Chapter 5

Risk and Return

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Learning Objectivs

After studying Chapter 5, you should be able to:

1. Understand the relationship (or "trade-off") between risk and return.
2. Define risk and return and show how to measure them by calculating expected return, standard deviation, and coefficient of variation.
3. Discuss the different types of investor attitudes toward risk.
4. Explain risk and return in a portfolio context, and distinguish between individual security and portfolio risk.
5. Distinguish between avoidable (unsystematic) risk and unavoidable (systematic) risk and explain how proper diversification can eliminate one of these risks.
6. Define and explain the capital-asset pricing model (CAPM), beta, and the characteristic line.
7. Calculate a required rate of return using the capital-asset pricing model (CAPM).
8. Demonstrate how the Security Market Line (SML) can be used to describe this relationship between expected rate of return and systematic risk.
9. Explain what is meant by an "efficient financial market" and describe the three levels (or forms) to market efficiency.

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Chapter Outline

- Defining Risk and Return
- Using Probability Distributions to Measure Risk
- Attitudes Toward Risk
- Risk and Return in a Portfolio Context
- Diversification
- The Capital Asset Pricing Model (CAPM)
- Efficient Financial Markets

Defining Return

Income received on an investment plus any change in market price, usually expressed as a percent of the beginning market price of the investment.

$$R = \frac{D_t + (P_t - P_{t-1})}{P_{t-1}} \qquad [5.1]$$

Return Example

The stock price for Stock A was $10 per share 1 year ago. The stock is currently trading at $9.50 per share and shareholders just received a $1 dividend. What return was earned over the past year?

$$R = \frac{\$1.00 + (\$9.50 - \$10.00)}{\$10.00} = 5\%$$

Defining Risk

The variability of returns from those that are expected.

- What rate of return do you expect on your investment (savings) this year?
- What rate will you actually earn?
- Does it matter if it is a bank CD or a share of stock?

Determining Expected Return (Discrete Dist.)

$$\overline{R} = \sum_{i=1}^{n} (R_i)(P_i) \qquad [5.2]$$

$\overline{R}$ is the expected return for the asset,

R_i is the return for the i^{th} possibility,

P_i is the probability of that return occurring,

n is the total number of possibilities.

How to Determine the Expected Return and Standard Deviation

Stock BW		
R_i	P_i	$(R_i)(P_i)$
-.15	.10	-.015
-.03	.20	-.006
.09	.40	.036
.21	.20	.042
.33	.10	.033
Sum	*1.00*	*.090*

The expected return, $\overline{R}$, for Stock BW is .09 or 9%

Determining Standard Deviation (Risk Measure)

$$\sigma = \sqrt{\sum_{i=1}^{n} (R_i - \bar{R})^2 (P_i)} \qquad [5.3]$$

Standard Deviation, σ, is a statistical measure of the variability of a distribution around its mean.

It is the square root of variance.

Note, this is for a discrete distribution.

How to Determine the Expected Return and Standard Deviation

Stock BW			
R_i	P_i	$(R_i)(P_i)$	$(R_i - \bar{R})^2(P_i)$
-.15	.10	-.015	.00576
-.03	.20	-.006	.00288
.09	.40	.036	.00000
.21	.20	.042	.00288
.33	.10	.033	.00576
Sum	*1.00*	*.090*	*.01728*

Determining Standard Deviation (Risk Measure)

$$\sigma = \sqrt{\sum_{i=1}^{n}(R_i - \bar{R})^2(P_i)}$$

$$= \sqrt{.01728}$$

$$= .1315$$

Coefficient of Variation

Coefficient of variation is the ratio of the *standard deviation* of a distribution to the *mean* of that distribution.

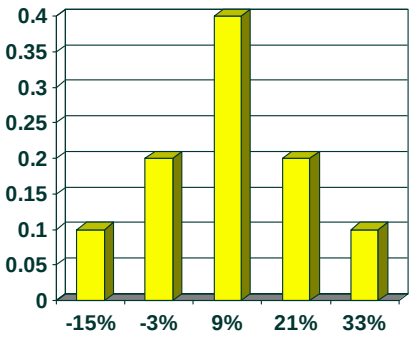
It is a measure of ***RELATIVE*** risk.

