EIN 5226

Errors, Power, Sample Size

Chapter 16 Sections 16.3
Chapter 17 Sections 17.3

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Rejecting the Null Hypothesis

In a hypothesis test, the null hypothesis is rejected when

- The p value is less than the acceptable α risk
- Z_{test} falls in the rejection area defined by $\pm Z_{\alpha/2}$ (for H_a: $\mu \neq \mu_0$)
- $\bar{x}$ falls outside the confidence interval (1-α) around μ_0 value

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Tests on Single Population Means
Large Sample Sizes (n≥30)

	Two-tailed	One-tailed	
Null hypothesis H_0	$\mu=\mu_0$		
Alternate H_a	$\mu\neq\mu_0$	$\mu>\mu_0$	$\mu<\mu_0$
Test statistic	$Z_{test} = \frac{\bar{x}-u_o}{s/\sqrt{n}}$		
P value (reject if p<α)	sum the areas in the tails, cut off by Z and –Z	Area to right of Z	Area to left of Z
Z – rejection region	$\lvert z_{test}\rvert > z_{\alpha/2}$	$z_{test} > +z_{\alpha}$	$z_{test} < -z_{\alpha}$

3

In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of 123 and a standard deviation of 5. Can we conclude, with 95% certainty, that the line is malfunctioning. (Assume normal distribution.)

H_0: $\mu_0 = 122$ H_a: $\mu \neq 122$

$$Z_{test} = \frac{\bar{x} - u_0}{s/\sqrt{n}}$$

What is the value of the test statistic?

A) 2.53 B) 1.26 C) 1.645 D) 1.35

What is the p value associated with the test statistic?

A) 0.103 B) 0.206 C) 0.100 D) 0.0114

T / F Based on this sample of 40 readings, there is not sufficient evidence to reject the null hypothesis and to conclude with 95% certainty that the process mean has shifted.

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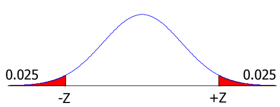
$$Z_{test} = \frac{\bar{x} - u_0}{s/\sqrt{n}}$$



What is the Z value associated with α= 5% for a two tailed test?

A) $\pm$ 2.326 B) $\pm$ 1.960 C) $\pm$1.645 D) $\pm$1.282

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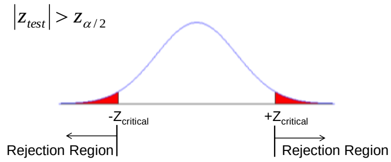
Rejection Region

- The region outside of $Z_{critical}$ is the rejection region.
- For H_a: $\mu \neq \mu_0$ it is the area defined by the two tails

$$|z_{test}| > z_{\alpha/2}$$



- For H_a: $\mu > \mu_0$ it is the area defined by $Z_{test} > +Z_\alpha$
- For H_a: $\mu < \mu_0$ it is the area defined by $Z_{test} < -Z_\alpha$

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Errors

When conducting a test at significance level α, two types of errors can be made.

- Type I: Reject H_0 when it is true. $\boldsymbol{\alpha}$
- Type II: Fail to reject H_0 when it is false. $\boldsymbol{\beta}$

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H_o: $\mu = 122$ Machine running at target

H_a: $\mu \neq 122$ Machine not running at target

		Truth	
		Did not change	**Changed**
Decision	**Null rejected** **Process Changed** **Fix machine**	**Type I error** **α**	✓
Made	**Not reject null** **Process unchanged** **Leave machine alone**	✓	**Type II error** **β**

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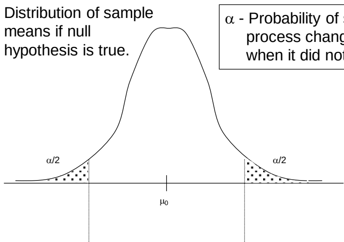
Type I Error

Distribution of sample means if null hypothesis is true.

