EIN 5226

Variable Control Charts

Part A

Chapter 10 Sections 9-14

Need:
Table of control chart factors
Calculator

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Statistical Process Control

SPC – using statistics to monitor processes and make decisions

Control charts - identify variation in process

- **Mechanics of establishing control limits**
- Planning steps
- Monitoring a process with control charts

Capability Analysis

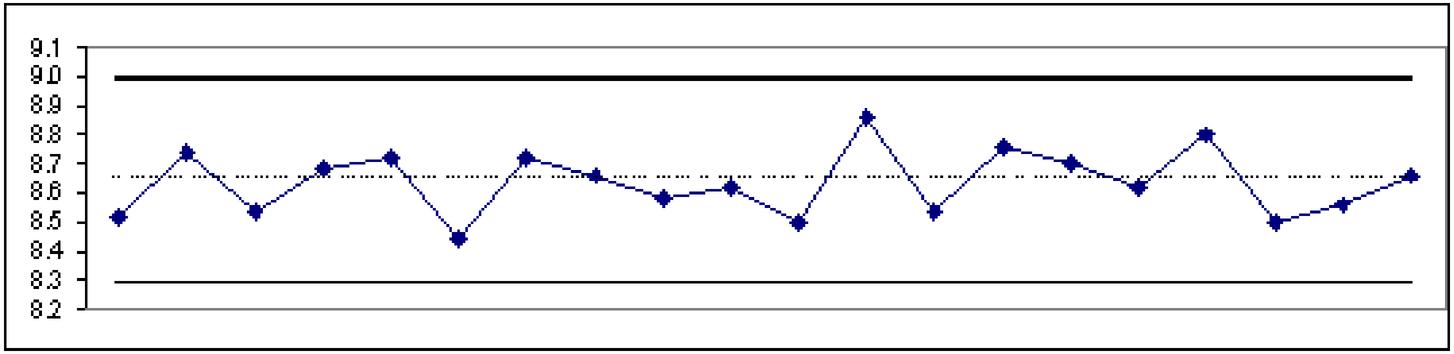
- Determine the ability of a stable process to meet requirements.

Variable Control Charts

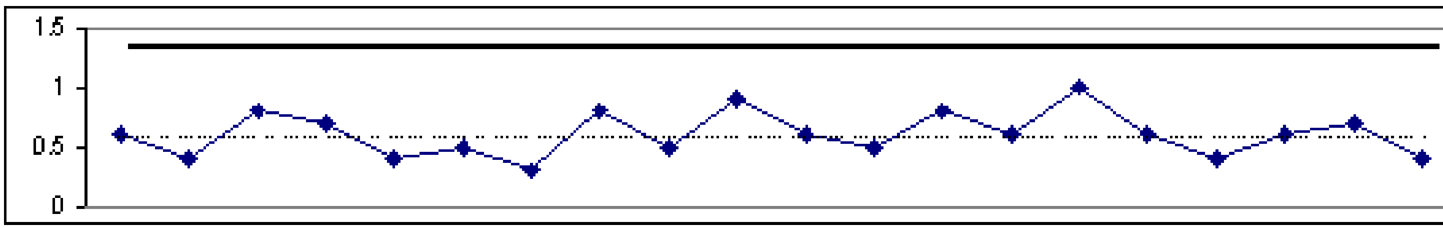
- Used when process output can be characterized by variable measurements
  - Examples: Size, weight, temperature, pressure
  - May be a part characteristic or a process parameter

- Two charts used. Most common are
  - For monitoring central tendency
    - $\bar{x}$ chart: Mean of the samples
  - For monitoring dispersion
    - r chart: range of samples
    - s chart: Standard deviation of samples

AVERAGES	$\bar{\bar{X}}$ = 8.64	UCL= 9.0 $(\bar{\bar{X}} + A_2\bar{R})$	LCL= 8.29 $(\bar{\bar{X}} - A_2\bar{R})$



RANGE	$\bar{R}$= 0.61	UCL = 1.28 $(D_4\bar{R})$	LCL = 0 (0 for Sample ≤ 5)

