7.21 Exercise 23

Fifty items are submitted for acceptance. If it is known that there are 4 defective items in the lot, determine the probability of finding exactly 1 defective item in a sample of 5. Determine the probability of finding less than 2 defective items in a sample of 5.

For this example,

$x = 1$
$N = 50$
$n = 5$
$m = 4$

The probability of finding exactly 1 defective item in a sample of 5 is

$$p(1,50,5,4) = \frac{\binom{4}{1}\binom{50-4}{5-1}}{\binom{50}{5}} = \frac{(4)(163,185)}{2,118,760} = 0.30808$$

This answer can be found using the Excel function

=HYPGEOMDIST(1,5,4,50)

The probability of finding less than 2 defective items in a sample of 5 is equal to the probability of exactly zero plus the probability of exactly 1. The probability of exactly zero is

$$p(0,50,5,4) = \frac{\binom{4}{0}\binom{50-4}{5-0}}{\binom{50}{5}} = \frac{(1)(1,370,754)}{2,118,760} = 0.64696$$

Therefore, the probability of finding less than 2 defective items in a sample of 5 is 0.30808 + 0.64696 = 0.95504. Unfortunately Excel does not have a cumulative form of the hypergeometric distribution.

The binomial distribution can be used to approximate the hypergeometric distribution when the population is large with respect to the sample size. When N is much larger than n, the change in the probability of success on a single trial is too small to significantly affect the results of the calculations. Again, it is silly to use approximations when exact solutions can be found with electronic spreadsheets. These approximations were useful for the engineer toiling with a slide rule, but are of little use now.