Chapter 4

The Valuation of Long-Term Securities

Learning Objectives

After studying Chapter 4, you should be able to:

1. Distinguish among the various terms used to express value.
2. Value bonds, preferred stocks, and common stocks.
3. Calculate the rates of return (or yields) of different types of long-term securities.
4. List and explain a number of observations regarding the behavior of bond prices.

Topics

- Distinctions Among Valuation Concepts
- Bond Valuation
- Preferred Stock Valuation
- Common Stock Valuation
- Rates of Return (or Yields)

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What is Value?

- Liquidation value represents the amount of money that could be realized if an asset or group of assets is sold separately from its operating organization.
- Going-concern value represents the amount a firm could be sold for as a continuing operating business.

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What is Value?

Book value represents either

(1) *an asset*: the accounting value of an asset -- the asset's cost minus its accumulated depreciation;

(2) *a firm*: total assets minus liabilities and preferred stock as listed on the balance sheet.

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What is Value?

- Market value represents the market price at which an asset trades.
- Intrinsic value represents the price a security "ought to have" based on all factors bearing on valuation.

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Bond Valuation

- Important Terms
- Types of Bonds
- Valuation of Bonds
- Handling Semiannual Compounding

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Important Bond Terms

- A bond is a long-term debt instrument issued by a corporation or government.
- The maturity value (MV) [or face value] of a bond is the stated value. In the case of a U.S. bond, the face value is usually $1,000.

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Important Bond Terms

- The bond's coupon rate is the stated rate of interest; the annual interest payment divided by the bond's face value.
- The discount rate (capitalization rate) is dependent on the risk of the bond and is composed of the risk-free rate plus a premium for risk.

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Different Types of Bonds

A perpetual bond is a bond that *never* matures. It has an infinite life.

$$V = \frac{I}{(1+k_d)^1} + \frac{I}{(1+k_d)^2} + \cdots + \frac{I}{(1+k_d)^\infty} \qquad [4.1]$$

$$= \sum_{t=1}^{\infty} \frac{I}{(1+k_d)^t} \qquad [4.2]$$

$$= \frac{I}{k_d} \qquad [4.3]$$

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Perpetual Bond Example

Bond P has a $1,000 face value and provides an 8% annual coupon. The appropriate discount rate is 10%. What is the value of the perpetual bond?

I = $1,000 (8%) = $80.

k_d = 10%.

V = I / k_d [*Reduced Form*]

= $80 / 10% = $800.

Different Types of Bonds

A non-zero coupon-paying bond is a coupon paying bond with a finite life.

$$V = \frac{I}{(1+k_d)^1} + \frac{I}{(1+k_d)^2} + \cdots + \frac{I}{(1+k_d)^n} + \frac{MV}{(1+k_d)^n}$$

$$= \sum_{t=1}^{n} \frac{I}{(1+k_d)^t} + \frac{MV}{(1+k_d)^n} \qquad [4.4]$$

$$= I(PVIFA_{k_d,n}) + MV(PVIF_{k_d,n}) \qquad [4.5]$$

$$= I(P/A, k_d, n) + MV(P/F, k_d, n)$$

Coupon Bond Example

Bond C has a $1,000 face value and provides an 8% annual coupon for 30 years. The appropriate discount rate is 10%. What is the value of the *coupon bond*?

$$V = \$80\ (PVIFA_{10\%,\ 30}) + \$1{,}000\ (PVIF_{10\%,\ 30})$$

$$= \$80\ (9.427) + \$1{,}000\ (.057)$$

$$= \$754.16 + \$57.00$$

$$= \$811.16.$$

Different Types of Bonds

A zero coupon bond is a bond that pays no interest but sells at a deep discount from its face value; it provides compensation to investors in the form of price appreciation.

$$V = \frac{MV}{(1+k_d)^n} \qquad [4.6]$$

$$= MV(PVIF_{k_d,n}) \qquad [4.7]$$

$$= MV(P/F, k_d, n)$$

Zero-Coupon Bond Example

Bond Z has a $1,000 face value and a 30 year life. The appropriate discount rate is 10%. What is the value of the *zero-coupon bond*?

$$V = \$1{,}000\ (PVIF_{10\%,\ 30})$$
$$= \$1{,}000\ (.057)$$
$$= \$57.00$$

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Semiannual Compounding

Most bonds *in the U.S.* pay interest twice a year (1/2 of the annual coupon).

