Probability Worksheet (1/31/14)

Part A. Basic Probability

The problem here, is designed to illustrate basic probability theory and approaches to solving problems.

Consider the following

> A process is producing items that may be misshapen (Type M defect) or poor color (type C defect) or both. The probability of a Type M defect is 0.05 and of a Type C defect is 0.04.

In drawing out one item there are 4 possible outcomes.

Good shape Good color	Good shape Bad color	Bad shape Good color	Bad shape Bad Color

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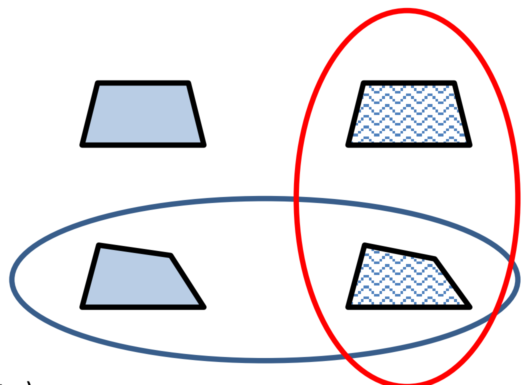
Considering the following:

Let M = Event of having defect Type M P(M)= .05
Let C = Event of having defect type C P(C) = .04

These events are not mutually exclusive as an item may have both. They are **independent**.

For independent outcomes P(A and B) = P (A∩B) = P(A)P(B)



This is known as the intersection of the two events.
(Note: the formula may be expanded to any number of independent events.)

We can use this formula to directly answer the following question:

What is the probability of a single item having both defects?

Since we know that the complement of Event A: $P(A^c) = P(not\ A) = 1 - P(A)$

We can answer this question using our knowledge about the probabilities of independent events:

What is the probability of a single item having not having defect type M?

Now determine the probabilities for each of the possible outcomes using what we have just reviewed. Complete the table. (One row is completed for you to provide an example of how to complete the table.)

Type M Defect (misshapen)	**Type C Defect (poor color)**	**Probability Calculation** For independent outcomes the general formula is P(A and B) = P (A∩B) = P(A)P(B)	**Probability**	**Outcome**
Yes	Yes			L
Yes	Not C	P(M) * P (not C) = .05 * (1-.04)	0.048	Z
Not M	Yes			N
Not M	Not C			R
		Total		

Notice the probabilities for each combination are calculated by multiplying because they are independent events.

While M and C are independent events, each combination is a **mutually exclusive outcome** when looked at from the perspective of the overall inspection "experiment". For discussion purposes they are noted as L, Z, N and R.

The sum of the probabilities of all possible outcomes must be equal to 1. Check your work above by verifying the total of the probabilities of the four possible outcomes is 1.

For mutually exclusive outcomes: $P(A \text{ } or \text{ } B) = P(A \cup B) = P(A) + P(B)$

This formula may be expanded to include any number of mutually exclusive events in an expirement

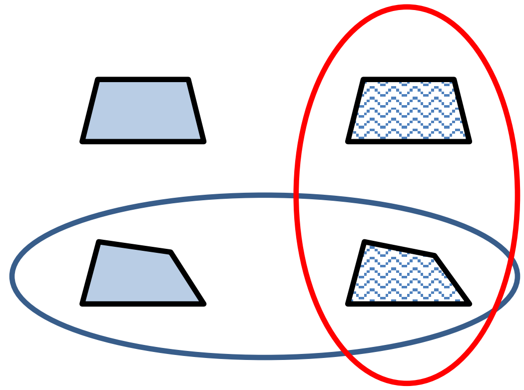
Our three mutually exclusive alternatives that have at least one defect are L, Z and N, therefore

What is the probability of having at least one defect?

Another way to work it is using: complement of Event A: $P(A^c) = P(not \text{ } A) = 1 - P(A)$

What is the probability of having at least one defect?

Another way to look at this problem is to realize that in the sample space it is the probability of A and B less the intersection.



Our formula is

$$P(A\ or\ B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

For our problem: P(M or C) = P(M) +P(C)- P(M∩C)

We previously found P(M and C) = P (A∩B) = P(M)P(C) = .05 *.04 = 0.002

What is the probability of having at least one defect?

Part B Conditional Probability

Assume we have manufactured 100 parts. 12 of these are known to be defective but the 12 bad ones got mixed up in a box with the good ones.

Recall that: If P(A) is the probability of event A occurring, then $P(A) = \frac{number\ of\ ways\ A\ can\ occur}{total\ number\ of\ possible\ outcomes}$

If we take one out of the box, what is the probability it will be defective?

Say the first part was defective and we take a second one out without putting the first back. That second event (taking out the second part) is not independent of the first event (taking out the first) because the number of ways the event can occur has changed and the number of possible outcomes has changed, thus changing the probability.

If we take a second one out, without putting the first back, what is the probability it will be defective?

Where events are not independent, the probability of both events happening is

P(A and B) = P (A) *P(B/A)

P(B/A) is the probability of B given that A has happened.

What is the probability of getting two defective parts in a row out of the box (assuming we do not put the first one back?

Part C. Problems

1) A process is producing material that is 30% defective. Four pieces are selected at random for inspection, what is the probability that there are no defective pieces in the sample?

Hint: The process is known to produce 30% bad parts so each part has the same probability of being good or bad – independent of each other. So this question could be phrased as "What is the probability of getting 4 good pieces in a row.?"

2) A process is producing 10% defective parts. If 10 parts are drawn randomly from the process, what is the probability that at least one part will have a defect?

Hint: Think about the complement?

3) Medication is to be dispensed in a hospital. In general, a doctor writes the prescription, the pharmacy fills it, then the nursing staff administers it to the patient. Say the doctor writes the prescription wrong 0.5% of the time, the pharmacy dispenses the incorrect medicine 0.5% of the time, and the nurse administers it incorrectly 0.5% of the time. What percent of the time will the patient's medication be appropriately taken as the doctor intended?

Hint: What are the probabilities of all three steps being done correctly? Are these mutually exclusive or independent events?

4) A bag contains 5 red marbles, 6 white marbles, 8 yellow marbles and 5 green marbles. What is the probability if one marble is drawn from the bag, that it will be either yellow or white?

5) Using the same bag of marbles as in the problem above, what is the probability of drawing a green marble then a yellow one?