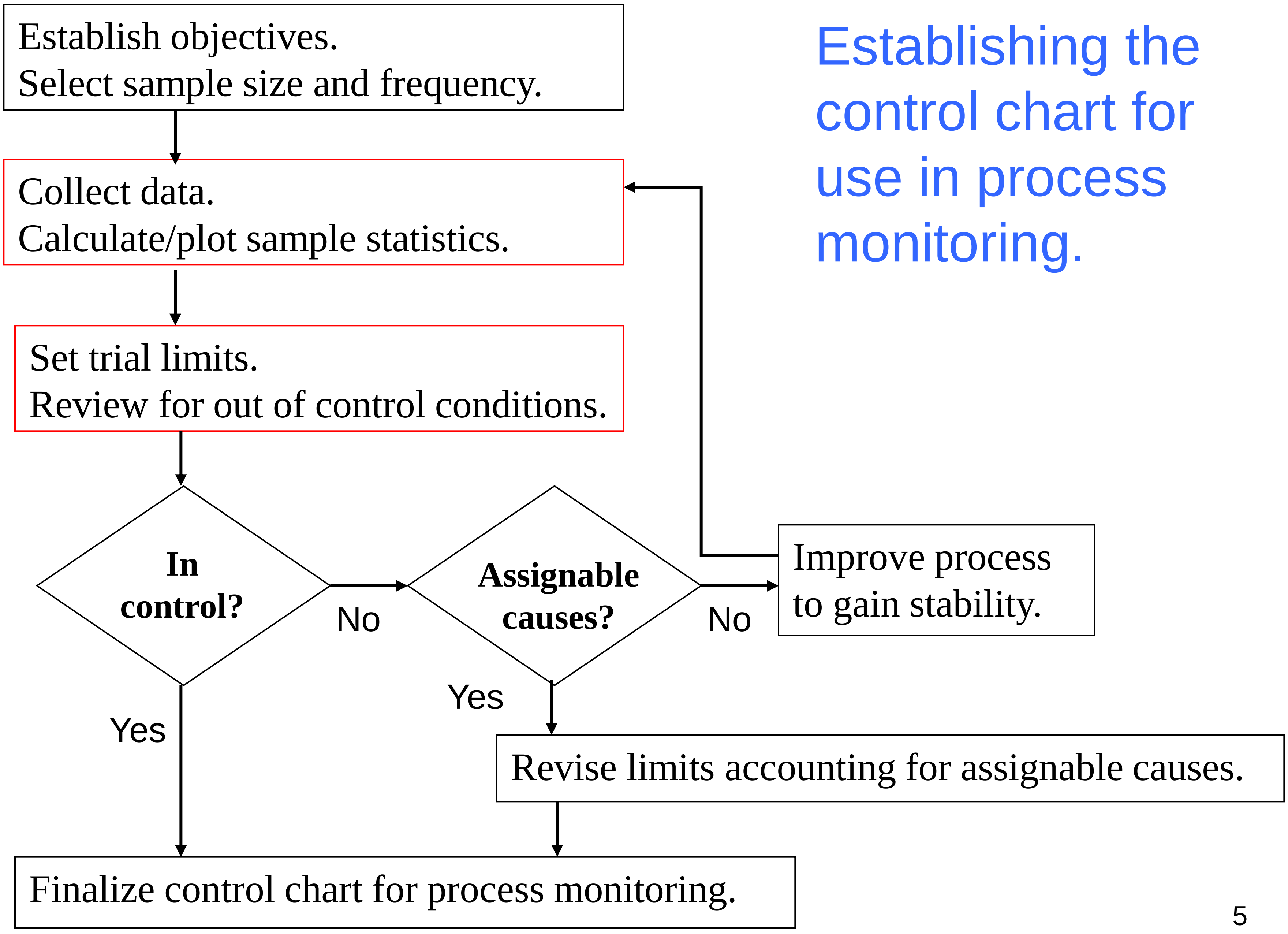


Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	8.7	8.8	8.1	9.1	8.7	8.5	8.8	8.7	8.6	9.1	8.4	8.9	8.6	8.9	8.5	8.6	8.6	8.6	8.2	8.9
2	8.5	8.9	8.5	8.8	8.8	8.1	8.7	8.3	8.4	8.8	8.6	9.1	8.9	9.1	8.1	8.8	9	8.8	8.6	8.7
3	8.8	8.5	8.6	8.5	8.9	8.4	8.9	8.5	8.5	8.6	8.2	8.9	8.5	8.5	8.9	8.9	8.9	8.3	8.9	8.5
4	8.4	8.9	8.9	8.6	8.5	8.6	8.6	8.7	8.5	8.2	8.5	8.6	8.1	8.7	9.1	8.5	8.8	8.2	8.6	8.6
5	8.2	8.6	8.6	8.4	8.7	8.6	8.6	9.1	8.9	8.4	8.8	8.8	8.6	8.6	8.9	8.3	8.7	8.6	8.5	8.6
Sum	42.6	43.7	42.7	43.4	43.6	42.2	43.6	43.3	42.9	43.1	42.5	44.3	42.7	43.8	43.5	43.1	44	42.5	42.8	43.3
$\bar{X}$	8.5	8.7	8.5	8.7	8.7	8.4	8.7	8.7	8.6	8.6	8.5	8.9	8.5	8.8	8.7	8.6	8.8	8.5	8.6	8.7
R	0.6	0.4	0.8	0.7	0.4	0.5	0.3	0.8	0.5	0.9	0.6	0.5	0.8	0.6	1.0	0.6	0.4	0.6	0.7	0.4

Constants for Subgroup Size = 5 : A2 = 0.58 D4 = 2.11

http://www.epplans.com/external/menu24/menu1.html

Establishing the control chart for use in process monitoring.



Variable control charts – data collection

$\overline{x}$ and r charts

Collect data at the process using control chart form

Need 25-30 subgroups to be statistically comfortable.

Terms used:

- n sample size (subgroup size)
- x value of single observation in sample
- $\overline{x}$ average of the x’s, readings in the sample
- m total number of samples/subgroups taken
- $\overline{\overline{x}}$ average of the sample averages
- r range within a subgroup
- $\overline{r}$ average of the ranges

Control Charts for Variables

Example problem:

Parts manufactured by an injection molding process are subjected to compressive strength test. Twenty-five samples of five parts each are collected and the compressive strengths with the data shown on the control chart.

.

Create the $\bar{x}$ and R charts for this process.

Why? Make sure the process is and stays stable.

Parts manufactured by an injection molding process are subjected to compressive strength test. Twenty-five samples of five parts each are collected and the compressive strengths with the data shown on the control chart.

Notation for variables control charts

- n - size of the sample (sometimes called a subgroup) chosen at a point in time

$$\boxed{n = 5}$$

- m - number of samples selected

$$\boxed{m = 25}$$

Notation for variables control charts

- For the first sample, we have values of x of

  81 77 80 81 80

- $\bar{x}_i$ = average of the observations in the ith sample (where i = 1, 2, ..., m)

for sample #1: $$\bar{x}_1 = \frac{81+77+80+81+80}{5} = 80$$

- $\bar{\bar{x}}$ = grand average or “average of the averages

from all samples: $$\bar{\bar{x}} = \frac{(80+81+.....+81)}{25} = 80.7$$

Notation and values

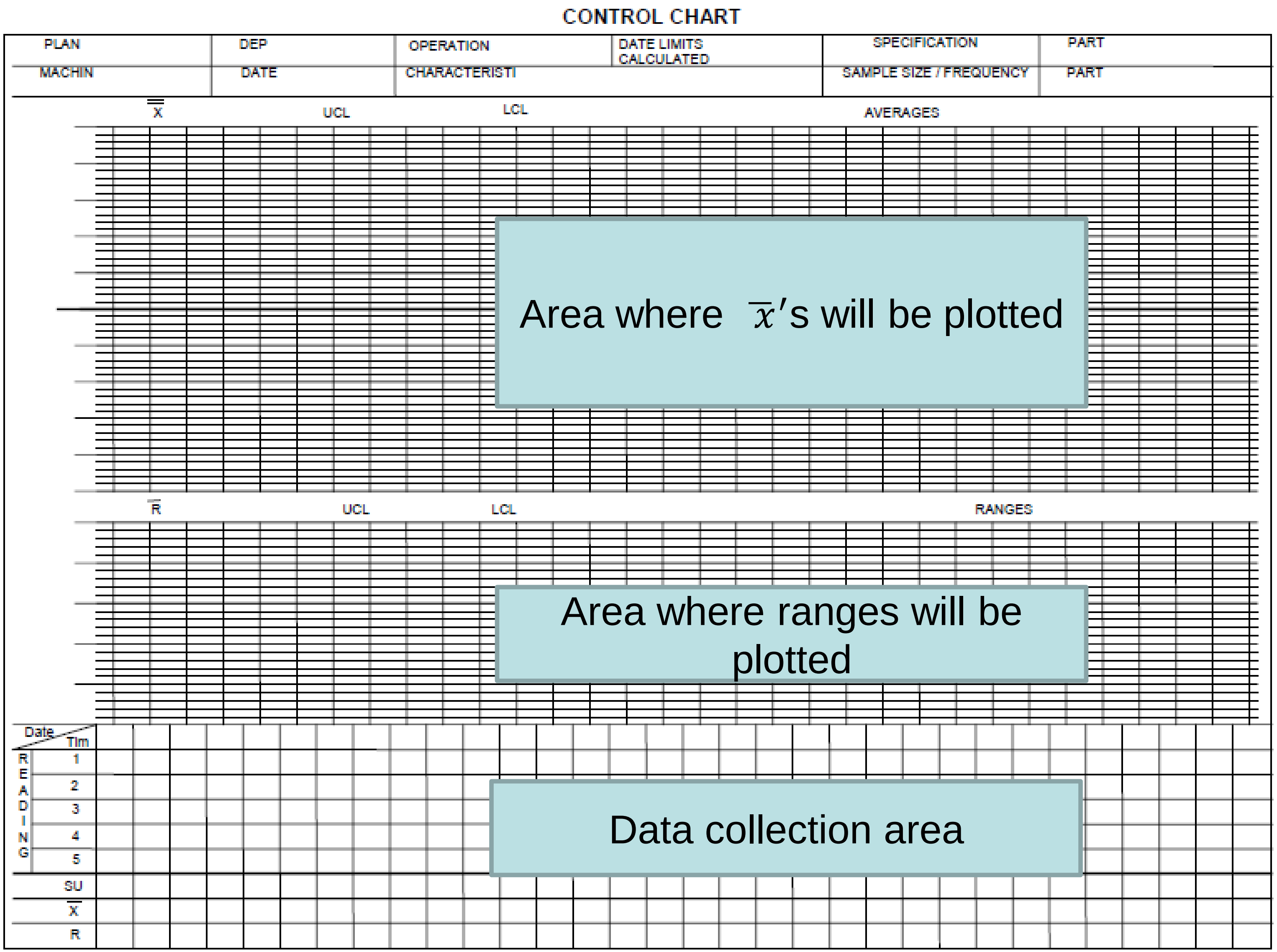
- R_i = range of the values in the ith sample