Adjustments needed:

(1) Divide k_d by 2

(2) Multiply n by 2

(3) Divide I by 2

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Semiannual Compounding

A non-zero coupon bond adjusted for semiannual compounding.

$$V = \sum_{t=1}^{2n} \frac{I/2}{(1+k_d/2)^t} + \frac{MV}{(1+k_d/2)^{2n}} \qquad [4.8]$$

$$= (I/2)(PVIFA_{k_d/2,2n}) + MV(PVIF_{k_d/2,2n}) \qquad [4.9]$$

$$= (I/2)(P/A, k_d/2, 2n) + MV(P/F, k_d/2, 2n)$$

Semiannual Coupon Bond Example

Bond C has a \$1,000 face value and provides an 8% semiannual coupon for 15 years. The appropriate discount rate is 10% (annual rate). What is the value of the *coupon bond*?

V = \$40 ($PVIFA_{5\%, 30}$) + \$1,000 ($PVIF_{5\%, 30}$)
= \$40 (15.373) + \$1,000 (.231)
= \$614.92 + \$231.00
= \$845.92

Preferred Stock Valuation

Preferred Stock is a type of stock that promises a (usually) fixed dividend, but at the discretion of the board of directors.

- Preferred Stock has preference over common stock in the payment of dividends and claims on assets.

Preferred Stock Valuation

$$V = \frac{D_P}{(1+k_P)^1} + \frac{D_P}{(1+k_P)^2} + \cdots + \frac{D_P}{(1+k_P)^\infty}$$

$$= \sum_{t=1}^{\infty} \frac{D_P}{(1+k_P)^t} = D_P(PVIFA_{k_d,\infty})$$

$$= \frac{D_P}{k_P} \qquad [4.10]$$

Preferred Stock Example

Stock PS has an 8%, $100 par value issue outstanding. The appropriate discount rate is 10%. What is the value of the preferred stock?

Div_P = $100 (8%) = $8.00.
k_P = 10%.
V = Div_P / k_P = $8.00 / 10%
= $80

Common Stock Valuation

Common stock represents a residual ownership position in the corporation.

- Pro rata share of future earnings after all other obligations of the firm (if any remain).
- Dividends *may* be paid out of the pro rata share of earnings.

Common Stock Valuation

What cash flows will a shareholder receive when owning shares of common stock?

(1) Future dividends

(2) Future sale of the common stock shares

Dividend Valuation Model

- Basic dividend valuation model accounts for the PV of all future dividends.

$$V = \frac{D_1}{(1+k_e)^1} + \frac{D_2}{(1+k_e)^2} + \cdots + \frac{D_\infty}{(1+k_e)^\infty} \qquad [4.11]$$

$$= \sum_{t=1}^{\infty} \frac{D_t}{(1+k_e)^t} \qquad [4.12]$$

D_t:	Cash Dividend at time t
k_e:	Equity investor's required return

Adjusted Dividend Valuation Model

- The basic dividend valuation model adjusted for the future stock sale.

$$V = \frac{D_1}{(1+k_e)^1} + \frac{D_2}{(1+k_e)^2} + \cdots + \frac{D_n + P_n}{(1+k_e)^n}$$

n: The year in which the firm's shares are expected to be sold.
P_n: The expected share price in year n.

Dividend Growth Pattern Assumptions

The dividend valuation model requires the forecast of *all* future dividends. The following dividend growth rate assumptions simplify the valuation process.

- Constant Growth
- No Growth
- Growth Phases

Constant Growth Model

- The constant growth model assumes that dividends will grow forever at the rate g.

$$V = \frac{D_0(1+g)}{(1+k_e)^1} + \frac{D_0(1+g)^2}{(1+k_e)^2} + \cdots + \frac{D_0(1+g)^\infty}{(1+k_e)^\infty} \quad [4.13]$$

$$= \frac{D_1}{(k_e - g)} \quad [4.14]$$

$$k_e = \frac{D_1}{V} + g \quad [4.15]$$

D_0:	Dividend paid at time 0.
g :	The constant growth rate.
k_e:	Investor's required return.

Constant Growth Model Example

Stock LVN's dividend per share at t=1 is expected to be $4, that it is expected to grow at a 6% rate forever, and that the appropriate discount rate is 14%. What is the value of the common stock?

