Probability

Chapter 6

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Probability

Experiment: Process that results in an outcome that cannot be predicted in advance

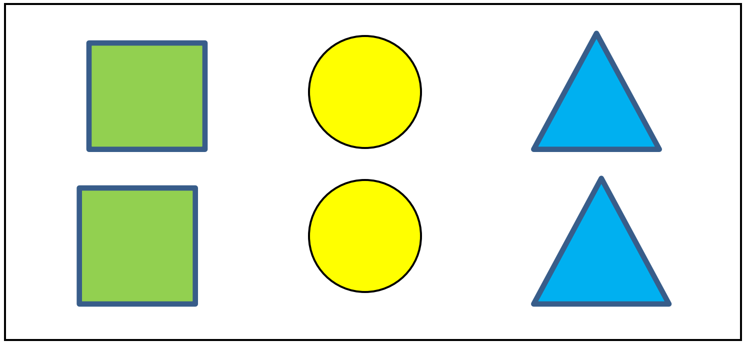
Sample space: Set of all possible outcomes of an experiment

Event: Subset of a sample space, Outcome of interest

Probability: How likely an event is to occur.

Probability

What is the probability of drawing a ball on a single blind draw from the box?



Experiment – drawing a shape from the sample space

Sample Space: block, ball, triangle

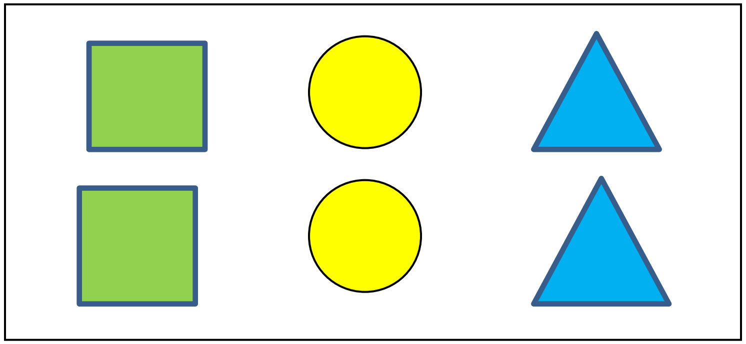
Event: draw a ball

Basic Probability

P(A) = probability of event A occurring

$$= \frac{\text{number of ways event A can occur}}{\text{total number of possible outcomes}}$$

Let A = outcome of drawing ball on a single blind draw from the box



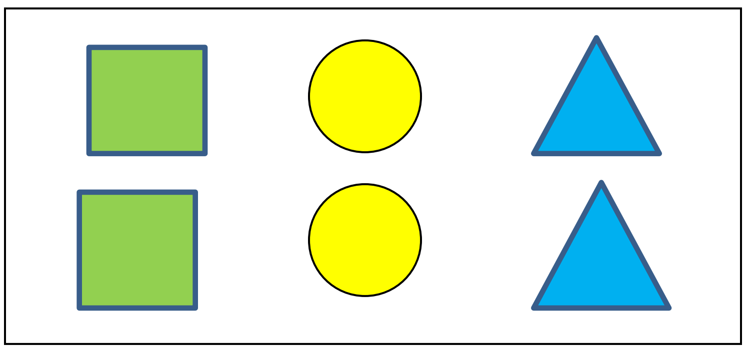
$$P(A) = \frac{2}{6} = .333 = 33.3\%$$

For any event A, 0≤ P(A) ≤ 1

Basic Probability

P(A) = probability of event (outcome A) occurring

$$= \frac{\text{number of ways outcome A can occur}}{\text{total number of possible outcomes}}$$



The probability of NOT drawing a ball is

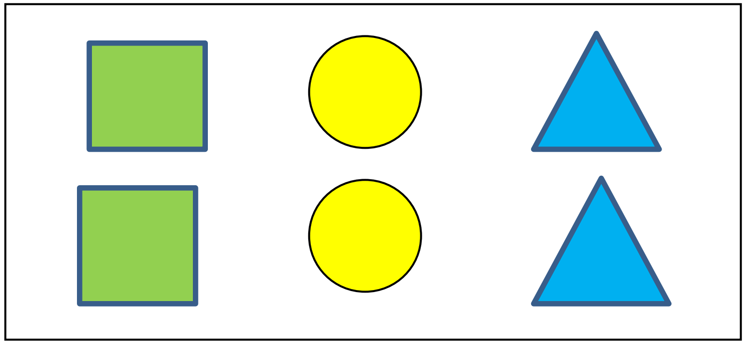
B. 2/6
C. 4/6
D. 1-P(drawing a ball)
E. C or D