α - Probability of saying a process changed when it did not



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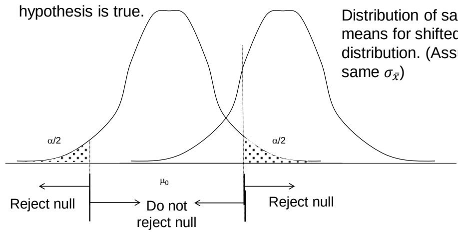
Type II Error

Distribution of sample means if null hypothesis is true.

Process mean has shifted. Null not true.

Distribution of sample means for shifted distribution. (Assume same $\sigma_{\bar{x}}$)



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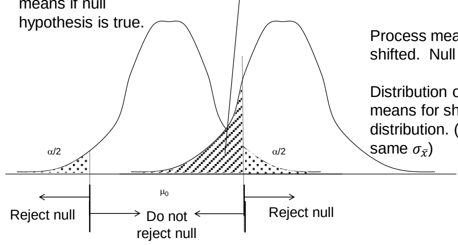
Type II Error

β - Probability of saying a process did not change when it did

Distribution of sample means if null hypothesis is true.

Process mean has shifted. Null not true.

Distribution of sample means for shifted distribution. (Assume same $\sigma_{\bar{x}}$)



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Decision Made		Truth: Did not change	Truth: Changed
	Null rejected (support H_a)	**Type I error** **α**	**Correct (1-β)** "Power" of the test
	Not reject null (Do not support H_a)	**Correct (1-α)** "Confidence level" of test	**Type II error** **β**

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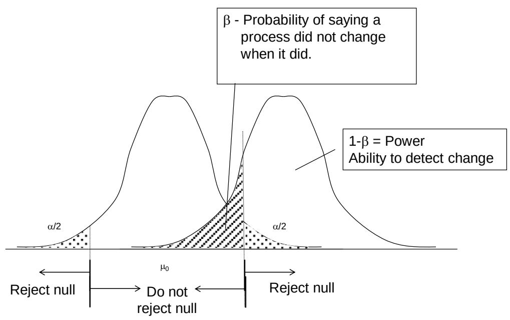
Type II Error

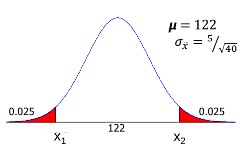


13

In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of 123 and a standard deviation of 5. Can we conclude, with 95% certainty, that the line is malfunctioning. (Assume normal distribution.)

H_0: $\mu_0 = 122$
H_a: $\mu \neq 122$

$\mu = 122$
$\sigma_{\bar{x}} = {}^{5}/_{\sqrt{40}}$



To determine the x values associated with the two tails I could

A) Use the 95% confidence interval formulas
B) Look up .025 in the body of the Z table, determine the associated Z then solve for x
C) Ask a classmate who knows how to work the problem
D) All of the above.

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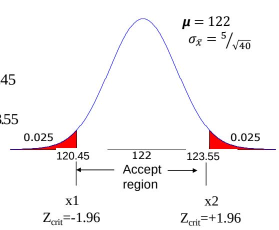
H_0: $\mu_0 = 122$ H_a: $\mu \neq 122$

From the tables, for 2.5% : Z = ±1.96
95% CI around *u*

$$x_1 = u - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 122 - 1.96\frac{5}{\sqrt{40}} = 120.45$$

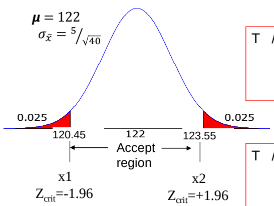
$$x_2 = u + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 122 + 1.96\frac{5}{\sqrt{40}} = 123.55$$

$\mu = 122$
$\sigma_{\bar{x}} = {}^{5}/_{\sqrt{40}}$



15

In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of **123** and a standard deviation of 5. Can we conclude, with 95% certainty, that the line is malfunctioning. (Assume normal distribution.)



T / F If I take a sample of size n=40 , with an sample average of 123 psi, my p value will be greater than 5%.

