EIN 5226

Confidence Intervals for Means –Sigma known

Chapter 16 Sections 16.1-2
Chapter 17 Sections 17.4-6

Need Z table

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Point Estimates

- From a sample, we calculate statistics and use them as **point estimates** of population parameters
  - sample mean (Normal Distribution)
  - sample proportion (Binomial Distribution)
- More useful are interval estimates, which are called **confidence intervals**.

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Confidence Intervals for Means

.The confidence interval

- Provides margin of error for the sample statistic to indicate how far off the true value the point estimate could be.
- Provides a range of values in which repeated values of a sample statistic are expected to fall a certain percentage of time

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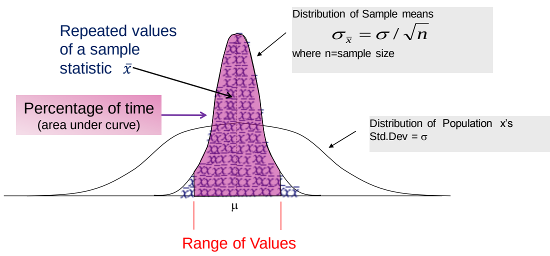
The Central Limit Theorem



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Confidence intervals

A confidence interval is a range of values within which repeated sample statistics are predicted to fall a certain percentage of the time.

α = risk that true population parameter will fail to fall in defined interval

Confidence level = 100 (1-α)%

Example: Risk $\boldsymbol{\alpha} = .05$ or 5%
Confidence Level= .95 or 95%

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Large Sample (n≥30)
Confidence Interval for Means

Needed

Sample mean, $\bar{x}$

Standard deviation of distribution of sample mean, $\sigma_{\bar{x}}$

Use s=$\hat{\sigma}$ to estimate $\sigma_{\bar{x}}$ with $^{s}/_{\sqrt{n}}$

Confidence level (1-α) or level of risk (α)

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A sample of size n=49 is taken from a population, we get a sample mean of 40.0 and sample standard deviation of standard deviation of 2.1. What is the 90% confidence interval for the true mean of the population?

Standard deviation of distribution of sample mean, $\sigma_{\bar{x}}$

Use s=$\hat{\sigma}$ to estimate $\sigma_{\bar{x}}$ with $^{s}/_{\sqrt{n}}$

What is the estimate of $\sigma_{\bar{x}}$ for the problem?

A. 2.100 B. 0.0429 C. 0.300 D. 0.420

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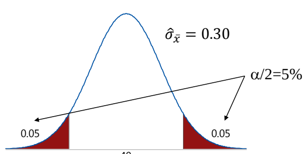
A sample of size n=49 is taken from a population, we get a sample mean of 40.0 and sample standard deviation of standard deviation of 2.1. What is the 90% confidence interval for the true mean of the population?

Distribution of sample averages assuming population mean is $\bar{x}$



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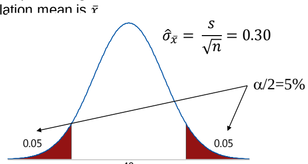
$$Z = \frac{x - \hat{u}}{\sigma/\sqrt{n}} = \frac{x - \bar{x}}{s/\sqrt{n}}$$

Distribution of sample averages assuming population mean is $\bar{x}$



What value of Z will correspond to the probabilities in the tails?

A) -3.30, +3.30 B) -2.575, +2.575 C) -1.645, +1.645 D) -2.578, +2.578

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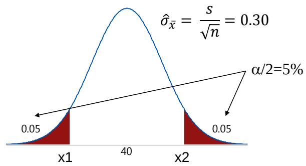
Distribution of sample averages
assuming population mean is $\bar{x}$

$$Z = \frac{x - \hat{u}}{\sigma/\sqrt{n}} = \frac{x - \bar{x}}{s/\sqrt{n}}$$

$$x = \bar{x} \pm Z\frac{s}{\sqrt{n}}$$



What value of X1?
A. 36.82 B. 39.51 C. 37.13 D. 38.12

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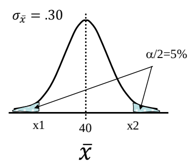
A sample of size n=49 is taken from a population, we get a sample mean of 40.0 and sample standard deviation of standard deviation of 2.1. What is the 90% confidence interval for the true mean of the population?

