Mark Scheme (Results)
January 2008
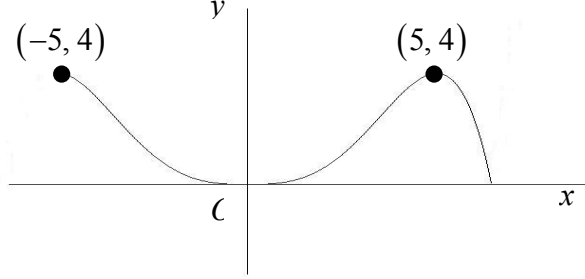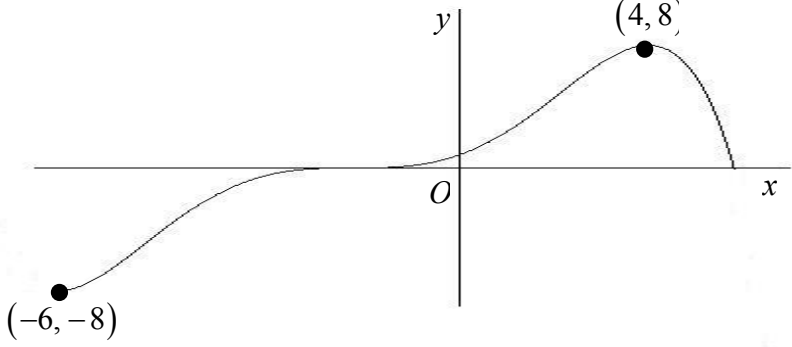
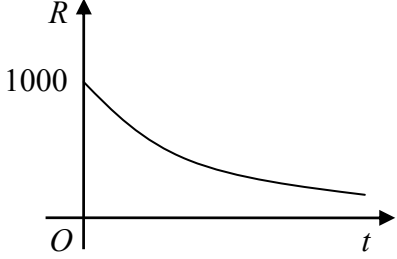**GCE**

GCE Mathematics (6665/01)

Question Number	Scheme	Marks
1.	$$x^2 - 1 \enclose{longdiv}{\begin{array}{l} \;\; 2x^2 \qquad\; -1 \\ \hline 2x^4 - 3x^2 + x + 1 \\ 2x^4 \;\; - 2x^2 \\ \hline \; -x^2 + x + 1 \\ \; -x^2 \quad\; +1 \\ \hline \; x \end{array}}$$ $a = 2$ stated or implied $c = -1$ stated or implied $$2x^2 - 1 + \frac{x}{x^2 - 1}$$ $a = 2, b = 0, c = -1, d = 1, e = 0$ $d = 1$ and $b = 0, e = 0$ stated or implied	M1 A1 A1 A1 **[4]**
2. (a)	$$\frac{dy}{dx} = 2e^{2x} \tan x + e^{2x} \sec^2 x$$ $$\frac{dy}{dx} = 0 \implies 2e^{2x} \tan x + e^{2x} \sec^2 x = 0$$ $$2\tan x + 1 + \tan^2 x = 0$$ $$(\tan x + 1)^2 = 0$$ $$\tan x = -1 \quad \ast \qquad\qquad \text{cso}$$	M1 A1+A1 M1 A1 A1 (6)
(b)	$$\left(\frac{dy}{dx}\right)_0 = 1$$ Equation of tangent at $(0, 0)$ is $y = x$	M1 A1 (2) **[8]**

Question Number	Scheme		Marks	
3.	(a) $\qquad \qquad f(2) = 0.38 \dots$			
	$\qquad \qquad f(3) = -0.39 \dots$		M1	
	Change of sign (and continuity) $\Rightarrow$ root in $(2, 3)$ ✶	cso	A1	(2)
	(b) $\qquad \qquad x_1 = \ln 4.5 + 1 \approx 2.50408$		M1	
	$\qquad \qquad x_2 \approx 2.50498$		A1	
	$\qquad \qquad x_3 \approx 2.50518$		A1	(3)
	(c) $\quad$ Selecting $[2.5045, 2.5055]$, or appropriate tighter range, and evaluating at both ends.		M1	
	$\qquad \qquad f(2.5045) \approx 6 \times 10^{-4}$			
	$\qquad \qquad f(2.5055) \approx -2 \times 10^{-4}$			
	Change of sign (and continuity) $\Rightarrow$ root $\in (2.5045, 2.5055\ )$			
	$\qquad \qquad \qquad \qquad \Rightarrow$ root $= 2.505$ to 3 dp ✶	cso	A1	(2)
				[7]
	Note: The root, correct to 5 dp, is 2.50524			

