EIN 5226

Hypergeometric Distribution

Note: Need Calculator

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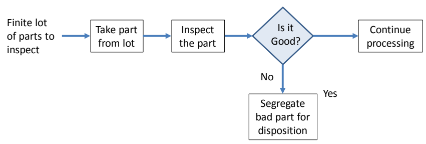
Quality Inspection



Inspection processes are typically sampling without replacement.

Hypergeometric used to get exact probabilities of occurrence of given number of defectives in a sample out of a lot of a defined size.

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Probability of Defective parts

There are 18 parts presented for inspection. 6 of the parts are defective and 12 of the parts are good. A sample of 5 parts is to be inspected.

What is the probability that all five are good?

a. (6/18)*(5/17)*(4/16)*(3/15)*(2/14) = 0.01%

b. (12/18)*(11/17)*(10/16)*(9/15)*(8/14) = 9.24%

c. $(12/18)^5$ = 13.17%

d. None of the above

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Using basic probability

P(A) = probability of event A occurring

= number of ways event A can occur / total number of possible outcomes

P(All 5 good) = # of outcomes that result all 5 good / # of possible samples of size 5

= (# combinations of 5 good) (# combinations of 0 defective) / (total # of possible samples/combinations of size 5)

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Combinations

Number of ways in which a larger group of items can be arranged into smaller subgroups

Combinations
- where order does not matter

$$\binom{n}{x} = nCx = \frac{n!}{x!\,(n-x)!}$$

Note: You need to learn to use your combinations function on your calculator.

Where n = size larger group
x = subgroup size

note: n! = (n)(n-1)(n-2)(n-3)….(3)(2)(1)

Using basic probability

There are **18** parts presented for inspection. 6 of the parts are defective and 12 of the parts are good. A sample of **5** parts is to be inspected.

P(All 5 good) = (# combinations of 5 good) (# combinations of 0 defective) / **(total # of possible samples/combinations of size 5)**

For the denominator

n = size larger group = 18

x = subgroup size = 5

$$\binom{n}{x} = nCx = \frac{n!}{x!\,(n-x)!}$$

$$\binom{18}{5} = {}_{18}C_5 = \frac{18!}{5!\,(18-5)!}$$

$$\binom{18}{5} = \frac{18*17*16*15*14*13*12*11*10*9*8*7*6*5*4*3*2*1}{(5*4*3*2*1)(13*12*11*10*9*8*7*6*5*4*3*2*1)}$$

$$\binom{18}{5} = \frac{18*17*16*15*14}{(5*4*3*2*1)} = \frac{1{,}028{,}160}{120} = 8560$$

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Using basic probability

There are 18 parts presented for inspection. **6** of the parts are defective and 12 of the parts are good. A sample of 5 parts is to be inspected.

P(All 5 good) = (# combinations of 5 good) **(# combinations of 0 defective)** / (total # of possible samples/combinations of size 5)

 # combinations of 0 defective in sample

n = size larger group = 6

x = subgroup size = 0

$$\binom{n}{x} = nCx = \frac{n!}{x!\,(n-x)!}$$

$$\binom{6}{0} = {}_{18}C_5 = \frac{6!}{0!\,(6-0)!}$$

Note: 0! = 1 by definition

$$\binom{6}{0} = \frac{6*5*4*3*2*1}{(1)(6*5*4*3*2*1)} = 1$$

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Using basic probability

$$\binom{n}{x} = nCx = \frac{n!}{x!\,(n-x)!}$$

There are 18 parts presented for inspection. **6** of the parts are defective and 12 of the parts are good. A sample of 5 parts is to be inspected.

P(All 5 good) = **(# combinations of 5 good)** (# combinations of 0 defective) / (total # of possible samples/combinations of size 5)

For the # combinations of 5 good in sample

n, the larger group that the good parts are selected from, is

A. 18 B. 12 C. 6 D. 5

x, the subset size of good parts of interest, is

A. 18 B. 12 C. 6 D. 5

The number of possible combinations of 5 good parts in the sample is

A. 689 B. 486 C. 792 D. 1024

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Using basic probability

There are **18** parts presented for inspection. 6 of the parts are defective and 12 of the parts are good. A sample of **5** parts is to be inspected.

