Confidence Intervals in Randomized Block Designs using R

Consider the following data corresponding to a randomized block design with $k = 4$ treatments and $b = 5$ blocks.

	Block 1	Block 2	Block 3	Block 4	Block 5
Treatment 1	-3.0	-5.0	-5.5	-6.4	-5.5
Treatment 2	-2.3	-2.3	-1.8	-2.8	-1.6
Treatment 3	-0.3	1.6	1.5	-1.6	0.3
Treatment 4	6.2	4.6	5.5	5.5	3.7

We calculated the following the sums of squares in an earlier example:

$$\begin{aligned} \text{CM} &= 4.232 & \text{Total SS} &= 296.788 \\ \text{SSB} &= 5.608 & \text{MSB} &= 1.402 \\ \text{SST} &= 278.628 & \text{MST} &= 92.876 \\ \text{SSE} &= 12.552 & \text{MSE} &= 1.046 \end{aligned}$$

Comparing treatments

Recall that a $100(1-\alpha)\%$-confidence for the difference between τ_1 and τ_2 is given by

$$\bar{Y}_{1\bullet} - \bar{Y}_{2\bullet} \pm t_{\alpha/2} S \sqrt{\frac{2}{b}},$$

where $t_{\alpha/2}$ is calculated using $n - b - k + 1 = 12$ degrees of freedom and $S = \sqrt{MSE}$ gives an unbiased estimate of σ^2.

We can calculate the observed estimate of $\tau_1 - \tau_2$ by using the `rowMeans` function to calculate $\bar{Y}_{i\bullet}$ for each $i = 1, 2, 3, 4$.

```
# Create matrix storing observations.
b <- 5
k <- 4
n <- b*k
y <- matrix(data = c(-3.0, -2.3, -0.3, 6.2,
                     -5.0, -2.3, 1.6, 4.6,
                     -5.5, -1.8, 1.5, 5.5,
                     -6.4, -2.8, -1.6, 5.5,
                     -5.5, -1.6, 0.3, 3.7),
            nrow = 4, ncol = 5)

# Calculate row averages.
rm <- rowMeans(y)
```

The first two entries of the output vector `rm` store $\bar{Y}_{1\bullet}$ and $\bar{Y}_{2\bullet}$ respectively. The following code yields the desired confidence interval for $\alpha = 0.05$.

```
alpha <- 0.10
rad <- qt(p = alpha/2, df = n - b - k + 1, lower.tail = FALSE)*sqrt(mse)*sqrt(2/b)
lb <- rm[1] - rm[2] - rad
ub <- rm[1] - rm[2] + rad
```

This gives the 90% confidence interval for τ

$$-5.08 + 2.16 \pm 1.3562(1.0227414)\sqrt{\frac{2}{5}}$$

or, equivalently

$$-4.0728521 < \tau_1 - \tau_2 < -1.7671479.$$

Note that we can conclude that there is a significant evidence that τ_1 and τ_2 differ (at the $\alpha = 0.05$ level) since 0 is outside this confidence interval.