EIN 5226

Poisson Applications in Quality

Chapter 7 Sections 8-9

Note: Need Calculator

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The Poisson Distribution

- Model of random phenomena that occurs on a "per unit" basis
- Can count when an event occurs, but cannot count when the event does not occur
- Common application: customer arrival rates
- Examples in quality control and continuous improvement:
  - Defects per 100 square feet of paper
  - Machine failures per month
  - Accidents per facility

Defects versus Defective

Defect: A product's or service's nonfulfillment of an intended requirement or reasonable expectation for use, including safety considerations.

There are four classes of defects:

- class 1, very serious, leads directly to severe injury or catastrophic economic loss;
- class 2, serious, leads directly to significant injury or significant economic loss;
- class 3, major, is related to major problems with respect to intended normal or reasonably foreseeable use; and
- class 4, minor, is related to minor problems with respect to intended normal or reasonably foreseeable use. Also see "blemish," "imperfection" and "nonconformity."

Defective: A defective unit; a unit of product that contains one or more defects with respect to the quality characteristic(s) under consideration.

https://asq.org/quality-resources/quality-glossary/

A defective unit may have multiple defects.

The Poisson Distribution

Probability Mass Function:

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

Where: x= number of events occurring per unit

λ= μ = population mean

= average number of events per unit

Mean and variance are the same: $\mu=\sigma^2$

Poisson Distribution $P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

Portable television sets are subjected to a final inspection for surface defects. Historically, the average number of minor defects per TV set is equal to 4.1.

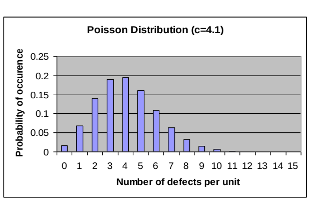
What is the probability that a TV set has exactly 4 defects?

$$P(X = 4) = \frac{e^{-4.1}4.1^{4}}{4!} = 0.1951$$

$\lambda = 4.1$
x = 4

Portable television sets are subjected to a final inspection for surface defects. Historically, the average number of minor defects per TV set is equal to 4.1.

$\mu = \lambda = c = 4.1$ $p(x) = \frac{e^{-c}c^{x}}{x!}$



P(0) = 0.0166
P(1) = 0.0679
P(2) = 0.1393
P(3) = 0.1904
P(4) = 0.1951
P(5) = 0.1600
P(6) = 0.0193
P(7) = 0.0641
P(8) = 0.0328
P(9) = 0.0510
P(10) = 0.0061
P(11) = 0.0023
P(12) = 0.0008
P(13) = 0.0002
P(14) = 0.0001

Poisson Distribution

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

Portable television sets are subjected to a final inspection for surface defects. Historically, the average number of minor defects per TV set is equal to 4.1.

What is the probability that a TV set has at least 1 defect?

$\lambda = 4.1$
x = 0

$$P(X > 0) = 1 - P(X = 0)$$

$$= 1 - \frac{e^{-4.1}4.1^{0}}{0!} = 1- 0.0166= 0.9834$$

Poisson Distribution

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

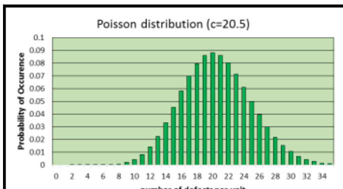
Portable television sets are subjected to a final inspection for surface defects. Historically, the average number of minor **defects per TV set is equal to 4.1.**

What is the probability that **when inspecting 5 TV sets**, a total of exactly 15 defects are found?

$\lambda = 4.1$ defects per 1 unit = **20.5 defects per 5 TV sets**
x = **15 defects per 5 TV sets**

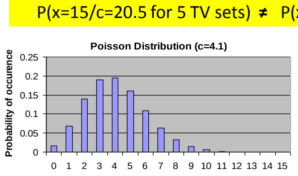
$$P(x = 15) = \frac{e^{-20.5}20.5^{15}}{15!} = 0.0454$$

Must change λ units to match x units.
Cannot change x units to match λ units



Unit defined as 5 TV sets
c=20.1
P(15) = 0.0454

Two very different distributions
P(x=15/c=20.5 for 5 TV sets) ≠ P(x=5/c=4.1 for 1 TV sets)



Unit defined as 1 TV set
c=4.1
P(5) = 0.1600

Poisson Problem

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

An electronics manufacturer has observed that historically the average number of solder defects per board is 6.3.

