# EIN 5226

# Inferences –Continuous Response

Chapter 16 Sections 16.3-4

Chapter 17 Sections 17.2-5

Need Z table and t table for this lecture

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# Hypothesis Testing

Hypothesis

An assumption or theory made about a population parameter or relationship between populations

Hypothesis test

Statistical test of hypothesis based on sample data collected from the population/s of interest

# Types of Hypothesis tests

- Tests for a single sample
  - One population's mean to external criterion (large sample size)
  - One population proportion to external criterion
  - One population's mean to external criterion (small sample size)
  - Populations expected distribution of outcomes to samples distribution of outcomes.

- Tests comparing two samples
  - Difference between two population means (large sample size)
  - Difference between two population proportions
  - Difference between two population means (small sample size)
  - Difference between two population variances

In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of 124 and a standard deviation of 5. Can we conclude, with 90% certainty, that the line is malfunctioning. (Assume normal distribution.)

<u>"Intuitive" approach</u>

Gather data and analyze sample data.

Compare it to where think it should be.

In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of 124 and a standard deviation of 5. Can we conclude, with 90% certainty, that the line is malfunctioning. (Assume normal distribution.)

"Intuitive" approach: Gather data and analyze sample data.
Compare it to where think it should be.

1. Collect sample data. Find the sample mean and estimate the population standard deviation.
2. Estimate the sigma of the sample means distribution
3. Set up a confidence limit about the value you want to compare it to ($\mu_0$).
4. Check – is the sample mean within the limits
5. If it is in the limits, you cannot conclude that it is different.

In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of 124 and a standard deviation of 5. Can we conclude, with 90% certainty, that the line is malfunctioning. (Assume normal distribution.)

1. Collect 40 sample measurements. Find the sample mean and estimate the population standard deviation.

Sample mean: $$\bar{x} = \frac{\sum x}{n} = 124$$

Estimate the population standard deviation with sample

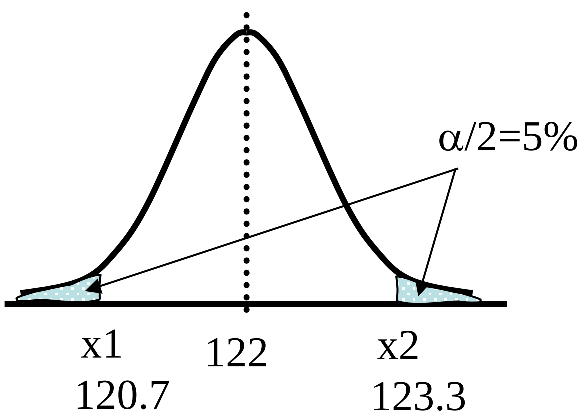
$$\hat{\sigma} = s = \sqrt{\frac{\sum (x - \bar{x})^2}{(n-1)}} = 5$$

2. Estimate the sigma of the distribution of sample means

$$\sigma_{\bar{x}} = {}^{\sigma}/_{\sqrt{n}} = {}^{5}/_{\sqrt{40}} = 0.791$$

In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of 124 and a standard deviation of 5. Can we conclude, with 90% certainty, that the line is malfunctioning. (Assume normal distribution.)

3. Set up a 90% confidence limit about the value you want to compare it to ($\mu_0$).




From the tables, for 5% : Z = ±1.645

90% CI around $u_0$

$$x_1 = u - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 122 - 1.645\frac{5}{\sqrt{40}} = 120.7$$

$$x_2 = u + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 122 + 1.645\frac{5}{\sqrt{40}} = 123.3$$

In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of 124 and a standard deviation of 5. Can we conclude, with 90% certainty, that the line is malfunctioning. (Assume normal distribution.)

4. Check – is it within the 90% limits

   Is 124 between 120.7 and 123.3? NO.

5. If it is not, you can state with 90% certainty that it is not 122.

   Can make decision – fix machine!

In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of 124 and a standard deviation of 5. Can we conclude, with 90% certainty, that the line is malfunctioning. (Assume normal distribution.)

What is the 90% confidence interval for <u>the true mean of the sample</u>?

From the tables, for 5% : Z = ±1.645

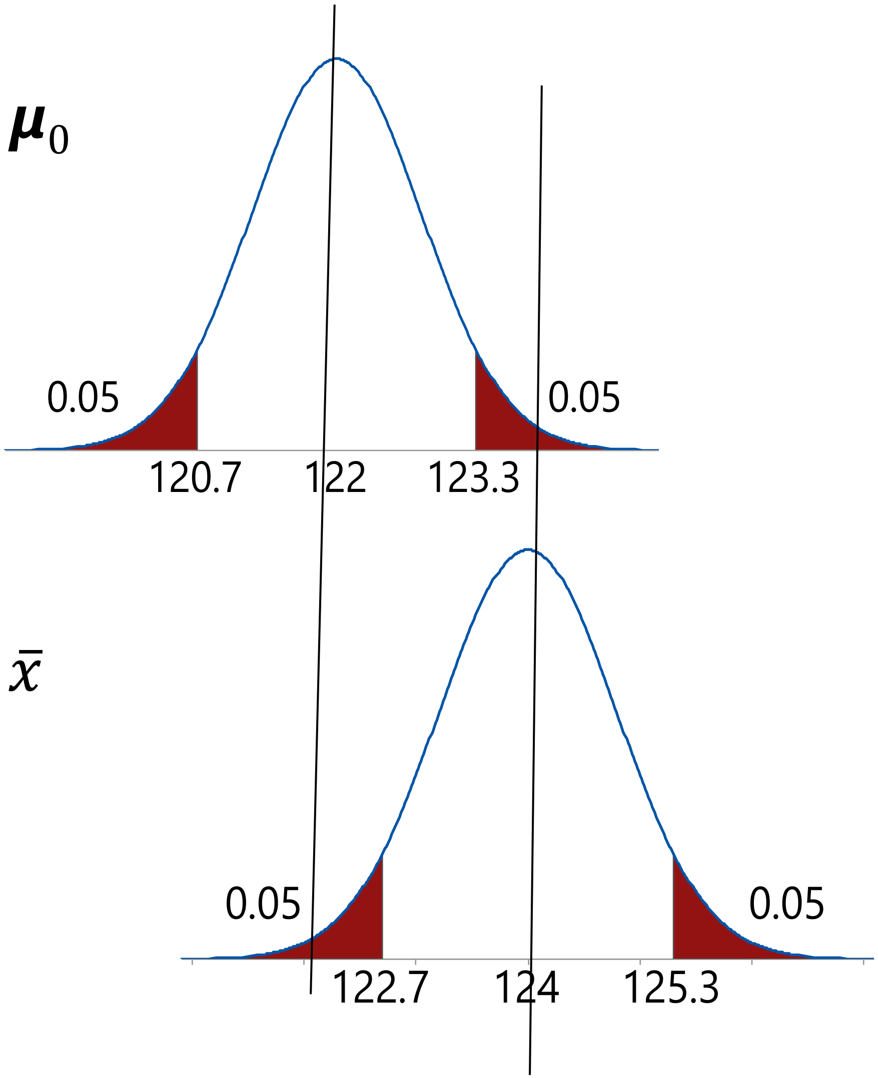
90% CI around $\bar{x}$

$$x_1 = \bar{x} - Z\alpha_{/2}\frac{\sigma}{\sqrt{n}} = 124 - 1.645\frac{5}{\sqrt{40}} = 122.7$$