P(All 5 good) = # of outcomes that result all 5 good / # of possible samples of size 5

= (# combinations of 5 good) (# combinations of 0 defective) / (total # of possible samples/combinations of size 5)

$$= \frac{\binom{12}{5}\binom{6}{0}}{\binom{18}{5}} = \frac{(792)*(1)}{8568} = 0.0924$$

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Using basic probability

P(All 5 good) = (# combinations of 5 good) (# combinations of 0 defective) / (total # of possible samples/combinations of size 5)

$$= \frac{\binom{12}{5}\binom{6}{0}}{\binom{18}{5}} = \frac{(792)*(1)}{8568} = 0.0924$$

$$\binom{12}{5} = \frac{12*11*10*9*8*7*6*5*4*3*2*1}{(5*4*3*2*1)*(7*6*5*3*4*2*1)} = \frac{12*11*10*9*8}{1*2*3*4*5}$$

$$\binom{18}{5} = \frac{18*17*16*15*14}{5*4*3*2*1}$$

$$\binom{12}{5} \Big/ \binom{18}{5} = \frac{12*11*10*9*8}{1*2*3*4*5} * \frac{5*4*3*2*1}{18*17*16*15*14} = \frac{12*11*10*9*8}{18*17*16*15*14}$$

Using Conditional Probabilities: $\underbrace{(12/18)*(11/17)*(10/16)*(9/15)*(8/14)}$ = 9.24%

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Hypergeometric Distribution

Hypergeometric assumptions

1. Outcome of each trial is either a “success” or “failure”.
2. The probability of a success is not constant.
   Sampling without replacement
3. Trials are not independent

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Understanding the assumptions

For each of these situations/questions, indicate if hypergeometric distribution is applicable

Yes / No	A coin is flipped 6 times. What is the probability of exactly 2 heads out of the 6 flips?
Yes / No	A box contains 30 shapes: 6 balls, 15 blocks, and 9 pyramids. Drawing 4 shapes out without replacement, what is the probability of exactly 3 blocks and 1 ball?
Yes / No	A box contains 30 shapes: 6 balls, 15 blocks, and 9 pyramids. Drawing 4 shapes out, without replacement. What is the probability of exactly 2 blocks out of the 4?

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Hypergeometric Distribution

Hypergeometric assumptions

1. Outcome of each trial is either a “success” or “failure”.
2. The probability of a success is not constant.
   Sampling without replacement
3. Trials are not independent

What type of numerical data are we dealing with when using the hypergeometric distribution?

A. Continuous B. Discrete C. It depends on the application

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Hypergeometic Distribution

18 parts are in a lot presented for inspection. The are 6 defective and 12 good parts are in the lot

A sample of 5 parts will be randomly drawn without replacement in accordance with inspection procedures.

What is the probability that there will be 2 defective and 3 good in the sample?

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Any sample of 5 parts is equally likely, therefore

P(2 defective and 3 good) =

$$\frac{\text{\# of outcomes that result in 2 defective and 3 good}}{\text{\# of possible samples of size 5}}$$

For the denominator:

\# of possible samples of size 5 is $\binom{18}{5} = 8568$

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Any sample of 5 parts is equally likely, therefore

P(2 defective and 3 good) =

# of outcomes that result in 2 defective and 3 good
# of possible samples of size 5

For the numerator

combinations resulting in 2 defective is $\binom{6}{2}$ = 15

combinations resulting in 3 good is $\binom{12}{3}$ = 220

Multiply to get the # combinations with 2 defective and 3 good = 3300

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Any sample of 5 parts is equally likely, therefore

P(2 defective and 3 good) =

# of outcomes that result in 2 defective and 3 good
# of possible samples of size 5

P(2 defective and 3 good) =

$$\frac{\binom{6}{2}\binom{12}{3}}{\binom{18}{5}} = \frac{15 * 220}{8568} = .385$$

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Hypergeometric Distribution

$$p(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

Where *N = total number of elements in the population*
r = number of success in the population
N-r = number of failures in the population
n = number of trials (sample size)
x = number of successes in trial
n-x= number of failures in n trials

Let p=r/N, then $\mu_x = np$, and $\sigma_x{}^2 = np(1-p)(\frac{N-n}{N-1})$.

