EIN 5226

Confidence Intervals and Hypotheses testing for means – sigma unknown -

Chapter 16 Sections 16.1-2
Chapter 17 Sections 17.4-6

Need t table for this lecture

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Student's t probability distribution

- Normal tables not as accurate for smaller sample sizes (n<30) where the standard deviation is unknown.
- Student's t distribution considers sample size - degrees of freedom (n-1)
- Developed by W.S Gosset in 1908

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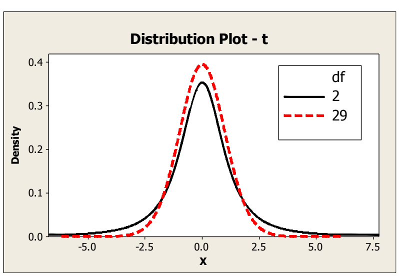
Student's t probability distribution



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Small Sample confidence interval for means (n≤30)

Must know:

Sample mean, $\bar{x}$

Standard deviation of the distribution of sample mean, $\sigma_{\bar{x}}$

Estimate $\sigma_{\bar{x}}$ with $^{s}/_{\sqrt{n}}$

Confidence Level (1-α) / level of risk (α)

Degrees of freedom, n-1

Confidence interval formulas:

Lower Limit

$$\bar{x} - t_{\alpha/2} \frac{s}{\sqrt{n}}$$

Upper limit

$$\bar{x} + t_{\alpha/2} \frac{s}{\sqrt{n}}$$

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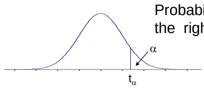
Portion of Table D (page 1043)
Probability Points of the *t* Distribution:
Single sided



Probabilities are in the right side tail

Degrees of freedom
v = n-1

Probabilities across the top α

v	0.40	0.30	0.20	0.10	0.05	0.025	0.01	0.005	0.001
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106	4.025
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055	3.930
13	0.259	0.538	0.870	1.354	1.771	2.160	2.650	3.012	3.852
14	0.258	0.537	0.868	1.771	1.761	2.145	2.624	2.997	3.787
15	0.258	0.536	0.866	1.761	1.753	2.131	2.602	2.947	3.733

T values are in the body of the table

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Small Sample CI $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$

A random sample of 14 bags of mulch are weighed, the average weight was found to be 49.4 lbs. with a standard deviation of 1.0 lbs. Obtain a 95% confidence interval for the true mean.

v	0.40	0.30	0.20	0.10	0.05	0.025
11	0.260	0.540	0.876	1.363	1.796	2.201
12	0.259	0.539	0.873	1.356	1.782	2.179
13	0.259	0.538	0.870	1.354	1.771	2.160
14	0.258	0.537	0.868	1.771	1.761	2.145
15	0.258	0.536	0.866	1.761	1.753	2.131

n=14
v=n-1=13
$\bar{x}$=49.4
s=1.0
$\boldsymbol{\alpha}$=1-.95=0.05
$\boldsymbol{\alpha}/2$=.025

$t_{.05/2,13} = t_{.025,13}$= 2.160

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Small Sample CI $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$

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$\bar{x}$=49.4
s=1.0
$\boldsymbol{\alpha}$=1-.95=0.05
$\boldsymbol{\alpha}/2$=.025
$t_{.05/2,13} = t_{.025,13}$= 2.160

Lower confidence limit =
$49.4 - 2.16\frac{1.0}{\sqrt{14}}$=49.82

Upper confidence limit =
$49.4 + 2.16\frac{1.0}{\sqrt{14}} = 49.98$

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Small Sample CI $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$

An oven is set to 350ºF. When the preheat buzzer sounds a second calibrated thermometer reading is taken.

The procedure is performed 6 times with an average temperature of 351.6 and standard deviation of 1.8. What is the 90% confidence interval for the true average temperature of the oven at preheat?

What t value is correct for this problem?

A) $t_{0.10,6}$= 1.440 B) $t_{0.05,5}$= 2.015
C) $t_{0.10,5}$= 1.476 D) $t_{0.05,6}$= 1.943

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Small Sample CI $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$

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The procedure is performed 6 times with an average temperature of 351.6 and standard deviation of 1.8.

What is the 90% confidence interval for the true average temperature of the oven at preheat?