$$Z = \frac{x-\hat{u}}{\sigma/\sqrt{n}} = \frac{x-\bar{x}}{s/\sqrt{n}} = \pm 1.645$$

$$x = \bar{x} \pm Z\frac{s}{\sqrt{n}}$$



Lower limit = x_1 =

$$\bar{x} - Z_{\alpha/2}\frac{s}{\sqrt{n}} = 40 - 1.645\frac{2.1}{\sqrt{49}} = 39.51$$

Upper limit = x_2 =

$$\bar{x} + Z_{\alpha/2}\frac{s}{\sqrt{n}} = 40 + 1.645\frac{2.1}{\sqrt{49}} = 40.49$$

Therefore with 90% confidence we can say the true mean of the population is between 39.51 and 40.49

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Large Sample (n≥30)
Confidence Interval for Means

Needed

Sample mean, $\bar{x}$

Standard deviation of distribution of sample mean, $\sigma_{\bar{x}}$

Use s=$\hat{\sigma}$ to estimate $\sigma_{\bar{x}}$ with $s/\sqrt{n}$

Confidence level (1-α) or level of risk (α)

Confidence interval formulas:

Lower Limit	Upper limit
$\bar{x} - z_{\alpha/2}\frac{s}{\sqrt{n}}$	$\bar{x} + z_{\alpha/2}\frac{s}{\sqrt{n}}$

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Practice
Large Sample, Interval for Means

$\sigma_{\bar{x}} = {}^{s}/_{\sqrt{n}}$

A dishwasher manufacturer studied the noise level on its new design by studying 55 units. The average decibel level was found to be 65 with a standard deviation of 12. What is the 95% confidence level for the new model's average noise level?

The risk level, $\boldsymbol{\alpha}$, for this problem is

A) 10% B) 5% C) 2.5% D) 12%

The standard error of the mean, $\sigma_{\bar{x}}$ applicable in this problem is

A) 12 B) 7.218 C) 1.618 D) 1.839

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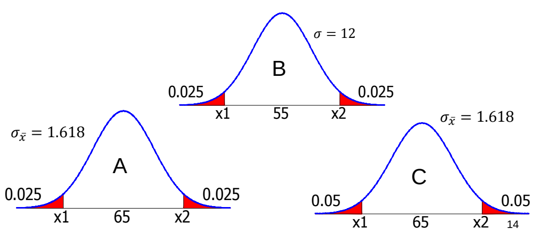
A dishwasher manufacturer studied the noise level on its new design with 55 units. The average decibel level was found to be 65 with a standard deviation of 12. What is the 95% confidence level for the new model's average noise level?

Which is the correct depiction of the problem being analyzed?



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Confidence interval formulas: Lower Limit $\bar{x} - z_{\alpha/2}\frac{s}{\sqrt{n}}$ Upper limit $\bar{x} + z_{\alpha/2}\frac{s}{\sqrt{n}}$

A dishwasher manufacturer studied the noise level on its new design by studying 55 units. The average decibel level was found to be 65 with a standard deviation of 12. What is the 95% confidence level for the new model's average noise level?

The Z value for this problem is

A) ± 1.645 B) ± 1.96 C) ± 2.81 D) ± 1.85

The lower and upper limits are

A) 61.83, 72.01 B) 60.83, 69.17

C) 52.08, 57.92 D) 61.83, 68.17

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Question?

Does this 95% confidence interval actually cover the true mean of the population, μ?

- The sample observations come from the middle 95% of the population distribution and then the true mean would be in the interval.
- If the sample mean was unusually large or small, the observations in it may have come from the outer 5% of the population. In this case, the true mean will not be in the interval.
- In the long run, if we repeated these confidence intervals over and over, then 95% of the samples will have means in the middle 95% of the population. Then 95% of the confidence intervals will cover the population mean.

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A dishwasher manufacturer studied the noise level on its new design by studying 55 units. The average decibel level was found to be 65 with a standard deviation of 12. Based on the sample, the 95% confidence level for the new model's average noise level was determined to be [61.8 and 68.2].

T / F The mean of the population is 65.

T / F The probability that the true mean is in the defined interval is 95%.

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Confidence Interval Simulation

Rice Virtual Lab in Statistics
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Confidence interval Simulation
stat_sim\RVLS_simulations\stat_sim\conf_interval\Contents.html

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A dishwasher manufacturer studied the noise level on its new design by studying 55 units. The average decibel level was found to be 65 with a standard deviation of 12. Based on the sample, the 95% confidence level for the new model's average noise level was determined to be [61.8 and 68.2].