T / F If I take samples of size n=40 and get an average less than 123.5 but greater than 120.45, I will not reject the null hypothesis and will not send someone to adjust the machine.

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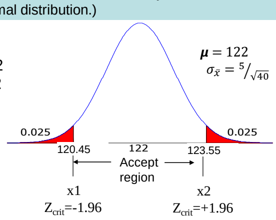
In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of **123** and a standard deviation of 5. Can we conclude, with 95% certainty, that the line is malfunctioning. (Assume normal distribution.)

H_0: $\mu_0 = 122$
H_a: $\mu \neq 122$



T / F If I do not reject my null hypothesis and my process really has shifted and the new mean is actually 123, I would have made a Type II error.

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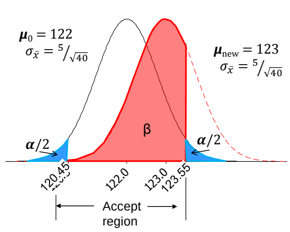
In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. Historical standard deviation is 5 psi.

If process has shifted to have average of 123, what is the probability of not detecting the shift, when taking 40 measurements and using a 95% confidence interval for the mean.



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In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. Historical standard deviation is 5 psi.

If process has shifted to have average of 123, what is the probability of not detecting the shift, when taking 40 measurements and using a 95% confidence interval for the mean. (Assume historical standard deviation)

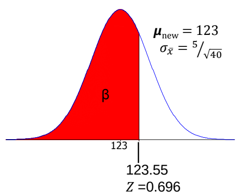
For the shifted distribution, when taking a sample of size 40, the probability of the average being less than 123. 5 is determined a follows:

$$Z = \frac{x-u}{\sigma/\sqrt{n}} = \frac{123.55-123}{5/\sqrt{40}} = 0.696$$

P(Z<.0.70) = .7580 (from table)

$\boldsymbol{\beta}$= .7580

75.8% of the time a shift will not be detected on the next sample.



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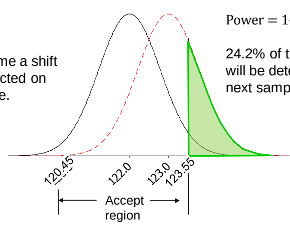
In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. Historical standard deviation is 5 psi.

If process has shifted to have average of 123, what is the probability of not detecting the shift, when taking 40 measurements and using a 95% confidence interval for the mean.

$\boldsymbol{\beta}$= .7580

75.8% of the time a shift will not be detected on the next sample.



Power = 1- $\boldsymbol{\beta}$= .242

24.2% of the time a shift will be detected on the next sample.

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Power

- A Type II Error is the failure to reject H_0 when it is false. The probability of a Type II error is $\boldsymbol{\beta}$.
- The **power** of the test is the probability of *rejecting* H_0 when it is false.

Power = 1 – $\boldsymbol{\beta}$

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Computing the Power

1. Compute the rejection region.
   Set up a confidence interval for given sample size and Type I risk.

2. Compute the probability that the test statistic falls in the rejection region for a given difference from the null. This is power.

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A manufacturer of plastic laminate sheets produces 0.750" sheets that have an historical standard deviation of .041". Samples are taken of 30 sheets to monitor the process and decide if adjustments should be made.

What are the null and alternative hypotheses inherent in the test being performed?

A. H_o: μ=0.750 H_a: μ≠0.750

B. H_o: μ=0.750 H_a: μ>0.750

C. H_o: μ=0.750 H_a: μ<0.750

23

A manufacturer of plastic laminate sheets produces 0.750" sheets that have an historical standard deviation of .041". Samples are taken of 30 sheets to monitor the process and decide if adjustments should be made.

If the acceptable level of α risk is 5%, what are the $Z_{critical}$ values needed to define the rejection region?

