

Optimal Throughput and Delay in Delay-tolerant Networks with Ballistic Mobility

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ABSTRACT

This work studies delay and throughput achievable in delay-tolerant networks with ballistic mobility – informally, when the average distance a node travels before changing direction does not become vanishingly small as the number of nodes in the deployment area grows. Ballistic mobility is a simple condition satisfied by a large number of well-studied mobility models, including the i.i.d. model, the random waypoint model, the uniform mobility model and Levy walks with exponent less than 1. Our contribution is twofold.

First, we show that, under some very mild and natural hypotheses satisfied by all models in the literature, ballistic mobility is strictly necessary to achieve simultaneously, as the number of nodes grows, a) per-node throughput that does not become vanishingly small and b) communication delay that does not become infinitely large. Any network whose nodes exhibit a more “local” mobility pattern (e.g. Levy walks with exponent greater than 1, or Brownian motion) must sacrifice either a) or b), regardless of the communication scheme adopted – even with network coding.

Second, we present a novel packet routing scheme. Our scheme is relatively simple and does not rely on centralized control, replication, or static base stations. At the same time it achieves both non-vanishing throughput and bounded delay as the number of nodes grows, on any network with ballistic mobility (i.e. whenever they can be simultaneously achieved), asymptotically outperforming any existing communication scheme that exploits node mobility to boost throughput.

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1. INTRODUCTION

This work identifies in *ballistic mobility* the key to obtaining simultaneously high per-node throughput and low delay in delay-tolerant mobile wireless networks. After a very brief recap of the intuition why mobility can boost throughput in delay-tolerant networks (Subsection 1.1), and of the large number of models developed for the problem (Subsection 1.2), this introduction provides a succinct review of the state of the art (Subsection 1.3) and a summary of our results and of the organization of the following sections (Subsection 1.4).

1.1 Interference and Mobility

In wireless networks, interference can seriously reduce the throughput available to each node. Consider a network of n randomly deployed, *static* nodes equipped with omnidirectional antennas, each transmitting to a random destination either directly or through any number of relays. Then the maximum average throughput achievable by each node is only a fraction $O(1/\sqrt{n})$ of the peak throughput achievable by two nodes communicating in isolation [9].

[8] observed that throughput can be considerably improved (to a constant fraction of the peak throughput) if a constant fraction of the nodes is mobile, by accepting potentially large communication delays (this would be a reasonable tradeoff e.g. in many sensor network scenarios). Informally, when a source node u_s generates a packet for a destination node u_d , it transmits the packet to the first mobile node u_r passing “sufficiently close” that the Signal-to-Noise ratio between the two allows nearly peak throughput; when u_r eventually wanders “sufficiently close” to the destination u_d , it can in turn relay the information nearly at peak throughput. A vast literature has since appeared analysing the delay-throughput tradeoffs in networks exploiting node mobility [7, 13, 14, 17, 11, 12, 19, 8, 2, 15, 5, 20, 22].

1.2 Too Many Models?

It turns out that the performance achievable strongly depends on the mobility pattern of the nodes, and on the communication model adopted. Unfortunately, different models are appropriate for different scenarios: for example the mobility of wild animals is different from that of microsensors in a fluid, and both are different from that of humans. Furthermore, one can often model the same situation with different degrees of accuracy vs. tractability, and this has further multiplied the number of models in the literature.

All models in the literature share, however, some common characteristics. All assume nodes moving in straight lines

at constant speed between waypoints in a deployment area that is either a circle, a square, or a spherical or toroidal surface. Once a waypoint is reached, possibly after a waiting time drawn from some distribution, a new direction, distance and speed are selected and determine the next waypoint. Almost all mobility patterns observed tend to match this very general framework: inanimate objects tend to move in straight lines at constant speed between physical events that change their velocity vector, and animals and humans tend to take the shortest path between “points of interest” [18, 16].

