**Applications of Solving Linear System**

**1st Project Report for MATH-237-005**

**Linear Algebra**

**Submitted by**

**Yichen Huang**

**11906882**

**A picture containing object

Description automatically generated**

**The University of Alabama**

**Tuscaloosa, Alabama 35487**

**February 28, 2020**

Table of Contents

[Problem 1 3](#_Toc33540086)

[Question 3](#_Toc33540087)

[Answer 3](#_Toc33540088)

[Problem 2 5](#_Toc33540089)

[Question 5](#_Toc33540090)

[Answer 6](#_Toc33540091)

[Problem 3 6](#_Toc33540092)

[Question 6](#_Toc33540093)

[Answer 7](#_Toc33540094)

[Appendixes 9](#_Toc33540095)

[Appendix A – Problem 1 9](#_Toc33540096)

[Appendix B – Problem 2 9](#_Toc33540097)

[Appendix C – Problem 3 11](#_Toc33540098)

# Problem 1

## Question

A State Fish and Game Department supplies three types of food to a lake that supports

four species of fish. The following matrix represents the weekly average need of foods

for each fish. For example, the 5 means that each fish of Species 2 consumes, each

week, an average of 5 units of Food 3.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Species 1 | Species 2 | Species 3 | Species 4 |
| Food 1 | 1 | 3 | 2 | 2 |
| Food 2 | 1 | 4 | 1 | 3 |
| Food 3 | 2 | 5 | 4 | 1 |

Suppose that each week 25, 000 units of Food 1, 20, 000 units of Food 2 and 40, 000

units of Food 3 are supplied to the lake. Assume that all the food is eaten.

(a) Set up a system of equations where the unknowns x1,x2 etc. are the numbers of

each species of fish that can coexist in the lake.

(b) Find the augmented matrix of the system and solve the system using the “rref”

command. Find your solutions in terms of free variables in matrix form.

(c) Assuming that each unknown (number of fish of each species) is non-negative

(that is xi >= 0), find restriction(s) on the values of the free variables. Describe

how such restriction(s) affects the values of the other unknowns.

(d) Determine the number of solutions in the solution set under the assumption in

(c).

## Answer

### (a)

Let’s denote x1 as the number of fish which are species 1 can live in the lake; x2 as the number of fish which are species 2 can live in the lake; x3 as the number of fish which are species 3 can live in the lake; x4 as the number of fish are species 4 can live in the lake.

Then, we can set up the system equation:

### (b)

After the step of section(a), we can get an augmented matrix that:

After the “rref” command, we can get:

Then, we can learn that x4 is free variable. Let’s set s4 is the solution of x4.

Finally, we can get the solution that:

### (c)

From the problem (c), we can learn that x1, x2, x3 and x4 are greater and equal to 0.

Then, we plug in the solution from problem (b), we can get the system equation:

After calculation, we can get the solution:

Finally, we can get the solution of free variable:

### (d)

Let’s denote D as number of solutions:

Therefore, there are 151 solutions.

# Problem 2

## Question

Enter the matrix

(a) In order to form the “super augmented" matrix use a command like super=[B

eye(5)]. Use the rref command to determine the inverse of the matrix A. Say a

little about what this command does.

(b) Use the inv command to find the inverse of B and store the inverse as D:

Explain what this command does. Explain, without doing the calculation, what

BD should be.

## Answer

### (a)

After entering the command “super = [B | eye(5)]”, we can get

Then, entering the command “ A = rref(super)”, we can get

Therefore, the command “rref” change the matrix super to reduced echelon form, the right half of the A is inverse of inverse of B(B-1).

### (b)

After entering the command “D = inv(B)”, we can get

Because the “inv” command let D become the inverse of B. Therefore, B \* D = B \* B-1 = I5.

# Problem 3

## Question

One process for encoding a secret message is to use certain matrices whose entries are

integers and whose inverse also have integer entries. To do this, take a message, assign

a number to each letter (e.g. a=1, b=2, etc. and space =27) and arrange the numbers

in a matrix from left to right in each row (see example 1 later), where the number of

entries in the row matches the size of the encoding matrix B: Then post multiply this

matrix by B (i.e. B is on the right), transcribe the message to a string of numbers

(reading left to right on each row, see example 2 later), and send the message. In short

we have

encoded message= (original message) \* B.

(a) Suppose that you know the encoding matrix B and have received a string of

numbers which represent an encoded message. What would need to be done now

to decode the message. i.e. how would one undo the encoding to find the original

message?

