL - \bar{\bar{x}}}{3S} \qquad P_{pl} = \frac{\bar{\bar{x}} - LSL}{3S}$$

- S is the “overall” standard deviation of all observations taken.
- Automotive Industry Action Group (AIAG) developed and promotes their use.
- Quality experts disagree among themselves about their appropriateness.


Process Capability - Compressive Strength
LSL
USL
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Target *
USL 89
Sample Mean 80.728
Sample N 125
StDev(Within) 3.3534
StDev(Overall) 3.44857
Within
Overall
Potential (Within) Capability
Cp 0.80
CPL 0.77
CPU 0.82
Cpk 0.77
Overall Capability
Pp 0.77
PPL 0.75
PPU 0.80
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Cpm *
72
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Observed Performance
PPM < LSL 16000.00
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PPM < LSL 10596.48
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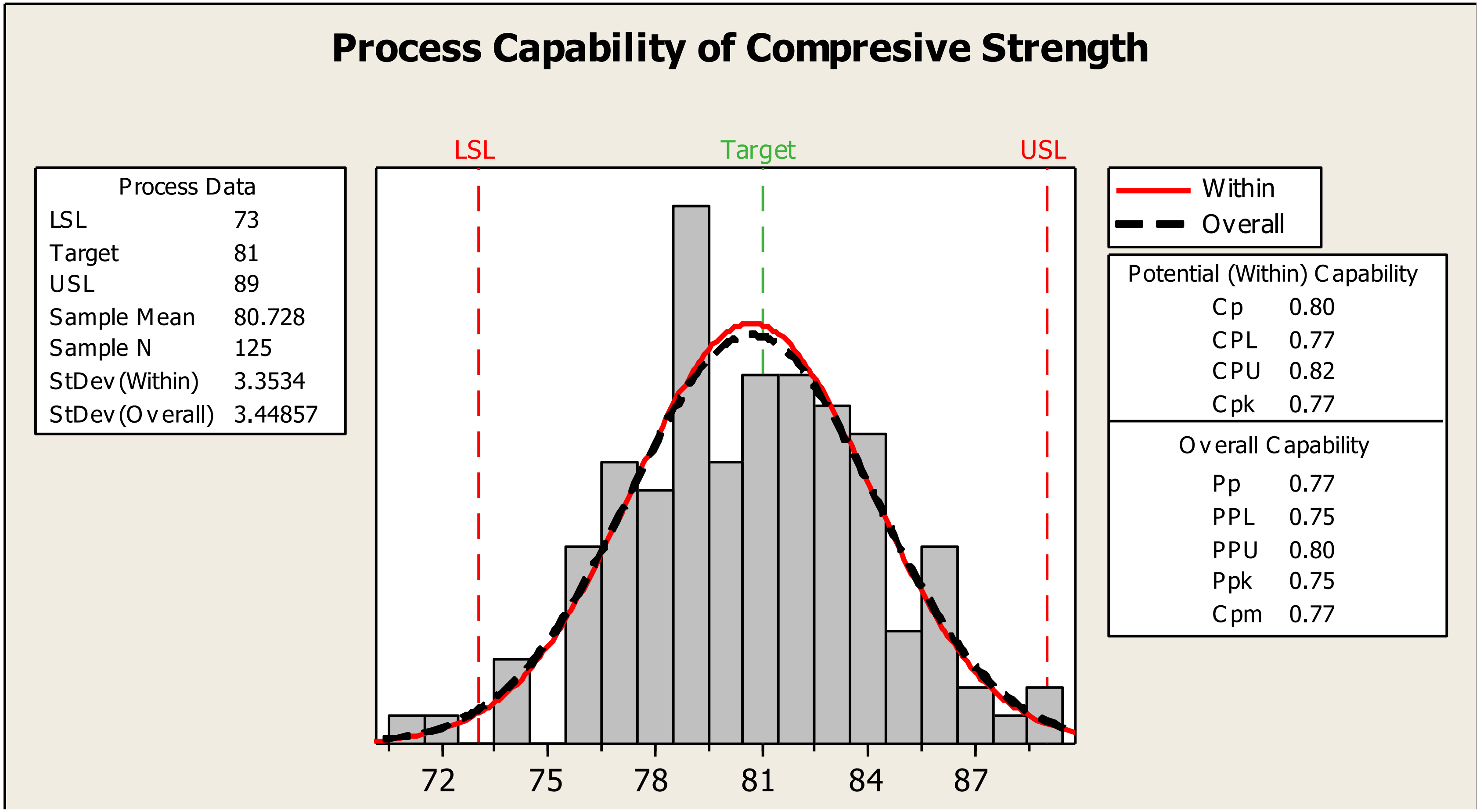
PPM Total 17413.70
Exp. Overall Performance
PPM < LSL 12515.33
PPM > USL 8227.24
PPM Total 20742.57