The difference between mobility models lies in how direction, distance and speed between waypoints are selected [3]. All models choose the next waypoint randomly, possibly with some bounds on the maximum distance travelled, or choose a direction uniformly at random (bouncing off or “wrapping around” the edges of the deployment area) and a distance within an interval according to some probability distribution (typically either uniform or a power law privileging shorter distances). Speed is chosen randomly according to a distribution that may depend on distance travelled, but is otherwise independent of the direction of travel. In virtually all models a node’s choices at each waypoint are independent from those at previous waypoints, and from those of other nodes (again, this seems a fair approximation of observed mobility patterns [18, 16]).

In terms of communication, the most widely used model is the Signal-to-Noise ratio (SNR) model [2, 8, 9, 15, 21, 24], where the critical parameter when a source u_s attempts to transmit to a destination u_d is the Signal-to-Noise ratio $\frac{S_{u_s}}{N_0 + \sum_{u \neq u_s} S_u}$ between the source’s signal power S_{u_s} at u_d , and the sum of the signal power S_u of every other transmitting node u plus a constant background noise N_0 . If the ratio exceeds some threshold, the source can communicate at peak throughput with the destination. Signal power is known to decay with distance from the source as a *piecewise* polynomial [10, 24], i.e. the ratio between the signal power received at distance d_1 and d_2 lies between $(d_1/d_2)^{-\gamma_{min}}$ and $(d_1/d_2)^{-\gamma_{max}}$. In practice, most of the literature assumes for simplicity strictly polynomial decay, i.e. $\gamma_{min} = \gamma_{max}$. Both γ_{min} and γ_{max} are generally assumed larger than 2, so that, if nodes are uniformly distributed and all transmit with identical power, the bulk of the noise afflicting a communication will come from the nodes closest to the receiver. This justifies the even simpler Relaxed Protocol (RP) model [7] where a source can transmit at peak throughput with a destination at distance d if and only if every other node is at distance greater than $(1 + \Delta)d$ from the destination.

The majority of the literature assumes that nodes can communicate only at waypoints (“end of flight”), and not while in transit between them (“in flight”). The only justification for this choice is that it makes the analysis far simpler; in almost every scenario assuming in-flight communication is more realistic. Note that if the distance between consecutive waypoints is of the same order as the average distance between a node and its closest neighbour (i.e. $\Theta(1/\sqrt{n})$ for n nodes distributed in a unit square), there is little difference between in flight and end of flight – informally because the distance between any two nodes is unlikely to change drastically between two waypoints. However, ignoring in-flight communication can lead to strongly suboptimal performance for nodes moving on longer trajectories [11].

1.3 A Performance Comparison

Table 1 compares the various delay/throughput bounds in the literature, and those provided in this work. To allow a meaningful comparison we have rescaled all results to a common time and space scale (so results in the table may at first sight appear different from those in the respective articles, which use different units of measure). In particular, we assume that the diameter of the deployment area is 1; that the speed of the fastest node is 1 (meaning that spanning the diameter of the deployment area at maximum speed takes time 1); and that peak throughput achievable between nodes in the absence of interference is 1.

The random waypoint (RWP) model [15, 19, 20] and the i.i.d. model [14, 15, 22, 17, 11] both assume that the next waypoint is chosen uniformly at random; the former assumes speed is independent of distance travelled (and thus travel time is proportional to it), while the latter assumes speed is proportional to distance travelled (and thus travel time is independent of it). Similarly, truncated Levy walks (LW) and Levy flights (LF) [11, 12] assume, respectively, independent speed and independent travel time, but choose a random direction and a travel distance d in the interval $\approx [1/\sqrt{n}, 1]$ with probability proportional to $d^{-(1+\alpha)}$. The uniform mobility (UM) model of [2, 5] is similar to a Levy walk in that direction is chosen uniformly and speed is independent of distance travelled, but the distance d is chosen with probability $\Theta(e^{-d/\mu})$, with $\mu = \Theta(1)$. The Brownian motion (BM) model [8, 7, 19, 20, 13] can be seen as a truncated Levy walk/flight with $\alpha \rightarrow \infty$: nodes travel distance $\approx 1/\sqrt{n}$ between consecutive waypoints, all at the same speed.