$V_{LVN} = D_1 / (k_e - g) = \$4 / (.14 - .06) = \$50$

Constant Growth Model

Conversion to an Earnings Multiplier Approach:

Let b = constant retained earning proportion

$$(1-b) = \frac{D_1}{E_1} \quad [4.16]$$

$$V = \frac{D_1}{(k_e - g)} = \frac{(1-b)E_1}{(k_e - g)} \quad [4.17]$$

$$\text{Earning Multiplier} = \frac{V}{E_1} = \frac{(1-b)}{(k_e - g)} \quad [4.18]$$

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Constant Growth Model Example

Stock LVN has a retention rate of 40% and earning per share for period 1 are expected to be $6.67 and growing at 6%. The appropriate discount rate is 14%. What is the value of the common stock?

$$V = \frac{(1-b)E_1}{(k_e - g)}$$

$$= \frac{(1-0.4)\$6.67}{(0.14-0.06)} = \$50$$

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Zero Growth Model

- The zero growth model assumes that dividends will grow forever at the rate g = 0.

$$V = \frac{D_0}{(1+k_e)^1} + \frac{D_0}{(1+k_e)^2} + \cdots + \frac{D_0}{(1+k_e)^\infty}$$

$$= \frac{D_1}{(k_e)}$$

[4.19]

D_1:	Dividend paid at time 1.
k_e:	Investor's required return.

Zero Growth Model Example

Stock ZG has an expected growth rate of 0%. Each share of stock just received an annual \$3.24 dividend per share. The appropriate discount rate is 15%. What is the value of the common stock?

D_1 = \$3.24 (1 + 0) = \$3.24

V_{ZG} = D_1 / (k_e - 0) = \$3.24 / (.15 - 0)
= \$21.60

Growth Phases Model

- The growth phases model assumes that dividends for each share will grow at two or more *different* growth rates.

$$V = \sum_{t=1}^{n} \frac{D_0(1+g_1)^t}{(1+k_e)^t} + \sum_{t=n+1}^{\infty} \frac{D_n(1+g_2)^{t-n}}{(1+k_e)^t} \quad [4.20]$$

$$= \sum_{t=1}^{n} \frac{D_0(1+g_1)^t}{(1+k_e)^t} + \left(\frac{1}{(1+k_e)^n}\right)\left(\frac{D_{n+1}}{k_e - g_2}\right) \quad [4.21]$$

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Growth Phases Model Example

Stock GP has an expected growth rate of 16% for the first 3 years and 8% thereafter. Each share of stock just received an annual $3.24 dividend per share. The appropriate discount rate is 15%. What is the value of the common stock under this scenario?

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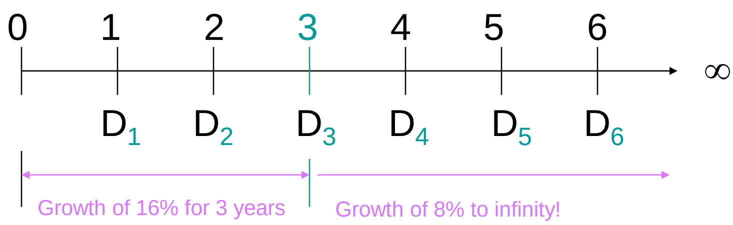
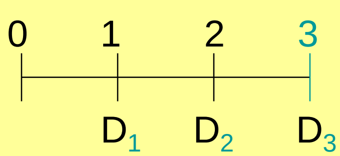
Growth Phases Model Example

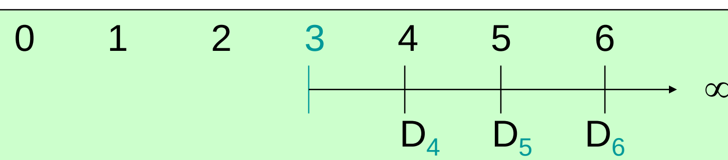


Stock GP has two phases of growth. The first, 16%, starts at time t=0 for 3 years and is followed by 8% thereafter starting at time t=3. We should view the time line as two separate time lines in the valuation.

Growth Phases Model Example



Growth Phase #1 plus the infinitely long Phase #2



Note that we can value Phase #2 using the *Constant Growth Model*

Growth Phases Model Example

$$V_3 = \frac{D_4}{k-g}$$

We can use this model because dividends grow at a constant 8% rate beginning at the end of Year 3.

