Formulas – Confidence Intervals and Inference Tests (Rev 1_16)

Standard error of the mean $s_{\bar{x}} = {}^{\sigma}/_{\sqrt{n}}$

In formulas, point estimator s may be substituted for $\boldsymbol{\sigma}$ as appropriate.

CI for Means- Large sample size

Two sided: $\bar{x} \pm z\alpha/_2 \frac{\sigma}{\sqrt{n}}$

` One sided: $\bar{x} + Z_\alpha \frac{\sigma}{\sqrt{n}}$ or $\bar{x} - Z_\alpha \frac{\sigma}{\sqrt{n}}$

Sample size: $n = \left(\frac{Z\alpha/_2\sigma}{w}\right)^2$

CI for Means – small sample size - substitute *t* for Z in above equations

Two sided: $\bar{x} \pm t\alpha/_2 \frac{s}{\sqrt{n}}$

` One sided: $\bar{x} + Z_\alpha \frac{\sigma}{\sqrt{n}}$ or $\bar{x} - t_\alpha \frac{s}{\sqrt{n}}$

Comparing 1 sample mean with a known μ.

	Null Hypothesis	**Alternate Hypothesis**	**Test statistic**	**Rejection Region**
Normal Distribution (Large Sample)	$\mu = \mu_0$	$\mu \neq \mu_0$	$Z_{test} = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$	$\lvert z_{test} \rvert > z_{\alpha/2}$
		$\mu < \mu_0$		$z_{test} < -z_\alpha$
		$\mu > \mu_0$		$z_{test} > z_\alpha$
Students t Distribution (σ unknown, Small sample) $v = n - 1$	$\mu = \mu_0$	$\mu \neq \mu_0$	$t_{test} = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$	$\lvert t_{test} \rvert > t_{\alpha/2,v}$
		$\mu < \mu_0$		$t_{test} < -t_{\alpha,v}$
		$\mu > \mu_0$		$t_{test} > t_{\alpha,v}$

Sample size for 1 sample Z: $n = \frac{\left(Z\alpha/_2 + Z_\beta\right)^2 \sigma^2}{(\mu - \mu_0)^2}$

two sample means

	Null Hypothesis	Alternate Hypothesis	Test statistic	Rejection Region
Normal Distribution (Large Sample)	$\mu_1 = \mu_2$	$\mu_1 \neq \mu_2$	$Z_{test} = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$	$\lvert z_{test} \rvert > z_{\alpha/2}$
		$\mu_1 < \mu_2$		$z_{test} < -z_{\alpha}$
		$\mu_1 > \mu_2$		$z_{test} > z_{\alpha}$
Students t Distribution (Small sample sizes, assumes Equal Variances) t-test with $v = n_1 + n_2 - 2$	$\mu_1 = \mu_2$	$\mu_1 \neq \mu_2$	$t_{test} = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_p^2(\dfrac{1}{n_1} + \dfrac{1}{n_2})}}$ with $s_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$	$\lvert t_{test} \rvert > t_{\alpha/2,v}$
		$\mu_1 < \mu_2$		$t_{test} < -t_{\alpha,v}$
		$\mu_1 > \mu_2$		$t_{test} > t_{\alpha,v}$
Students t Distribution (Small sample sizes, assumes Variances not equal)	$\mu_1 = \mu_2$	$\mu_1 \neq \mu_2$	$t_{test} = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1{}^2}{n_1} + \dfrac{s_2{}^2}{n_2}}}$ where $v = \dfrac{\left[(s_X^2/n_X) + (s_Y^2/n_Y)\right]^2}{\left[\left[(s_X^2/n_X)^2/(n_X - 1)\right] + \left[(s_Y^2/n_Y)^2/(n_Y - 1)\right]\right]}$	$\lvert t_{test} \rvert > t_{\alpha/2,v}$
		$\mu_1 < \mu_2$		$t_{test} < -t_{\alpha,v}$
		$\mu_1 > \mu_2$		$t_{test} > t_{\alpha,v}$
Students t distribution Paired comparison t- test with $v = n - 1$	$\mu_d = \mu_0$	$\mu_d \neq \mu_0$	$t_{test} = \dfrac{\bar{D} - \mu_0}{S_d / \sqrt{n}}$ Where $\bar{D}$= average difference S_d= standard deviation of the differences	$\lvert t_{test} \rvert > t_{\alpha/2,v}$
		$\mu_d < \mu_0$		$t_{test} < -t_{\alpha,v}$
		$\mu_d > \mu_0$		$t_{test} > t_{\alpha,v}$

CI for variance $\dfrac{(n-1)s^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq \dfrac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}}$

Comparison of Normal Distribution Variances

	Null Hypothesis	Alternate Hypothesis	Test statistic	Rejection Region
2 sample variances F Distribution $v_1 = n_1 - 1$ $v_2 = n_2 - 1$	$\sigma_1^2 = \sigma_2^2$	$\sigma_1^2 \neq \sigma_2^2$	$F_{test} = \dfrac{s_1^2}{s_2^2}$ (larger sample variance always on top)	$F_{test} > F_{\alpha/2,v_1,v_2}$
		$\sigma_1^2 > \sigma_2^2$		$F_{test} > F_{\alpha,v_1,v_2}$

CI for proportions $p \pm Z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$

Sample size for CI, p known $n = \frac{z_{\alpha/2}^2 \, p(1-p)}{w^2}$

Sample size for CI, p unknown $n = \frac{z_{\alpha/2}^2}{4w^2}$

Comparing One proportion with a known p

Binomial Distribution (Large Sample)	$p = p_0$	$p \neq p_0$	$Z_{test} = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$	$\lvert z_{test}\rvert > z_{\alpha/2}$
		$p < p_0$		$z_{test} < -z_{\alpha}$
		$p > p_0$	with $\hat{p} = \dfrac{x}{n}$	$z_{test} > z_{\alpha}$

Comparing two proportions

Binomial Distribution Large Sample	$p_1 = p_2$	$p_1 \neq p_2$	$Z_{test} = \dfrac{\hat{p}_1 - \hat{p}_2}{\sigma_{p_1 - p_2}}$	$\lvert z_{test}\rvert > z_{\alpha/2}$
		$p_1 < p_2$	with	$z_{test} < -z_{\alpha}$
		$p_1 > p_2$	$\sigma_{p_1-p_2} = \sqrt{\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$	$z_{test} > z_{\alpha}$

Chi Squared tests for attribute data

	Null Hypothesis	**Alternate Hypothesis**	**Test statistic**	**Rejection Region**
χ2 test for goodness of fit ν = k-1	k cell probabilities are $p_1, p_2, p_3, \ldots p_k$	At least one cell probability is different than in H_0	$\chi^2 = \sum_{i=1}^{k} \dfrac{(O_i - E_i)^2}{E_i}$	$\chi^2_{test} > \chi^2_{\nu,\alpha}$
χ2 test Contingency table ν = (r-1)(c-1)	Two variables are independent	Two variables are dependent	$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c} \frac{(O_{ij}-E_{ij})^2}{E_{ij}}$ where $E_{ij} = \frac{(R_i)(C_j)}{n}$	$\chi^2_{test} > \chi^2_{\nu,\alpha}$