DSP HW3

January 2022

Q1

$h[n] = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega n}\, d\omega - \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega n}\, d\omega$
$= \delta[n] - h_{lowpass}$
$= \delta[n] - \frac{sin\omega_c n}{\pi n}$

Q2

a

$\sum_{n=0}^{N-1} e^{-j2\pi n/N} = \frac{1-e^{-j2\pi}}{1-e^{-j2\pi/N}}$
$\because e^{-j2\pi} = 1$
$\Rightarrow \frac{1-e^{-j2\pi}}{1-e^{-j2\pi/N}} = 0$

b

Following the notation in "Course 16 - DFT" page 13/30, $X[k] = \sum_{n=0}^{N-1} W_N^{kn}$.
$k = 0 \implies X[0] = \sum_{n=0}^{N-1} 1 = N$
$k \neq 0 \implies X[k] = 0$, this can be proved in the same fashion as (a).

Q3

a

$2*1+0*0+1*0+0*-1 = 2$
$2*-1+0*1+1*0+0*0 = -2$
$2*0+0*-1+1*1+0*0 = 1$
$2*0+0*0+1*-1+0*1 = -1$
Output=[2,-2,1,-1]

b

$X[k] = [3, 2+W_N^2, 2+W_N^4, 2+W_N^6] = [3,1,3,1]$
$Y[k] = [0, 1-W_N, 1-W_N^2, 1-W_N^3] = [0, 1+j, 2, 1-j]$

c

$Z[k] = [0, 1+j, 6, 1-j]$

d

$z[n] = \frac{1}{4}[(0{+}1{+}j{+}6{+}1{-}j), (j{-}1{-}6{-}j{-}1), (-1{-}j{+}6{-}1{+}j), (-j{+}1{-}6{+}j{+}1)]$
$= [2, -2, 1, -1]$

Q4

$V[k] = \sum_{n=0}^{2N-1} v[n] W_{2N}^{kn}$
$G[k] = \sum_{n=0}^{N-1} v[2n] W_N^{kn}$
$H[k] = \sum_{n=0}^{N-1} v[2n+1] W_N^{kn}$
Observe that $H[k]$ would need to be multiplied by W_{2N}^{k} to match up with the odd terms in $V[k]$.
$f[k] = W_{2N}^{k}$