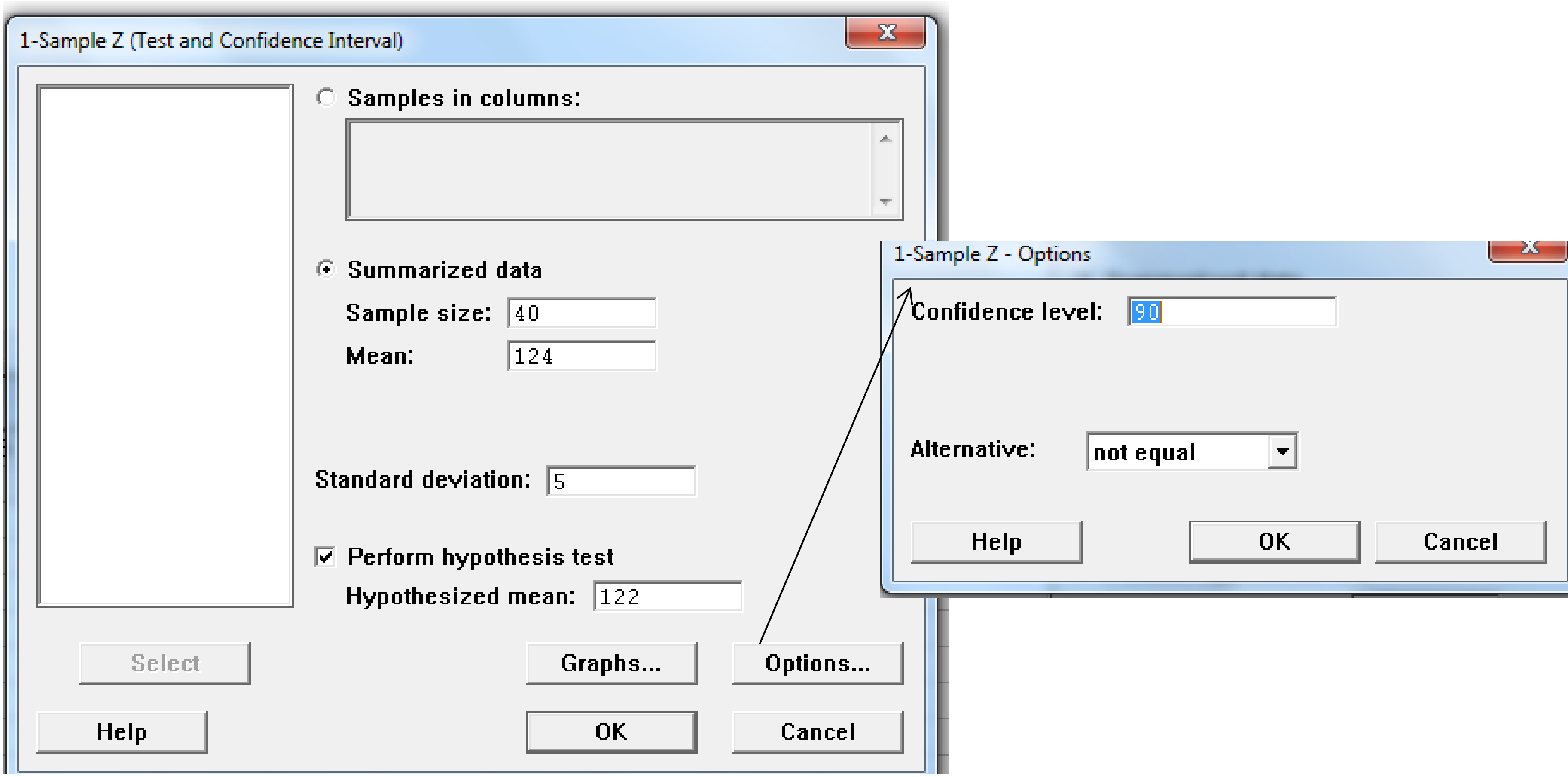
$$x_1 = \bar{x} + Z\alpha_{/2}\frac{\sigma}{\sqrt{n}} = 124 + 1.645\frac{5}{\sqrt{40}} = 125.3$$

90% CI around $\boldsymbol{\mu}_0$

90% CI around $\bar{x}$




Checking to see if $\boldsymbol{\mu}_0$ is in the interval around $\bar{x}$ gives you the same answer as if you checked to see if $\bar{x}$ is in the interval around $\boldsymbol{\mu}_0$




# One-Sample Z

Test of mu = 122 vs not = 122

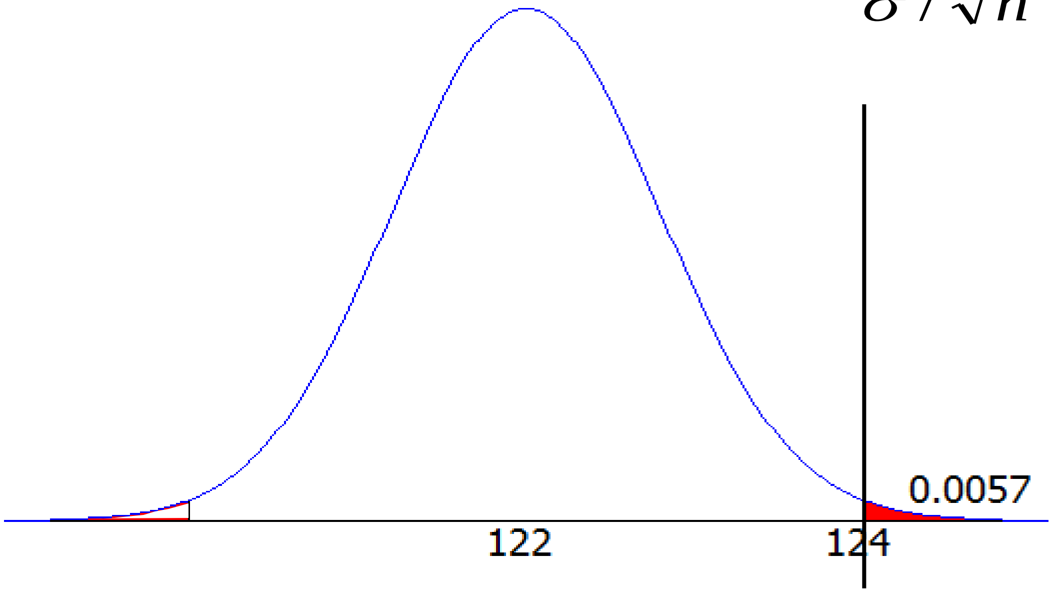
The assumed standard deviation = 5

| N | Mean | SE Mean | 90% CI | Z | P |
|---|---|---|---|---|---|
| 40 | 124.000 | 0.791 | (122.700, 125.300) | 2.53 | 0.011 |