Roughly speaking, *without in-flight communication, all schemes in the literature achieving $\Theta(1)$ throughput per node incur delay equal to the time required by a node to change direction $\Omega(n)$ times – i.e. delay $\Omega(\sqrt{n})$ to $\Omega(n)$ depending on the mobility model used.* Reducing throughput to $\lambda < 1$ allows a reduction in delay by a factor between λ to λ^3 depending on the model and the scheme used (including the possibility of diversity routing over multiple paths and/or of a centralized scheduler coordinating transmissions [6]).

In-flight communication increases performance, but *no previous work achieves $\Theta(1)$ throughput with less than $\Theta(\sqrt{n})$ delay.* $\Theta(1)$ throughput is achieved with $O(\sqrt{n})$ delay in the i.i.d. model, in the BM model, in the RWP model and in the LF model with $\alpha \leq 0$. In the LW model with $\alpha > 1$ and in the LF model with $\alpha > 0$ throughput $\Theta(1)$ is only achieved with larger delays, respectively $\Theta(n^{\frac{\alpha}{2}})$ and $\Theta(n^{\frac{1+\alpha}{2}})$. Again, reducing throughput to $\lambda < 1$ allows a reduction of delay by a factor between $\lambda^{\frac{1}{2}}$ and $\lambda^{\frac{3}{2}}$. The only work presenting a considerably better tradeoff is [2]: in a network where mobile nodes represent a fraction η of the node population and move according to the UM model, $\Theta\left(\eta \frac{1}{\log^2 n (\log n + 1/\mu)}\right)$ throughput is achieved with $\Theta(1)$ delay. However [2] assumes that *all destinations are static, with all their positions known to every mobile node*, and states that achieving a better tradeoff than [8] with mobile destinations seems impossible.

1.4 Our Results

This work presents the first scheme that achieves $\Theta(1)$ throughput with only $\Theta(1)$ delay. This is a $\Theta(\sqrt{n})$ improvement in delay over the best results with mobile destinations in the literature. Our scheme works in both the SNR and the RP communication models (with in-flight communica-

Table 1: Throughput and delay in different models. Coloured entries use in-flight communication.

Work	Mobility	Throughput λ	Delay D	Notes
[8]	BM	$\lambda = \Theta(1)$	$D = O(\inf)$	2-hop routing
[2]	UM	$\lambda = \Omega\left(\frac{\min(m,n)}{n \log^3 n}\right)$	$D = \Theta(1)$	m static nodes
[7]	BM	$\frac{1}{n} \leq \lambda \leq 1$	$D(\lambda) = O(\sqrt{n}\lambda)$	Multi-hop routing
[14], [15]	i.i.d.	$\frac{1}{n} \leq \lambda \leq 1$	$D(\lambda) = O\left(\frac{n\lambda^3}{\log^{9/2} n}\right)$	Centralized scheduler. Delay $\leq D$ for <i>each</i> packet w.h.p.
[15]	RWP	$\frac{1}{n} \leq \lambda \leq 1$	$D(\lambda) = \Theta\left(\frac{\sqrt{n}\lambda^2}{\log^{3/2} n}\right)$	Partially obtained by simulation
[5]	UM	$\lambda = \Theta(1)$	$D(\lambda) = O(n)$	Replication
[19], [20]	BM	$\lambda = \Theta(1)$	$D = O(\sqrt{n} \log^2 n)$	2-hop routing
[19], [20]	RWP	$\lambda = \Theta(1)$	$D = \Theta(\sqrt{n})$	Only simulation results
[22]	i.i.d.	$\lambda = \Theta\left(\frac{1}{n^{\frac{1-d}{2} \log \frac{5}{2} n}}\right)$	$D(\lambda) = O(n^d)$	Delay $\leq D$ for <i>each</i> packet w.h.p.
[17]	i.i.d.	$\frac{1}{\sqrt{n}} \leq \lambda \leq 1$	$D(\lambda) = O(n\lambda)$	Replication
[13]	BM	$\lambda = \Theta\left(\frac{1}{\sqrt{n}}\right)$	$D = \Theta(1)$	Centralized scheduler
[12]	LW	$\lambda = \omega\left(\frac{1}{\sqrt{n}}\right)$	$D = \Omega\left(n^{\frac{\max(1,\alpha)}{2}}\right)$	Only critical delay analysis
[12]	LF	$\lambda = \omega\left(\frac{1}{\sqrt{n}}\right)$	$D = \Omega\left(n^{\frac{\alpha}{2}}\right)$	Only critical delay analysis
[11]	i.i.d.	$\frac{1}{\sqrt{n}} \leq \lambda \leq 1$	$D(\lambda) = O(\sqrt{\max(1, n\lambda^3)})$	2-hop, multiple path routing
[11]	LF	$\frac{1}{\sqrt{n}} \leq \lambda \leq 1$	$D(\lambda) = O(\sqrt{\min(n^{1+\alpha}\lambda, n^2)})$	2-hop, multiple path routing
This work	Non-ballistic (includes BM, $\text{LW}_{\alpha \geq 1}, \text{LF}_{\alpha \geq 0}$)	$\lambda = \Theta(1)$	$D = \omega(1)$	Regardless of replication, network coding or centralized scheduling
This work	Ballistic (includes i.i.d., RWP, UM, $\text{LW}_{\alpha < 1}, \text{LF}_{\alpha < 0}$)	$\lambda = \Theta(1)$	$D = \Theta(1)$	Multi-hop routing, no replication, no network coding, no centralized scheduler

