EIN 5226

Normal Distribution Quick Review

Chapter 7 Sections 7.2, 7.3

Note: Need Calculator & Z table Handout for lecture

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What type of data is it?

- Discrete – attribute data

- Raw data can only logically be whole numbers
- Examples: Defect count, % repeat customers
- Can be a finite set or an infinite set of values
- Examples
  - Any counted data, i.e., counts of defects
  - Rating scale can only take on the values 1, 2, 3, 4, and 5

- Continuous – variable data

- Exact value of variable can take on any value in an interval
- Possible outcomes are theoretically infinite
- Typically a measurement
- Examples:
  - weight,
  - temperature,
  - linear measurements

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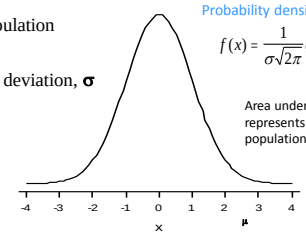
The Normal Distribution

Normal distribution - Important **continuous** distribution
- Symmetric and bell-shaped.

For the population
- Mean, **μ**
- Standard deviation, **σ**

Probability density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-u)^2/2\sigma^2}$$



Area under the curve represents 100% of the population

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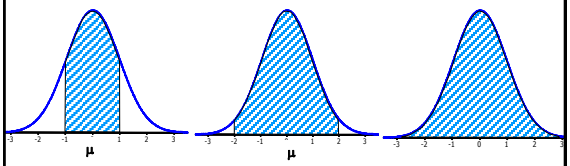
The Normal Distribution



~ 68% of the data lie within 1 standard deviation of the mean.

~ 95% of the data lie within 2 standard deviations of the mean.

~ 99.7% of the data lie within 3 standard deviations of the mean.

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Standard Normal Tables

- The proportion of a normal population that is within a given distance to the mean is the same for any normal population.
- Standard normal tables - summarize the probabilities of being less than Z units from the mean where $\mu = 0$ and $\sigma = 1$.
- To use the tables, we must convert from the units in which the population items were originally measured to the number of **standard units** denoted by Z.

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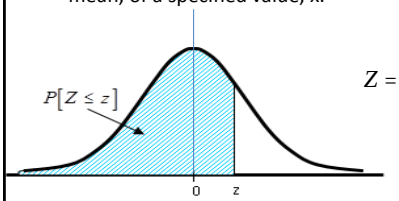
Standard normal distribution

μ= 0 and σ = 1
Z: *Standardized* normal random variable

Z = number of standard deviations away from the mean, of a specified value, x.



$$Z = \frac{x - \mu}{\sigma}$$

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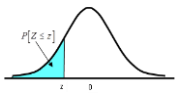
Reading the Z table

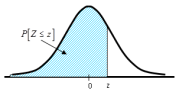


Standard Normal Probabilities

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010

Example: P(Z<-3.17) = .0008

Reading the Z table

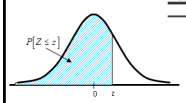


Standard Normal Probabilities

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224

Example: P(Z< +0.48) = 0.6844

Reading the Z table



Standard Normal Probabilities

z	0	0.01	0.02	0.03	0.04	0.05	0.06
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636
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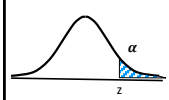
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123

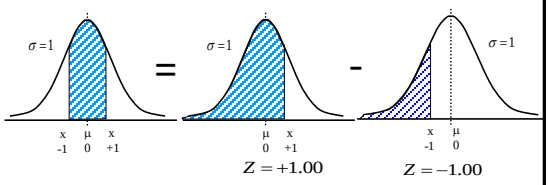
Table A. page 1040



z	0	0.01	0.02	0.03	0.04	0.05	0.06
0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877

Includes only positive z values.
Gives area to the right of z. $\alpha = p(Z>z)$
Since the normal curve is symmetric, $\alpha = p(Z< -z)$

Area between two values



$Z = +1.00$ $Z = -1.00$

P(Z < +1.00) = 0.8413 P(Z<-1.00) = .1587

P(-1< Z < +1) = P(Z<+1) – P(Z—1) = 0.8413 - .1587 = .6826 = 68.26%

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Normal Distribution Problem

$$Z = \frac{x - \mu}{\sigma}$$

A process characteristic has a mean of 3.1 and standard deviation of 1.2.

What % of the parts from the process will fall below 2.5?

Z used to work this problem is
A. +0.50 B. -0.50 C. -1.45 D. +1.45

The percent of the parts below 2.5 is
A. 51.99% B. 69.15% C. 48.01% D. 30.85%

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Normal Distribution Problem

$$Z = \frac{x - \mu}{\sigma}$$

A process characteristic has a mean of 3.1 and standard deviation of 1.2.

What % of the parts from the process be greater than 3.7?

Z used to work this problem is
A. +0.50 B. -0.50 C. -1.45 D. +1.45

The percent of the parts greater than is
A. 51.99% B. 69.15% C. 48.01% D. 30.85%

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Normal Distribution Problem

$$Z = \frac{x - \mu}{\sigma}$$

A process characteristic has a mean of 3.1 and standard deviation of 1.2.

What % of the parts from the process will be between 2.5 and 3.7?

A. 56.22% B. 35.12% C. 38.30% D. 40.23%

What % of the parts from the process will be between 2.5 and 3.1?

A. 51.99% B. 19.15% C. 48.01% D. 30.85%

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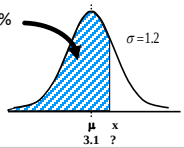
Given a process mean of 3.1 and standard deviation of 1.2, below what value will 60% of the observations fall?

Area = 60%

What are we looking for?



What do we know? $P(Z < +z) = 0.600$ $z = \frac{x - 3.1}{1.2}$

Look for probability in body of table, then over to find Z.

From Table: z = .25

→ Plug into equation and solve for x.

$$0.25 = \frac{x - 3.1}{1.2}$$

x=(.25*1.2)+3.1 = 3.4

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Normal Distribution Problem

$$Z = \frac{x - \mu}{\sigma}$$

A process characteristic has a mean of 3.1 and standard deviation of 1.2.

(use closest in table to determine)

Below what number will 90% of the parts fall?

A. 4.64 B. 5.21 C. 3.68 D. 4.56

Above what number will 80% of the parts fall?

A. 1.78 B. 2.17 C. 3.01 D. 2.09

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Independent events

When sampling from a distribution that is assumed to be normal, probabilities do not change.

When taking multiple observations from a process, each is an independent event.

For independent events: P(A and B) = P(A) * P(B)

A process characteristic has a mean of 3.1 and standard deviation of 1.2.

What is the probability that 2 parts in a row taken from the process are below 1.9?

A. 0.1480 B. 0.3164 C. 0.0252 D. 0.1587

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Related Assignments

See Blackboard for related assignments