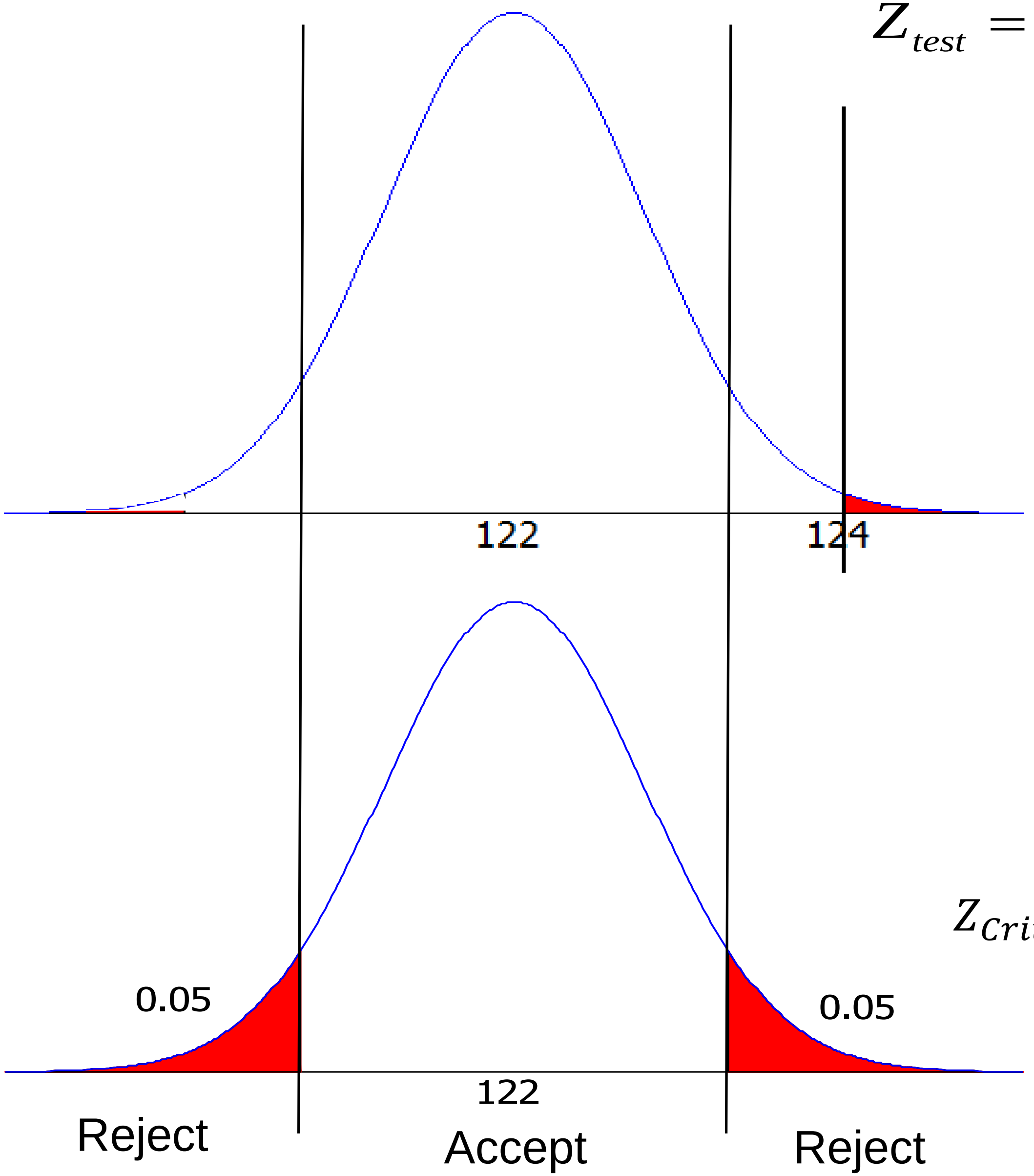
Instead of calculating the Z values for interval, and finding the upper and lower confidence limits, and comparing the sample mean to the interval:

Calculate Z for the sample mean, determine the probability in the tail from the table , then compare that probability to what we are allowed.

$$Z_{test} = \pm \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \pm \frac{124 - 122}{5 / \sqrt{40}} = \pm 2.53$$



$$Z_{test} = +2.53$$




Compare $Z_{test}$ to $Z_{critical}$ defined rejection area.

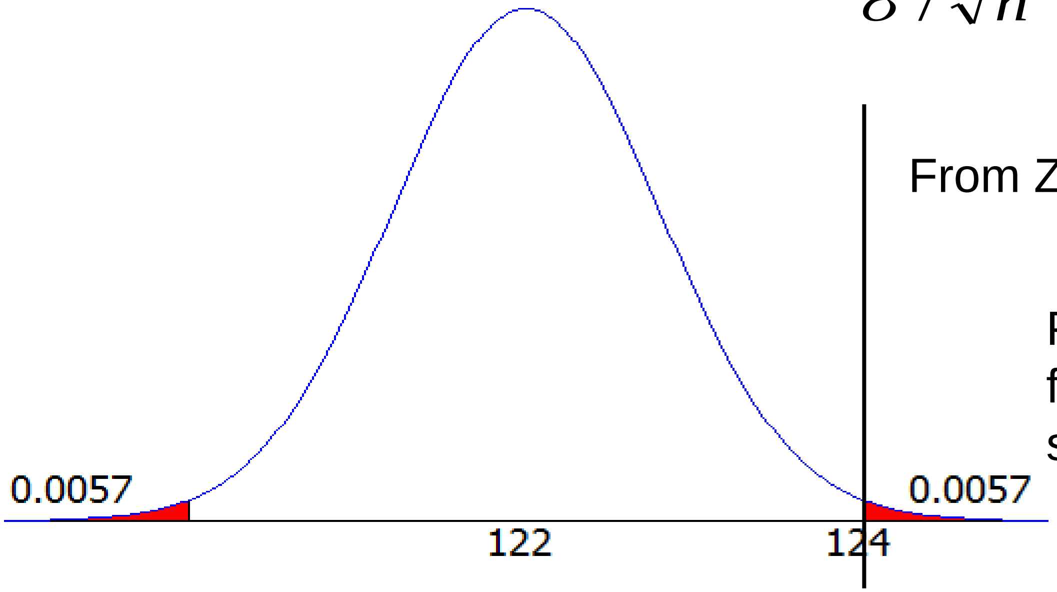
Reject if Z test in rejection area

$$Z_{Critical} = Z_{\alpha/2} = Z_{.05} = \pm 1.645$$

Instead of calculating the Z values for interval, and finding the upper and lower confidence limits, and comparing the sample mean to the interval:

Calculate Z for the sample mean, determine the probability in the tail from the table , then compare that probability to what we are allowed.