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Hypergeometic Distribution

There are 18 parts presented for inspection. 6 of the parts are defective and 12 of the parts are good. A sample of 5 parts is to be inspected.

What is the probability that there will be 2 defective and 3 good parts in the sample?

Define: Defective part as success
Good part as failure.

N = 18 *total number of elements in the population*
r = 6 *number of success in the population*
N-r = 12 *number of failures in the population*
n = 5 *number of trials (sample size)*
x = 2 *number of successes in trial*
n-x = 3 *number of failures in n trials*

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Hypergeometic Distribution

There are 18 parts presented for inspection. 6 of the parts are defective and 12 of the parts are good. A sample of 5 parts is to be inspected.

What is the probability that there will be 2 defective and 3 good parts in the sample?

Define: Defective part as success
Good part as failure.

N = 18
r = 6
N-r = 12
n = 5
x = 2
n-x = 3

$$p(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

$$\text{P}(2) = \frac{\binom{6}{2}\binom{12}{3}}{\binom{18}{5}} = \frac{15*220}{8568} = .385$$

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There are 18 parts presented for inspection. 6 of the parts are defective and 12 of the parts are good. A sample of 5 parts is to be inspected.

What is the probability that there will be 2 defective and 3 good parts in the sample?

Define: Defective part as success
Good part as failure.

	Successes	Failures
Population		
Sample		

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There are 18 parts presented for inspection. 6 of the parts are defective and 12 of the parts are good. A sample of 5 parts is to be inspected.

What is the probability that there will be 2 defective and 3 good parts in the sample?

Define: Defective part as success
Good part as failure.

	Successes *# Defective*	Failures *# Good*
Population	6	12
Sample		

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What is the probability that there will be 2 defective and 3 good parts in the sample?

Define: Defective part as success
Good part as failure.

	Successes *# Defective*	Failures *# good*
Population	6	12
Sample	2	3

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There are 18 parts presented for inspection. 6 of the parts are defective and 12 of the parts are good. A sample of 5 parts is to be inspected.

What is the probability that there will be 2 defective and 3 good parts in the sample?

Define: Defective part as success
Good part as failure.

	Successes # Defective	Failures # Good
Population	6	12
Sample	2	3

Totals	
Population	18
Sample	5

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There are 18 parts presented for inspection. 6 of the parts are defective and 12 of the parts are good. A sample of 5 parts is to be inspected.

What is the probability that there will be 2 defective and 3 good parts in the sample?

Define: Defective part as success
Good part as failure.

	Successes	Failures
Population / Sample	$\binom{6}{2}$	$\binom{12}{3}$

Totals: Population / Sample $\binom{18}{5}$

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N = 18
r = 6
N-r = 12
n = 5
x = 2
n-x = 3

$$p(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

$$P(2) = \frac{\binom{6}{2}\binom{12}{3}}{\binom{18}{5}} = \frac{15*220}{8568} = .385$$

	Successes	Failures
Population / Sample	$\binom{6}{2}$	$\binom{12}{3}$

Totals: Population / Sample $\binom{18}{5}$

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Hypergeometic Distribution

There are 18 parts presented for inspection. 6 of the parts are defective and 12 of the parts are good. A sample of 5 parts is to be inspected.

On average, how many defective parts would be in the sample?

N = 18
r = 6
N-r = 12
n = 5
x = 2
n-x = 3

p=r/N = 6/18 = .333

$\mu_x = np$ = (5)(.333) = 1.665

$\sigma_x{}^2 = np(1-p)(\frac{N-n}{N-1})$.= .849

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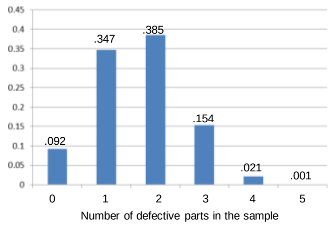
There are 18 parts presented for inspection. 6 of the parts are defective and 12 of the parts are good. A sample of 5 parts is to be inspected.