Complement of Event A: $P(A^c) = P(not\ A) = 1 - P(A)$

Basic Probability

Let A = drawing ball

Let B = drawing block



What is the probability of drawing a ball or a block on a single blind draw from the box?

$$\frac{2}{6}+\frac{2}{6}=\frac{4}{6}=0.667=66.7\%$$

For mutually exclusive outcomes:

$$P(A\ or\ B)=P(A\cup B)=P(A)+P(B)$$

- In a given sample space, Events A and B are said to be **mutually exclusive** if they have no outcomes in common.
- The **union** of the two subsets is AUB which means "A or B "
- For mutually exclusive events

$$P(A\ or\ B) = P(A \cup B) = P(A) + P(B)$$

Or more generally

$$P(A\ or\ B\ or\ C\ or\ \ldots) = P(A \cup B \cup C \cup ..)$$
$$= P(A) + P(B) + P(C) + \ldots$$

Are the following mutually exclusive outcomes where the formula is applicable?

$$P(A\ or\ B\ or\ C\ or\ \ldots) = P(A \cup B \cup C \cup \cdots.)$$
$$= P(A) + P(B) + P(C) + \ldots$$

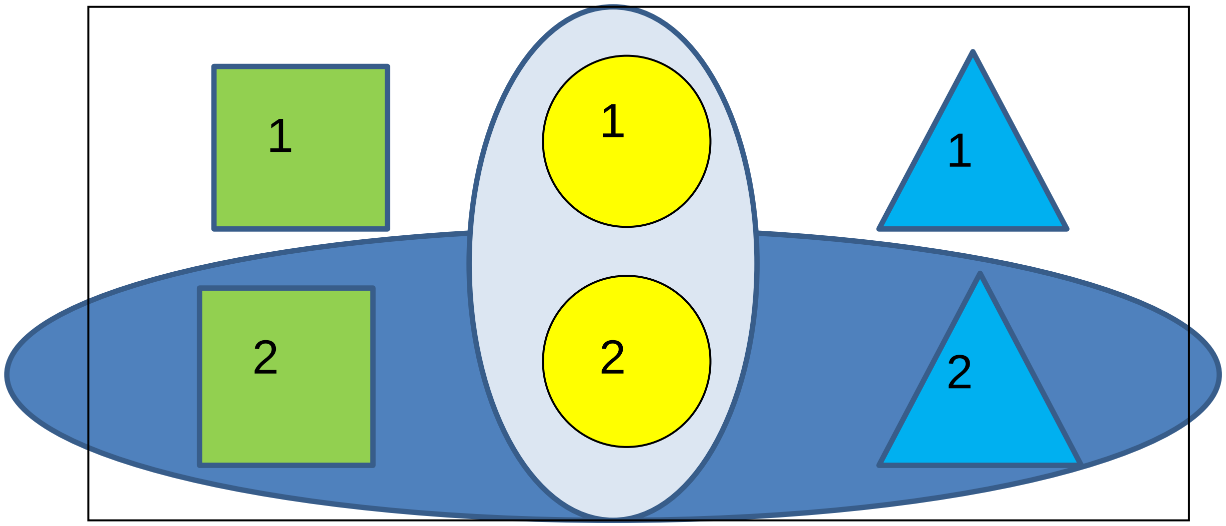
Yes / No In a class re. final exam: Students earning A /Students earning B/ students earning C

Yes / No Dog owners in Texas/Cat owners in Texas

Yes / No Players on football team who have thrown a touchdown/ Players on baseball team who have hit a home run.