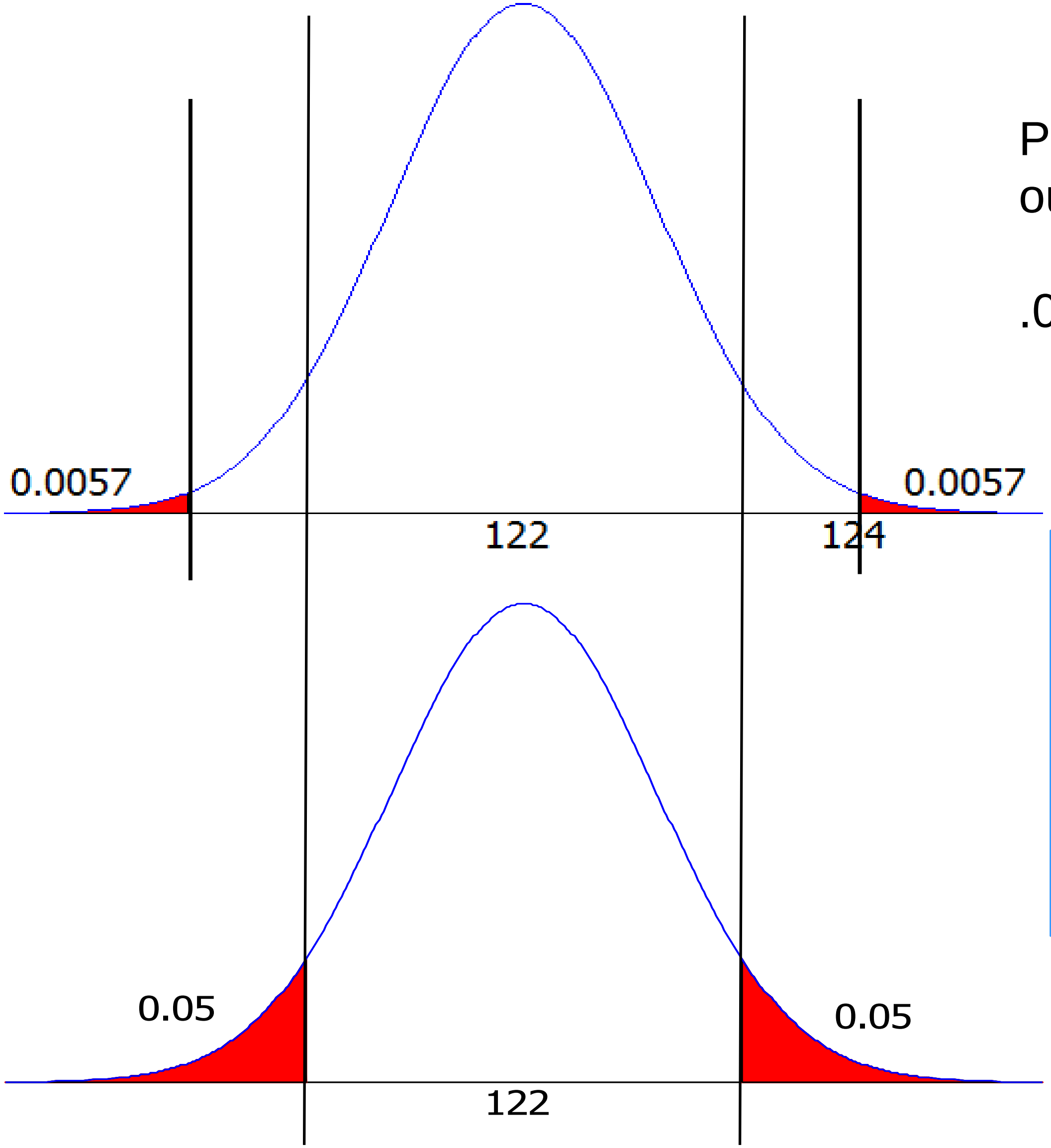
$$Z_{test} = \pm\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} = \pm\frac{124-122}{5/\sqrt{40}} = \pm 2.53$$




From Z table P(Z<-2.53) = .0057

Probability of getting a value further out than our test statistic for high side tail.

# Calculating the P Value




Probability of value further out either side:

.0057+ .0057 = .0114 = p

Compare p to allowed $\alpha$

If lower, is outside of the interval.

## Use of P-Values in Hypothesis Testing

P-value

- *probability* associated with the test statistic, $Z_{test}$.
- smallest level of $\alpha$ risk that would lead to rejection of the null hypothesis $H_0$
- probability of getting a value further out than your test values

# Statistical Inference
# - Using p values

Steps in Hypothesis Testing

1. Identify your objective
2. State the null hypothesis, $H_0$
3. State the alternative hypotheses, $H_a$.
4. Calculate the appropriate test statistic
5. Compute the p value of the test statistic
6. Determine the acceptable risk
7. Compare the p value of test statistic to the to the acceptable risk.

In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of 124 and a standard deviation of 5. Can we conclude, with 90% certainty, that the line is malfunctioning. (Assume normal distribution.)

1) Identify your objective

*For our problem we want to know if we could conclude that the population mean had shifted, if it was no longer equal to 122*. Do I need to fix machine?

Comparison of 1 sample mean to a defined $\mu$

In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of 124 and a standard deviation of 5. Can we conclude, with 90% certainty, that the line is malfunctioning. (Assume normal distribution.)

2) Formulate a null hypothesis.

$H_0$ – Null hypothesis. Always arrange the null claim such that it contains the condition of equality.

$H_0$: $\mu = \mu_0$ $\mu = 122$

The null claim is what you are comparing the sample data to.

In the problem, we are comparing the sample data to 122

In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of 124 and a standard deviation of 5. Can we conclude, with 90% certainty, that the line is malfunctioning. (Assume normal distribution.)

3) Formulate an alternative hypothesis

$H_a$ – Alternative hypothesis. The alternative will involve exactly one of three signs, <, >,or $\neq$. If the alternative is $\neq$, then the test will be two-tailed. If < or >, it will be one tailed.

$H_a$: $\mu \neq \mu_0$ $\mu \neq 122$

| Two tailed test | One tailed test |
|---|---|
| $\alpha$ split between tails, | $\alpha$ all in one tail, |
| Equal to or not equal to | Less than or greater than |
| Results supported in either direction | Other side ignored |

## Note

For one-tailed tests, some textbooks will use < or > signs in the null hypothesis. This is not really wrong, but is not necessary. It makes it somewhat more difficult to establish the null and alternative hypothesis.

For purposes of this class, the null hypothesis will always be the equality.

When we get to examples, I will show you a technique that will help in the setting up of these.

In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of 124 and a standard deviation of 5. Can we conclude, with 90% certainty, that the line is malfunctioning. (Assume normal distribution.)

4) Calculate a test statistic from the sample information.

$$Z_{test} = \frac{\bar{x} - u_0}{\sigma/\sqrt{n}} = \frac{124 - 122}{5/\sqrt{40}} = \pm 2.53$$

Possible test statistics: Z, t, $\aleph^2$, F

For a two tailed test, $Z_{test}$ is $\pm$

In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of 124 and a standard deviation of 5. Can we conclude, with 90% certainty, that the line is malfunctioning. (Assume normal distribution.)

5) Determine probability, p, that a value will be further out than the sample value.

$$P(|Z| < 2.53 = .0057 + .0057 = .0114 = 1.14\% = p$$

For $H_1 = \mu \neq \mu_0,$ sum the areas in the tails, cut off by Z and –Z

$H_1 = \mu > \mu_0,$ p value is area to the right of Z

$H_1 = \mu < \mu_0,$ p value is area to the left of Z

In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of 124 and a standard deviation of 5. Can we conclude, with 90% certainty, that the line is malfunctioning. (Assume normal distribution.)

