MA 237-005
Assignment 1
Due Monday March 9, 2020

APPLICATIONS OF SOLVING LINEAR SYSTEMS

Turn in a hardcopy of your diary record which meets the guidelines for homework submissions. Also turn in the master MATLAB (.mat) file as an appendix:

1. Put your names, section number, date and assignment number at the top of the first page.
2. Label the problem number and separate different parts of a problem by a row of asterisks.
3. Staple the pages together in upper left corner.
4. Put comments in your work, using an editor if necessary. Avoid hand written material. For mathematical formulas, you may use either MATLAB code or ordinary mathematical notation, or a mixture of the two.
5. Try to produce a document which is neat and readable. Make believe that you are preparing a presentation for a major client.

1. A State Fish and Game Department supplies three types of food to a lake that supports four species of fish. The following matrix represents the weekly average need of foods for each fish. For example, the 5 means that each fish of Species 2 consumes, each week, an average of 5 units of Food 3.

	Species 1	Species 2	Species 3	Species 4
Food 1	1	3	2	2
Food 2	1	4	1	3
Food 3	2	5	4	1

Suppose that each week $25,000$ units of Food 1, $20,000$ units of Food 2 and $40,000$ units of Food 3 are supplied to the lake. Assume that all the food is eaten.

(a) Set up a system of equations where the unknowns x_1, x_2 etc are the numbers of each species of fish that can coexist in the lake.

(b) Find the augmented matrix of the system and solve the system using the **rref** command. Find your solutions in terms of free variables in matrix form.

(c) Assuming that each unknown (number of fish of each species) is non-negative (that is $x_i \geq 0$), find restriction(s) on the values of the free variables. Describe how such restriction(s) affects the values of the other unknowns.

(d) Determine the number of solutions in the solution set under the assumption in (c).

2. Enter the matrix

$$B = \begin{bmatrix} 1 & 2 & -3 & 4 & 5 \\ -2 & -5 & 8 & -8 & -9 \\ 1 & 2 & -2 & 7 & 9 \\ 1 & 1 & 0 & 6 & 12 \\ 2 & 4 & -6 & 8 & 11 \end{bmatrix}.$$

3. (a) In order to form the "super augmented" matrix use a command like **super=[B eye(5)]**. Use the **rref** command to determine the inverse of the matrix A. Say a little about what this command does.

(b) Use the **inv** command to find the inverse of B and store the inverse as D. Explain what this command does. Explain, without doing the calculation, what BD should be.

4. One process for encoding a secret message is to use certain matrices whose entries are integers and whose inverse also has integer entries. To do this, take a message, assign a number to each letter (eg. a=1, b=2, etc. and space =27) and arrange the numbers in a matrix from left to right in each row (see example 1 later), where the number of entries in the row matches the size of the encoding matrix B. Then postmultiply this matrix by B (ie. B is *on the right*), transcribe the message to a string of numbers (reading left to right on each row, see example 2 later), and send the message. In short we have

encoded message= (original message)$*B$.

(a) Suppose that you know the encoding matrix B and have received a string of numbers which represent an encoded message. What would need to be done now to decode the message. ie. how would one undo the encoding to find the original message?

(b) Suppose the encoding matrix is the matrix B entered earlier. Suppose that you receive the following message that was encoded using the matrix B. Decode it and interpret the message.

$$47, 49, -19, 257, 487, 10, -9, 63, 137, 236, 79, 142, -184, 372, 536, 59, 70, -40, 332, 588.$$

To get started you have to arrange the encoded message in a matrix with 5 columns (the size of the matrix B). Then perform the operation you need to decode the message, transfer the message back to row form and translate the message using $a = 1$ etc.

There is no need to include these examples in your hand-in version. They are just to help you with some of the problems.

Example 1. Arrange the row R=[1 2 3 4 5 6 7 8 9 10 11 12] into a 4×3 matrix M:

for $j = 1 : 4$

$M(j, :) = R(:, (j-1) * 3 + 1 : j * 3);$

end

Example 2. Arrange the 4×3 matrix M into a row R:

for $j = 1 : 4$

$R(:, (j-1) * 3 + 1 : j * 3) = M(j, :);$

end

Example 3. Convert the number message V=[19 5 5 27 25 15 21] to a letter message:

Alphabet='abcdefghijklmnopqrstuvwxyz', message=Alphabet(V)

you will obtain

Alphabet=

abcdefghijklmnopqrstuvwxyz

message=

see you