$$CV = \frac{\sigma}{\bar{R}} \qquad [5.5]$$

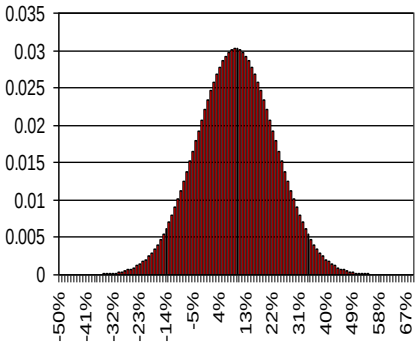
CV of BW = .1315 / .09 = 1.46

Discrete vs. Continuous Distributions

Discrete



Continuous



Risk Attitudes

Certainty Equivalent (*CE*) is the amount of cash someone would require with certainty at a point in time to make the individual indifferent between that certain amount and an amount expected to be received with risk at the same point in time.

Risk Attitudes

Certainty equivalent > Expected value
Risk Preference

Certainty equivalent = Expected value
Risk Indifference

Certainty equivalent < Expected value
Risk Aversion

Most individuals are Risk Averse.

Risk Attitude Example

You have the choice between (1) a guaranteed dollar reward or (2) a coin-flip gamble of $100,000 (50% chance) or $0 (50% chance). The expected value of the gamble is $50,000.

- Mary requires a guaranteed $25,000, or more, to call off the gamble.
- Raleigh is just as happy to take $50,000 or take the risky gamble.
- Shannon requires at least $52,000 to call off the gamble.

Risk Attitude Example

What are the Risk Attitude tendencies of each?

Mary shows "risk aversion" because her "certainty equivalent" < the expected value of the gamble.

Raleigh exhibits "risk indifference" because her "certainty equivalent" equals the expected value of the gamble.

Shannon reveals a "risk preference" because her "certainty equivalent" > the expected value of the gamble.

Determining Portfolio Expected Return

$$\overline{R}_P = \sum_{j=1}^{m} (W_j)(\overline{R}_j) \qquad [5.6]$$

$\overline{R}_P$ is the expected return for the portfolio,

W_j is the weight (investment proportion) for the j^{th} asset in the portfolio,

$\overline{R}_j$ is the expected return of the j^{th} asset,

m is the total number of assets in the portfolio.

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Determining Portfolio Standard Deviation

$$\sigma_P = \sqrt{\sum_{j=1}^{m}\sum_{k=1}^{m} W_j W_k \sigma_{jk}}$$

W_j is the weight (investment proportion) for the j^{th} asset in the portfolio,

W_k is the weight (investment proportion) for the k^{th} asset in the portfolio,

σ_{jk} is the covariance between returns for the j^{th} and k^{th} assets in the portfolio.

What is Covariance?

$$\sigma_{jk} = \sigma_j \sigma_k r_{jk}$$

σ_j is the standard deviation of the j^{th} asset in the portfolio,

σ_k is the standard deviation of the k^{th} asset in the portfolio,

r_{jk} is the correlation coefficient between the j^{th} and k^{th} assets in the portfolio.

Correlation Coefficient

- *A standardized statistical measure of the linear relationship between two variables.*
- Its range is from *-1.0* (perfect negative correlation), through *0* (no correlation), to *+1.0* (perfect positive correlation).

Variance - Covariance Matrix

A three asset portfolio:

	Col 1	Col 2	Col 3
Row 1	$W_1W_1\sigma_{1,1}$	$W_1W_2\sigma_{1,2}$	$W_1W_3\sigma_{1,3}$
Row 2	$W_2W_1\sigma_{2,1}$	$W_2W_2\sigma_{2,2}$	$W_2W_3\sigma_{2,3}$
Row 3	$W_3W_1\sigma_{3,1}$	$W_3W_2\sigma_{3,2}$	$W_3W_3\sigma_{3,3}$

$\sigma_{j,k}$ = is the covariance between returns for the j^{th} and k^{th} assets in the portfolio.

Portfolio Risk and Expected Return Example

You are creating a portfolio of Stock D and Stock BW (from earlier). You are investing $2,000 in Stock BW and $3,000 in Stock D. Remember that the expected return and standard deviation of Stock BW is 9% and 13.15% respectively. The expected return and standard deviation of Stock D is 8% and 10.65% respectively. The correlation coefficient between BW and D is 0.75.

What is the expected return and standard deviation of the portfolio?