6) Determine the acceptable risk.

Can we conclude, with 90% certainty, that the line is malfunctioning

$\boldsymbol{\alpha} = 10\%$

(two tailed test)

In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of 124 and a standard deviation of 5. Can we conclude, with 90% certainty, that the line is malfunctioning. (Assume normal distribution.)

6) Compare your p value to the acceptable risk to make an inference about the population.

$p = 1.14\%$ $\qquad\qquad$ $\boldsymbol{\alpha} = 10\%$

$p < \boldsymbol{\alpha}$

Therefore reject the hypothesis that $\mu = 122$

"If p is low, null must go.

If p is high the null will fly."

# Drawing Conclusions from the Results of Hypothesis Tests

- Statistically, there are only two possible conclusions
  - Reject $H_0$: Conclude that $H_0$ is false.
  - Do not reject $H_0$: Conclude $H_0$ is plausible. Our evidence is not strong enough to reject it.
- One can never conclude that $H_0$ is true. We can just conclude that $H_0$ might be true.
- The smaller the $P$-value, the stronger the evidence is against $H_0$.

# More on the *P*-value

- The P-value should always be reported with the results of a hypothesis test.
- The smaller the *P*-value, the more certain we can be that $H_0$ is false.
- The larger the *P*-value, the more plausible $H_0$ becomes but we can never be certain that $H_0$ is true.
- If you just report the confidence level, you do not know how strong your conclusion is.

# Tests on Single Population Means
## Large Sample Sizes ($n \geq 30$)

| | Two-tailed | One-tailed | |
|---|---|---|---|
| Null hypothesis $H_0$ | $\mu = \mu_0$ | | |
| Alternate $H_a$ | $\mu \neq \mu_0$ | $\mu > \mu_0$ | $\mu < \mu_0$ |
| Test statistic | $Z_{test} = \dfrac{\bar{x} - u_o}{s / \sqrt{n}}$ | | |
| p<br><br>(reject if $p < \alpha$ ) | sum the areas in the tails, cut off by Z and –Z | Area to right of Z | Area to left of Z |

In a power generating plant, a certain line is supposed to maintain an average of 122 psi over any 8 hour period. In one period, 40 measurements were taken with a mean of 124 and a standard deviation of 5. Can we conclude, with 90% certainty, that the line is malfunctioning. (Assume normal distribution.)

$H_0: \mu = \mu_0$ $\mu_0$ is what is claimed, what is being tested against.

$H_0: \mu = 122$

$H_a: \mu \neq 122$

What happens if it is equal?

Do not fix machine

What is the alternative?

Fix machine

For alternative to happen, is it $<$, $>$ or $\neq$ ?

60 specimens of rubber are tested resulting in an average hardness of 62.40 and standard deviation of 1.7. The hardness is claimed to be 63. Is there reason to suspect that the actual hardness is less than claimed? (Use a 99% confidence.)

$H_0: \mu = \mu_0$ $\mu_0$ is what is claimed, what is being tested against.

$H_0: \mu = 63$

$H_a: \mu < 63$

What happens if it is equal?

Not suspect

What is the alternative?

Is suspect.

For alternative to happen, is it $<$, $>$ or $\neq$ ?

60 specimens of rubber are tested resulting in an average hardness of 62.40 and standard deviation of 1.7. The hardness is claimed to be 63. Is there reason to suspect that the actual hardness is less than claimed? (Use a 99% confidence.)

$H_0$: μ = 63 $H_a$: μ < 63

α = 1%

Test value: $$Z_{test} = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{62.4-63}{1.7/\sqrt{60}} = -2.73$$

The p value associated with the $Z_{test}$ is 0.0032.= 0.32%

(one tailed test, do not add the other tail)

p< α, therefore reject the null hypothesis.

# Practice 1a

A manufacturer of sports equipment has developed a new fishing line that he claims has a mean breaking strength of 20 lbs.  A sample of 50 lines is tested and found to have a mean breaking strength of 19.6 lbs. with a standard deviation of 1.0 lbs.  With a 99% confidence, is there sufficient evidence to state that the claim is true?

1. What are the null and alternate hypotheses for this problem?

A) $H_0$: $\mu = 19.6$  $H_a$: $\mu < 19.6$

B) $H_0$: $\mu = 20$  $H_a$: $\mu < 20$

C) $H_0$: $\mu = 19.6$  $H_a$: $\mu \neq 19.6$

D) $H_0$: $\mu = 20$  $H_a$: $\mu \neq 20$

# Practice 1b

$$Z_{test} = \frac{\bar{x} - u_o}{s/\sqrt{n}}$$

A manufacturer of sports equipment has developed a new fishing line that he claims has a mean breaking strength of 20 lbs. A sample of 50 lines is tested and found to have a mean breaking strength of 19.6 lbs. with a standard deviation of 1.0 lbs. With a 99% confidence, is there sufficient evidence to state that the claim is true?

2. $Z_{test}$ is

A) -2.828 B) -2.257 C) 3.25 D) -3.20

# Practice 1c

$$Z_{test} = \frac{\bar{x} - u_o}{s / \sqrt{n}}$$

A manufacturer of sports equipment has developed a new fishing line that he claims has a mean breaking strength of 20 lbs. A sample of 50 lines is tested and found to have a mean breaking strength of 19.6 lbs. with a standard deviation of 1.0 lbs. With a 99% confidence, is there sufficient evidence to state that the claim is true?

3. The p value for this problem is

A) .0060 B) .0046 C) .0023 D) .0038

# Practice 1d

A manufacturer of sports equipment has developed a new fishing line that he claims has a mean breaking strength of 20 lbs. A sample of 50 lines is tested and found to have a mean breaking strength of 19.6 lbs. with a standard deviation of 1.0 lbs. With a 99% confidence, is there sufficient evidence to state that the claim is true?

This analysis would result in the following conclusion:

T / F 4. p is less than $\alpha$, therefore we reject the null hypothesis.

T / F 5. The null hypothesis is rejected and we conclude that the mean breaking strength is not 20 lbs.