tion) for all *ballistic* mobility models: roughly speaking, all mobility models in which the positions of a node between time t and time $t+1$ have $\Omega(1)$ probability of being $\Omega(1)$ distance apart. Ballistic mobility thus encompasses the random waypoint, i.i.d. and uniform mobility models, Levy walks with $\alpha < 1$, and Levy flights with $\alpha < 0$. The key to our scheme’s performance is exploiting information about node positions for packet forwarding. Our scheme does not require centralized control, network coding or packet replication, and each node needs only have access to an accurate clock and gauge the position and velocity vectors *relative to itself* of nodes it successfully communicates with (a GPS is sufficient to guarantee this, but is not necessary).

Furthermore, we prove our result is strict in the sense that without ballistic mobility it is impossible to simultaneously achieve $\Theta(1)$ throughput and $\Theta(1)$ delay whether or not one uses in-flight communication, centralized omniscient control, multiple message copies, or network coding (as long as transmissions are omnidirectional).

The rest of this work is organized as follows. Section 2 introduces our mobility and communication models, formalizing the crucial notion of ballistic mobility. Section 3 proves that ballistic mobility is necessary to achieve simultaneously $\Theta(1)$ throughput and $\Theta(1)$ delay. Section 4 presents and analyses our routing scheme. Finally, Section 5 discusses the significance of our results as well as possible directions of future work.

2. A GENERAL NETWORK MODEL

This section describes our mobile network model. Our purpose is to make it as general as possible, so as to en-

compass the widest variety of “natural” models, including all models already in the literature. Subsections 2.1 and 2.2, below, describe respectively the mobility and communication models of the nodes. Subsection 2.3 introduces the crucial notion of ballistic mobility.

2.1 Mobility Model

n mobile nodes u_1, u_2, \dots, u_n move within a deployment area A that is either a square, a circle, a toroidal surface or a spherical surface, with diameter 1. We denote the position of node u_i at any time t by the vector $\mathbf{x}_i(t) = \langle x_i^1(t), x_i^2(t) \rangle$; given a pair of nodes u_i and u_j , we denote their distance at time t by $d_{i,j}(t)$. Every node travels in a straight line between intermediate waypoints; we refer to each such segment as a *trajectory*. Once at a waypoint, a new waypoint for that node is selected by choosing at random (independently from other waypoint choices for the same node or other nodes):

1. a direction uniformly in $[0, 2\pi[$
2. independently, a distance d according to some distribution D_d within the interval $[0, 1]$.
3. independently of the direction but not necessarily of the distance, a travel time (or wait time if $d = 0$). This induces a speed $s(d)$ according to some distribution $D_{s(d)}$ with support in $[0, 1]$ (i.e. maximum speed is 1).