Determining Portfolio Expected Return

W_{BW} = $2,000 / $5,000 = .4

W_D = $3,000 / $5,000 = .6

$$\bar{R}_P = (W_{BW})(\bar{R}_{BW}) + (W_D)(\bar{R}_D)$$
$$= (.4)(9\%) + (.6)(8\%)$$
$$= 8.4\%$$

Determining Portfolio Standard Deviation

Two-asset portfolio:

	Col 1	Col 2
Row 1	$W_{BW} W_{BW} \sigma_{BW,BW}$	$W_{BW} W_D \sigma_{BW,D}$
Row 2	$W_D W_{BW} \sigma_{D,BW}$	$W_D W_D \sigma_{D,D}$

This represents the variance - covariance matrix for the two-asset portfolio.

Determining Portfolio Standard Deviation

Two-asset portfolio:

	Col 1	Col 2
Row 1	(.4)(.4)(.0173)	(.4)(.6)(.0105)
Row 2	(.6)(.4)(.0105)	(.6)(.6)(.0113)

This represents substitution into the variance - covariance matrix.

Determining Portfolio Standard Deviation

Two-asset portfolio:

	Col 1	Col 2
Row 1	(.0028)	(.0025)
Row 2	(.0025)	(.0041)

This represents the actual element values in the variance - covariance matrix.

Determining Portfolio Standard Deviation

$$\sigma_P = \sqrt{.0028 + .0025 + .0025 + .0041}$$
$$= \sqrt{.0119}$$
$$= .1091$$

A weighted average of the individual standard deviations is *INCORRECT*.

Determining Portfolio Standard Deviation

The WRONG way to calculate is a weighted average like:

σ_P = .4 (13.15%) + .6(10.65%)

σ_P = 5.26 + 6.39 = 11.65%

10.91% ≠ 11.65%

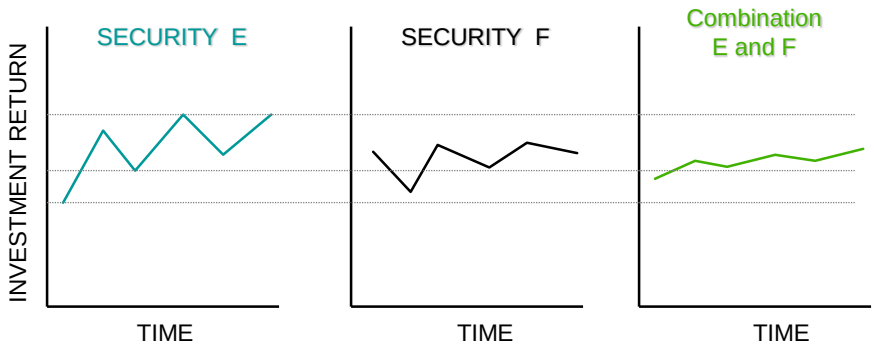
This is *INCORRECT*.

Summary of the Portfolio Return and Risk Calculation

	Stock C	Stock D	Portfolio
Return	9.00%	8.00%	8.64%
Stand. Dev.	13.15%	10.65%	10.91%
CV	1.46	1.33	1.26

The portfolio has the *LOWEST* coefficient of variation due to diversification.

Diversification and the Correlation Coefficient



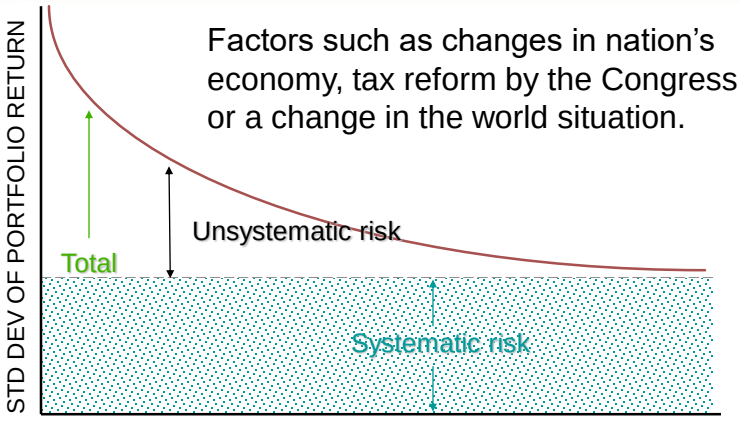
Combining securities that are not perfectly, positively correlated reduces risk.

