EIN 5226

Applications of Basic Probability in Quality
Part A

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Probability

Experiment:	Process that results in an outcome that cannot be predicted in advance
Sample space:	Set of all possible outcomes of an experiment
Event:	Subset of a sample space, Outcome of interest
Probability:	How likely an event is to occur.

P(A) = probability of event A occurring

$= \frac{\text{number of ways event A can occur}}{\text{total number of possible outcomes}}$

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Probability – Reminders

- For any event A, 0≤ P(A) ≤ 1
- For any sample space, the sum of probabilities of possible outcomes must equal 1.
- The probability of event A not happening is 1-P(A).

Complement of Event A:

$$P(A^c) = P(not\ A) = 1 - P(A)$$

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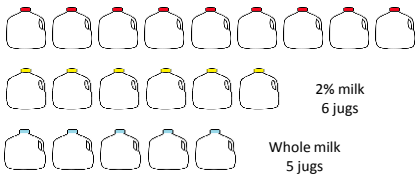
At a convenience store there are 20 jugs of milk in the cooler:



1% milk
9 jugs

2% milk
6 jugs

Whole milk
5 jugs

milk jug images from MyCuteGraphics.com

If a jug of milk is selected at random, what is the probability that it is whole milk?

a. 9/20 b. 5/20 c. 5/11 d. 15/20

If a jug of milk is selected at random, what is the probability that it is not 1% milk?

a. 0.350 b. 0.500 c. 0.4500 d. 0.5500

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- In a given sample space, Events A and B are said to be **mutually exclusive** if they have no outcomes in common.
- The **union** of the two subsets is A∪B which means "A or B "
- For mutually exclusive events

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$$

Or more generally

$$P(A \text{ or } B \text{ or } C \text{ or } \ldots) = P(A \cup B \cup C \cup \ldots) = P(A) + P(B) + P(C) + \ldots$$

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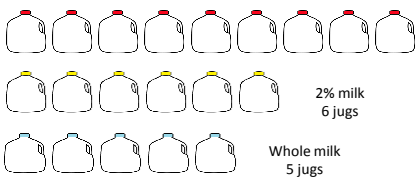
At a convenience store there are 20 jugs of milk in the cooler:



1% milk
9 jugs

2% milk
6 jugs

Whole milk
5 jugs

milk jug images from MyCuteGraphics.com

If a jug of milk is selected at random, what is the probability that it is 1% OR 2% milk?

a. 15/20 b. 0.750 c. 75% d. All of the answers.

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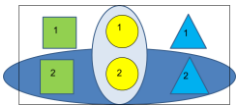
Basic Probability

The **intersection** of the two subsets is A∩B which means "A and B "

Let A = drawing a ball

Let B = drawing a "2"



What is the probability of drawing a ball or a "2" on a single blind draw from the box?

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A\_or\_B) = \frac{2}{6} + \frac{3}{6} - \frac{1}{6} = \frac{4}{6} = .6667 = 66.67\%$$

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At a convenience store there are 20 jugs of milk in the cooler.

1% milk
9 jugs
3 expired

2% milk
6 jugs, 2 expired

Whole milk
5 jugs, 1 expired

milk jug images from MyCuteGraphics.com

Quality control is quite lax at the store. Some of the jugs, as shown are past the expiration date.

If a jug of milk is selected at random, what is the probability that the jug is either 1% milk OR expired?

a. 0.500 b. 0.600 c. 0.400 d. 0.700

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Basic Probability

<u>Toss of coin, once</u>

Outcome	Probability
Heads	½=50%
Tails	½=50%

<u>Toss of coin, multiple times</u>

Two events A and B are **independent** if the probability of each event remains the same whether or not the other occurs.

Each time a coin is tossed it is an independent event.

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Basic Probability

Let A = toss of heads on 1st coin flip

Let B = toss of tails on the 2nd flip

What is the probability of tossing a heads and then a tail?

For Independent outcomes

$$P(A \text{ and } B) = P(A \cap B) = P(A)P(B)$$

$$P(heads\ on\ first\ trial\ and\ tails\ on\ second) = P(heads)P(tails) = (0.50)(0.50) = 0.25$$

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Probability of Defective parts

A process produces 12% defective parts. Each time a part is taken from the process, it is considered an independent event.