A. ± 1.96 B. ± 1.645 C. ± 2.575

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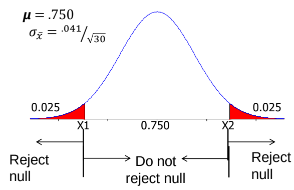
Confidence interval $\mu \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

A manufacturer of plastic laminate sheets produces 0.750" sheets that have an historical standard deviation of .041". Samples are taken of 30 sheets to monitor the process and decide if adjustments should be made.



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Confidence interval $\mu \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

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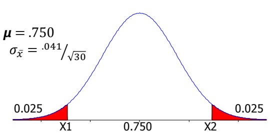

What are the values that define the rejection region if $\boldsymbol{\alpha}$= 5%?

A) [0.738, 0.762] B) [0.735, 0.765] C) [0.747, 0.753]

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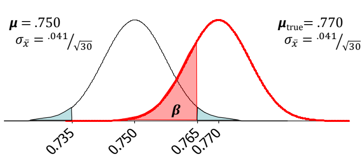
A manufacturer of plastic laminate sheets produces 0.750" sheets that have an historical standard deviation of .041".

If the true thickness of the sheets is 0.770, what is the power of a sample of 30 sheets to detect the difference? Assume $\boldsymbol{\alpha}$= 5%.



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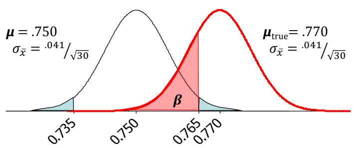
A manufacturer of plastic laminate sheets produces 0.750” sheets that have an historical standard deviation of .041”. **If the true thickness of the sheets is 0.770,** what is the power of a sample of 30 sheets to detect the difference? Assume α= 5%.



What is the Z value used to calculate the probability β?,
A) +2.67 B) -1.220 C) -.853 D) -0.668

The probability, β, is approximately
A) 0.2514 B) 0.0475 C) 0.3120 D) 0.2944

The power of the test to detect the shift is
A) 0.7486 B) 0.4817 C) 0.9500 D) 0.05

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More on Power

β and power probabilities are dependent on:

- Distance of the shifted distribution from the null distribution.
  Would have to calculate for all possible shifts
- The sample size
- Allowable type I error, α

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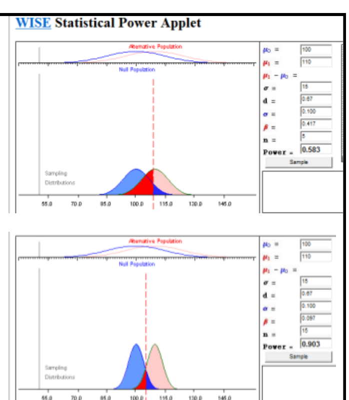
n Impact on Power
- Constant α, shift -

n= 5
Power = 0.583

n=15
Power = 0.903



Charts created with -
http://wise.cgu.edu/power/powerapplet1.html

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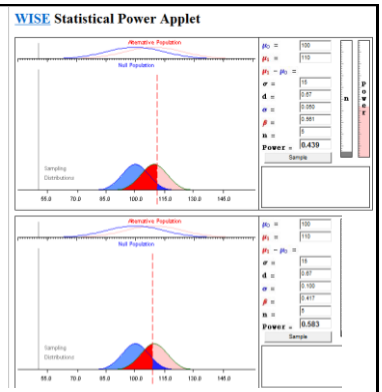
α Impact on Power
- Constant n, shift -

$\alpha = .05$
Power = 0.439

$\alpha = .10$
Power = 0.583



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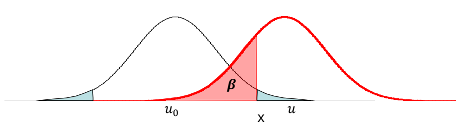
Determining Sample Size with specified α and β

α and β may be specified for a given shift $(u_0\ to\ u)$ in the process.