We also require the speed distribution to satisfy a mild technical condition to avoid some pathological mobility patterns: if s has a probability bounded away from 0 of exceeding a certain threshold \bar{s} , then it has probability density at least $\Omega(1/\bar{s})$ over an open interval of speeds of size $\Omega(\bar{s})$

containing \bar{s} (this condition essentially prohibits node speeds from having infinitely small modes).

If the choices above would yield a waypoint beyond the boundaries of the deployment area (which is never the case for a toroidal or spherical surface, but may be the case with a square or circle), we allow the node to take an arbitrary path of length bounded by 1 at an arbitrary speed bounded away from 0 (including a path that leaves the deployment area at one point and re-enters at some other point); we call these arbitrary paths *out of boundary*, in order to distinguish them from *normal* trajectories. In this case we only place two mild technical conditions to avoid pathological situations: that at any given time the fraction of nodes on out-of-boundary paths be bounded away from 1, and that the steady state probability density of finding a node at any given point be roughly uniform across the deployment region, save possibly near the borders (where it can approach 0). This can be formalized using the definition of ρ -uniform distribution:

DEFINITION 1. Consider a probability density function $p(x)$ with support X and average $\bar{p} = \frac{1}{|X|} \int_X p(x)$. Given $\rho > 0$, $p(x)$ is ρ -uniform if $\rho\bar{p} \leq p(x) \leq \frac{1}{\rho}\bar{p}$ for every $x \in X$.

We then require the existence of some arbitrarily small ρ bounded away from 0, such that the node probability density is ρ -uniform at distance at least ρ from the boundaries of the deployment area. Note that the absence of out-of-boundary paths automatically guarantees ρ -uniformity.

It is easy to verify that this very general model encompasses all mobility models we described in the previous section, including the i.i.d., RWP, UM, LW, LF, and BM models. But it can also encompass additional/more general mobility models. For example, experimental studies [18] suggest that the “correct” mobility model for long-range human mobility is a hybrid between the Levy walk and the Levy flight, that we call the *Levy run* (LR). In a Levy run, the probability of selecting the next waypoint at distance d is still proportional to $d^{-(1+\alpha)}$, but speed grows with the distance as d^{α^*} for some non-negative $\alpha^* \leq 1$ (Levy walks and flights are then the two extreme cases of $\alpha^* = 0$ and $\alpha^* = 1$).

2.2 Communication Model

We assume all nodes are identical and can freely and continuously adjust their transmission power between 0 and a given maximum (we remark that in our routing scheme in Section 4 nodes do not take advantage of this flexibility, and always transmit at maximum power or not at all). We denote by $T_i(t)$ the ratio between the transmission power of node u_i at time t and its maximum transmission power (so $0 \leq T_i(t) \leq 1$), and by $S_{i,j}(t)$ the power of the signal from node u_i received at time t by node u_j . We assume that $S_{i,j}(t)$ is proportional to the transmission power of u_i , and decreases with distance $d_{i,j}(t)$ between u_i and u_j according to a piecewise power law. More formally, we assume the existence of two constants γ_{min} and γ_{max} with $2 < \gamma_{min} \leq \gamma_{max}$ such that, for any pair of nodes u_h and u_i with $T_h(t), T_i(t) > 0$, and any other node u_j with $d_{h,j}(t) \leq d_{i,j}(t)$, we have:

$$\frac{T_h(t)}{T_i(t)} \left(\frac{d_{h,j}(t)}{d_{i,j}(t)} \right)^{-\gamma_{min}} \leq \frac{S_{h,j}(t)}{S_{i,j}(t)} \leq \frac{T_h(t)}{T_i(t)} \left(\frac{d_{h,j}(t)}{d_{i,j}(t)} \right)^{-\gamma_{max}} \quad (1)$$