0 1 2 3 4 5 6 ∞

D_4 D_5 D_6

Note that we can now replace <u>all</u> dividends from year 4 to infinity with the *value* at time t=3, V_3! Simpler!!

Growth Phases Model Example

0 1 2 3

D_1 D_2 D_3

New Time Line

0 1 2 3

V_3

Where $V_3 = \dfrac{D_4}{k-g}$

Now we only need to find the first four dividends to calculate the necessary cash flows.

Growth Phases Model Example

Determine the annual dividends.

D_0 = \$3.24 (this has been paid already)

$D_1 = D_0(1+g_1)^1 = \$3.24(1.16)^1 = \$3.76$

$D_2 = D_0(1+g_1)^2 = \$3.24(1.16)^2 = \$4.36$

$D_3 = D_0(1+g_1)^3 = \$3.24(1.16)^3 = \$5.06$

$D_4 = D_3(1+g_2)^1 = \$5.06(1.08)^1 = \$5.46$

Growth Phases Model Example

0	1	2	3
	3.76	4.36	5.06

Actual Values

0	1	2	3
			78

Where $\$78 = \dfrac{5.46}{.15-.08}$

Now we need to find the present value of the cash flows.

Growth Phases Model Example

We determine the PV of cash flows.

$PV(D_1) = D_1(PVIF_{15\%,\,1}) = \$3.76\ (.870) = \underline{\$3.27}$

$PV(D_2) = D_2(PVIF_{15\%,\,2}) = \$4.36\ (.756) = \underline{\$3.30}$

$PV(D_3) = D_3(PVIF_{15\%,\,3}) = \$5.06\ (.658) = \underline{\$3.33}$

$P_3 = \$5.46\ /\ (.15 - .08) = \$78$ [CG Model]

$PV(P_3) = P_3(PVIF_{15\%,\,3}) = \$78\ (.658) = \underline{\$51.32}$

Growth Phases Model Example

Finally, we calculate the *intrinsic value* by summing all of cash flow present values.

V = $3.27 + $3.30 + $3.33 + $51.32

V = $61.22

$$V = \sum_{t=1}^{3} \frac{D_0(1+.16)^t}{(1+.15)^t} + \left(\frac{1}{(1+.15)^3}\right)\left(\frac{D_4}{.15-.08}\right)$$

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Calculating Rates of Return (or Yields)

Steps to calculate the rate of return (or Yield)

1. Determine the expected cash flows.
2. Replace the intrinsic value (V) with the market price (P_0).
3. Solve for the *market required rate of return* that equates the discounted cash flows to the market price.

Determining Bond YTM

Determine the Yield-to-Maturity (YTM) for the annual coupon paying bond with a finite life.

$$P_0 = \sum_{t=1}^{n} \frac{I}{(1+k_d)^t} + \frac{MV}{(1+k_d)^n} \qquad [4.22]$$

$$= I(PVIFA_{k_d,n}) + MV(PVIF_{k_d,n})$$

$$= I(P/A, k_d, n) + MV(P/F, k_d, n)$$

k_d = YTM

Determining the YTM

Julie Miller want to determine the YTM for an issue of outstanding bonds at *Basket Wonders (BW)*. *BW* has an issue of 10% annual coupon bonds with 15 years left to maturity. The bonds have a current market value of *$1,250*. *What is the YTM?*

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YTM Solution (Try 9%)

$1,250 = $100(PVIFA$_{9\%,15}$) + $1,000(PVIF$_{9\%, 15}$)

$1,250 = $100(8.061) + $1,000(.275)

$1,250 = $806.10 + $275.00 = $1,081.10

[*Rate is too high!*]

$1,250 = $100(PVIFA$_{7\%,15}$) + $1,000(PVIF$_{7\%, 15}$)

$1,250 = $100(9.108) + $1,000(.362)

$1,250 = $910.80 + $362.00 = $1,272.80

[*Rate is too low!*]

YTM = .07 + .0024 = .0724 or 7.24%

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Determining Semiannual Coupon Bond YTM

Determine the Yield-to-Maturity (YTM) for the semiannual coupon paying bond with a finite life.

$$P_0 = \sum_{t=1}^{2n} \frac{I/2}{(1+k_d/2)^t} + \frac{MV}{(1+k_d/2)^{2n}} \quad [4.23]$$

$$= (I/2)(PVIFA_{k_d/2, 2n}) + MV(PVIF_{k_d/2, 2n})$$

$$= (I/2)(P/A, k_d/2, 2n) + MV(P/F, k_d/2, 2n)$$

$[1 + (k_d / 2)]^2 - 1 = YTM$

Determining the Semiannual Coupon Bond YTM

Julie Miller want to determine the YTM for another issue of outstanding bonds. *The firm* has an issue of 8% semiannual coupon bonds with 20 years left to maturity. The bonds have a current market value of *$950*. *What is the YTM?*

- Determine the Yield-to-Maturity (YTM) for the semiannual coupon paying bond with a finite life.

