Minitab Exercise 5 Poisson. Rev 12/14

In this this exercise you will

- Practice solving classic Poisson problems by hand and with Minitab
- Gain a better understanding of application of approximations with Poisson, binomial and normal distributions.

Part A. Poisson by hand

A "classic" Poisson problem is one where you can count the number of items that occur, but cannot count the number of items that do not occur.

The probability mass function for Poisson is $P(X) = \frac{e^{-\lambda}\lambda^x}{x!}$ where λ is the rate of occurrence and x is the number of occurrences

The units on the rate of occurrence λ must match the units on the occurrences x. If they are not the same, you must convert the units on the rate to match the x.

Work the following problems by hand. (Carry probability calculations to 4 decimal places.)

1) A chemical processor claims that a certain suspension contains at least 2 particles per ml. $\lambda = 2$ particles/ml

a. Assuming this is true and a 1 ml sample is taken, what is the probability of finding no particles in the sample?

$$P(0) = \frac{e^{-2} 2^0}{0!} = \frac{e^{-2}(1)}{(1)} = e^{-2} = 0.1353$$

b. What is the probability of finding 2 or fewer particles in the 1 ml sample? $P(x \leq 2)$

$$P(x \leq 2) = P(0) + P(1) + P(2) = .1353 + .2707 + .2707 = 0.6767$$

$$P(1) = \frac{e^{-2} 2^1}{1!} = \frac{0.1353(2)}{1} = .2707$$

$$P(2) = \frac{e^{-2} 2^2}{2!} = \frac{0.1353(4)}{2} = .2707$$

c. What is the probability of finding between 1 and 3 particles (inclusive) in the 1 ml sample?

$$P(1 \leq x \leq 3) = P(1) + P(2) + P(3) = .2707 + .2707 + .1804 = 0.7218$$

$$P(x = 3) = \frac{e^{-2} 2^3}{3!} = \frac{(.1353)(8)}{6} = .1804$$

Part B. Poisson with Minitab.

Check your work. For problem 1)a:

Calc>Probability Distributions>Poisson
Check **Probability**
Mean: **2**
Input constant: **0**
OK

For 1)b, you want the **Cumulative probability**, and the input constant is **2**

For part 1)c you could add probabilities or subtract cumulative probabilities, but let's try another way:

> **Graph>Probability Distribution Plot**
> Select **View Probability**
> Distribution: **Poisson** (Use the drop down to select)
> Mean: **2** (for distribution 1)
> Click on Shaded Area
> Select X value
> Select Middle
> X value 1: 1 X value 2: 3
> OK

Work the problems with Minitab check your manual calculations. Correct your calculations as required

Part C. More Poisson problems.

2) The number of hits on a website follows a Poisson distribution with a mean rate of 6 hits per hour $\lambda = 6$ hits/hour

a. What is the probability that exactly 4 hits are received in one hour?

$$P(4) = \frac{e^{-6}\, 6^{4}}{4!} = 0.13385$$

b. What is the probability that exactly 1 hit is received in 15 minutes? (Not the same as part a – what is the mean hit rate per 15 minutes?)

$$\lambda = 6 \text{ hits}/60\text{ min} = 1.5 \text{ hits}/15\text{ min}$$

$$P(1) = \frac{e^{-1.5}\, 1.5^{1}}{1!} = 0.3347$$

For question 2), let's see why they are not the same problem. Let's create a histogram to show the probability distribution of each

> **Graph>Probability Distribution Plot**
> Select **Two Distributions**
> Distributions: **Poisson** (Use the drop down to select for both distributions)
> Mean: **6** (for distribution 1) **1.5** (for distribution 2)
> Click on **Multiple graphs**
> Select **In separate panels of the same graph**
> Leave the same scales for the graphs
> OK twice.

You can check your calculations by clicking on the bar for the number of occurrences that you were looking for.

Looking at the graph, you can see that they are not the same distribution at all. Look at the spread of the data

For λ=6 hit /hour, the number of occurrences range from __0__ to __15__ (scale on the bottom)

For λ=1 hit /15 minutes, the number of occurrences range from __0__ to __6__ (scale on the bottom)

Why does this make sense?

The range of the potential # of particles is smaller for the smaller time span. So the total probability is spread out over fewer x's, which would mean many of those x's would have to have higher probabilities.

So that is why you always make your rate in terms of the units that you are interested in.

c. Assuming the mean hit rate is constant at 6 per hour across the shift, what is the mean hit rate for an 8 hour shift? What is the probability of getting exactly 50 hits for the ~~day~~? shift ?

$$\lambda = 6 \text{ hits/hour} = 48 \text{ hits/8 hour shift}$$

$$P(50) = \frac{e^{-48}\, 48^{50}}{50!} = .0541$$

Let's create one more Poisson graph to check your calculation and compare to the other graphs.

Graph>Probability Distribution Plot
Select **View Single**
Distributions: **Poisson**
Mean: **48**
OK

If you compare the three distribution plots for λ=1.5, λ=6 and λ=48 you observe that the larger λ is, the more it looks like a __normal__ distribution.

Before statistical software packages, it was accepted practice to approximate the Poisson distribution with the normal distribution when λ was at least 5.

Part D. Poisson Approximation of Binomial

Poisson is also known as the distribution of rare events, because the potential for occurrences is theoretically infinite, but actual occurrences are relatively small.