$$R_i = x_{max} - x_{min}$$

$$\boxed{R_1 = 81 - 77 = 4}$$

- $\overline{R}$ = average range for all m samples

$$\boxed{\overline{R} = \frac{(4+4+.....+8)}{25} = 7.8}$$


CONTROL CHART
PLAN
DEP
OPERATION
DATE LIMITS CALCULATED
SPECIFICATION
PART
MACHIN
DATE
CHARACTERISTI
SAMPLE SIZE / FREQUENCY
PART
$\overline{\overline{X}}$
UCL
LCL
AVERAGES
Area where $\overline{x}'$s will be plotted
$\overline{R}$
UCL
LCL
RANGES
Area where ranges will be plotted
Date
Tim
READING
1
2
3
4
5
SU
$\overline{X}$
R
Data collection area

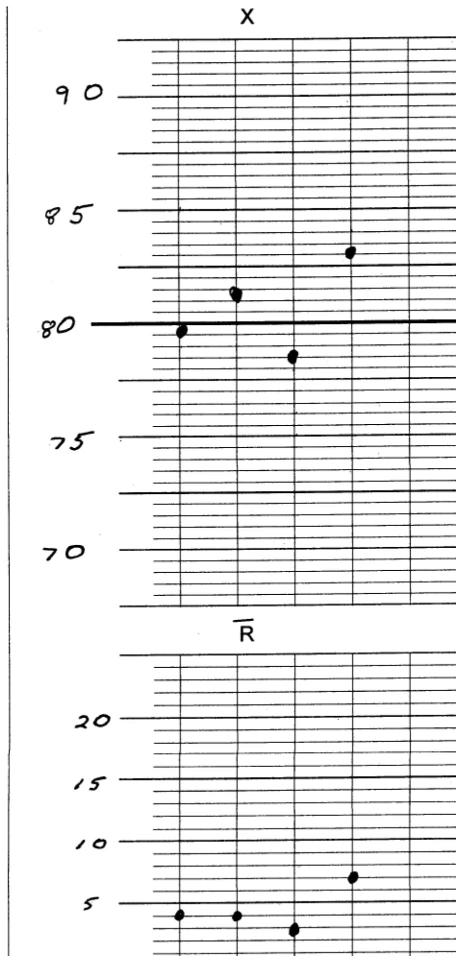
Sample # 1

Sample size n=5,
Five values entered

Date / Time		1	2	3	4			
READING	1	81	83	78	83			
	2	77	81	77	79			
	3	80	82	80	83			
	4	81	79	78	84			
	5	80	81	80	86			
	Sum							
	$\overline{X}$	79.8	81.2	78.6	83.0			
	R	4	4	3	7			

Sample#1 mean

Range
Sample #1



Date / Time		1	2	3	4	
READING	1	81	83	78	83	
	2	77	81	77	79	
	3	80	82	80	83	
	4	81	79	78	84	
	5	80	81	80	86	
	Sum					
	$\overline{X}$	79.8	81.2	78.6	83.0	
	R	4	4	3	7	

After getting 10-12 sample means, figure out what scale you need for the x bar and r graphs. Label the scales and start plotting points.

After all observations are made for the initial chart set-up, Calculate $\bar{R}$ and $\bar{\bar{x}}$ and draw center lines through the graph.

Next step is calculation of the control limits.

Control chart – General model

$$UCL = \mu_W + 3\sigma_W$$

$$Center \quad Line = \mu_W$$

$$LCL = \mu_W - 3\sigma_W$$

where

μ_W = mean of the sample statistic

σ_W = standard deviation of the statistic

Control chart – General model - $\bar{x}$ chart

$$UCL_{\bar{x}} = \mu_{\bar{x}} + 3\sigma_{\bar{x}} = \bar{\bar{x}} + 3\sigma_{\bar{x}}$$

$$CL_{\bar{x}} = \mu_{\bar{x}} = \bar{\bar{x}}$$

$$LCL_{\bar{x}} = \mu_{\bar{x}} - 3\sigma_{\bar{x}} = \bar{\bar{x}} - 3\sigma_{\bar{x}}$$

For a control chart of the sample mean

The sample statistic w is $\bar{x}$

$\mu_{\bar{x}}$ is estimated with $\bar{\bar{x}}$

The standard deviation of the sample statistic is $\sigma_{\bar{x}}$

An estimate of the standard deviation of the population can be made with the sample average range:

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$ where d2 is a constant

Therefore we can calculate the standard error of the means,

$$\hat{\sigma}_{\bar{x}} = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{\bar{R}}{d_2 \sqrt{n}}$$

Common Control Chart Factors

n	A_2	d_2	D_3	D_4
2	1.880	1.128	--	3.267
3	1.023	1.693	--	2.574
4	0.729	2.059	--	2.282
5	0.577	2.326	--	2.114
6	0.483	2.534	--	2.004
7	0.419	2.704	0.076	1.924
8	0.373	2.847	0.136	1.864
9	0.337	2.970	0.184	1.816
10	0.308	3.078	0.223	1.777

Estimating the Process Standard Deviation

For the injection molding problem, we decided to set up our control chart based on taking 25 samples of size 5.

$$\boxed{n = 5}$$

We took our samples and found

$$\boxed{\bar{\bar{x}} = 80.7} \qquad \boxed{\bar{R} = 7.8}$$

We can then estimate σ for the population, where n=5, d_2=2.326

$$\boxed{\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{7.8}{2.326} = 3.35}$$