What is the probability that a board will have exactly three solder defects?

A) 0.1533 B) 0.1023 C) 0.0765 D) 0.0618

Poisson Distribution

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

The average number of flaws in 10 feet of material is 12.

What is the probability that there will be exactly 8 flaws in 10 feet of the material?

A. 0.0655 B. 0.0455 C. 0.1315 D. 0.1624

What is the probability that there will be exactly 28 flaws in 20 feet of the material?

A. 0.0655 B. 0.0455 C. 0.1315 D. 0.1624

The Poisson Distribution

- **The Poisson distribution** can be used as an approximation to the **binomial distribution** when n is large and p is small.
- When n is large and p is small the mass function depends almost entirely on the mean np, and very little on the specific values of n and p.
- We approximate the binomial mass function with a quantity $\lambda = np$ in the Poisson distribution.

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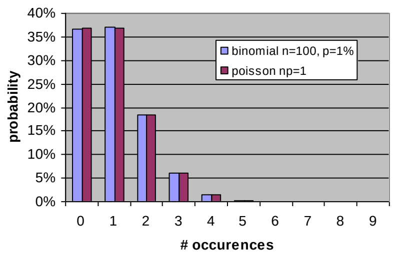
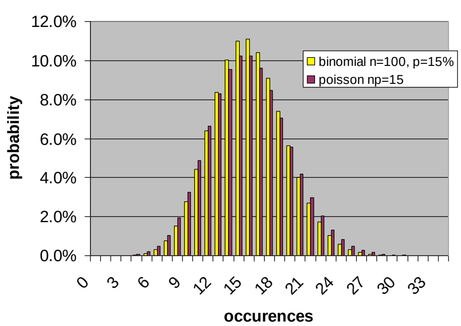
Can use Poisson to approximate binomial when

sample size is large (>50) and p is small (<10%)

µ (or np) < 7





Poisson Distribution $\quad P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!}$

If 2% of the light bulbs manufactured by a company are defective, what is the estimate of the probability that in a lot of 100 bulbs , 4 bulbs are defective?

n=100
p=.02
x=4

$$P(X=4)=\binom{4}{100}(0.02)^4(0.98)^{96}$$
$$=0.090208$$

λ=np=2 $\quad P(X=4)=\frac{e^{-2}2^4}{4!}=0.090224$

Poisson Distribution $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Given that 5% of people are left handed, estimate the probability that a random sample of 50 people contains 2 or more left handed people.

λ for this problem is

A) 0.05 B) 0.10 C) 2.5 D) 5.0

The probability that 2 or more of the 50 people are left handed is.

A) 0.713 B) 0.325 C) 0.877 D) .456

Poisson Problem

$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

A manufacturer claims his product is only 0.5% defective. At receiving inspection you inspect 50 parts.

If the claim is true, what is the probability that you will find any defective product during the inspection?

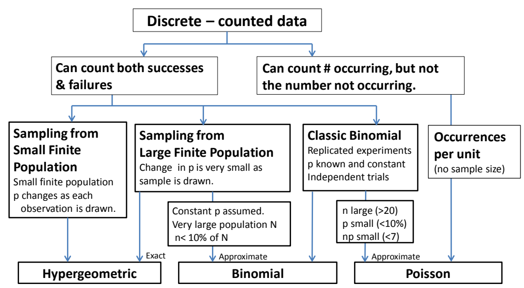
A) 0.7788 B) 0.1635 C) 0.2212 D) 0.8365

Binomial versus Poisson

Binomial - Can count both successes and failures
Given the probability of success
Given sample size
An exact probability is needed.

Poisson - Can count what is occurring, not what is not occurring
Given average per unit (no sample size)

Poisson to estimate binomial
n is large(>50), p is small(<10%), np is small(< 7)



Related Assignments

Please see Blackboard for related assignments