Total Risk = Systematic Risk + Unsystematic Risk

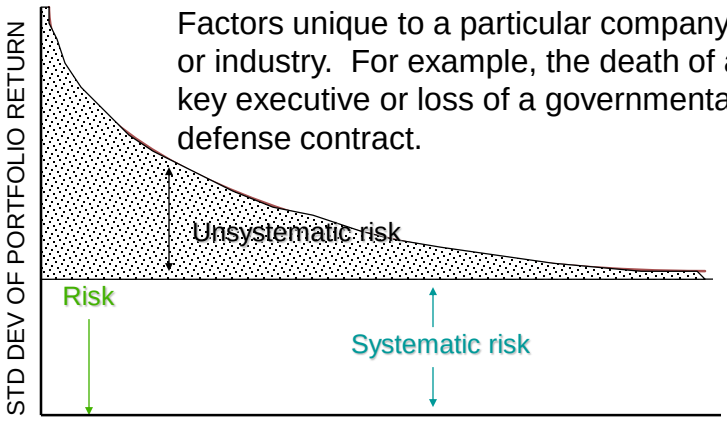
Total Risk = *Systematic Risk* + *Unsystematic Risk* [5.7]

- *Systematic Risk* is the variability of return on stocks or portfolios associated with changes in return on the market as a whole.
- *Unsystematic Risk* is the variability of return on stocks or portfolios not explained by general market movements. It is avoidable through diversification.

Total Risk = Systematic Risk + Unsystematic Risk



Total Risk = Systematic Risk + Unsystematic Risk



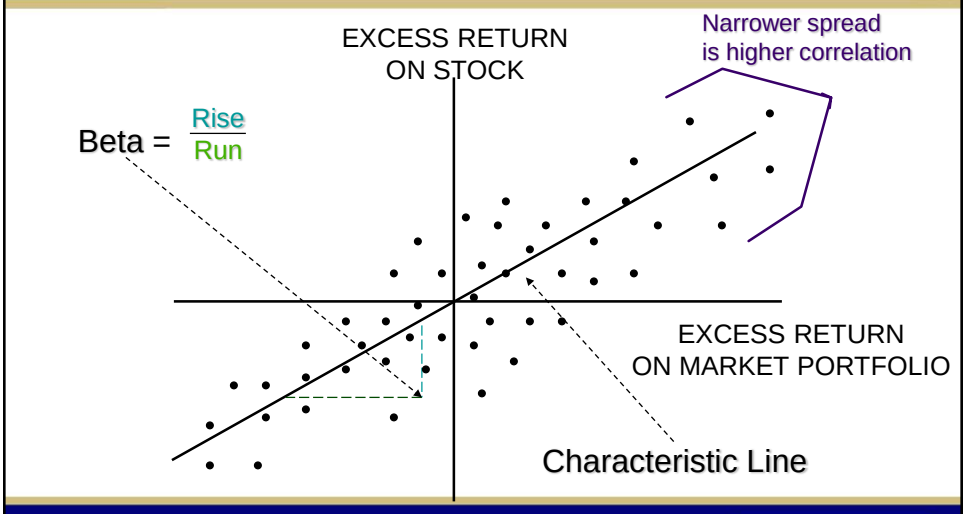
Capital Asset Pricing Model (CAPM)

CAPM is a model describes the *relationship* between risk and expected (required) return; in this model, a security's expected (required) return is the risk-free rate plus a premium based on the *systematic risk* of the security.

CAPM Assumptions

1. Most investors want to avoid risk (risk averse)
2. Capital markets are efficient.
3. Homogeneous investor expectations over a given period (all investors have equal access to information)
4. *Risk-free* asset return is certain (use short-to intermediate-term Treasuries as a proxy).
5. Market portfolio contains *only* *systematic risk* (use S&P 500 Index or similar as a proxy).
6. There are no transactional costs or taxation and assets and securities are divisible into small little packets.
7. Investors are not limited in their borrowing and lending under the risk free rate of interest.