Now we have

$$n = 5$$

$$\bar{\bar{x}} = 80.7$$

$$\bar{R} = 7.8$$

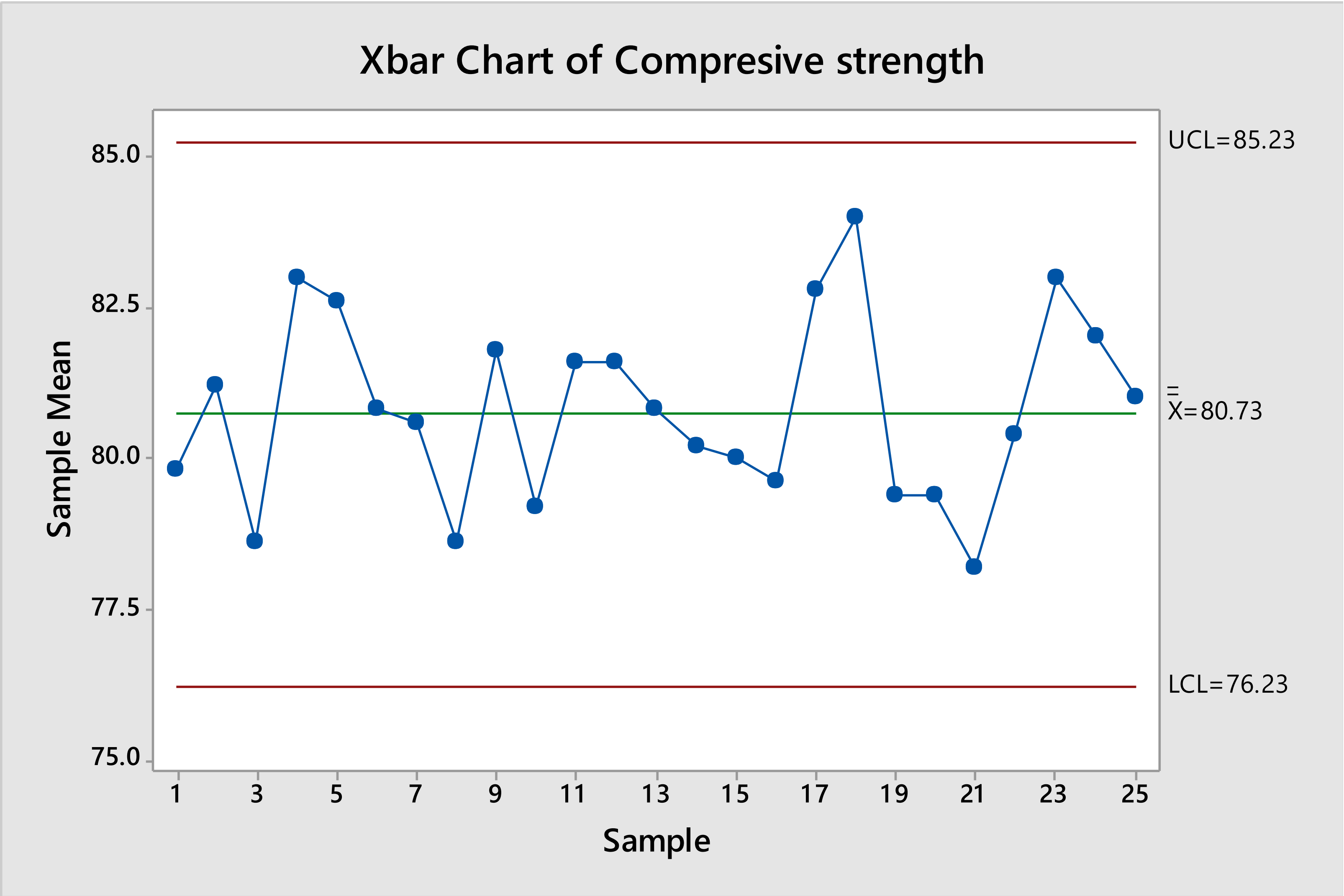
$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{7.8}{2.326} = 3.35$$

So we can determine

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3.35}{\sqrt{5}} = 1.50$$

Our control limits are then

$$UCL_{\bar{x}} = \bar{\bar{x}} + 3\sigma_{\bar{x}} = 80.7 + (3)(1.50) = 85.2$$
$$CL_{\bar{x}} = \bar{\bar{x}} = 80.7$$
$$LCL_{\bar{x}} = \bar{\bar{x}} - 3\sigma_{\bar{x}} = 80.7 - (3)(1.50) = 76.2$$


Xbar Chart of Compresive strength
UCL=85.23
$\bar{\bar{X}}$=80.73
LCL=76.23
85.0
82.5
80.0
77.5
75.0
Sample Mean
1
3
5
7
9
11
13
15
17
19
21
23
25
Sample

Comprehension Questions

In setting the $\bar{x}$ control chart limits, which of the following is true?

T / F A point estimate of the mean of the population, μ_x, is made using the sample data.

T / F The measure of dispersion, $\bar{R}$, can be used to estimate the population standard deviation.

Comprehension Questions

In setting the $\bar{x}$ control chart limits, which of the following is true?

T / F An estimate of the standard error of the sample means, $\sigma_{\bar{x}}$ was needed to calculate the 3 $\sigma_{\bar{x}}$ limits.

T / F If I already have established limits, at one sample size, and a decision is made to change the sample size, it would be necessary to collect a new set of data to calculate the $\bar{x}$ control limits.

We have $\boxed{n = 5}$ $\boxed{\bar{\bar{x}} = 80.7}$ $\boxed{\bar{R} = 7.8}$

$$\boxed{\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{7.8}{2.326} = 3.35}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3.35}{\sqrt{5}} = 1.50$$

Our control limits are

$$UCL_{\bar{x}} = \bar{\bar{x}} + 3\sigma_{\bar{x}} = 80.7 + (3)(1.50) = 85.2$$
$$CL_{\bar{x}} = \bar{\bar{x}} = 80.7$$
$$LCL_{\bar{x}} = \bar{\bar{x}} - 3\sigma_{\bar{x}} = 80.7 - (3)(1.50) = 76.2$$

If we let $A_2 = \dfrac{3}{d_2\sqrt{n}}$

Notice that $3\sigma_{\bar{x}} = \dfrac{3\sigma}{\sqrt{n}} = \dfrac{3}{\sqrt{n}}\dfrac{\bar{R}}{d_2} = A_2\bar{R}$

Common Control Chart Factors

n	A_2	d_2	D_3	D_4
2	1.880	1.128	--	3.267
3	1.023	1.693	--	2.574
4	0.729	2.059	--	2.282
5	0.577	2.326	--	2.114
6	0.483	2.534	--	2.004
7	0.419	2.704	0.076	1.924
8	0.373	2.847	0.136	1.864
9	0.337	2.970	0.184	1.816
10	0.308	3.078	0.223	1.777

$$A_2 = \frac{3}{d_2\sqrt{n}}$$

$$A_2 = \frac{3}{2.326\sqrt{5}} = 0.577$$

For the injection molding problem, we decided to set up our control chart based on taking 25 samples of size 5.