In the SNR model, denoting by N_0 the background noise and by β the minimum Signal-to-Noise ratio for successful transmissions, the transmission from u_i is received by u_j if:

$$\frac{S_{i,j}(t)}{N_0 + \sum_{h \neq i} S_{h,j}(t)} \geq \beta \quad (2)$$

In the simplified RP model [7], at any given time any node is either silent or transmitting at full power (i.e. $T_i(t) \in \{0, 1\} \quad \forall i, t$); the transmission from u_i is received by u_j if for every other node u_h transmitting at t :

$$d_{h,j}(t) \geq (1 + \Delta)d_{i,j}(t) \quad (3)$$

where $\Delta > 0$ is a system parameter. In either model, the peak throughput between two nodes in the absence of interference is 1, i.e. up to 1 unit of information can be exchanged in 1 unit of time – subdivided into packets we assume of identical size P for simplicity. We also make the realistic assumption that 1 unit of information amounts to at least $k_{com}\sqrt{n}$ packets (so $P \ll 1$) each of at least $k_{id} \log n$ bits for two appropriate constants $k_{com}, k_{id} = \Theta(1)$: informally, each packet must hold sufficient information to identify its sender and be short enough to be exchanged by two nodes during the time ($\approx 1/\sqrt{n}$) they remain closest neighbours. Note that $\Omega(n)$ simultaneous packet exchanges, and thus $\Omega(1)$ per-node throughput (regardless of delay), require most transmissions to originate from one of the $O(1)$ closest nodes to the receiver – so this “short packets” assumption is shared, implicitly or explicitly, by most previous work.

Each node *generates* packets at a rate of at most λ units of information per unit of time; each packet must be delivered (possibly through a number of intermediate relays) to a destination chosen uniformly and independently at random. We are interested in the maximum λ sustainable while guaranteeing a given delivery delay for at least a constant fraction of the packets [2]. More precisely, we say that a communication scheme *achieves throughput* $\lambda(n)$ *and delay* $D(n)$ if there exists a probability $p > 0$ independent of n such that, if every node generates $\lambda < \lambda(n)/p$ information per unit of time, each packet has probability at least p of being delivered within time $D(n)$ of its generation.

2.3 Ballistic Mobility

Despite the generality of our mobility model, we prove in the following sections that a very simple condition is both necessary and sufficient to guarantee simultaneously $\Omega(1)$ throughput and $O(1)$ delay: *ballistic mobility*. Informally, mobility is ballistic if, as the number of nodes n grows, at any given time a non-vanishing fraction of them is headed with non-vanishing speed towards a waypoint at non-vanishing distance. More formally:

DEFINITION 2. A mobility model is *ballistic* if there exists some $\epsilon > 0$ independent of n such that at any given time, choosing a node uniformly at random, with probability at least ϵ that node is headed with speed at least ϵ towards a waypoint at distance at least ϵ .

It is immediate to verify that the i.i.d., random waypoint, and uniform mobility models, as well as Levy flights with $\alpha < 0$, are ballistic. Similarly, it is immediate to verify that Brownian motion is *not* ballistic, since the distance between consecutive waypoints is $\Theta(1/\sqrt{n})$.

It is less obvious that Levy walks with $\alpha < 1$ are ballistic, since for any given $\epsilon > 0$ the probability that a node's next waypoint is selected at distance ϵ or greater is $\Theta(n^{-\alpha})$, which becomes vanishingly small with n . However, nodes selecting a distant waypoint spend a longer time to reach it, and thus the fraction of nodes on such "long" trajectories at any given time remains bounded away from 0 as long as $\alpha < 1$.

Similarly, it is easy to verify that Levy runs with $\alpha + \alpha^* < 1$ are ballistic, and that Levy runs with $\alpha + \alpha^* \geq 1$ (and thus Levy walks with $\alpha \geq 1$ and Levy flights with $\alpha \geq 0$) are not ballistic. Table 2 summarizes our classification between ballistic and non-ballistic models.

Table 2: Ballistic vs. non-ballistic models.