Basic Probability

The **intersection** of the two subsets is A∩B which means "A and B "



Let A = drawing a ball   Let B = drawing a "2"

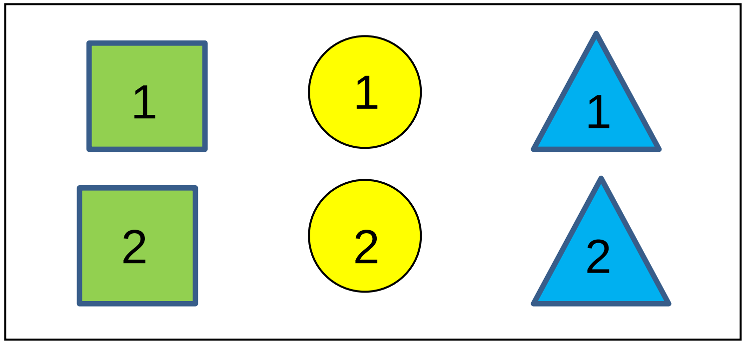
P (A and B) = P(A∩B)

=(items in intersection/total items = 1/6

Basic Probability

Let A = drawing a ball

Let B = drawing a "2"



What is the probability of drawing a ball or a "2" on a single blind draw from the box?

$$P(A\ or\ B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A\_or\_B) = \frac{2}{6} + \frac{3}{6} - \frac{1}{6} = \frac{4}{6} = .6667 = 66.67\%$$

Basic Probability

Toss of coin, once

Outcome	Probability
Heads	½=50%
Tails	½=50%

Toss of coin, multiple times

Two events A and B are **independent** if the probability of each event remains the same whether or not the other occurs.

Each time a coin is tossed it is an independent event.

Basic Probability

P(A) = probability of event occurring

$$= \frac{\text{number of ways outcome can occur}}{\text{total number of possible outcomes}}$$

<u>Toss of coin, once</u>

Outcome	Probability
Heads	½=50%
Tails	½=50%

<u>Toss of coin, twice</u>

Outcomes	Probability
Heads/Heads	¼=.25%
Heads/Tails	¼=.25%
Tails/Heads	¼=.25%
Tails/Tails	¼=.25%

Basic Probability

Let A = toss of heads on 1st coin flip

Let B = toss of tails on the 2nd flip

What is the probability of tossing a heads and then a tail?

For Independent outcomes

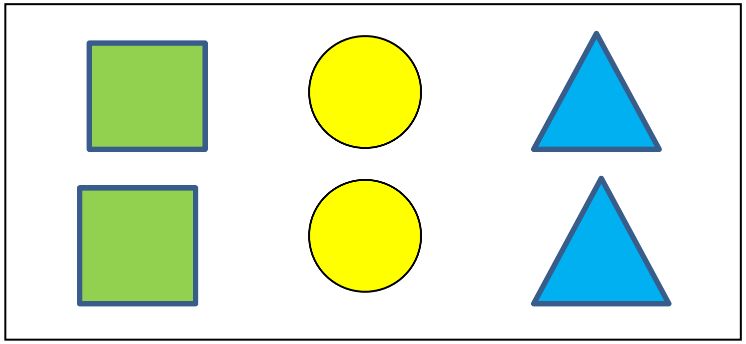
$$P(A\ and\ B) = P(A \cap B) = P(A)P(B)$$

$$P(heads\ on\ first\ trial\ and\ tails\ on\ second) \\ = P(heads)P(tails) = (0.50)(0.50) = 0.25$$

Basic Probability

Let A = drawing ball on the 1st draw

Let B = drawing block on the 2nd draw



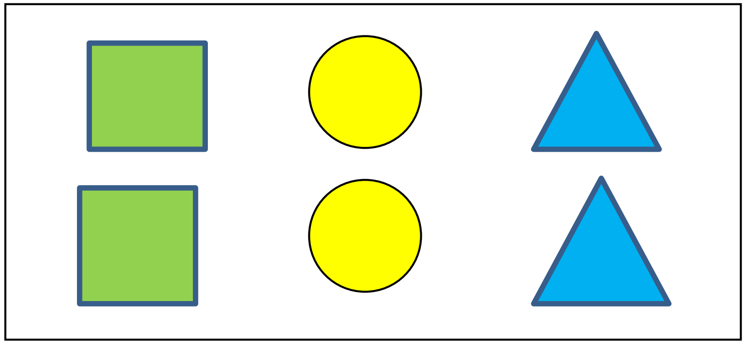
What is the probability of drawing a ball and then a block (without replacement)?

- Outcome of 1st trial impacts outcome of 2nd trial
- Therefore they are not independent events.

Basic Probability

Let A = drawing ball on the 1st draw

Let B = drawing block on the 2nd draw



What is the probability of drawing a ball and then a block (without replacement)?

$$P(A\_and\_B) = P(A) * P(B/A)$$

$$P(A\_and\_B) = \frac{2}{6} * \frac{2}{5} = .1333 = 13.33\%$$

Probability – Reminders

- For any event A, $0 \leq P(A) \leq 1$
- For any sample space, the sum of probabilities of possible outcomes must equal 1.
- The probability of event A not happening is $1-P(A)$.