$$\boxed{n = 5}$$

We took our samples and found

$$\boxed{\bar{\bar{x}} = 80.7} \qquad \boxed{\bar{R} = 7.8}$$

where n=5, A_2=0.577

Using A2 we find the control limits as

$$UCL = \bar{\bar{x}} + A_2\bar{R} = 80.7 + 0.577(7.8) = 85.2$$

$$Center \quad Line = \bar{\bar{x}} = 80.7$$

$$LCL = \bar{\bar{x}} - A_2\bar{R} = 80.7 - 0.577(7.8) = 76.2$$

Dependency on Dispersion

$$UCL_{\bar{x}} = \mu_{\bar{x}} + 3\sigma_{\bar{x}} = \bar{\bar{x}} + \frac{3\sigma}{\sqrt{n}} = \bar{\bar{x}} + \frac{3}{\sqrt{n}}\frac{\bar{R}}{d_2} = \bar{\bar{x}} + A_2\bar{R}$$

$$CL_{\bar{x}} = \mu_{\bar{x}} = \bar{\bar{x}}$$

$$LCL_{\bar{x}} = \mu_{\bar{x}} - 3\sigma_{\bar{x}} = \bar{\bar{x}} - \frac{3\sigma}{\sqrt{n}} = \bar{\bar{x}} - \frac{3}{\sqrt{n}}\frac{\bar{R}}{d_2} = \bar{\bar{x}} - A_2\bar{R}$$

- The $\bar{x}$ control chart limits are dependent on the process variability.
- Must verify the process variability is stable prior to validating $\bar{x}$ control chart limits

Control Limits for the R chart

$$UCL_R = \overline{R} + 3\sigma_R$$
$$CL_R = \overline{R}$$
$$LCL_R = \overline{R} - 3\sigma_R$$

$$UCL_R = D_4\overline{R}$$
$$CL_R = \overline{R}$$
$$LCL_R = D_3\overline{R}$$

- D3 and D4 are found in tables for various values of *n*.
- The lower control limit may not go below zero, so the chart may not be symmetric.

Common Control Chart Factors

n	A_2	d_2	D_3	D_4
2	1.880	1.128	--	3.267
3	1.023	1.693	--	2.574
4	0.729	2.059	--	2.282
5	0.577	2.326	--	2.114
6	0.483	2.534	--	2.004
7	0.419	2.704	0.076	1.924
8	0.373	2.847	0.136	1.864
9	0.337	2.970	0.184	1.816
10	0.308	3.078	0.223	1.777

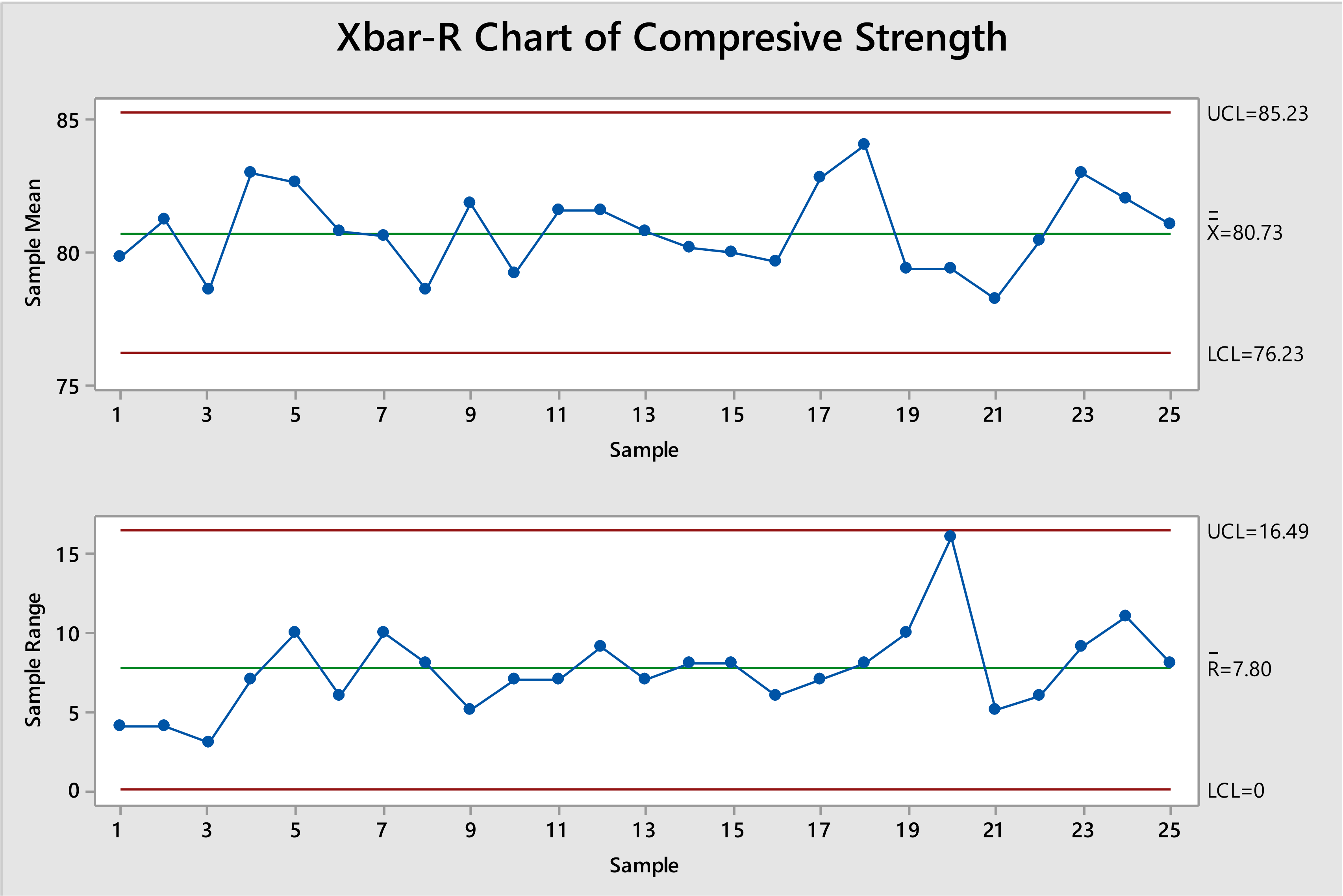
Where there is a "–" in the table, that means the factor is not defined. Therefore the lower limit will be zero.