(b) Suppose the encoding matrix is the matrix B entered earlier. Suppose that you

receive the following message that was encoded using the matrix B: Decode it

and interpret the message.

47; 49;-19; 257; 487; 10;-9; 63; 137; 236; 79; 142;-184; 372; 536; 59; 70;-40; 332; 588:

To get started you have to arrange the encoded message in a matrix with 5 columns

(the size of the matrix B). Then perform the operation you need to decode the

message, transfer the message back to row form and translate the message using

a = 1 etc.

## Answer

### (a)

To decode the message; we firstly change the encoded matrix to a new matrix which have 5 rows. Then, we let the new matrix multiply the inverse of B and change to the one row matrix. The one row matrix is the original message.

To encode the message; firstly, we need to change the string of the number to a matrix which the number of colum is 5. Because the number of rows in matrix B is 5. Then, we change the new matrix to 1 row matrix.

### (b)

To solve the problem, we firstly denote a matrix Encode to store the encoded number.

Then, we change the matrix Encode to a 4 × 5 matrix M.

Then, we let M multiply the inverse of B to get matrix P

After we get matrix P, we can change P to one row matrix R to get the decoded message

Finally, we can get the decoded the message is “are you having fun ”

# Appendixes

## Appendix A – Problem 1

>> A = [1 3 2 2 25000;1 4 1 3 20000;2 5 4 1 40000]

A =

1 3 2 2 25000

1 4 1 3 20000

2 5 4 1 40000

>> B = rref(A)

B =

1 0 0 -11 -35000

0 1 0 3 10000

0 0 1 2 15000

## Appendix B – Problem 2

>> B = [1 2 -3 4 5;-2 -5 8 -8 -9;1 2 -2 7 9;1 1 0 6 12;2 4 -6 8 11]

B =

1 2 -3 4 5

-2 -5 8 -8 -9

1 2 -2 7 9

1 1 0 6 12

2 4 -6 8 11

>> super = [B eye(5)]

super =

1 2 -3 4 5 1 0 0 0 0

-2 -5 8 -8 -9 0 1 0 0 0

1 2 -2 7 9 0 0 1 0 0

1 1 0 6 12 0 0 0 1 0

2 4 -6 8 11 0 0 0 0 1

>> A = rref(super)

A =

1 0 0 0 0 14 1 -2 1 -5

0 1 0 0 0 22 -7 -4 6 -19

0 0 1 0 0 13 -3 -2 3 -10

0 0 0 1 0 -2 1 1 -1 2

0 0 0 0 1 -2 0 0 0 1

>> D = inv(B)

D =

14.0000 1.0000 -2.0000 1.0000 -5.0000

22.0000 -7.0000 -4.0000 6.0000 -19.0000

13.0000 -3.0000 -2.0000 3.0000 -10.0000

-2.0000 1.0000 1.0000 -1.0000 2.0000

-2.0000 0 0.0000 0.0000 1.0000

## Appendix C – Problem 3

>> Encode = [47 49 -19 257 487 10 -9 63 137 236 79 142 -184 372 536 59 70 -40 332 588]

Encode =

Columns 1 through 14

47 49 -19 257 487 10 -9 63 137 236 79 142 -184 372

Columns 15 through 20

536 59 70 -40 332 588

>> for j = 1 : 4

M(j,:) = Encode(:,(j-1)\*5+1:j\*5);

end

>> M

M =

47 49 -19 257 487

10 -9 63 137 236

79 142 -184 372 536

59 70 -40 332 588

>> B = [1 2 -3 4 5;-2 -5 8 -8 -9;1 2 -2 7 9;1 1 0 6 12;2 4 -6 8 11]

B =

1 2 -3 4 5

-2 -5 8 -8 -9

1 2 -2 7 9

1 1 0 6 12

2 4 -6 8 11

>> P = M \* inv(B)

P =

1.0000 18.0000 5.0000 27.0000 25.0000

15.0000 21.0000 27.0000 8.0000 1.0000

22.0000 9.0000 14.0000 7.0000 27.0000

6.0000 21.0000 14.0000 27.0000 27.0000

>> for j=1:4

R(:,(j-1)\*5+1:j\*5) = P(j,:);

end

>> R

R =

Columns 1 through 8

1.0000 18.0000 5.0000 27.0000 25.0000 15.0000 21.0000 27.0000

Columns 9 through 16

8.0000 1.0000 22.0000 9.0000 14.0000 7.0000 27.0000 6.0000

Columns 17 through 20

21.0000 14.0000 27.0000 27.0000