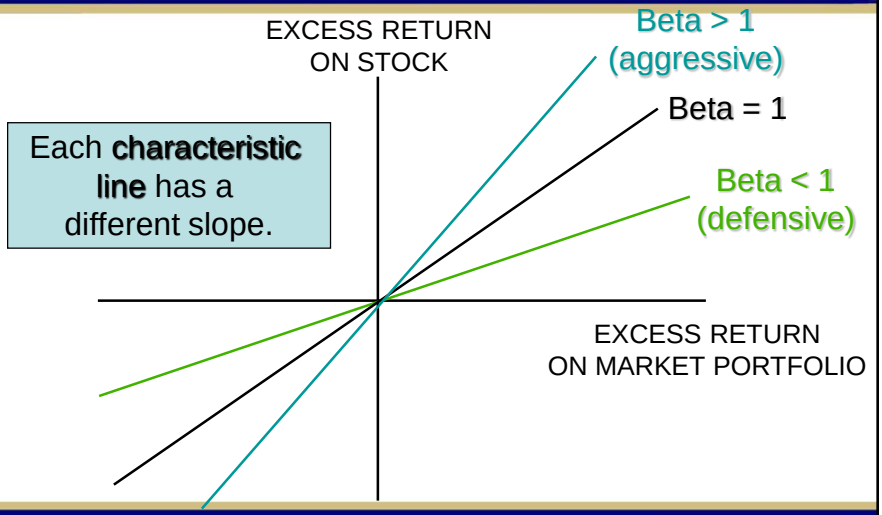
Characteristic Line



What is Beta?

- An index of *systematic risk*.
- It measures the *sensitivity* of a stock's returns to changes in returns on the market portfolio.
- The *beta* for a portfolio is simply a weighted average of the individual stock betas in the portfolio.

Characteristic Lines and Different Betas



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Security Market Line

$$\bar{R}_j = R_f + \beta_j(\bar{R}_M - R_f) \qquad [5.8]$$

$\bar{R}_j$ is the required rate of return for stock j,

R_f is the risk-free rate of return,

β_j is the beta of stock j (measures systematic risk of stock j),

$\bar{R}_M$ is the expected return for the market portfolio.

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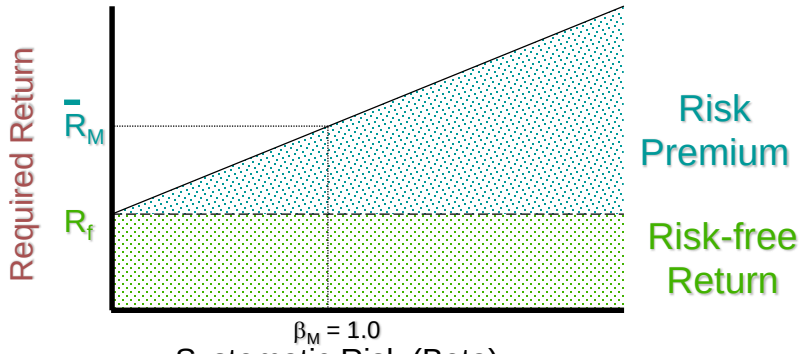
Security Market Line

$$\bar{R}_j = R_f + \beta_j(\bar{R}_M - R_f)$$



Security Market Line

- Obtaining Betas
  - Can use historical data if past best represents the expectations of the future
  - Can also utilize services like Value Line, Ibbotson Associates, etc.
- Adjusted Beta
  - Betas have a tendency to revert to the mean of 1.0
  - Can utilize combination of recent beta and mean

    2.22 (.7) + 1.00 (.3) = 1.554 + 0.300 = 1.854 estimate

Determination of the Required Rate of Return

Lisa Miller at *Basket Wonders* is attempting to determine the rate of return required by their stock investors. Lisa is using a 6% R_f and a long-term market expected rate of return of 10%. A stock analyst following the firm has calculated that the firm beta is 1.2. What is the *required rate of return* on the stock of *Basket Wonders*?

BWs Required Rate of Return

$$\bar{R}_{BW} = R_f + \beta_j(\bar{R}_M - R_f)$$

$$\bar{R}_{BW} = 6\% + 1.2(10\% - 6\%)$$

$$\bar{R}_{BW} = 10.8\%$$

The required rate of return exceeds the market rate of return as BW's beta exceeds the market beta (1.0).

Determination of the Intrinsic Value of BW

Lisa Miller at BW is also attempting to determine the intrinsic value of the stock. She is using the constant growth model. Lisa estimates that the dividend next period will be $0.50 and that BW will grow at a constant rate of 5.8%. The stock is currently selling for $15.

What is the intrinsic value of the stock? Is the stock over or underpriced?