Ballistic	Non-ballistic
i.i.d., UM, RWP, $\text{LW}_{\alpha < 1}$, $\text{LF}_{\alpha < 0}$, $\text{LR}_{\alpha + \alpha^* < 1}$	BM, $\text{LW}_{\alpha \geq 1}$, $\text{LF}_{\alpha \geq 0}$ $\text{LR}_{\alpha + \alpha^* \geq 1}$

3. BALLISTIC MOBILITY IS NECESSARY FOR $\Theta(1)$ THROUGHPUT AND DELAY

This section is devoted to the proof of the following:

THEOREM 1. *Without ballistic mobility, no communication scheme can simultaneously achieve $\Theta(1)$ delay and $\Theta(1)$ average per-node throughput – even assuming centralized control, perfect knowledge of all future node movements and packet destinations, and arbitrary network coding.*

PROOF. The cornerstone of our proof is the fact that, in the absence of ballistic mobility, as n grows nodes tend to remain within a progressively smaller area during any given time interval. More formally:

LEMMA 1. *In the absence of ballistic mobility, for any arbitrarily small $\epsilon > 0$ there exists a sufficiently large \bar{n} such that, with $n > \bar{n}$ nodes, choosing a node u_i uniformly at random the expected maximum distance between any two positions of u_i in the interval $[t, t + 1]$ is strictly less than ϵ .*

PROOF. Consider any arbitrarily small $\delta > 0$. One difficulty in the proof of the lemma is dealing with "out-of-boundary" paths (in deployment areas with boundaries). Denote by \mathcal{E}_{out} the event that u_i is within distance δ of the boundaries (a region of area $\Theta(\delta)$) at time t . By our mobility model, $\Pr(\mathcal{E}_{out}) < k_{out}\delta$ for some constant k_{out} independent of n .

Suppose \mathcal{E}_{out} does not take place, so that u_i starts at distance greater than δ from the boundaries (this is automatically satisfied in deployment areas without boundaries such as spherical surfaces). Since mobility is not ballistic, for any arbitrarily small $\gamma > 0$ the event \mathcal{E}_{long} that during an interval of duration 1 u_i finds itself on a trajectory of length greater than γ can be given a probability smaller than δ by choosing a sufficiently large n . We want to prove that, if neither \mathcal{E}_{out} nor \mathcal{E}_{long} take place, the probability of the event \mathcal{E}_{max} that u_i achieves a maximum distance δ or greater from its starting point is less than δ (which avoids "out-of-boundary" paths, and proves the lemma). We do so by proving that with probability greater than $1 - \delta$ the maximum distance during the (possibly shorter) interval $[t', t'']$ from the first to the last waypoint reached during $[t, t + 1]$ is

less than $\delta - 2\gamma$ (since on the way to its first waypoint, and while moving away from the last waypoint, the node covers less than 2γ total distance).

Simple geometric considerations show that the furthest point in a segment AB from a given point C (that may or may not belong to AB) is either A or B. Denote by $\Delta x_i^1, \Delta x_i^2$ the difference between the coordinates of the i^{th} and $(i + 1)^{th}$ waypoint during $[t', t'']$. To prove the bound on the maximum distance moved during $[t', t'']$, we can then just prove that:

$$\max_i \sum_{j=1}^i \Delta x_i^1 \leq \frac{\delta - 2\gamma}{2} \quad (4)$$

and an identical bound on $\max_i \sum_{j=1}^i \Delta x_i^2$. Since we are considering at least $\frac{1-2\gamma}{\gamma}$ independent variables, with mean 0 and variance bounded by γ , by the law of iterated logarithm the left-hand term of Equation 4 is in expectation $\Theta(\sqrt{\gamma \log \log(1/\gamma)})$. Since the probability that a non-negative variable exceeds m times its expectation is less than $1/m$, by choosing a sufficiently small γ the probability that Equation 4 is not satisfied can be made smaller than δ .

Then, for any arbitrarily small $\delta > 0$, for all sufficiently large n the probability that u_i moves further than δ from its starting point during an interval of duration 1 is at most $\Pr(\mathcal{E}_{out}) + \Pr(\mathcal{E}_{long}) + \Pr(\mathcal{E}_{max}) < (k_{out} + 2)\delta$, proving the lemma. \square

A simple consequence of Lemma 1 is the following:

COROLLARY 1. *In a deployment area of diameter 1, with an expected node density at most $\rho n = O(n)$ at any given point, in the absence of ballistic mobility in expectation $o(n)$ nodes enter or leave any given convex cell of the deployment area during any interval $[t, t + \Delta t]$ with $\Delta t = O(1)$.*