A) [349.9, 353.3] B) [348.7, 354.5]
C) [350.1, 353.1] D) [350.6, 352.6]

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Tests on Single Population Means
Small Sample Sizes (n<30), σ unknown

	Two-tailed	One-tailed	
Null hypothesis H_0	$\mu=\mu_0$		
Alternate H_a	$\mu\neq\mu_0$	$\mu>\mu_0$	$\mu<\mu_0$
Test statistic	$t_{test} = \dfrac{\bar{x}-u_o}{s/\sqrt{n}}$		
p (reject if p<α)	sum the areas in the tails, cut off by t_{test} and $-t_{test}$	Area to right of t_{test}	Area to left of t_{test}

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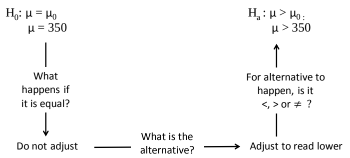
An oven is set to 350ºF. When the preheat buzzer sounds a second calibrated thermometer reading is taken.

The procedure is performed 6 times with an average temperature of 351.6 and standard deviation of 1.8.

With 90% confidence, should the oven's internal thermometer be adjusted to read lower?



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An oven is set to 350ºF. When the preheat buzzer sounds a second calibrated thermometer reading is taken.

The procedure is performed 6 times with an average temperature of 351.6 and standard deviation of 1.8.

With 90% confidence, should the oven's internal thermometer be adjusted to read lower?

$$t_{test} = \frac{\bar{x}-u_o}{s/\sqrt{n}}$$

H_0: $\mu = 350$ H_a : $\mu > 350$

n=6 v=6-1=5 α = 0.10 (one tailed test)

Test value: $t_{test} = \frac{351.6-350}{1.8/\sqrt{6}} = 2.18$

From t table, v=5 : P(t<2.015)=0.05 and P(t<2.571)=0.025
therefore .025 < P(t < 2.18) <0.05

Conclusion: Reject the null hypothesis.
Conclude device should be adjusted lower.

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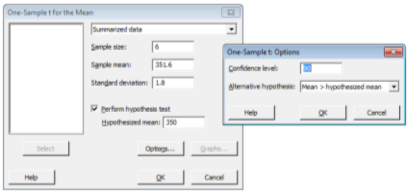

One-Sample T

Test of μ = 350 vs > 350

N	Mean	StDev	SE Mean	90% Lower Bound	T	P
6	351.60	1.80	0.735	350.515	2.18	0.041

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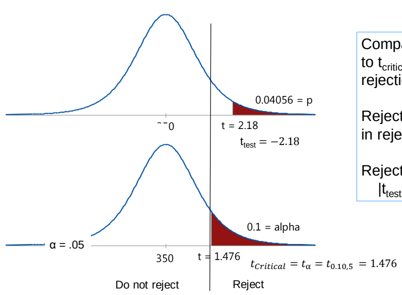

Compare t_{test} to $t_{critical}$ defined rejection area.

Reject null if t_{test} in rejection area

Reject null if $|t_{test}| > t_{critical}$

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$$t_{test} = \frac{\bar{x} - u_o}{s/\sqrt{n}}$$

An oven is set to 350ºF. When the preheat buzzer sounds a second calibrated thermometer reading is taken.

The procedure is performed 6 times with an average temperature of 351.6 and standard deviation of 1.8.

With 90% confidence, should the oven's internal thermometer be adjusted to read lower?

H_0: μ = 350 H_a : μ > 350

n=6 v=6-1=5 α = 0.10 (one tailed test)

Test value: $t_{test} = \frac{351.6-350}{1.8/\sqrt{6}} = 2.18$

T critical: $t_{critical} = t_{\alpha,\upsilon} = t_{.10,5} = 1.476$

Conclusion: $|t_{test}| > t_{critical}$ >>> Reject the null hypothesis.
Conclude device should be adjusted lower.

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Restock

A vending machine company wants to improve its efficiency in restocking its machines. Past history has shown that the machines reach 30% stocked in 14 days on average. Going below this level does not provide customers with sufficient selection reducing customer satisfaction and causing lost sales. Stocking too soon increases costs.

The company has decided to analyze individual routes and make adjustments, increasing or decreasing the pick-up interval as needed.

They will use the same standardized hypothesis test for routes analyzed, and make adjustments if they conclude with 95% confidence that the stock level is different from the 30% target?

What are the null and alternate hypotheses will the company use?

A) H_0: $\mu = 30$ H_a: $\mu < 30$

B) H_0: $\mu = 30$ H_a: $\mu \neq 30$

D) Cannot be determined with data given.