If 3 pieces are selected at random from the process, what is the probability that all three are defective?

a. 0.17% b. 36% c. 2.5% d. 1.2%

If 3 pieces are selected at random from the process, what is the probability that none are defective?

a. 52% b. 76% c. 88% d. 68 %

If 3 pieces are selected at random from the process, what is the probability that at least 1 defective?

a. 42% b. 24% c. 12% d. 32 %

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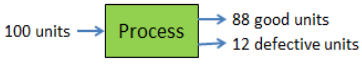
First Pass Yield

First Pass Yield:

- Also referred to as the quality rate. The percentage of units that completes a process and meets quality guidelines without being scrapped, rerun, retested, returned or diverted into an off-line repair area.
- FPY is calculated by divided the units entering the process minus the defective units by the total number of units entering the process (ASQ Quality Glossary)



FPY = (100 – 12)/100 = 0.88 = 88%

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Rolled Throughput Yield

Rolled Throughput Yield (RTY)

- Cumulative calculation of yield or defects through multiple process steps.
- RTY = $p_1 * p_2 * p_3 * \ldots * p_k$
  - Where p_i is the yield from the individual process and there are K processes.
  - And processes are sequential and independent

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First Pass Yield

Part Insertion
- Putting components on the board.
- Yield = 488/500 = 0.976

Wave Solder
- Automated soldering of components on the board.
- Yield = 451/488 = 0.924

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First Pass Yield

What is the first pass yield for the test process?

A. 1.06 B. 0.850 C. 0.942 D. 0.671

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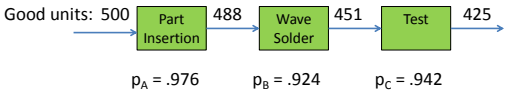
Rolled Throughput Yield



$p_A = .976$ $p_B = .924$ $p_C = .942$

Asking same thing

What is the probability of a unit making it through all three steps without a problem?

What is the rolled throughput yield for the above sequence of operations?

A. 1.06 B. 0.850 C. 0.942 D. 0.671

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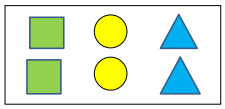
Conditional Probability

Let A = drawing ball on the 1st draw

Let B = drawing block on the 2nd draw



What is the probability of drawing a ball and then a block (without replacement)?

- Outcome of 1st trial impacts outcome of 2nd trial
- Therefore they are not independent events.

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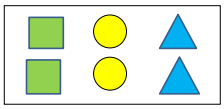
Conditional Probability

Let A = drawing ball on the 1st draw

Let B = drawing block on the 2nd draw



What is the probability of drawing a ball and then a block (without replacement)?

*P(A and B) = P (A ∩ B) = P(A) * P(B/A)*

$$P(A\_and\_B) = \frac{2}{6} * \frac{2}{5} = .1333 = 13.33\%$$

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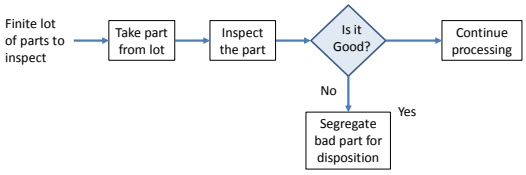
Quality Inspection



Inspection processes are typically sampling without replacement.

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Whole milk
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milk jug images from MyCuteGraphics.com

Quality control is quite lax at the store. Some of the jugs, as shown are past the expiration date.

Two jugs of milk are selected at random. What is the probability that both jugs are expired?

a. 7.5% b. 7.9% c. 9.0% d. 8.3%

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At a convenience store there are 20 jugs of milk in the cooler.

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Quality control is quite lax at the store. Some of the jugs, as shown are past the expiration date.

Three jugs of milk are selected at random. What is the probability that none of the three are expired?

a. 0.3193 b. 0.3466 c. 0.4011 d. 0.2730

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Another Conditional

*P(A and B) = P(A) * P(B/A)*

A doctor has very poor handwriting and writes prescriptions.

- If the regular (R) pharmacist fills an order for the doctor, she must call the doctors office for clarification (C) about 5% of the time.
- If the substitute (S) pharmacist is filling the order, he must call for clarification (C) about 20% of the time.
- The regular pharmacist is on duty is on duty 70% of the time with a substitute on 30% of the time.

Four possible scenarios:

R – regular fills the order P(R) = 0.70	needs clarification	P(C/R) = 0.05
	without clarification	
S – substitute fills the order P(S) = 0.30	needs clarification	P(C/S) = 0.20
	without clarification	

The probability of the regular pharmacist getting the order and needing clarification is P(R and C) = P(R) * P(C/R) = 0.70 * 0.05 = .035

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Another Conditional

R fills the order P(R) = 0.70	needs clarification	P(C/R) = 0.05
	without clarification	
S fills the order P(S) P(S) = 0.30	needs clarification	P(C/S) = 0.20
	without clarification	

What is the probability S fills the order AND it needs clarification?
a. 0.300 b. 0.060 c. 0.006 d. 0.035

What is the probability R fills the order AND does not need clarification?
a. 0.525 b. 0.450 c. 0.665 d. 0.700

What is the probability an order will need clarification?
a. 0.250 b. 0.125 c. 0.035 d. 0.095

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Related Assignments

Please continue to Part B.

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