# Practice 2 a

A company wants to reduce the assembly time for one of its products that is produced at multiple manufacturing facilities. The current assembly time averages 25.0 minutes. A new fixture was implemented at one of the facilities. 35 products were assembled with an average time of 23.5 minutes and standard deviation of 3.6 minutes. Would you recommend implementing the fixture at other facilities? (Use $\alpha = .05$)

6. What are the null and alternate hypotheses for this problem?

A) $H_0$: $\mu = 25.0$ $H_a$: $\mu < 25.0$

B) $H_0$: $\mu = 23.5$ $H_a$: $\mu < 25.0$

C) $H_0$: $\mu = 23.5$ $H_a$: $\mu \neq 23.5$

D) $H_0$: $\mu = 25.0$ $H_a$: $\mu \neq 25.0$

# Practice 2 b

$$Z_{test} = \frac{\bar{x} - u_o}{s/\sqrt{n}}$$

A company wants to reduce the assembly time for one of its products that is produced at multiple manufacturing facilities. The current assembly time averages 25.0 minutes. A low cost method changed was implemented at one of the facilities. 35 products were assembled with an average time of 23.5 minutes and standard deviation of 3.6 minutes. Would you recommend implementing the method at other facilities? (Use $\alpha$=.05)

7. $Z_{test}$ is

A) -2.81 B) -2.47 C) + 2.47 D) -3.26

8. The p value for this problem is

A) .0055 B) .0089 C) .0068 D) .9932

# Practice 2 c

$$Z_{test} = \frac{\bar{x} - u_o}{s/\sqrt{n}}$$

A company wants to reduce the assembly time for one of its products that is produced at multiple manufacturing facilities. The current assembly time averages 25.0 minutes. A low cost method changed was implemented at one of the facilities. 35 products were assembled with an average time of 23.5 minutes and standard deviation of 3.6 minutes. Would you recommend implementing the method at other facilities? (Use $\alpha = .05$)

From the hypothesis test which of the following are true statements of conclusion.

T / F 9. The null hypothesis, u = 25.0, is rejected.

T / F 10. The time for assembly is now less than 25 minutes.

T / F 11. The time for assembly is now 23.5 minutes

T / F 12. Assuming the cost to implement is sufficiently low, we should implement at the other facilities.

# Significance

- Even though a result may be statistically significant, common sense needs to be applied in taking action based on the result.

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- Sometimes statistically significant results do not have any scientific or practical importance.

# Tests on Single Population Means
# Small Sample Sizes (n<30), $\sigma$ unknown

| | Two-tailed | One-tailed | |
|---|---|---|---|
| Null hypothesis $H_0$ | $\mu=\mu_0$ | | |
| Alternate $H_a$ | $\mu\neq\mu_0$ | $\mu>\mu_0$ | $\mu<\mu_0$ |
| Test statistic | $t_{test} = \dfrac{\bar{x} - u_o}{s/\sqrt{n}}$ | | |
| p<br><br>(reject if p<$\alpha$ ) | sum the areas in the tails, cut off by $t_{test}$ and $-t_{test}$ | Area to right of $t_{test}$ | Area to left of $t_{test}$ |

The average time for students to register for classes at a certain college has been 7 minutes. The response time on the registration system has recently been upgraded. A random sample of 21 students was taken with the average time at 6.4 minutes and a standard deviation of 1. Can you, with 95% certainty, state that the upgrade has reduced the time to register?

$H_0$: $\mu = \mu_0$

$\mu = 7$

What happens if it is equal?

Conclude upgrade did not reduce time

What is the alternative?

Conclude upgrade reduced time

For alternative to happen, is it <, > or $\neq$ ?

$H_a$ : $\mu < \mu_0$ :

$\mu < 7$

$$t_{test} = \frac{\bar{x} - u_o}{s/\sqrt{n}}$$

The average time for students to register for classes at a certain college has been 7 minutes. The response time on the registration system has recently been upgraded. A random sample of 21 students was taken with the average time at 6.4 minutes and a standard deviation of 1. Can you, with 95% certainty, state that the upgrade has reduced the time to register?

$H_0$: $\mu = 7$ $H_a$: $\mu < 7$

n=21 v=21-1=20 $\alpha = .05$ (one tailed test)

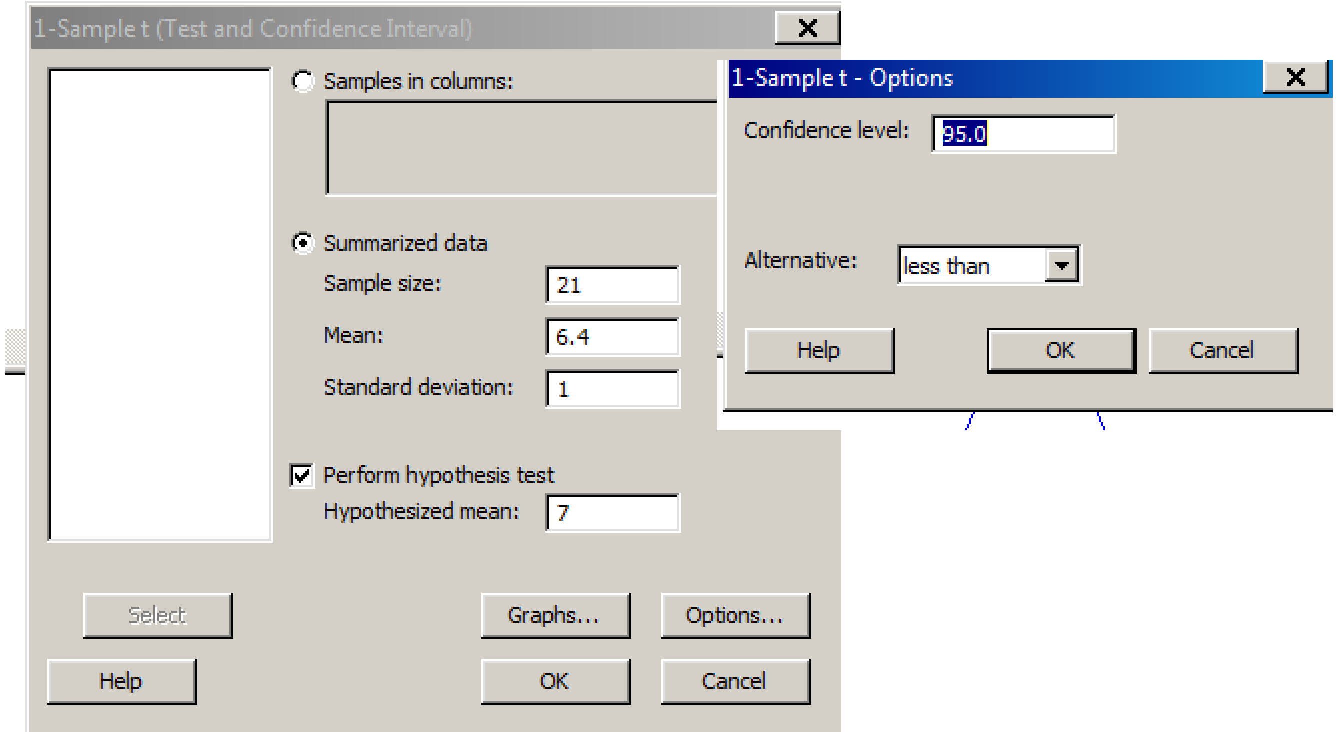
Test value: $$t_{test} = \frac{\bar{x} - u_o}{s/\sqrt{n}} = \frac{6.4 - 7}{1/\sqrt{21}} = -2.75$$

From t table, v=20 : P(t<2.528)=0.01 and P(t<2.845)=0.005

therefore .005 < P(t < 2.75) <0.01

Conclusion: Reject the null hypothesis.