For the hypothesis test the upper limit critical point is

$$x \;=\; u_0 + Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

For the shifted distribution, *the x value associated with β in the tail is*

$$x \;= u - Z_\beta\left({}^{\sigma}/_{\sqrt{n}}\right)$$

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A manufacturer of plastic laminate sheets produces 0.750" sheets that have an historical standard deviation of .041".

Suppose we want the power to detect a shift to .770 to be 90%. What sample size is needed to get this power if alpha is maintained at 5%?

T / F The formula for the x value for the upper critical point for the hypothesis test is

$$x_2 \;=\; u_0 + Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) = 0.750 + 1.96\left(\frac{.041}{\sqrt{n}}\right)$$

T / F In determining β, where a shift is greater than u_0, one must use the area under the shifted curve to determine the probability of being less than the upper critical point (x_2).

33

A manufacturer of plastic laminate sheets produces 0.750" sheets that have an historical standard deviation of .041".

Suppose we want the power to detect a shift to .770 to be 90%. What sample size is needed to get this power if alpha is maintained at 5%?

If the power of the test is desired at 90%, the associated $\boldsymbol{\beta}$ is
A) 0.10 B) 0.05 C) 0.01 D) 0.15

The Z value associated with a probability of 10% in a left hand tail is
A) -1.645 B) -1.28 C) -2.33 D) -1.56

T / F The formula for the Z value is

$$Z = \frac{x-u}{\sigma/\sqrt{n}}$$ and can be rearranged as $x = u + Z\left(\sigma/\sqrt{n}\right)$

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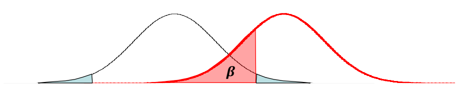

For the hypothesis test the upper limit critical point is

$$x = u_0 + Z\alpha_{/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

We know
u_0 = .750 $Z\alpha_{/2}$= 1.96 $\boldsymbol{\sigma} = .041$

For the shifted distribution*, the x value associated with* $\boldsymbol{\beta}$ *in the tail is*

$$x = u - Z_\beta\left(\sigma/\sqrt{n}\right)$$

We know
$\boldsymbol{\mu}$= .770 Z_β= 1.28 $\boldsymbol{\sigma} = .041$

Set the x equations equal to each other and solve for n.

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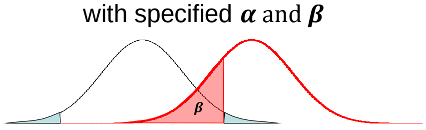
Determining Sample Size with specified $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$



For the hypothesis test the upper limit critical point is

$$x = u_0 + Z\alpha_{/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

For the shifted distribution, *the x value associated with* $\boldsymbol{\beta}$ *in the tail is*

$$x = u - Z_\beta\left(\sigma/\sqrt{n}\right)$$

Setting the two equations equal to each other, then solving for n:
(must round up!)

$$n = \frac{\left(Z\alpha_{/2} + Z_\beta\right)^2 \sigma^2}{(\mu - \mu_0)^2}$$

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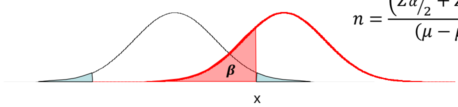
$$n = \frac{\left(Z_{\alpha/2} + Z_\beta\right)^2 \sigma^2}{(\mu - \mu_0)^2}$$



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$$x = u_0 + Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

We know
u_0 = .750 $Z_{\alpha/2}$= 1.96 $\boldsymbol{\sigma} = .041$

For the shifted distribution, *the x value associated with* $\boldsymbol{\beta}$ *in the tail is*

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We know
$\boldsymbol{\mu}$= .770 Z_β= 1.28 $\boldsymbol{\sigma} = .041$

A manufacturer of plastic laminate sheets produces 0.750" sheets that have an historical standard deviation of .041".

Suppose we want the power to detect a shift to .770 to be 90%. What sample size is needed to get this power if alpha is maintained at 5%?

A) 36 B) 35 C) 45 D) 44

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Related Assignments

Please see Blackboard for related assignments

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