T / F If I increase the sample size and calculate a new 95% confidence interval, the probability of true being in the interval will improve.

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Confidence interval formulas: Lower Limit $\bar{x} - z_{\alpha/2}\frac{s}{\sqrt{n}}$ Upper limit $\bar{x} + z_{\alpha/2}\frac{s}{\sqrt{n}}$

A dishwasher manufacturer studied the noise level on its new design by studying **100** units. The average decibel level was found to be 65 with a standard deviation of 12. What is the 95% confidence level for the new model's average noise level?

The Z value for this problem is

A) ± 1.645 B) ± 1.96 C) ± 2.81 D) ± 1.85

The lower and upper limits are

A) 61.83, 68.17 B) 63.10, 66.9

C) 63.12, 66.88 D) 62.65, 67.35

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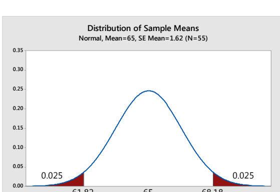

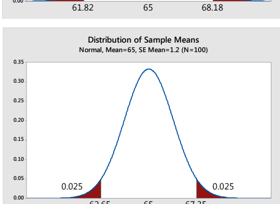

The larger the sample size, the more **precise** the confidence interval is said to be.

For the given sample size, you will still always have the same risk, α, used to compute the interval.

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Estimating Sample Sizes

Larger Sample Size

= Smaller Confidence Interval

= Greater Accuracy & Precision

Estimate sample size for mean:

$$n = \left(\frac{Z_{\alpha/2}\sigma}{w}\right)^2$$

Always round up or you will get less than your desired level of confidence

where w = desired width (±w)

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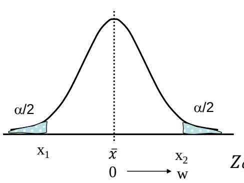

$$Z_{\alpha/2} = \frac{x - u}{\sigma/\sqrt{n}} = \frac{w - 0}{\sigma/\sqrt{n}}$$

Formula for estimating sample size: $n = \left(\frac{Z_{\alpha/2}\sigma}{w}\right)^2$

To use need: Level of risk willing to take (to determine Z)
Population standard deviation (or estimate)
w: desired ± interval

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Sample Size - Means

A dishwasher manufacturer studied the noise level on its new design with 55 units. The average decibel level was found to be 65 with a standard deviation of 12.

A minor design change was made and an additional study is required. How many units must be sampled so that a **99% confidence interval** specifies the mean to **within 10 decibels**

$Z_{(0.01/2)} = Z_{(.005)} = 2.575$

$\hat{\sigma} = s = 12$

$w = \pm 10$

$$n = \left(\frac{Z_{\alpha/2}\sigma}{w}\right)^2$$

$$n = \left(\frac{(2.575)(12)}{10}\right)^2 = 9.55 >> 10$$

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Sample Size - Means

$$n = \left(\frac{Z_{\alpha/2}\sigma}{w}\right)^2$$

A dishwasher manufacturer studied the noise level on its new design with 55 units. The average decibel level was found to be 65 with a standard deviation of 12.

The company president thinks the interval previously used is too large. How many units must be sampled so that a 95% confidence interval specifies the mean to within 5 decibels.

A) 39 B) 5 C) 22 D) 23

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Sample Size - Means

$$n = \left(\frac{Z_{\alpha/2}\sigma}{w}\right)^2$$

A manufacturer specifies that the length of a purchased part be 2.50 ± 0.15. For the type of process used to produce the part a process variance of 0.0016 is typical (use to estimate sigma). If the manufacture wants to have 95% certainty that a supplier has a process mean between 2.49 and 2.51, how many parts should be measured at receiving inspection?

For the sample size calculation, σ =

A) 0.0016 B) 0.050 C) 0.150 D) 0.040

For the sample size calculation, w =

A) 0.150 B) 0.020 C) 0.025 D) 0.010

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Sample Size - Means

$$n = \left(\frac{Z_{\alpha/2}\sigma}{w}\right)^2$$

A manufacturer specifies that the length of a purchased part be 2.50 ± 0.15. For the type of process used to produce the part a process variance of 0.0016 is typical (use to estimate sigma). If the manufacture wants to have 95% certainty that a supplier has a process mean between 2.49 and 2.51, how many parts should be measured at receiving inspection?

