The Matched Pairs Experiment

Consider the following data, which corresponds to pairs of observations of two normally distributed random variables (x_i, y_i), $i = 1, 2, \ldots, 16$.

Group 1 (X)	Group 2 (Y)
-0.75	-0.01
-0.20	2.13
0.18	0.54
-1.49	3.29
0.73	-4.32
1.57	2.50
-0.78	1.37
0.92	2.94
0.23	0.89
-2.39	0.02
0.37	0.18
0.94	1.71
-0.03	0.25
1.24	1.86
-0.67	-0.40
0.27	2.26

Let's test the null hypothesis $H_0 : \mu_1 = \mu_2$ against the alternative hypothesis $H_a : \mu_1 \neq \mu_2$. Under the assumption that the differences $D_i = X_i - Y_i$ are normally distributed, we can use the test statistic

$$t = \frac{\bar{D}}{S_D/\sqrt{n}},$$

which has t-distribution with $n - 1$ degrees of freedom.

We can calculate the observed value of t for the observed data using the following code.

```r
# Store x and y as vectors.
x <-  c(-0.75, -0.20,  0.18, -1.49,  0.73,  1.57, -0.78, 0.92,
        0.23, -2.39,  0.37,  0.94, -0.03, 1.24, -0.67,  0.27)
y <- c(-0.01,  2.13,  0.54,  3.29, -4.32,  2.50,  1.37,  2.94,
       0.89,  0.02,  0.18,  1.71,  0.25,  1.86, -0.40,  2.26)
n <- length(x)

# Calculate mean difference.
Dbar <- mean(x - y)

# Calculate sample standard deviation of differences.
SD <- sd(x - y)

# Calculate value of t.
tval <- Dbar/(SD/sqrt(n))
```

This yields

$$\bar{D} = -0.941875, \qquad S_D^2 = 4.0799496, \qquad S_D = 2.0198885,$$

which gives the observed value of the test statistic

$$t = -1.865202.$$

For a two-sided t-test with $\alpha = 0.05$, we use the rejection region

$$|t| > t_{0.025} = 2.1314495.$$

Since the observed value of t is outside the rejection region, we do not have sufficient evidence to reject H_0.

Using the `t.test` function

Alternately, we could have performed this hypothesis test using the `t.test` function and the argument `paired = TRUE`:

```
t.test(x,y, paired = TRUE, alternative = "two.sided")
```

```
##
##  Paired t-test
##
## data:  x and y
## t = -1.8652, df = 15, p-value = 0.08184
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -2.0181976  0.1344476
## sample estimates:
## mean of the differences
##               -0.941875
```

This code returns a summary of the hypothesis test. In particular, it provides the observed value of the test statistic t (which agrees with that calculated above), as well as a 95% confidence interval for $\mu_1 - \mu_2$ based on the paired observations. Since the p-value (0.08184) is greater than 0.05, we cannot reject H_0.