

DSP HW2

December 2021

1 Q1

1.1 a

$$\begin{aligned}y_1[n] &= x_1[n] + x_1[n-1] = x_1[n] * (\delta[n] + \delta[n-1]) = x_1[n] * h_1[n] \\y_2[n] &= x_2[n] + 2x_2[n-1] - x_2[n-2] = x_2[n] * (\delta[n] + 2\delta[n-1] - \delta[n-2]) = \\&= x_2[n] * h_2[n] \\y_3[n] &= x_3[n-1] + x_3[n-2] = x_3[n] * (\delta[n-1] + \delta[n-2]) = x_3[n] * h_3[n] \\&\text{Therefore, in the cascading system, } y[n] = x[n] * h[n] \\&\Rightarrow y[n] = x[n] * (h_1[n] * h_2[n] * h_3[n]) \\&= x[n] * (\delta[n-1] + 4\delta[n-2] + 4\delta[n-3] - \delta[n-5]) \\&\Rightarrow \text{The impulse response of the system:} \\h[n] &= (\delta[n-1] + 4\delta[n-2] + 4\delta[n-3] - \delta[n-5])\end{aligned}$$

1.2 b

Yes, it is both FIR and IIR.

This system can be represented in the general form of FIR:

$$y[n] = \sum_{k=0}^M b_k x[n-k] = x[n-1] + 4x[n-2] + 4x[n-3] - x[n-5]$$

And it is obvious that it can also be represented in the general form of IIR:

$$y[n] = \sum_{l=1}^N a_l y[n-l] + \sum_{k=0}^M b_k x[n-k]$$

2 Q2

2.1 c

$$\begin{aligned}\text{Frequency response } H(e^{j\omega}) &= \sum_{k=-\infty}^{\infty} h[k] e^{-j\omega k} \\&= \sum_{k=-\infty}^{\infty} (\delta[k-1] + 4\delta[k-2] + 4\delta[k-3] - \delta[k-5]) e^{-j\omega k} \\&= e^{-j\omega} + 4e^{-2j\omega} + 4e^{-3j\omega} - e^{-5j\omega}\end{aligned}$$

2.2 d

Yes. By definition taught in class, *a system is causal if it does not depend on future inputs*. Since the mentioned system only depends on previous inputs, it is then a causal system.