$$Cpm = \frac{USL - LSL}{6\sqrt{(\mu - T)^2 + \sigma^2}}$$

Cpm is an alternative metric that considers how far off the mean is from the target value as well as the specification limits.

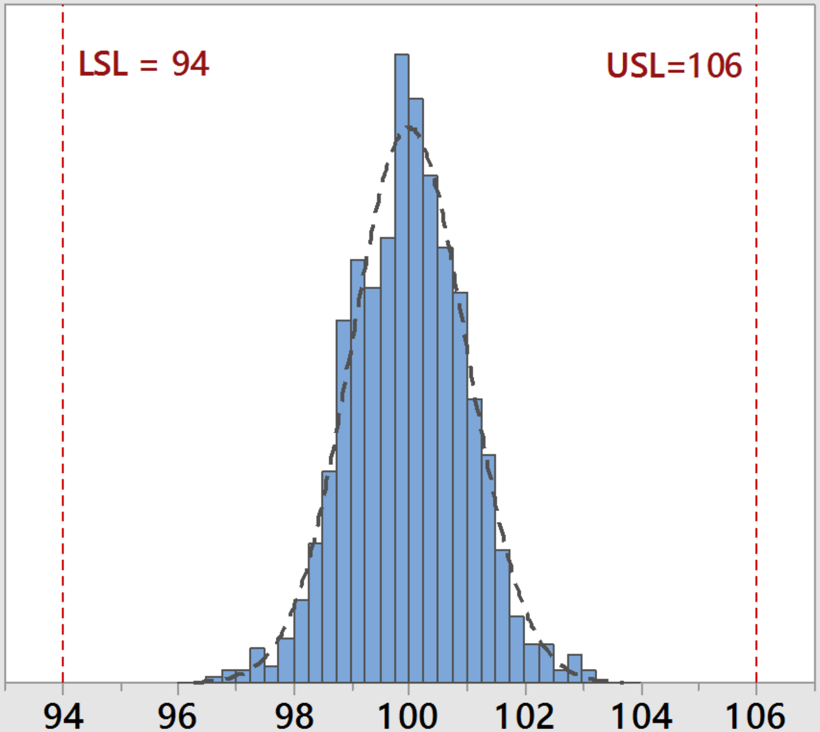
A Six Sigma Process

Motorola: “6 sigma process” is defined at
3.4 defects per million opportunities
(or better)

- Controversial - not intuitive statistically

“Common” interpretation is
0.002 defects per million opportunities
(or better)

Process Capability: mean=100



Process Data

LSL	94
Target	100
USL	106
Sample Mean #	100
Sample N	1000
StDev(Within) #	1

Potential (Within) Capability

Cp	2.00
CPL	2.00
CPU	2.00
Cpk	2.00

Performance

	Observed	Expected Within
PPM < LSL	0.00	0.001
PPM > USL	0.00	0.001
PPM Total	0.00	0.002