Practice Problems (A)

Consider the probability of drawing a queen or a jack when drawing one card from a standard deck of cards*.

True or false?

T / F   The outcomes are mutually exclusive.

T / F   The formula to solve for the probability is

$$P(A \cup B) = P(A) + P(B)$$

T / F   The probability is 15.38%

*A standard deck of cards contains 52 cards:. There are 4 suits: hearts, spades, clubs and diamonds. In each suit there will be one each: Ace, King, Queen, Jack, and numbers 2 through 10. So there are 4 queens and 4 jacks, one of each suit.

Practice Problems (B)

Two cards are dealt from an ordinary deck of cards (without replacement).

1) What is the probability that the first card will be a king?

a) 1/52 b) 4/52 c) 13/52 d) 26/52

2) What is the probability that second card will be a king?

a) 4/52 b) 4/51 c) 3/51 d) it depends on what the first card was

3) What is the probability that you will draw two kings?

a) 13.57% b) 0.443% c) 0.603% d) 0.452%

Practice Problems (C)

A coin is tossed multiple times. Which of the following statements are true regarding the 3rd and 4th and 5th tosses of the coin?

True or false?

T / F The outcomes are independent.

T / F The formula to solve for the probability of the 3rd and 4th and 5th tosses all being heads is

$$P(A \cap B \cap C) = P(A) * P(B) * P(C)$$

The probability of the 3rd and 4th and 5th tosses all being heads is

a) 25% b) 12.5% c) 75% d) 3.1%

A little more complicated

Experiment: A fair coin is flipped three times.

Sample space: Set of all possible outcomes

Heads>Heads>Heads
Heads>Heads>Tails
Heads>Tails>Heads
Heads>Tails>Tails
Tails>Heads>Heads
Tails>Heads>Tails
Tails>Tails>Heads
Tails>Tails>Tails

8 possible outcomes from flipping a coin 3 times

A little more complicated

Experiment: A fair coin is flipped three times.
Sample space: Set of all possible outcomes

Heads>Heads>Heads
Heads>Heads>Tails
Heads>Tails>Heads
Heads>Tails>Tails
Tails>Heads>Heads
Tails>Heads>Tails
Tails>Tails>Heads
Tails>Tails>Tails

8 possible outcomes from flipping a coin 3 times

T / F Each of these outcomes is mutually exclusive.

T / F The sum of the probabilities of these outcomes must equal to 1.

A little more complicated

Experiment: A fair coin is flipped three times.
Sample space: Set of all possible outcomes

Heads>Heads>Heads	Heads>Heads>Tails
Heads>Tails>Heads	Heads>Tails>Tails
Tails>Heads>Heads	Tails>Heads>Tails
Tails>Tails>Heads	Tails>Tails>Tails

Now consider the probability of an individual outcome.

Assuming a fair coin, what is the probability of getting three heads in a row?

A. 0.10 B. 0.125 C. 0.25 D. 0.5

T / F To determine the probability of each outcome, I consider that there are three independent events and multiply the individual event probabilities.

A little more complicated

Experiment: A fair coin is flipped three times.
Sample space: Set of all possible outcomes

Heads>Heads>Heads
Heads>Heads>Tails
Heads>Tails>Heads
Heads>Tails>Tails
Tails>Heads>Heads
Tails>Heads>Tails
Tails>Tails>Heads
Tails>Tails>Tails

8 mutually exclusive outcomes

Each outcome is a series of independent events

A little more complicated

Experiment: A fair coin is flipped three times.
Sample space: Set of all possible outcomes

Outcome	Probability	
Heads>Heads>Heads	0.5 x 0.5 x 0.5 =	0.125
Heads>Heads>Tails	0.5 x 0.5 x 0.5 =	0.125
Heads>Tails>Heads	0.5 x 0.5 x 0.5 =	0.125
Heads>Tails>Tails	0.5 x 0.5 x 0.5 =	0.125
Tails>Heads>Heads	0.5 x 0.5 x 0.5 =	0.125
Tails>Heads>Tails	0.5 x 0.5 x 0.5 =	0.125
Tails>Tails>Heads	0.5 x 0.5 x 0.5 =	0.125
Tails>Tails>Tails	0.5 x 0.5 x 0.5 =	0.125
	Total	1.000