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A piggy bank contains 3 pennies and 5 dimes. If you shake out 4 coins, what is the probability that you will get exactly 2 dimes?

The population size, N is

A) 2 B) 3 C) 4 D) 5 E) 8

The sample size, n is

A) 2 B) 3 C) 4 D) 5 E) 8

If "success" is defined as getting a dime, how many successes are we looking for in the sample?

A) x=2 B) x=3 C) x= 4 D) x=5

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A piggy bank contains 3 pennies and 5 dimes. If you shake out 4 coins, what is the probability that you will get exactly 2 dimes?

Using "hyper geometric without formulas", how would this problem be set up? (Assume dimes=success)

A)

	Successes	Failures
Population / Sample	$\binom{5}{2}$	$\binom{2}{3}$
Totals (Population / Sample)	$\binom{8}{4}$	

B)

	Successes	Failures
Population / Sample	$\binom{5}{2}$	$\binom{3}{2}$
Totals (Population / Sample)	$\binom{8}{4}$	

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$$p(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

A piggy bank contains 3 pennies and 5 dimes. If you shake out 4 coins, what is the probability that you will get exactly 2 dimes?

A) 0.3581 B) 0.6122 C) 0.5013 D) 0.4286

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A piggy bank contains 3 pennies and 5 dimes. If you shake out 4 coins, what is the probability that you will get exactly 2 dimes?

$$p(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

T / F when working the problem, the combination $\binom{8}{4}$ gives the number of ways the coins could be drawn from the bank that have two dimes in it.

T / F When working the problem, the combination $\binom{5}{2}$ gives the total number of ways that the five dimes could arranged when two of them are in a sample.

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A piggy bank contains 3 pennies and 5 dimes. If you shake out 4 coins, what is the probability that you will get exactly 2 dimes?

$$p(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

T / F When you multiply the combinations $\binom{5}{2} x \binom{3}{2}$ this represents the total number of ways that the coins could be drawn from the bank that have two dimes in it.

T / F The formula divides the number of outcomes with 2 dimes in a sample of 4, by the total number of possible samples of 4.

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There are 12 barrels of chemicals sitting on a dock. A tipster calls the office and tells them 3 of the barrels are contaminated. The manager suspects a hoax, but decides to check 4 barrels.

The population size, N is
A) 3 B) 4 C) 12 D) 16

The sample size, n is A) 2 B) 3 C) 4

Assuming it is not a hoax, If "success" is defined as finding a contaminated barrel, how many successes are in the population?

A) 2 B) 3 C) 4

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There are 12 barrels of chemicals sitting on a dock. A tipster calls the office and tells them 3 of the barrels are contaminated. The manager suspects a hoax, but decides to check 4 barrels.

If the threat really is true, what is the probability that there will be no contaminated barrels in the sample?

Assuming contamination=success, the "without formulas set up for this problem is

A)

	Successes	Failures
Population / Sample	$\binom{3}{0}$	$\binom{9}{4}$
Totals (Population / Sample)	$\binom{12}{4}$	

B)

	Successes	Failures
Population / Sample	$\binom{4}{0}$	$\binom{9}{3}$
Totals (Population / Sample)	$\binom{12}{4}$	

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$$p(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$$

There are 12 barrels of chemicals sitting on a dock. A tipster calls the office and tells them 3 of the barrels are contaminated. The manager suspects a hoax, but decides to check 4 barrels.

If the threat really is true, what is the probability that there will be no contaminated barrels in the sample?

A. 0.311 B. 0.248 C. 0.255 D. 0.287

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More Problems

A box contains the 30 shapes: 20% of the shapes are squares, 20% are circles, and 60% are diamonds.

What is the probability of getting exactly 4 diamonds when selecting 10 shapes at random from the box?

a. 0.3058 b. 0.0941 c. 0.0541 d. 0.1244

What is the probability of getting exactly 3 squares when selecting 10 shapes at random from the box?

a. 0.3058 b. 0.0941 c. 0.0541 d. 0.2304

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End of lecture