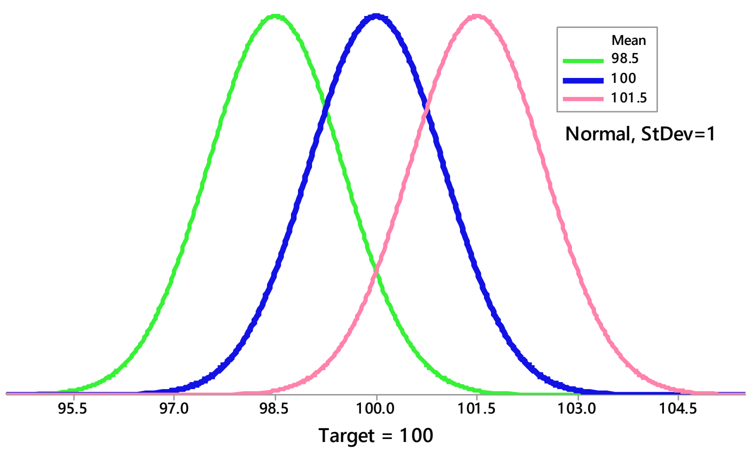
"Common" interpretation of what is meant by 6 sigma process.

- Specification limits are at plus or minus 6 sigma
- Process centered between spec limits
- Expected defects per million is 0.002

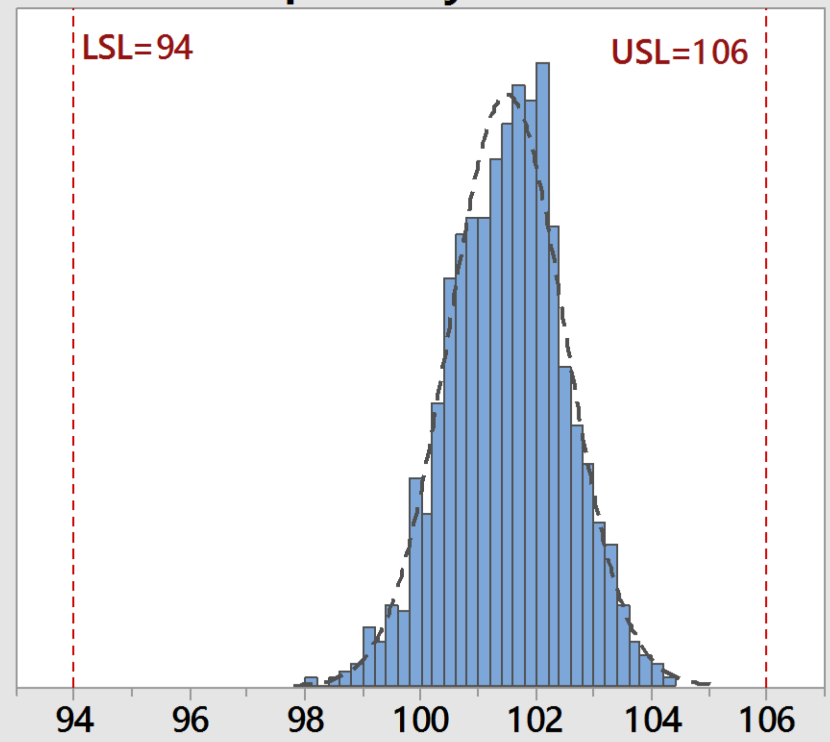
Mean = target = 100
Process standard deviation, σ = 1
Specs at mean -6σ = 94
and mean +6 σ=106

Original Motorola interpretation of what is meant by 6 sigma process.

- Specification limits are at plus or minus 6 sigma
- Process shift up to 1.5 sigma allowed. Recognizes processes do shift.
- Expected defects per million is 3.4



Process Capability: mean=101.5



Process Data

LSL	94
Target	*100
USL	106
Sample Mean #	101.5
Sample N	1000
StDev(Within) #	1

Potential (Within) Capability

Cp	2.00
CPL	2.50
CPU	1.50
Cpk	1.50

Performance

	Observed	Expected Within
PPM < LSL	0.00	0.00
PPM > USL	0.00	3.40
PPM Total	0.00	3.40

Related Assignments

Please see Blackboard for related assignments