COT 5407 Introduction to Algorithms
Homework 1

Due *in class* on Monday, September 11, 2017

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1. **[20 points]** You are given a stack of n pancakes that have different sizes. You need to sort the pancakes in such a way that the smaller pancakes are on top of the larger pancakes. The operation that you can perform is a **flip**. In a flip you insert a spatula under the top k pancakes for some integer k between 1 and n and flip them all over.

   (a) Describe an algorithm that sorts any stack of n pancakes with few flips as possible in the worst case.

   (b) Exactly how many flips does your algorithm do in the worst case?

2. **[20 points]** An inversion in an array $A[1..n]$ is defined as a two of indices (i, j) such that $i < j$ and $A[i] > A[j]$. The number of inversions on an array of size n is between 0 (when the array is sorted) and $\binom{n}{2}$ (when the array is sorted in reverse). Design and analyze an algorithm to count the number of inversions in an array with n elements in $O(nlog\,n)$ time. *Hint: Try modifying the Mergesort algorithm*.

3. **[15 points]**

   Sort the following functions in the table below from asymptotically smallest to asymptotically largest (indicate if there are ties). Do not write the proofs (but do them for practice), just the sorted list of functions. Remember than we use the convention $lg\,n = log_2\,n$.

n	$lg\,n$	$\sqrt{n}$	3^n
$\sqrt{lg\,n}$	$lg\,\sqrt{n}$	$3^{\sqrt{n}}$	$\sqrt{3^n}$
$3^{lg\,n}$	$lg\,(3^n)$	$3^{lg\,\sqrt{n}}$	$3^{\sqrt{lg\,n}}$
$\sqrt{3^{lg\,n}}$	$lg\,(3^{\sqrt{n}})$	$lg\,(\sqrt{3^n})$	$\sqrt{(lg\,(3^n))}$

4. **[15 points]** Give a solution to the following recurrences. Give tight asymptotic bounds in the form of $\Theta(f(n))$ for some recognizable function $f(n)$. If your solution needs a particular base case mention it. :

   - $T(n) = T(\sqrt{n}) + n$
   - $T(n) = T(n-1) + lg\,n$
   - $T(n) = 4T(n/8) + \sqrt{n}$

- $T(n) = 4T(n/2) + nlg\, n$

5. **[10 points]**

   Prove that for any positive integer, it is possible to write it as a sum of distinct powers of 2. Here are some examples $40 = 2^5 + 2^3$, $25 = 2^4 + 2^3 + 2^0$, $17 = 2^4 + 2^0$.

6. **[5 points]** Read chapter 1 from Cormen 3rd edition and briefly (a paragraph) answer questions **1.1-1, 1.1-3, 1.1-4**. For the reading part do not write anything in your solution.

7. **[15 points]** Carefully read pages 18-20 and 31-33 from Cormen 3rd edition where the correctness of insertion sort and merge sort are presented. Solve problem **2-2** (pg 40). For the reading part do not write anything in your solution.