For our problem: $\boxed{n = 5}$ $\boxed{\overline{R} = 7.8}$

From the table: $\boxed{D_4 = 2.114}$ $\boxed{D_3 = 0}$

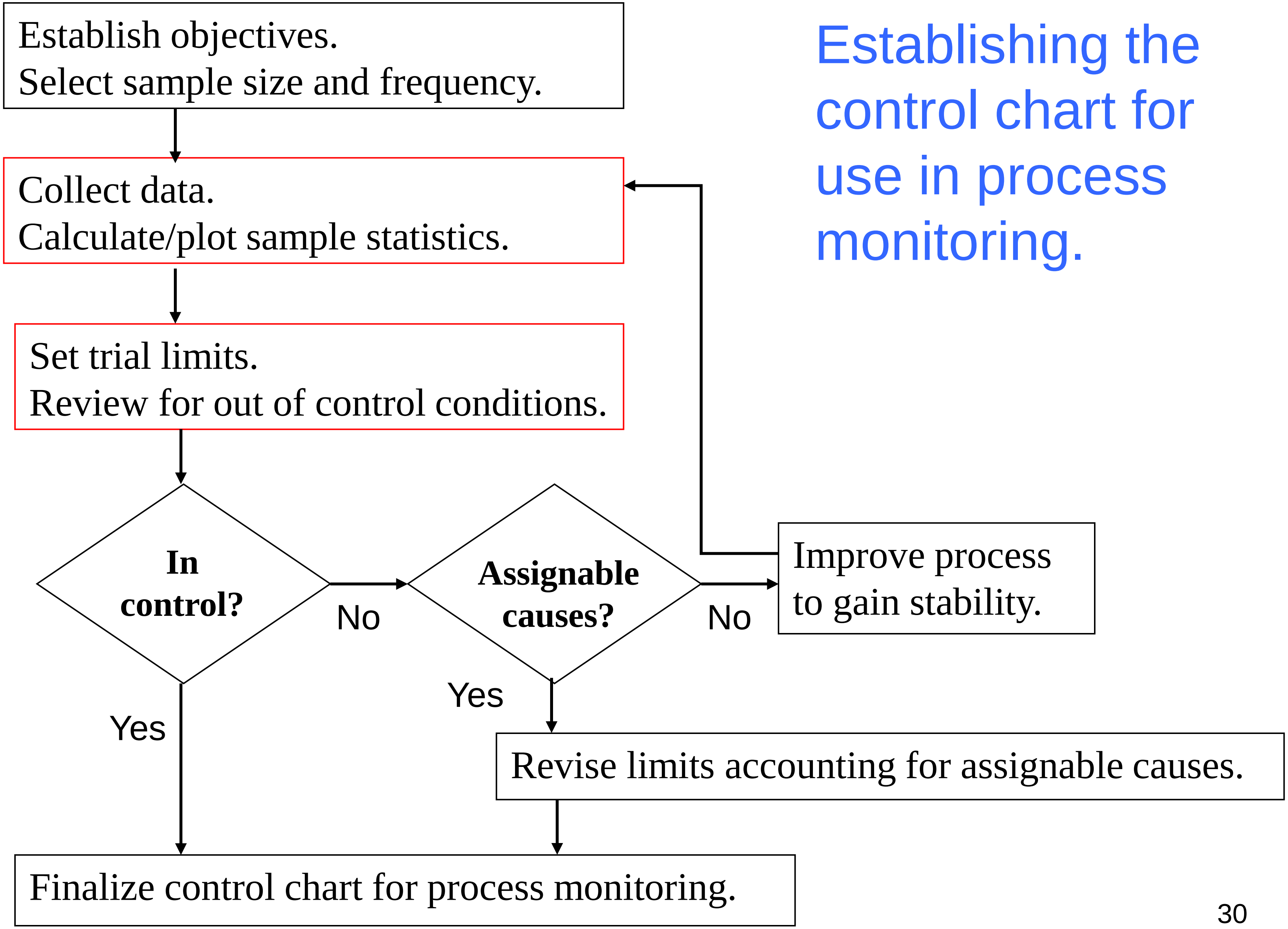
Range Chart Control Limits

$$\boxed{\begin{aligned}
&UCL_R = D_4\overline{R} = (2.114)(7.8) = 16.5\\
&CL_R = \overline{R} = 7.8\\
&LCL_R = D_3\overline{R} = (0)(7.8) = 0
\end{aligned}}$$



Xbar-R Chart of Compresive Strength
Sample Mean
85
80
75
UCL=85.23
X̿=80.73
LCL=76.23
1 3 5 7 9 11 13 15 17 19 21 23 25
Sample
Sample Range
15
10
5
0
UCL=16.49
R̄=7.80
LCL=0
1 3 5 7 9 11 13 15 17 19 21 23 25
Sample

Establishing the control chart for use in process monitoring.



$\bar{x} - R$ Control Chart Equations

$\bar{x}$ Chart

$$UCL_{\bar{x}} = \mu_{\bar{x}} + 3\sigma_{\bar{x}} = \bar{\bar{x}} + \frac{3\sigma}{\sqrt{n}} = \bar{\bar{x}} + \frac{3}{\sqrt{n}}\frac{\bar{R}}{d_2} = \bar{\bar{x}} + A_2\bar{R}$$

$$CL_{\bar{x}} = \mu_{\bar{x}} = \bar{\bar{x}}$$

$$LCL_{\bar{x}} = \mu_{\bar{x}} - 3\sigma_{\bar{x}} = \bar{\bar{x}} - \frac{3\sigma}{\sqrt{n}} = \bar{\bar{x}} - \frac{3}{\sqrt{n}}\frac{\bar{R}}{d_2} = \bar{\bar{x}} - A_2\bar{R}$$

R Chart

$$UCL_R = \overline{R} + 3\sigma_R \qquad UCL_R = D_4\overline{R}$$

$$CL_R = \overline{R} \qquad CL_R = \overline{R}$$

$$LCL_R = \overline{R} - 3\sigma_R \qquad LCL_R = D_3\overline{R}$$

Practice

The quality control plan for a certain production process is to be developed taking samples of size 4. 25 samples are taken with the results.

The sum of the $\bar{x}$'s for the samples is $\sum \bar{x} = 850$.

The sum of the r's for the samples is $\sum r = 160$.

What are $\bar{\bar{X}}$ and $\bar{R}$?

A) 212, 40 B) 35, 18.3 C) 8.5, 1.6 D) 34.0, 6.4

What is the estimate of sigma for the population based on $\bar{R}$?

A) 3.35 B) 2.79 C) 3.11 D) 6.81

Practice Problem

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} \qquad UCL_R = D_4\bar{R}$$

$$CL_{\bar{x}} = \bar{\bar{x}} \qquad CL_R = \bar{R}$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} \qquad LCL_R = D_3\bar{R}$$

The quality control plan for a certain production process is to be developed taking samples of size 4. 25 samples are taken with the results. $\bar{\bar{X}}$ and $\bar{R}$ were determined to be 34.0 and 6.4, respectively.

What is the factor for A_2 to be used in calculating the xbar chart limits?

A) 0.153 B) 1.023 C) 0.729 D) 1.628

The upper and lower control limits on the xbar chart are

	$UCL_{\bar{x}}$	$LCL_{\bar{x}}$		$UCL_{\bar{x}}$	$LCL_{\bar{x}}$
A)	38.67	29.33	B)	36.43	30.02
C)	38.67	29.95	D)	36.43	29.33

Practice Problem

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} \qquad UCL_R = D_4\bar{R}$$

$$CL_{\bar{x}} = \bar{\bar{x}} \qquad CL_R = \bar{R}$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} \qquad LCL_R = D_3\bar{R}$$

The quality control plan for a certain production process is to be developed taking samples of size 4. 25 samples are taken with the results. $\bar{\bar{X}}$ and $\bar{R}$ were determined to be 34.0 and 6.4, respectively.

What is the factor to be used in calculating the r chart upper limit?

A) 0 B) 0.459 C) 1.541 D) 2.282

The upper and lower control limits on the R chart are

	UCL_R	LCL_R		UCL_R	LCL_R
A)	0	14.60	B)	14.60	0
C)	77.67	0	D)	13.31	0

S Chart

- The S chart can be used in place of the *R* chart. Where R appears on the control chart form, s would appear – calculate the standard deviation for each subgroup.