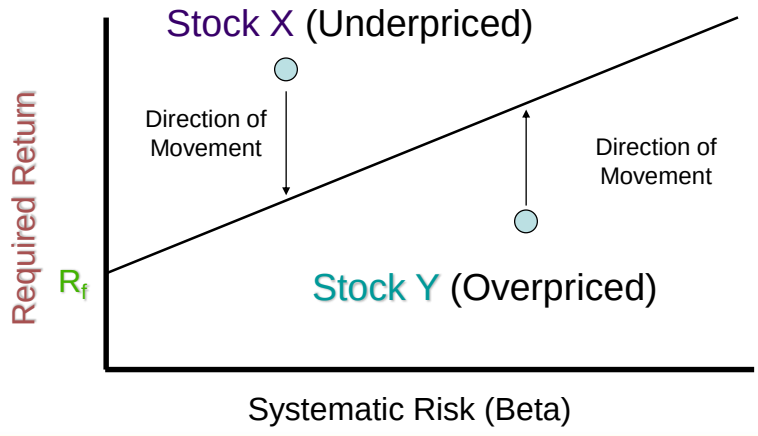
Determination of the Intrinsic Value of BW

$$\text{Intrinsic Value} = \frac{\$0.50}{10.8\% - 5.8\%}$$

$$= \$10$$

The stock is *OVERVALUED* as the market price ($15) exceeds the intrinsic value ($10).

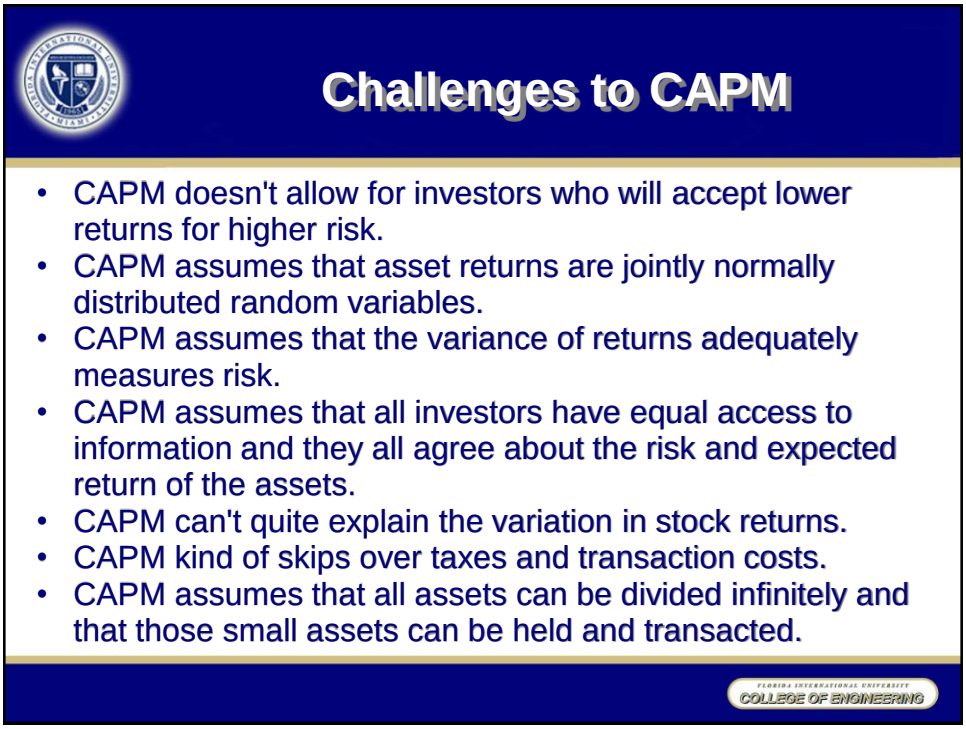
Security Market Line



Determination of the Required Rate of Return

- Small-firm Effect
- Price / Earnings Effect
- January Effect

- These anomalies have presented serious challenges to the CAPM theory.


Challenges to CAPM
• CAPM doesn't allow for investors who will accept lower returns for higher risk.
• CAPM assumes that asset returns are jointly normally distributed random variables.
• CAPM assumes that the variance of returns adequately measures risk.
• CAPM assumes that all investors have equal access to information and they all agree about the risk and expected return of the assets.
• CAPM can't quite explain the variation in stock returns.
• CAPM kind of skips over taxes and transaction costs.
• CAPM assumes that all assets can be divided infinitely and that those small assets can be held and transacted.
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