Three flip sample space
Possible outcomes

Heads/Heads/Heads	Tails/Tails/Tails
Heads/Heads/Tails	Tails/Tails/Heads
Heads/Tails/Heads	Tails/Heads/Tails
Heads/Tails/Tails	Tails/Heads/Heads

8 possible outcomes.
P(any one of the outcomes) = 0.125

Three flip sample space

Event: Subset of a sample space, Outcome of interest

Heads/Heads/Heads	Tails/Tails/Tails
Heads/Heads/Tails	Tails/Tails/Heads
Heads/Tails/Heads	Tails/Heads/Tails
Heads/Tails/Tails	Tails/Heads/Heads

Event: Getting at least two heads in the three flips

A little more complicated

Experiment: A fair coin is flipped three times.

Outcome	Probability	
Heads>Heads>Heads	0.5 x 0.5 x 0.5 =	0.125
Heads>Heads>Tails	0.5 x 0.5 x 0.5 =	0.125
Heads>Tails>Heads	0.5 x 0.5 x 0.5 =	0.125
Heads>Tails>Tails	0.5 x 0.5 x 0.5 =	0.125
Tails>Heads>Heads	0.5 x 0.5 x 0.5 =	0.125
Tails>Heads>Tails	0.5 x 0.5 x 0.5 =	0.125
Tails>Tails>Heads	0.5 x 0.5 x 0.5 =	0.125
Tails>Tails>Tails	0.5 x 0.5 x 0.5 =	0.125
	Total	1.000

In the experiment, what is the probability of the event of getting at least two heads in the three flips?

A. 0.125 B. 0.500 C. 0.750 D. 0.875

A little more complicated

Experiment: A fair coin is flipped three times.

Outcome	Probability	
Heads>Heads>Heads	0.5 x 0.5 x 0.5 =	0.125
Heads>Heads>Tails	0.5 x 0.5 x 0.5 =	0.125
Heads>Tails>Heads	0.5 x 0.5 x 0.5 =	0.125
Heads>Tails>Tails	0.5 x 0.5 x 0.5 =	0.125
Tails>Heads>Heads	0.5 x 0.5 x 0.5 =	0.125
Tails>Heads>Tails	0.5 x 0.5 x 0.5 =	0.125
Tails>Tails>Heads	0.5 x 0.5 x 0.5 =	0.125
Tails>Tails>Tails	0.5 x 0.5 x 0.5 =	0.125
	Total	1.000

In the experiment, what is the probability of the event of getting at least one heads in the three flips?

A. 0.125 B. 0.500 C. 0.750 D. 0.875

Three flip sample space

Event: Getting at least one heads in the three flips

Heads/Heads/Heads	Tails/Tails/Tails
Heads/Heads/Tails	Tails/Tails/Heads
Heads/Tails/Heads	Tails/Heads/Tails
Heads/Tails/Tails	Tails/Heads/Heads

Sum of all possible outcomes must equal 1.
Only outcome where there is not a head is **Tails/Tails/Tails**
Therefore:

P(**at least 1 heads**) = 1 – P(**Tails/Tails/Tails**)

Three flip sample space

Event: Getting at least one heads in the three flips

Heads/Heads/Heads	Tails/Tails/Tails
Heads/Heads/Tails	Tails/Tails/Heads
Heads/Tails/Heads	Tails/Heads/Tails
Heads/Tails/Tails	Tails/Heads/Heads

Sum of all possible outcomes must equal 1.
Only outcome where there is not a head is **Tails/Tails/Tails**
Therefore:

P(**at least 1 heads**) = 1 – P(**Tails/Tails/Tails**)

P(not **Tails/Tails/Tails**) = 1 – P(**Tails/Tails/Tails**)

Complement of Event A: $P(A^c) = P(not\ A) = 1 - P(A)$

Order Fulfillment Problem

An on-line retailer ships orders in packages containing one part number in the package with multiple quantities of that item (part number).

Customer complaints triggered an investigation and it appears one warehouse has significant problems.

For any order from the warehouse, there is

- A 8% probability that an order will the incorrect version of the item in it
- A 10% probability that the package will contain the incorrect quantity for the item.

Order Fulfillment Problem

For any order, there is

- A 8% probability that an order will the incorrect version of the item in it
- A 10% probability that the package will contain the incorrect quantity for the item.