The second type of problem that is regularly solved with Poisson is for a special case of the binomial distribution – where the sample size, n, is large and the probability of "success", p, is small. This can then be thought of as comparable to the Poisson, and since the mathematics for Poisson is easier than for binomial, these problems have traditionally been worked with the Poisson distribution.

One rule of thumb is: If N>20 and p<.05, a Poisson approximation of the binomial is acceptable. (Different books have different rules of thumb.)

Let's work a problem both ways – first by hand.

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Part B

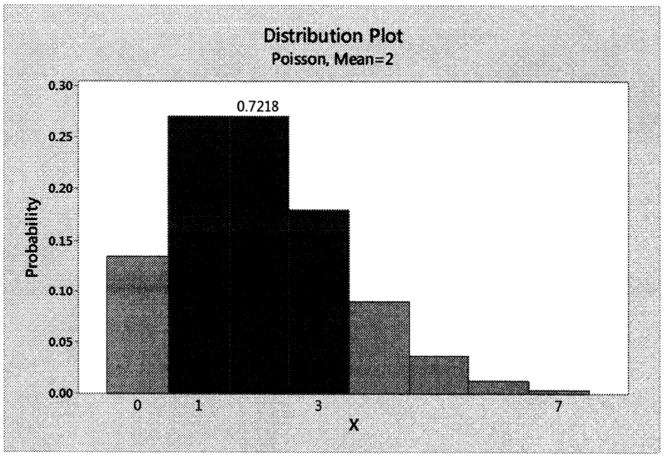
Probability Density Function

```
Poisson with mean = 2

x  P( X = x )
0    0.135335
```

Cumulative Distribution Function

```
Poisson with mean = 2

x  P( X ≤ x )
2    0.676676
```

Distribution Plot



Part C

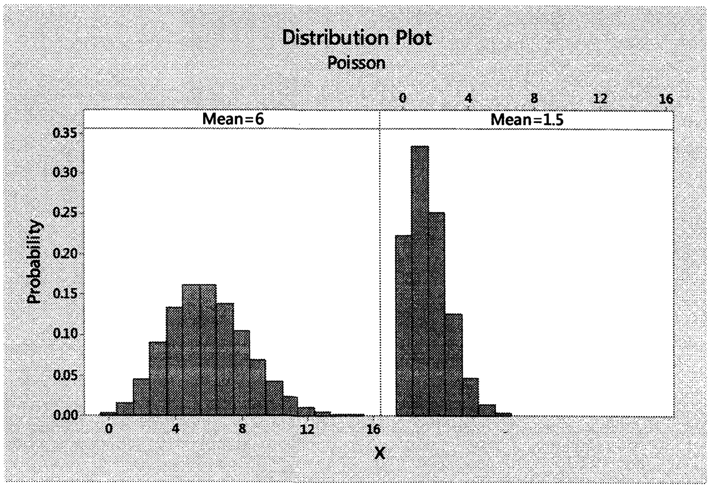
Probability Density Function

```
Poisson with mean = 6

x  P( X = x )
4    0.133853
```

Probability Density Function

```
Poisson with mean = 1.5

x  P( X = x )
1    0.334695
```

Distribution Plot



The Problem: A process is known to produce 4% defective parts. A sample of size 50 is selected. What is the probability of getting exactly 1 defective part in the sample?

$P(1)$

With binomial: $p(x) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{50}{1} (.04)^1 (1-.04)^{50-1} = 0.2706$

For this process, if we took repeated samples of size 50, on average, how many defective parts would we expect?

$\mu = np = (50)(.04) = 2$

This is the same as saying we would expect __2__ defects per 50 parts. This is the rate, λ, we will use when work the problem with Poisson. (λ=μ=np)

$P(1)$

With Poisson: $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-2} 2^1}{1!} = .1353(2) = 0.2707$

Check your work in Minitab. (The answers with binomial and Poisson do not differ until the 4th decimal place.)

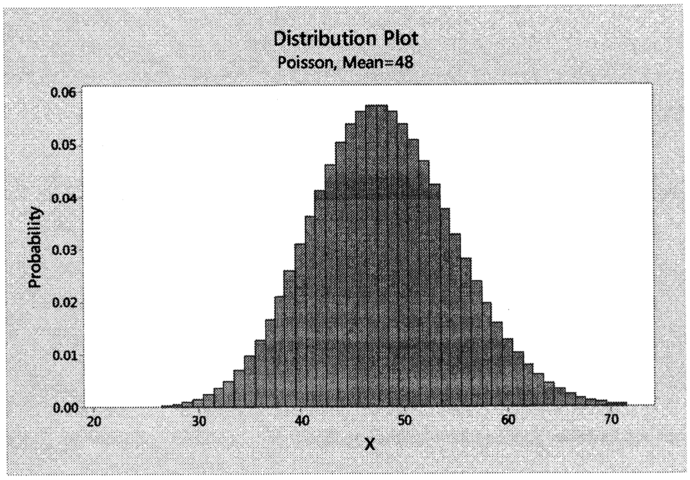
You can compare the two distributions using the **Graph>Probability Distribution Plot** for **Two Distributions** like you did in Part c.)

Probability Density Function

Poisson with mean = 48

x	P(X = x)
50	0.0540570

Distribution Plot



Part D

Probability Density Function

Binomial with n = 50 and p = 0.04

x	P(X = x)
1	0.270595

Probability Density Function

Poisson with mean = 2

x	P(X = x)
1	0.270671

Distribution Plot