$[1 + (k_d / 2)^2] - 1 = YTM$

$[1 + (.042626)^2] - 1 = .0871$ or 8.71%

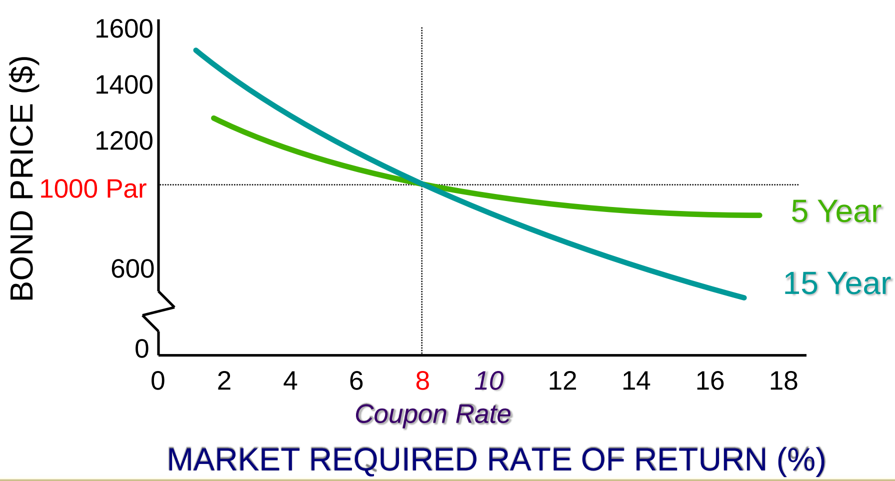
Bond Price - Yield Relationship

Discount Bond -- The market required rate of return exceeds the coupon rate (Par > P_0).

Premium Bond -- The coupon rate exceeds the market required rate of return (P_0 > Par).

Par Bond -- The coupon rate equals the market required rate of return (P_0 = Par).

Bond Price - Yield Relationship

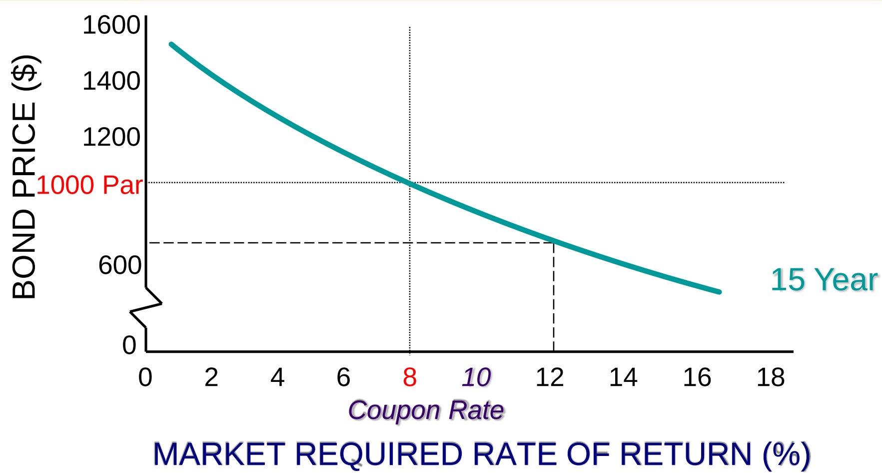


Bond Price-Yield Relationship

When interest rates ***rise***, then the market required rates of return ***rise*** and bond prices will ***fall***.

Assume that the required rate of return on a 15 year, 8% annual coupon paying bond *rises* from 8% to 12%. What happens to the bond price?