T / F The event that a package will have the incorrect version of the item and the event that the package will have the incorrect quantity are mutually exclusive.

T / F The event that a package will have the incorrect version of the item and the event that the package will have the incorrect quantity are independent events.

Order Fulfillment Problem

For any order, there is

- A 8% probability that an order will the incorrect version of the item in it
- A 10% probability that the package will contain the incorrect quantity for the item.

What percent of packages will have both the wrong version and incorrect quantity?

A. 18%   B. 0.8%   C. 9%   D. 1.8%.

Order Fulfillment Problem

For any order from the warehouse, there is

- A 8% probability that an order will the incorrect version of the item in it

  Let Y = event of incorrect version in package

  P(incorrect version) = P(Y) = 0.08

- A 10% probability that the package will contain the incorrect quantity for the item.

  Let Z = event of wrong quantity in package

  P(wrong quantity) = P(Z) = 0.10

<u>For independent events Y and Z</u>

P (incorrect version AND wrong Quantity) = P(Y and Z)

$$= P(Y \cap Z) = P(Y) \times P(Z) = .08 * .10 = .008$$

Order Fulfillment Problem

For any order, there is

- A 8% probability that an order will the incorrect version of the item in it (example, shipped blue color rather than the red which was ordered)
- A 10% probability that the package will contain the incorrect quantity for the item.

What is the probability of a package having the correct item version?

A. 8%	B. 10%	C. 90%	D. 92%.

Order Fulfillment Problem

For any order from the warehouse, there is

- A 8% probability that an order will the incorrect version of the item in it

  Let Y = event of incorrect version in package

  P(incorrect version) = P(Y) = 0.08

  P (correct version) = P (not Y) = 1- P(Y) = 1-.08 = .92

- A 10% probability that the package will contain the incorrect quantity for the item.

  Let Z = event of wrong quantity in package

  P(wrong quantity) = P(Z) = 0.10

  P (correct quantity) = P(not Z) = 1 – P(Z) = 1-.10 = .90

Complement of Event A: $P(A^c) = P(not\ A) = 1 - P(A)$

Order Fulfillment Problem

For any order, there is

- A 8% probability that an order will the incorrect version of the item in it (example, shipped blue color rather than the red which was ordered)
- A 10% probability that the package will contain the incorrect quantity for the item.

What percent of packages will have both the correct version and correct quantity?

A. 80% B. 81% C. 82% D. 83%.

Order Fulfillment Problem

For any order from the warehouse, there is

- A 8% probability that an order will the incorrect version of the item in it

  P(incorrect version) = P(Y) = 0.08

  P (correct version) = P (not Y) = 1- P(Y) = 1-.08 = .92

- A 10% probability that the package will contain the incorrect quantity for the item.

  P(wrong quantity) = P(Z) = 0.10

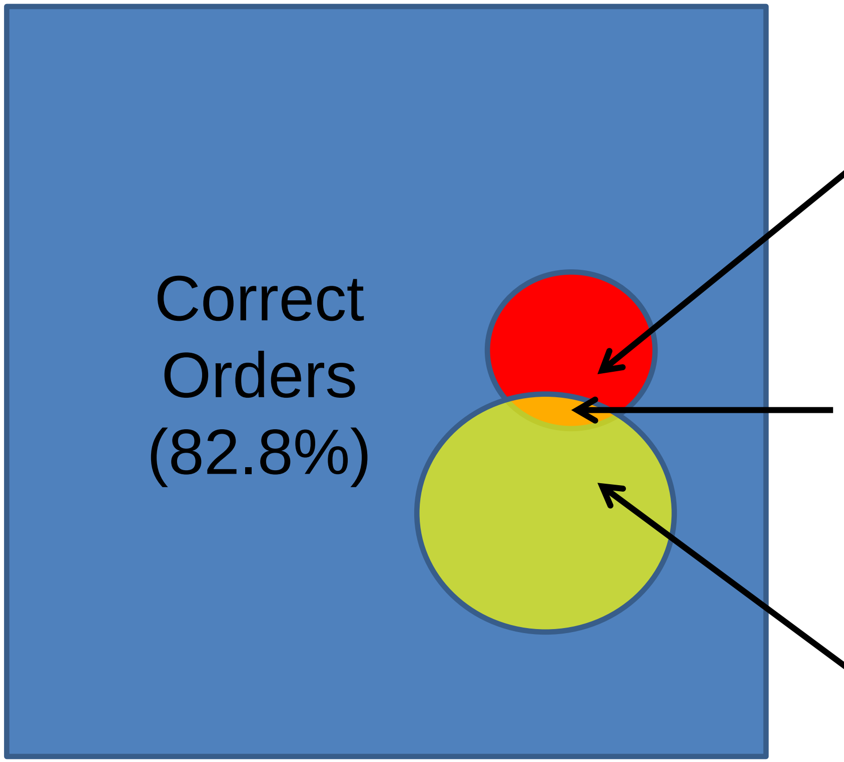
  P (correct quantity) = P(not Z) = 1 – P(Z) = 1-.10 = .90