For the sample size calculation, $Z_{\alpha/2}$ is

A) 1.96 B) 1.645 C) 2.575 D) 0.3085

The sample size, n, needed is

A) 61 B) 62 C) 7 D) 8

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One-Sided Confidence Intervals

(Often called bounds rather than intervals)

- Often situations require only an upper or a lower bound. A one sided confidence interval is used.
- With the same conditions as with the two-sided CI, the level 100(1-α)%
  - A lower confidence bound for μ is
    $$\overline{X} - z_\alpha \sigma_{\overline{X}}.$$
  - An upper confidence bound for μ is
    $$\overline{X} + z_\alpha \sigma_{\overline{X}}.$$
- All the risk, $\boldsymbol{\alpha}$, is included in the one tail.

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One-sided Confidence Bounds

Lower: $x = \bar{x} - Z_\alpha \frac{s}{\sqrt{n}}$ Upper: $x = \bar{x} + Z_\alpha \frac{s}{\sqrt{n}}$

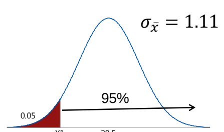
A company is trying to do a better job of projecting labor requirements. A manual assembly task was performed by 30 operators. The average time to complete the task was 20.5 minutes with a standard deviation of 6.1 minutes.

What is the 95% lower confidence bound for the average time?

$$\sigma_{\bar{x}} = {}^{s}/_{\sqrt{n}} = {}^{6.1}/_{\sqrt{30}} = 1.11$$

$Z_{.05} = 1.645$ (from table)

$$x = \bar{x} - Z_\alpha \frac{s}{\sqrt{n}}$$
$$= 94 - (1.645)(1.11) = 18.67$$



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A company is trying to do a better job of projecting labor requirements. A manual assembly task was performed by 30 operators. The average time to complete the task was 20.5 minutes with a standard deviation of 6.1 minutes.

Above what time would 95% of individual operators be expected to complete the task?

Which of the following statements is correct about working this problem?

A) This problem would be worked the same way as the last one.

B) The applicable standard error of the mean is
$$\sigma_{\bar{x}} = {}^{s}/_{\sqrt{n}} = {}^{6.1}/_{\sqrt{30}} = 1.11$$

C) You would not use the distribution of the means as the question is about individual observations .

D) The sample size requires use of a double sided approach.

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One-sided Confidence Bounds

Lower: $x = \bar{x} - Z_\alpha \frac{s}{\sqrt{n}}$ Upper: $x = \bar{x} + Z_\alpha \frac{s}{\sqrt{n}}$

Mercury levels in fish are considered detrimental for human consumption when over 0.3 parts per million. This level is considered a “trigger point” for posting of fishing advisories warning dangers of fish consumption.

36 fish were tested at a sampling station at a popular fishing location. The mean mercury level was 0.275 ppm with a standard deviation of 0.081

What is the 95% upper confidence bound for the true mean mercury ppm at the location?

A. 0.292 ppm B. 0.248 ppm C. 0.323 ppm D. 0.278 ppm

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Alternate Notation Confidence Interval Equations

Common notation, two sided

Lower limit Upper limit

$\bar{x} - z_{\alpha/2} \frac{s}{\sqrt{n}}$ $\bar{x} + z_{\alpha/2} \frac{s}{\sqrt{n}}$

Textbook notation, two sided

Lower limit Upper limit

$\bar{x} - U_\alpha \frac{s}{\sqrt{n}}$ $\bar{x} + U_\alpha \frac{s}{\sqrt{n}}$

U values found in Table C

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My notation: Where
α = total risk in problem,
use Z table with
$Z_{\alpha/2}$ to work problems

If risk α=5%, and test is two tailed test, use Z table for $Z_{.05/2}$ = $Z_{.025}$= 1.96

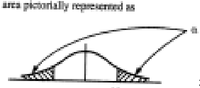
Books notation: Alpha is area under the curve – same on both sides.

If risk α=5%, and test is two tailed test, use Table C for U with α=5%,.
$U_{.05}$= 1.96

TABLE C Probability Points of the Normal Distribution: Double-Sided (Variance Known)

α only	U	α only	U
0.001	3.291	0.100	1.645
0.005	2.807	0.150	1.440
0.010	2.576	0.200	1.282
0.015	2.432	0.300	1.036
0.020	2.326	0.400	0.842
0.025	2.241	0.500	0.675
0.050	1.960	0.600	0.524

Note 1: The same information can be obtained from Table A; however, this table format is different.
Note 2: In this text the tabular value corresponds to U_α, where α is the value of probability associated with the distribution area pictorially represented as



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