10

DSP HW1

October 2021

1 Q1

Let $x_1(t) = A\cos(\omega t + \phi_1)$, $x_2(t) = B\cos(\omega t + \phi_2)$, being any two sinusoids of a same frequency. Since a cosine function can be expressed as: $\cos(\theta) = \frac{e^{j\theta}+e^{-j\theta}}{2}$, the sum of $x_1(t)$ and $x_2(t)$ is shown as follows:

$$x_1(t) = A\cos(\omega t + \phi_1) = A\frac{e^{j(\omega t+\phi_1)} + e^{-j(\omega t+\phi_1)}}{2}$$
$$x_2(t) = B\cos(\omega t + \phi_2) = B\frac{e^{j(\omega t+\phi_2)} + e^{-j(\omega t+\phi_2)}}{2}$$

$$\begin{aligned}
\to x_1(t) + x_2(t) &= \frac{1}{2}(Ae^{j(\omega t+\phi_1)} + Ae^{-j(\omega t+\phi_1)} + Be^{j(\omega t+\phi_2)} + Be^{-j(\omega t+\phi_2)}) \\
&= \frac{1}{2}(e^{j\omega t}(Ae^{\phi_1} + Be^{\phi_2}) + e^{-j\omega t}(Ae^{\phi_1} + Be^{\phi_2})) \\
&= \frac{1}{2}(e^{j\omega t} + e^{-j\omega t})(Ae^{\phi_1} + Be^{\phi_2}) \\
&= (Ae^{\phi_1} + Be^{\phi_2})\cos(\omega t)
\end{aligned}$$

Based on the above, we can conclude that **the sum of sinusoids of equal frequencies is still a sinusoid of the same frequency**.

2 Q2

Since the DC component is simply the average value in one period, instead of integrating it formally, simply calculate the area of the triangle in $[0, T_0]$, and then divide it by the time span T_0.

$$\frac{0.04 * 1}{2} * \frac{1}{0.04} = 0.5$$

The DC component of the signal is 0.5.

3 Q3

$$
\begin{aligned}
F(j\omega) &= \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \\
&= \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t}dt \\
&= \int_{0}^{\infty} e^{-(a+j\omega)t}dt \\
&= \frac{1}{a+j\omega}
\end{aligned}
$$

4 Q4

Let a Gaussian function be $f(t) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-t^2}{2\sigma^2}}$, its continuous Fourier transform pair $F(j\omega)$ can be derived as below:

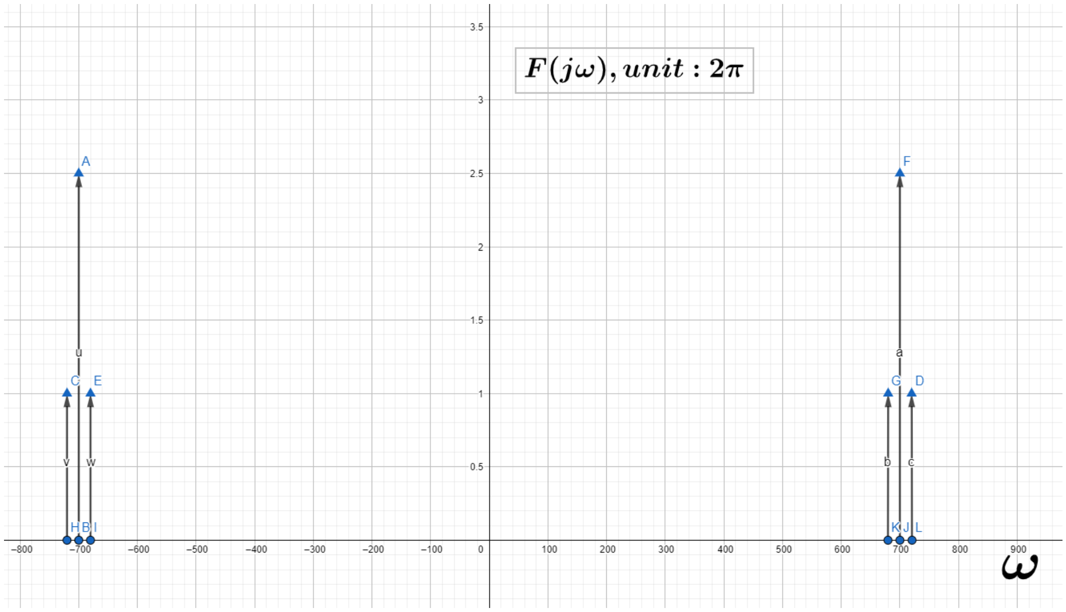
$$
\begin{aligned}
F(j\omega) &= \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \\
&= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{\frac{-t^2}{2\sigma^2}} \cdot e^{-j\omega t}dt \\
&= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{\frac{-1}{2\sigma^2}(t+\sigma^2 j\omega)^2}dt \cdot e^{\frac{(\sigma^2 j\omega)^2}{2\sigma^2}} \\
&= \frac{1}{\sqrt{2\pi\sigma^2}}[\sqrt{2\pi\sigma^2}]e^{(\frac{-\sigma^2\ \omega^2}{2})} = e^{(\frac{-\sigma^2\ \omega^2}{2})}
\end{aligned}
$$

$e^{(\frac{-\sigma^2\ \omega^2}{2})}$ is also a Gaussian function.

5 Q5

$$
\begin{aligned}
x(t) &= (5 + 4\cos(2\pi 20t))\cos(2\pi 700t) \\
&= (5 + 2(e^{(2\pi j20t)} + e^{(2\pi - j20t)}))\frac{1}{2}(e^{(2\pi j700t)} + e^{(2\pi - j700t)}) \\
&= \frac{5}{2}(e^{(2\pi j700t)} + e^{(2\pi - j700t)}) + e^{(2\pi j720t)} + e^{(2\pi - j720t)} + e^{(2\pi j680t)} + e^{(2\pi - j680t)}
\end{aligned}
$$

Since $\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{\pm j\omega t}dt = \delta(\omega)$, the spectrum of $x(t)$ can be illustrated as below:


$F(j\omega), unit: 2\pi$
A
F
C
E
G
D
u
a
v
w
b
c
H B I
K J L
3.5
3
2.5
2
1.5
1
0.5
0
−800
−700
−600
−500
−400
−300
−200
−100
100
200
300
400
500
600
700
800
900
ω