<u>For independent events (not Y) and (not Z)</u>

P (correct version AND correct Quantity) = P(not Y and not Z)

= $P(\text{not } Y \cap \text{not } Z)$ = P(not Y) x P(not Z) = .92 * .90 = .828

Package Sample Space



Orders with incorrect version, but correct quantity

Orders with incorrect version and wrong quantity (0.8%)

Orders with wrong quantity but correct part number.

Order Fulfillment Problem

For any order, there is

- 8% probability incorrect version $P(Y) = .08$
- 10% probability incorrect quantity $P(Z) = .10$

Outcome	Independent event Formulas	Calculation	Prob.
Wrong version, wrong quantity	P(Y) x P(Z)	(0.08)*(0.10)	0.008
Correct version, correct quantity	P(not Y) x P(not Z)	(1-.08)*(1-.10)	0.828
Wrong version, correct quantity	P(Y) x P(not Z)	(0.08)*(1-.10)	0.072
Correct version, wrong quantity	?	?	?

Order Fulfillment Problem

For any order, there is

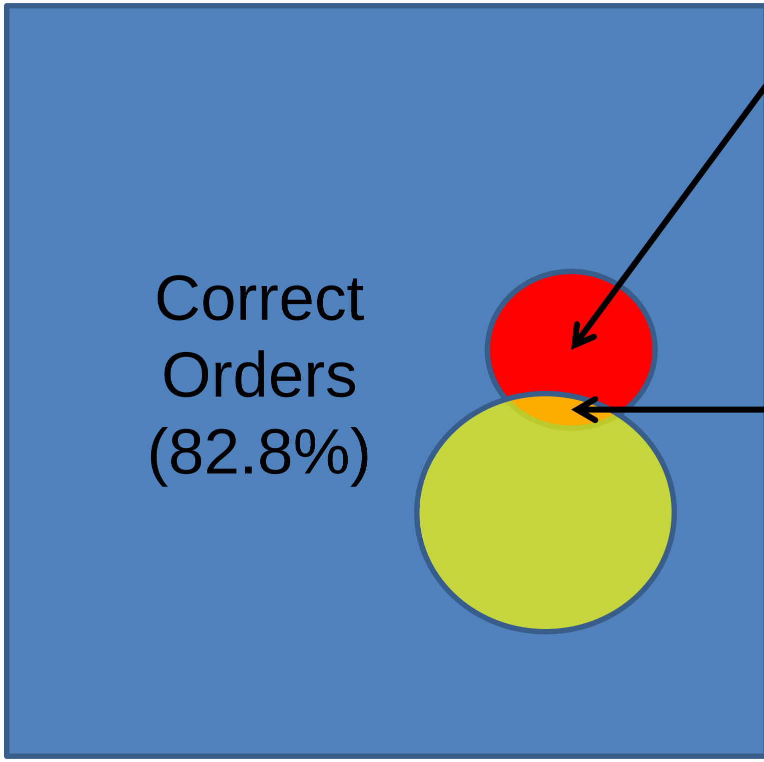
- 8% probability incorrect version $P(Y) = .08$
- 10% probability incorrect quantity $P(Z) = .10$

Outcome	Independent event Formulas	Calculation	Prob.
Wrong version, wrong quantity	P(Y) x P(Z)	(0.08)*(0.10)	0.008
Correct version, correct quantity	P(not Y) x P(not Z)	(1-.08)*(1-.10)	0.828
Wrong version, correct quantity	P(Y) x P(not Z)	(0.08)*(1-.10)	0.072
Correct version, wrong quantity	?	?	?

What percent of packages will have the correct version and wrong quantity?