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Restock

Route A currently follows the 14 day restocking schedule.

Over three months the stock level was noted on Route A as

28% 25% 31% 35% 23% 24% 33% 30% 27% 25%

n=10 $\bar{x}$= 28.1 s= 4.04

Using the standardized hypothesis test (95% level of confidence, stock level is different than the 30% target), would you recommend that changes be made to the schedule for this route?,

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Restock

$$t_{test} = \frac{\bar{x} - u_o}{s/\sqrt{n}}$$

The three month sample data for Route A is summarized as follows

n=10 $\bar{x}$= 28.1 s= 4.04

Using the standardized hypothesis test (95% level of confidence, stock level is different than the 30% target), would you recommend that changes be made to the schedule for this route?

t_{test} is

A) -0.56 B) -2.16 C) -1.49 D) -1.56

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Restock

$$t_{test} = \frac{\bar{x} - u_o}{s/\sqrt{n}}$$

The three month sample data for Route A is summarized as follows

n=10 $\bar{x}$= 28.1 s= 4.04

Using the standardized hypothesis test (95% level of confidence, stock level is different than the 30% target), would you recommend that changes be made to the schedule for this route?,

$t_{critical}$ is

A) $t_{0.05,\ 10}$ B) $t_{0.05,\ 9}$ C) $t_{0.025,\ 10}$ D) $t_{0.025,\ 9}$

$t_{critical}$ is

A) 1.833 B) 2.262 C) 3.690 D) 2.228

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Restock

The three month sample data for Route A is summarized as follows

n=10 $\bar{x}$= 28.1 s= 4.04

Using the standardized hypothesis test (95% level of confidence, stock level is different than the 30% target), would you recommend that changes be made to the schedule for this route?,

From this hypothesis test I

A) Reject the null hypothesis since $|t_{test}| > t_{critical}$

B) Do not reject the null hypothesis that μ=30

C) Accept the alternative hypothesis.

D) Both B and C.

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Restock

The three month sample data for Route A is summarized as follows

n=10 $\bar{x}$= 28.1 s= 4.04

Using the standardized hypothesis test (95% level of confidence, stock level is different than the 30% target), would you recommend that changes be made to the schedule for this route?,

The practical conclusion of the test is

A) Increase the interval between restocking

B) Decrease the interval between restocking

C) Make no changes to the Route A schedule.

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v	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587

α Probability of a greater value

t_{test} = -1.49 with 9 degrees of freedom

Area in the tail will be between 0.05 and 0.10.
This is two tailed test so p value would be between 0.10 and 0.20

One-Sample T

Test of μ = 30 vs ≠ 30

N	Mean	StDev	SE Mean	95% CI	T	P
10	28.10	4.04	1.28	(25.21, 30.99)	-1.49	0.171

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Restock

The three month sample data for Route A is summarized as follows

n=10 $\bar{x}$= 28.1 s= 4.04

Using the standardized hypothesis test (95% level of confidence, stock level is different than the 30% target), would you recommend that changes be made to the schedule for this route?,

After determining p = 0.171 for this problem, I can say

T / F If the true mean of the Route A level is 30% with standard deviation of 4.04, the probability of getting an average sample (n=10) value further from the mean than 28.1 is 17.1%

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Comparing 1 sample mean with a known μ.

	Null Hypothesis	Alternate Hypothesis	Test statistic	Rejection Region
Normal Distribution (Large Sample)	$\mu = \mu_0$	$\mu \neq \mu_0$	$Z_{test} = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$	$\lvert z_{test} \rvert > z_{\alpha/2}$
		$\mu < \mu_0$		$z_{test} < -z_{\alpha}$
		$\mu > \mu_0$		$z_{test} > z_{\alpha}$
Students t Distribution (σ unknown, Small sample) t- test with $v = n - 1$	$\mu = \mu_0$	$\mu \neq \mu_0$	$t_{test} = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$	$\lvert t_{test} \rvert > t_{\alpha/2,v}$
		$\mu < \mu_0$		$t_{test} < -t_{\alpha,v}$
		$\mu > \mu_0$		$t_{test} > t_{\alpha,v}$

Reminders –

- The assumption with the Z test is that $\boldsymbol{\sigma}$ is known or can be reasonably estimated with the sample data.
- Use the t test with sample sizes less than 30. $\boldsymbol{\sigma}$ is considered unknown and not estimated accurately with s.

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End of lecture

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