- The center line and the 3σ upper and lower control limits are given by

$$3\sigma \text{ upper limit} = B_4\bar{s}$$

$$\text{Center line} = \bar{s}$$

$$3\sigma \text{ lower limit} = B_3\bar{s}$$

- The values B_3 and B_4 depend on the sample size. Values are found in the Control Chart Factors Table.

S Charts

Notation and values

- S_i = standard deviation of the values in the i^{th} sample

$$s_i = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

- $\bar{S}$ = average of the sample standard deviations

$$\bar{S} = \frac{1}{m}\sum_{i=1}^{m} S_i$$

For our problem: $\boxed{n = 5}$ $\boxed{\bar{S} = 3.244}$

From the table: $\boxed{B_4 = 2.089}$ $\boxed{B_3 = 0}$

S Chart Control Limits

$$\boxed{\begin{aligned} UCL_S &= B_4\bar{S} = (2.089)(3.244) = 6.777 \\ CL_S &= \bar{S} = 3.244 \\ LCL_S &= B_3\bar{S} = (0)(3.244) = 0 \end{aligned}}$$

Variables Control Charts

<u>Control chart of the sample mean</u>

The process standard deviation, σ can be estimated by

$$\hat{\sigma} = \frac{\overline{S}}{c_4}$$

Control chart constant A_3 has have been developed for calculating the associated $\bar{x}$ chart lines for the 3 σ limits.

$$UCL_{\bar{x}} = u_{\bar{x}} + 3\sigma_{\bar{x}} = \bar{\bar{x}} + 3\sigma_{\bar{x}} = \bar{\bar{x}} + A_3\bar{S}$$

$$CL_{\bar{x}} = u_{\bar{x}} = \bar{\bar{x}}$$

$$LCL_{\bar{x}} = u_{\bar{x}} - 3\sigma_{\bar{x}} = \bar{\bar{x}} - 3\sigma_{\bar{x}} = \bar{\bar{x}} - A_3\bar{S}$$

For the injection molding problem, we decided to set up our control chart based on taking 20 samples of size 5. $\boxed{n = 5}$

We took our samples and found

$$\boxed{\bar{\bar{x}} = 80.73} \qquad \boxed{\bar{S} = 3.244}$$

We can estimate the population standard deviation as

$$\hat{\sigma} = \frac{\bar{S}}{c_4} = \frac{3.244}{.9400} = 3.45$$

For the injection molding problem, we decided to set up our control chart based on taking 20 samples of size 5. $\boxed{n = 5}$

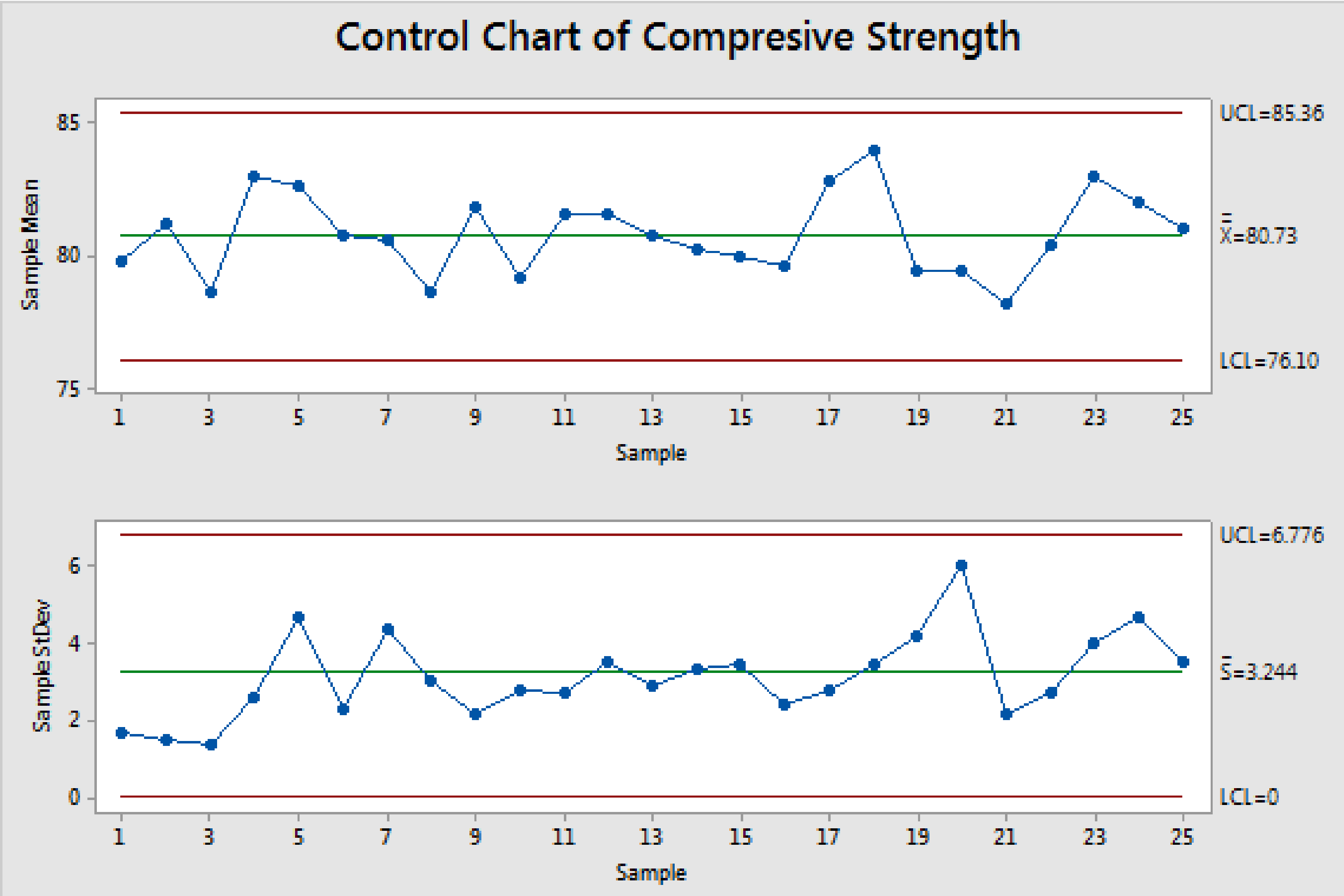
We took our samples and found

$\boxed{\bar{\bar{x}} = 80.73}$ $\boxed{\bar{S} = 3.244}$

where n=5, A_3=1.427

Using A_3 we find the control limits as

$$\boxed{\begin{aligned} UCL &= \bar{\bar{x}} + A_3\bar{S} = 80.73 + 1.427(3.244) = 85.36 \\ Center \quad Line &= \bar{\bar{x}} = 80.73 \\ LCL &= \bar{\bar{x}} - A_3\bar{S} = 80.73 - 1.427(3.244) = 76.10 \end{aligned}}$$


Control Chart of Compresive Strength
Sample Mean
85
80
75
UCL=85.36
$\bar{\bar{X}}$=80.73
LCL=76.10
1
3
5
7
9
11
13
15
17
19
21
23
25
Sample
Sample StDev
6
4
2
0
UCL=6.776
$\bar{S}$=3.244
LCL=0
1
3
5
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Sample

Practice Problem

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_3\bar{S} \qquad UCL_S = B_4\bar{S}$$

$$CL_{\bar{x}} = u_{\bar{x}} = \bar{\bar{x}} \qquad CL_S = \bar{S}$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_3\bar{S} \qquad LCL_S = B_3\bar{S}$$

The quality control plan for a certain production process is to be developed taking samples of size 4. 25 samples are taken with the results. $\bar{\bar{X}}$ and $\bar{S}$ were determined to be 34.0 and 2.863, respectively.