Question Number	Scheme	Marks
4.	(a)	
	Shape B1	
	$(5, 4)$ B1	
	$(-5, 4)$ B1 (3)	
	(b) For the purpose of marking this paper, the graph is identical to (a)	
	Shape B1	
	$(5, 4)$ B1	
	$(-5, 4)$ B1 (3)	
	(c)	
	General shape – unchanged B1	
	Translation to left B1	
	$(4, 8)$ B1	
	$(-6, -8)$ B1 (4)	
	In all parts of this question ignore any drawing outside the domains shown in the diagrams above.	**[10]**

Question Number	Scheme	Marks	
5.	(a) 1000	B1	(1)
	(b) $1000\,e^{-5730c} = 500$	M1	
	$e^{-5730c} = \dfrac{1}{2}$	A1	
	$-5730c = \ln\dfrac{1}{2}$	M1	
	$c = 0.000121$ cao	A1	(4)
	(c) $R = 1000\,e^{-22920c} = 62.5$ Accept 62-63	M1 A1	(2)
	(d) Shape 1000	B1 B1	(2) [9]

Question Number	Scheme	Marks
6.	(a) $\cos(2x+x) = \cos 2x \cos x - \sin 2x \sin x$	M1
	$\qquad = (2\cos^2 x - 1)\cos x - (2\sin x \cos x)\sin x$	M1
	$\qquad = (2\cos^2 x - 1)\cos x - 2(1 - \cos^2 x)\cos x$　any correct expression	A1
	$\qquad = 4\cos^3 x - 3\cos x$	A1　　(4)
	(b)(i)　$\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x} = \dfrac{\cos^2 x + (1+\sin x)^2}{(1+\sin x)\cos x}$	M1
	$\qquad\qquad = \dfrac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x)\cos x}$	A1
	$\qquad\qquad = \dfrac{2(1+\sin x)}{(1+\sin x)\cos x}$	M1
	$\qquad\qquad = \dfrac{2}{\cos x} = 2\sec x$　$\ast$　　　　　　cso	A1　　(4)
	(c)　　　$\sec x = 2$　or　$\cos x = \dfrac{1}{2}$	M1
	$\qquad\qquad x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$　　　　　accept awrt 1.05, 5.24	A1, A1　(3)
		[11]
7.	(a)　$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 6\cos 2x - 8\sin 2x$	M1 A1
	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_0 = 6$	B1
	$y - 4 = -\dfrac{1}{6}x$　　　　　　　　　or equivalent	M1 A1　(5)
	(b)　$R = \sqrt{(3^2 + 4^2)} = 5$	M1 A1
	$\tan\alpha = \dfrac{4}{3}$,　$\alpha \approx 0.927$　　　　　　awrt 0.927	M1 A1　(4)
	(c)　$\sin(2x + \text{ their } \alpha) = 0$	M1
	$\qquad x = -2.03, -0.46, 1.11, 2.68$	A1 A1 A1 (4)
	First A1 any correct solution; second A1 a second correct solution; third A1 all four correct and to the specified accuracy or better. Ignore the y-coordinate.	**[13]**

Question Number	Scheme	Marks
8.	(a) $x = 1 - 2y^3 \Rightarrow y = \left(\dfrac{1-x}{2}\right)^{\frac{1}{3}}$ or $\sqrt[3]{\dfrac{1-x}{2}}$	M1 A1 (2)

8.

(a) $x = 1 - 2y^3 \Rightarrow y = \left(\dfrac{1-x}{2}\right)^{\frac{1}{3}}$ or $\sqrt[3]{\dfrac{1-x}{2}}$ — M1 A1 (2)

$f^{-1} : x \mapsto \left(\dfrac{1-x}{2}\right)^{\frac{1}{3}}$ — Ignore domain

(b) $gf(x) = \dfrac{3}{1-2x^3} - 4$ — M1 A1

$= \dfrac{3 - 4(1-2x^3)}{1-2x^3}$ — M1

$= \dfrac{8x^3 - 1}{1-2x^3}$ ✻ — cso — A1 (4)

$gf : x \mapsto \dfrac{8x^3 - 1}{1-2x^3}$ — Ignore domain

(c) $8x^3 - 1 = 0$ — Attempting solution of numerator $= 0$ — M1

$x = \dfrac{1}{2}$ — Correct answer and no additional answers — A1 (2)

(d) $\dfrac{dy}{dx} = \dfrac{(1-2x^3) \times 24x^2 + (8x^3 - 1) \times 6x^2}{(1-2x^3)^2}$ — M1 A1

$= \dfrac{18x^2}{(1-2x^3)^2}$ — A1

Solving their numerator $= 0$ and substituting to find y. — M1

$x = 0, \ y = -1$ — A1 (5)

[13]