A. 10.0% B. 8.2% C. 7.2% D. 9.2%.

Package Sample Space



Orders with incorrect version, but correct quantity (7.2 %)

Red plus orange =7.2 +0.8 =8%
= P(incorrect version) = P(Y)

Orders with incorrect version and wrong quantity (0.8%)

$P(Y \text{ and } Z) = P(Y \cap Z)$
$= P(Y) \times P(Z) = .08 * .10 = .008$

$P(\text{correct version, incorrect quantity}) = P(Y) - P(Y \text{ and } Z)$
$= .08 - .008 = .072$

Order Fulfillment Problem

Outcome	Independent event Formulas	Calculation	Prob.
Wrong version, wrong quantity	P(Y) x P(Z)	(0.08)*(0.10)	0.008
Correct version, correct quantity	P(not Y) x P(not Z)	(1-.08)*(1-.10)	0.828
Wrong version, correct quantity	P(Y) x P(not Z)	(0.08)*(1-.10)	0.072
Correct version, wrong quantity	P(not Y) x P(Z)	(1-.08) * (0.10)	0.092

What percent of packages will have at least one thing wrong with it?

A. 18.0% B. 8.2% C. 17.2% D. 16.4%.

Order Fulfillment Problem

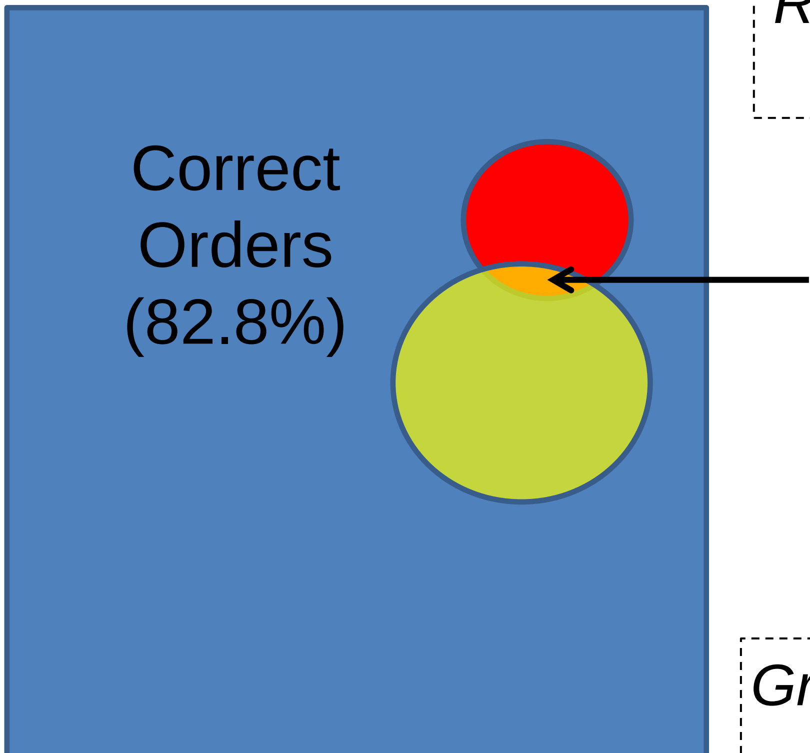
Outcome	Independent event Formulas	Calculation	Prob.
Wrong version, wrong quantity	P(Y) x P(Z)	(0.08)*(0.10)	0.008
Correct version, correct quantity	P(not Y) x P(not Z)	(1-.08)*(1-.10)	0.828
Wrong version, correct quantity	P(Y) x P(not Z)	(0.08)*(1-.10)	0.072
Correct version, wrong quantity	P(not Y) x P(Z)	(1-.08) * (0.10)	0.092

What percent of packages will have at least one thing wrong with it?

P(at least one thing wrong) = 1 – P(nothing wrong)

Complement of Event A: $P(A^c) = P(not\ A) = 1 - P(A)$

Another way to calculate P(one or more)



Red plus orange =7.2 +0.8 =8%
= P(incorrect version)= P(Y)

Orders with incorrect version and wrong quantity (0.8%)

P(Y and Z) = P(Y∩Z)
= P(Y) x P(Z) = .08 * .10 = .008

Green plus orange = 9.2+0.8 =10%
=P(incorrect quantity) = P(Z)

P(Y or Z) = P(Y **U** Z) = P(Y) + P(Z) - P(Y∩Z)
= 0.08 + 0.10 - 0.008 = .172

Related Assignments

Please see Blackboard for related assignments