What is the factor for A_3 to be used in calculating the xbar chart limits?

A) 0.606 B) 1.427 C) 0.729 D) 1.628

The upper and lower control limits on the xbar chart are

	$UCL_{\bar{x}}$	$LCL_{\bar{x}}$		$UCL_{\bar{x}}$	$LCL_{\bar{x}}$
A)	38.66	29.34	B)	36.43	30.10
C)	38.67	29.95	D)	36.43	29.43

Practice Problem

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R}$$
$$CL_{\bar{x}} = \bar{\bar{x}}$$
$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R}$$

$$UCL_S = B_4\bar{S}$$
$$CL_S = \bar{S}$$
$$LCL_S = B_3\bar{S}$$

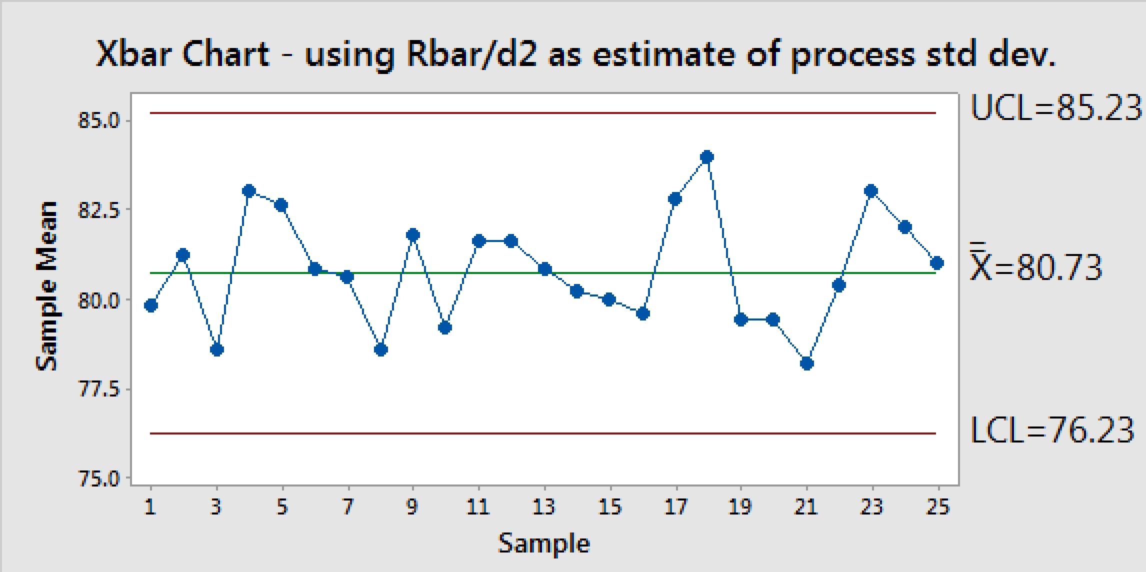
The quality control plan for a certain production process is to be developed taking samples of size 4. 25 samples are taken with the results. $\bar{\bar{X}}$ and $\bar{S}$ were determined to be 34.0 and 6.4, respectively.

What is the factor to be used in calculating the S chart upper limit?

A) 0 B) 2.282 C) 1.541 D) 2.266

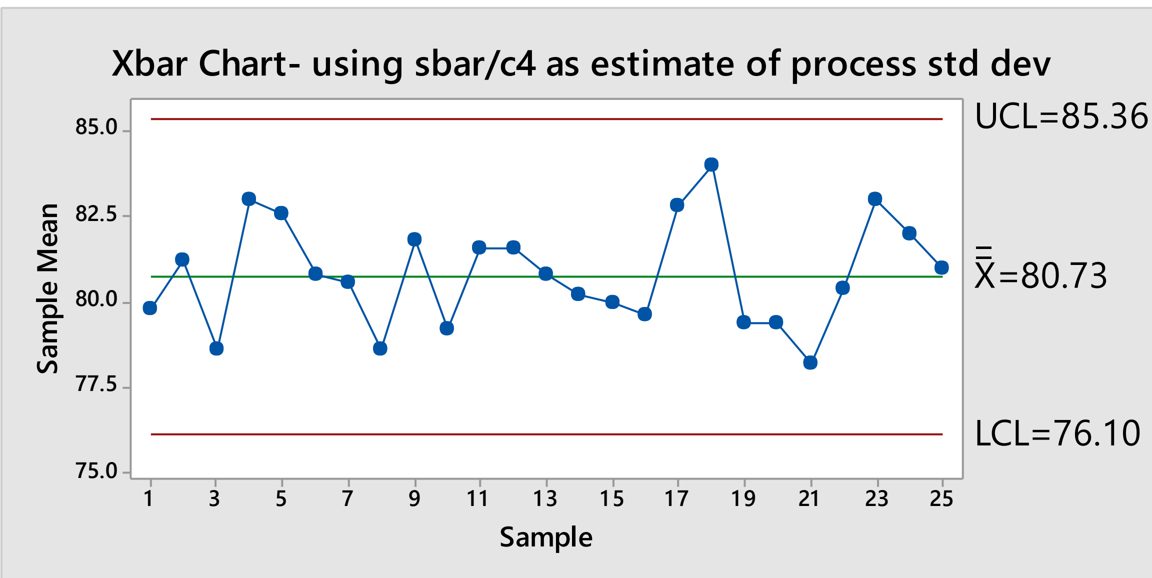
The upper and lower control limits on the S chart are

	UCL_s	LCL_s		UCL_s	LCL_s
A)	0	14.50	B)	14.50	0
C)	77.67	0	D)	14.62	0



$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{7.8}{2.326} = 3.35$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3.35}{\sqrt{5}} = 1.50$$



$$\hat{\sigma} = \frac{\bar{S}}{c_4} = \frac{3.244}{.9400} = 3.45$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3.45}{\sqrt{5}} = 1.54$$

Which is better to use, R or S chart?

- The range and standard deviation are both measures of dispersion and both can be used to estimate the process standard deviation.
- When the population is normal, s is a more precise estimate of the process standard deviation than R, because it has smaller uncertainty.
- The improvement in the precision obtained with s as opposed to R increases as the sample size increases.
- The range is typically used for small sample sizes and the standard deviation for large sample sizes.

Related Assignments

Please continue to Part B of this lecture.