Bond Price - Yield Relationship

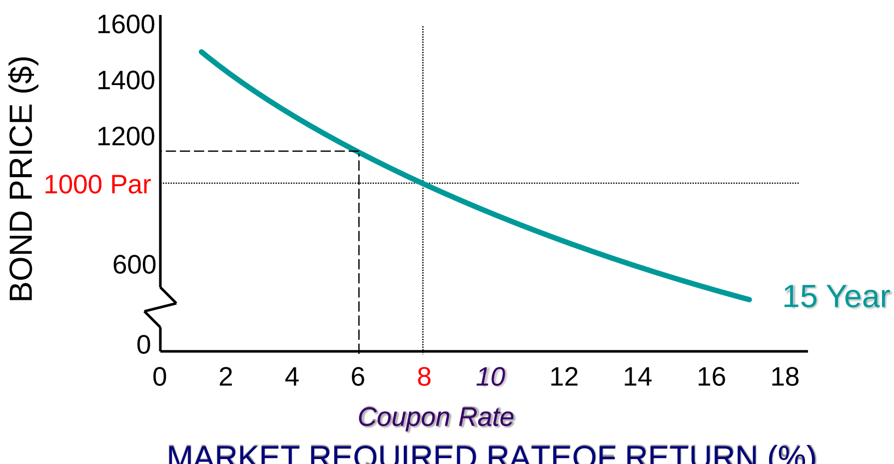


Bond Price-Yield Relationship

When interest rates *fall*, then the market required rates of return *fall* and bond prices will ***rise***.

Assume that the required rate of return on a 15 year, 8% annual coupon paying bond *falls* from 8% to 6%. What happens to the bond price?

Bond Price - Yield Relationship

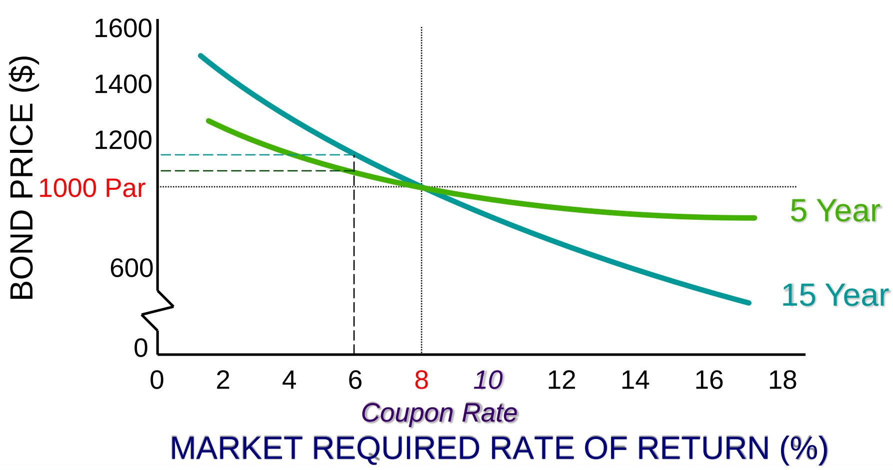


The Role of Bond Maturity

The longer the bond maturity, the greater the change in bond price for a given change in the market required rate of return.

Assume that the required rate of return on both the 5 and 15 year, 8% annual coupon paying bonds *fall* from 8% to 6%. What happens to the changes in bond prices?

Bond Price - Yield Relationship



The Role of the Coupon Rate

For a given change in the market required rate of return, the price of a bond will change by proportionally more, *the* ***lower*** *the coupon rate*.

Determining the Yield on Preferred Stock

Determine the yield for preferred stock with an infinite life.

$$P_0 = \frac{D_P}{k_P} \qquad [4.24]$$

Solving for k_P such that

$$k_P = \frac{D_P}{P_0} \qquad [4.25]$$

Preferred Stock Yield Example

Assume that the annual dividend on each share of preferred stock is *$10*. Each share of preferred stock is currently trading at *$100*. What is the *yield* on preferred stock?

$$k_P = \$10 / \$100.$$

$$k_P = 10\%.$$

Determining the Yield on Common Stock

Assume the constant growth model is appropriate. Determine the yield on the common stock.

$$P_0 = \frac{D_1}{k_e - g} \qquad [4.26]$$

Solving for k_e such that

$$k_e = \frac{D_1}{P_0} + g \qquad [4.27]$$

Common Stock Yield Example

Assume that the expected dividend (D_1) on each share of common stock is *$3*. Each share of common stock is currently trading at *$30* and has an expected growth rate of *5%*. What is the *yield* on common stock?

$$k_e = (\$3 / \$30) + 5\%$$

$$k_e = 10\% + 5\% = 15\%$$

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