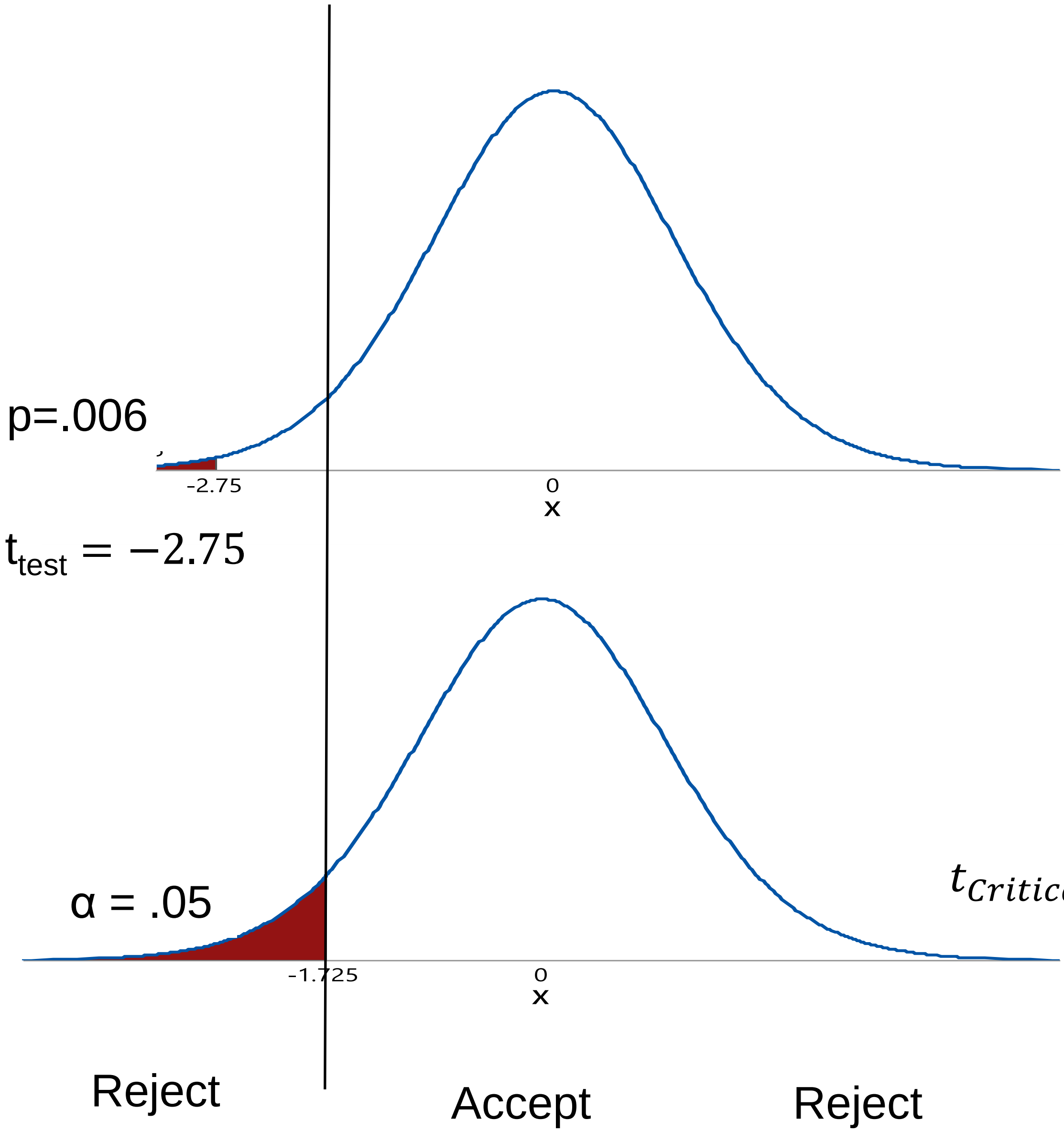
Conclude improvement has occurred.




# One-Sample T

Test of mu = 7 vs < 7

| N | Mean | StDev | SE Mean | 95% Upper Bound | T | P |
|---|---|---|---|---|---|---|
| 21 | 6.400 | 1.000 | 0.218 | 6.776 | -2.75 | 0.006 |




Compare $t_{test}$ to $t_{critical}$ defined rejection area.

Reject null if $t_{test}$ in rejection area

Reject null if $|t_{test}| > t_{critical}$

$$t_{Critical} = t_{\alpha} = t_{.05} = 1.725$$

$$t_{test} = \frac{\bar{x} - u_o}{s/\sqrt{n}}$$

The average time for students to register for classes at a certain college has been 7 minutes. The response time on the registration system has recently been upgraded. A random sample of 21 students was taken with the average time at 6.4 minutes and a standard deviation of 1. Can you, with 95% certainty, state that the upgrade has reduced the time to register?

$H_0$: μ = 7 $\qquad$ $H_a$ : μ < 7 :

n=21 $\qquad$ v=21-1=20 $\qquad$ α = .05 (one tailed test)

Test value: $$t_{test} = \frac{\bar{x} - u_o}{s/\sqrt{n}} = \frac{6.4 - 7}{1/\sqrt{21}} = -2.75$$

T critical: $t_{critical} = t_{\alpha,\upsilon} =$ $\qquad$ $t_{.05,20} = 1.725$

Conclusion: $|t_{test}| > t_{critical}$

Reject the null hypothesis.
Conclude improvement has occurred.

The average time for students to register for classes at a certain college has been 7 minutes. The response time on the registration system has recently been upgraded. A random sample of 21 students was taken with the average time at 6.4 minutes and a standard deviation of 1. Can you, with 95% certainty, state that the upgrade has reduced the time to register?

$H_0$: μ = 7 $H_a$ : μ < 7 :

n=21 v=21-1=20 α = .05 (one tailed test)

Test value:

$$t_{test} = \frac{\bar{x} - u_o}{s/\sqrt{n}} = \frac{6.4 - 7}{1/\sqrt{21}} = -2.75$$

T critical: $t_{critical} = t_{\alpha,\upsilon} =$ $t_{.05,20} = 1.725$

Conclusion: $|t_{test}| > t_{critical}$ Reject the null hypothesis.
Conclude improvement has occurred.

**One-Sample T**

Test of mu = 7 vs < 7

| N | Mean | StDev | SE Mean | 95% Upper Bound | T | P |
|---|---|---|---|---|---|---|
| 21 | 6.400 | 1.000 | 0.218 | 6.776 | -2.75 | 0.006 |

# Practice

The stress resistance of a certain plastic is specified to be 32 psi. The results from a test of 15 specimens resulted in an average of 31.6 psi and a standard deviation of 1.0 psi. Is there reason to doubt the specification? (Use a 95% confidence.)

13. What are the null and alternate hypotheses for this problem?

A) $H_0$: $\mu = 32$ $H_a$: $\mu < 32$

B) $H_0$: $\mu = 31.6$ $H_a$: $\mu < 31.6$

C) $H_0$: $\mu = 32$ $H_a$: $\mu \neq 32$

D) $H_0$: $\mu = 31.6$ $H_a$: $\mu \neq 31.6$

## Practice

$$t_{test} = \frac{\bar{x} - u_o}{s / \sqrt{n}}$$

The stress resistance of a certain plastic is specified to be 32 psi. The results from a test of 15 specimens resulted in an average of 31.6 psi and a standard deviation of 1.0 psi. Is there reason to doubt the specification? (Use a 95% confidence.)

