EIN 5226

Lognormal Distribution

Chapter 7 Sections 7.14

Note: Need Calculator & Z table Handout for lecture

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Uses of lognormal

Often used where the distribution is skewed

- Reliability Analysis
  - Time or cycles to failure in fatigue
  - Material strengths and loading variables in design
- Financial analysis
  - Stock price per share, rate of return on stocks, Earnings per share, option pricing,
- Other problems related to time where the distribution best fits the data under investigation.

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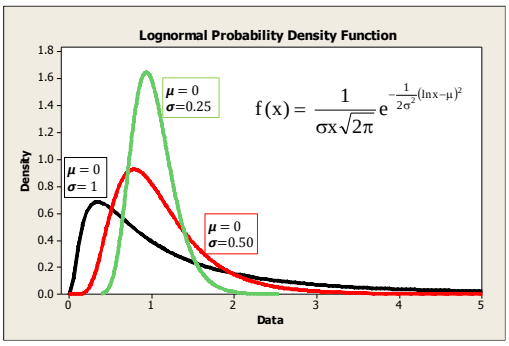

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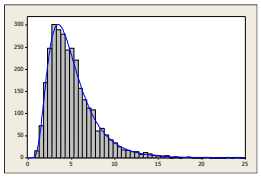
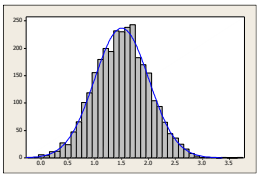
Lognormal Distribution





3000 observations: **y**

Lognormal Distribution
Parameters $\mu = 1.5$ $\sigma = 0.5$

Same 3000 observations: x = **ln y**

Normal Distribution
$\mu = 1.5$ $\sigma = 0.5$

If Y has the lognormal distribution with parameters μ and σ^2, then the random variable $X = \ln Y$ has the $N(\mu, \sigma^2)$ distribution.

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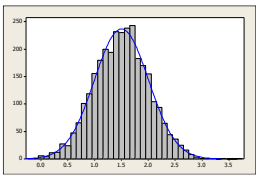
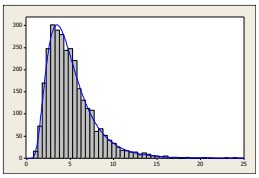
Lognormal Distribution





Same 3000 observations: x

Normal Distribution
$\mu = 1.5$ $\sigma = 0.5$

3000 observations: $\mathbf{y = e^x}$

Lognormal Distribution
Parameters $\mu = 1.5$ $\sigma = 0.5$

If $X \sim N(\mu, \sigma^2)$, then the random variable $Y = e^X$ has the lognormal distribution with parameters μ and σ^2

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Lognormal Problem

A process is know to follow a lognormal distribution with parameters $\boldsymbol{\mu} = 1.5$ and $\boldsymbol{\sigma} = 0.5$. What is the probability that an observed value will be less than 5?

P(y<5) = P(X< ln 5)
where X is normally distributed with $\boldsymbol{\mu} = 1.5$ and $\boldsymbol{\sigma} = 0.5$.

ln 5 = 1.609

To get P(x<1.609), calculate z and use the Z table.

$$z = \frac{x - \mu}{\sigma} = \frac{1.609 - 1.5}{0.5} = +.22$$

P(x<1.609) = P(Z<+.22) = .5871 *(from standard normal table)*

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Lognormal Distribution

- Mean or Expected Value of Y

$$E(Y) = e^{\mu + \frac{1}{2}\sigma^2}$$

- Variance Y

$$V(Y) = e^{2\mu+2\sigma^2} - e^{2\mu+\sigma^2}$$

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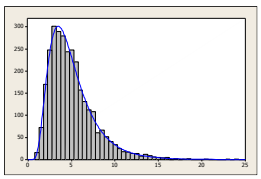
Lognormal Distribution



3000 observations - y

Lognormal Distribution
Parameters $\mu = 1.5$ $\sigma = 0.5$

$$E(Y) = e^{\mu + \frac{1}{2}\sigma^2}$$

$$= e^{1.5 + \frac{1}{2}(.5)^2} = 5.08$$

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Lognormal Problem A1

$$E(Y) = e^{\mu + \frac{1}{2}\sigma^2}$$

The time, in minutes, to repair an automatic component insertion machine used in circuit card assembly is know to follow a log normal distribution with parameters $\mu = 5$ and $\sigma = 1.5$.

What is the expected time for a repair?

A. 1408 B 314 C. 108 D. 457

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Lognormal Problem A2

The time, in minutes, to repair an automatic component insertion machine used in circuit card assembly is know to follow a log normal distribution with parameters $\boldsymbol{\mu} = 5$ and $\boldsymbol{\sigma} = 1.5$.

What percent of the time can a repair be performed in under 45 minutes?

A. 79% B 21% C. 40% D. 33%

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Lognormal Problem B1

$$E(Y) = e^{\mu + \frac{1}{2}\sigma^2}$$

A popular local restaurant does not take reservations and customers usually have to wait for a table.

Suppose that the wait time, Y, (in minutes) for a table follows a lognormal distribution with parameters $\mu = 3.4$ and $\sigma = 0.30$.

What is the expected wait time (in minutes) for a table?

A. 31.3 B. 32.8 C. 34.8 D. 36.9

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Lognormal Problem B2

A popular local restaurant does not take reservations and customers usually have to wait for a table.

Suppose that the wait time, Y, (in minutes) for a table follows a lognormal distribution with parameters $\mu = 3.4$ and $\sigma = 0.30$.

What percent of customers are seated within 20 minutes?

A. 7% B. 9% C. 12% D. 25%

What percent of customers must wait longer than 44 minutes?

A. 32% B. 10% C. 18% D. 8%

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Related Assignments

See Blackboard for related assignments