14. $t_{critical}$ is

A) $t_{.05,\ 15}$ B) $t_{.05,\ 14}$ C) $t_{.025,\ 15}$ D) $t_{.025,\ 14}$

15. $t_{critical}$ is

A) 2.145 B) 2.131 C) 1.753 D) 1.725

# Practice

$$t_{test} = \frac{\bar{x} - u_o}{s/\sqrt{n}}$$

The stress resistance of a certain plastic is specified to be 32 psi. The results from a test of 15 specimens resulted in an average of 31.6 psi and a standard deviation of 1.0 psi. Is there reason to doubt the specification? (Use a 95% confidence.)

16. $t_{test}$ is

A)-.56 B) -1.549 C) 3.12 D) -1.88

## Practice

$$t_{test} = \frac{\bar{x} - u_o}{s/\sqrt{n}}$$

The stress resistance of a certain plastic is specified to be 32 psi. The results from a test of 15 specimens resulted in an average of 31.6 psi and a standard deviation of 1.0 psi. Is there reason to doubt the specification? (Use a 95% confidence.)

17. From this hypothesis test I

A) Reject the null hypothesis since $|t_{test}| > t_{critical}$
B) Do not reject the null hypothesis that μ=32
C) Accept the alternative hypothesis.
D) Both B and C.

# Practice

$$t_{test} = \frac{\bar{x} - u_o}{s/\sqrt{n}}$$

The stress resistance of a certain plastic is specified to be 32 psi. The results from a test of 15 specimens resulted in an average of 31.6 psi and a standard deviation of 1.0 psi. Is there reason to doubt the specification? (Use a 95% confidence.)

18. Between what appropriate comparison t values in the t table*, does the test statistic fall?

A) $t_{.10,\,14} = 1.345$ $t_{.05,\,14} = 1.761$

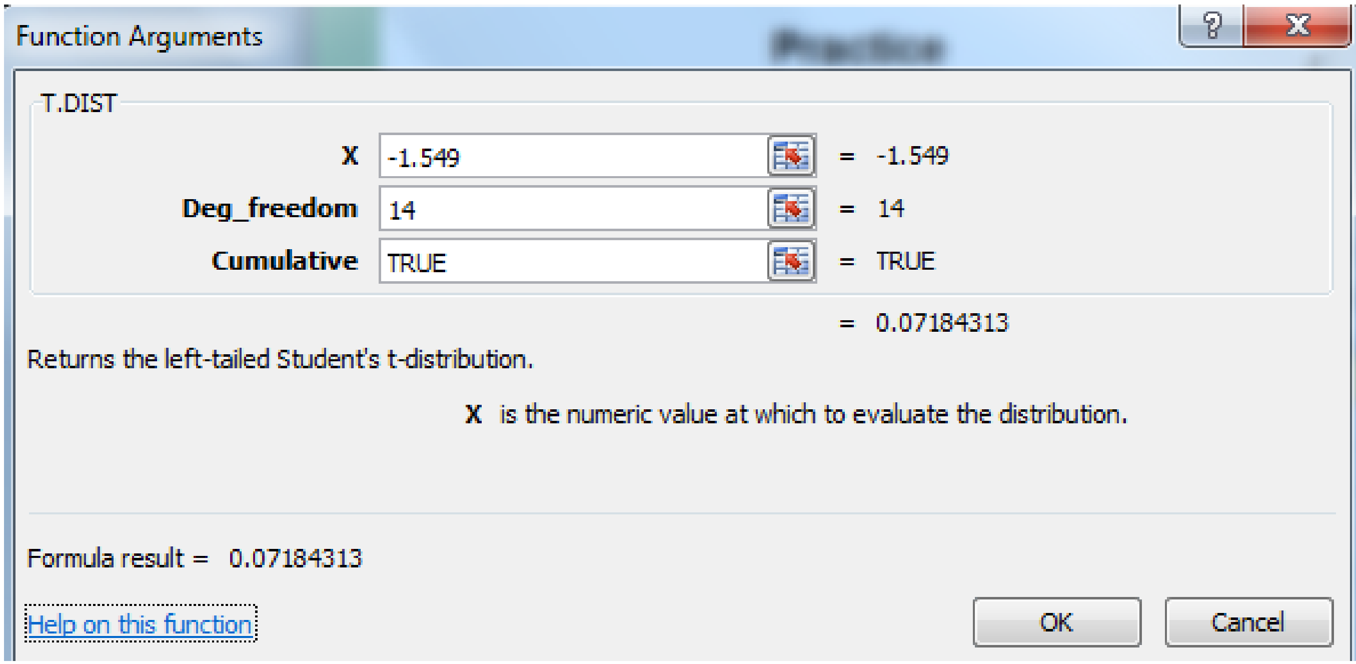
B) $t_{.25,\,15} = 0.691$ $t_{.10,\,15} = 1.341$

C) $t_{.25,\,14} = 0.694$ $t_{.10,\,14} = 1.350$

D) $t_{.10,\,14} = 1.345$, $t_{.25,\,14} = 1.761$

* Note that the t table only has positive values. Since the distribution is symmetrical you are essentially looking up the absolute value of $t_{test}$.

# Excel calculation for this problem:




Remember this is a two tailed test so the p value for the problem = 2 times .0718 = .1436

# Practice

The stress resistance of a certain plastic is specified to be 32 psi. The results from a test of 15 specimens resulted in an average of 31.6 psi and a standard deviation of 1.0 psi. Is there reason to doubt the specification? (Use a 95% confidence.)

After determining p = .1436 for this problem, I can say

T / F 19. If the true mean of the process is 32 psi with standard deviation of 1, the probability of getting a value of further from the mean than 31.6 is 14.36%

T / F 20. The p value of .1436 is greater the acceptable risk, $\alpha = .05$, Therefore I reject the null hypothesis that the mean psi is 32, and express my doubts about the specification.

**Comparing 1 sample mean with a known μ.**

| | Null Hypothesis | Alternate Hypothesis | Test statistic | Rejection Region |
|---|---|---|---|---|
| Normal Distribution (Large Sample) | $\mu = \mu_0$ | $\mu \neq \mu_0$ | $Z_{test} = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$ | $\lvert z_{test} \rvert > z_{\alpha/2}$ |
| | | $\mu < \mu_0$ | | $z_{test} < -z_{\alpha}$ |
| | | $\mu > \mu_0$ | | $z_{test} > z_{\alpha}$ |
| Students t Distribution (σ unknown, Small sample) t- test with $v = n - 1$ | $\mu = \mu_0$ | $\mu \neq \mu_0$ | $t_{test} = \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}}$ | $\lvert t_{test} \rvert > t_{\alpha/2,v}$ |
| | | $\mu < \mu_0$ | | $t_{test} < -t_{\alpha,v}$ |
| | | $\mu > \mu_0$ | | $t_{test} > t_{\alpha,v}$ |

Reminders –

- The assumption with the Z test is that ***σ*** is known or can be reasonably estimated with the sample data.
- Use the t test with sample sizes less than 30. ***σ*** is considered unknown and not estimated accurately with s.

# Related Assignments

Please see Blackboard for related assignments