PROOF. Simple geometric considerations show that for any η , within a deployment region of diameter 1, the area within distance η of the frontier of a generic convex cell is less than $3.1416 \cdot 2\eta$. By Lemma 1, for any arbitrarily small $\eta > 0$ and all sufficiently large n , the expected number of nodes further than η from the frontier at t that cross the frontier before $t + \Delta t$ is less than ηn ; and obviously the number of nodes closer to the frontier than η at t is less than $\rho \cdot 3.1416 \cdot 2\eta \cdot n$. \square

We now prove the main theorem by contradiction. Assume without loss of generality that the memory capacity M of each node is some integer multiple of P , and call *early* all throughput/packets with delay at most $1/\epsilon$, for some constant $\epsilon > 0$. Suppose that one can achieve in the absence of ballistic mobility for all n at least ϵ average per-node early throughput during an interval $[t, t + \frac{2}{\epsilon}]$ (note that all early packets generated during the first half of the interval must arrive before the interval's end).

Considering only early packets generated and delivered during $[t, t + \frac{2}{\epsilon}]$, partition the deployment area into convex cells of diameter $r = \Theta(\sqrt{\epsilon})$ and area $\Theta(r^2) = \Theta(\epsilon)$ sufficiently small that, for any given cell, the probability of a packet's destination being within that cell's r -neighbourhood (i.e. in that cell or within distance r of it) at any time during $[t, t + \frac{2}{\epsilon}]$ is less than $\frac{1}{4}\epsilon$ – this is possible by Corollary 1. For any such cell A , denote by \mathcal{S}_A the set of nodes always within A and by \mathcal{D}_A the set of nodes never inside the

r -neighbourhood of A during $[t, t + \frac{2}{\epsilon}]$. Then, with probability converging to 1 as n grows, one such cell A exists such that $|\mathcal{S}_A| \geq k_r \epsilon n$ for some $k_r = \Theta(1)$ (again, due to Corollary 1), the average per-node early throughput from \mathcal{S}_A is at least $\frac{3}{4}\epsilon$, and the average per-node throughput to destinations outside \mathcal{D}_A is at most $\frac{2}{4}\epsilon$; so the average per-node early throughput from \mathcal{S}_A to \mathcal{D}_A must be at least $\frac{1}{4}\epsilon$.

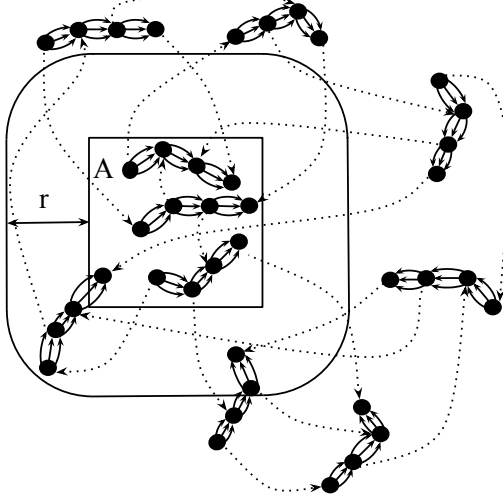


Figure 1: A convex cell A and its r -neighbourhood ($|\mathcal{S}_A| = 3$, $|\mathcal{D}_A| = 5$). Dotted and straight arcs model in-the-air and piggybacking information transfer, respectively.

Note that all information is transferred either “in the air” or by “piggybacking” on mobile nodes. We can model this with a communication multigraph having a vertex at each point in the deployment area where a mobile node transmits or receives a packet, an arc from any transmission vertex to the corresponding reception vertex (modelling in-the-air information transfer), and M/P arcs from any vertex to the vertex corresponding to the next transmission/reception from/to that same mobile node (modelling piggybacking information) – see Figure 1. As an arc marks a transfer of P information units, the number of arcs in the minimum cut separating the \mathcal{S}_A -vertices from the \mathcal{D}_A -vertices equals:

$$\text{mincut}(\mathcal{S}_A, \mathcal{D}_A) \geq k_r \epsilon n \cdot \frac{1}{4} \epsilon \cdot \frac{1}{P} = \frac{k_r}{4} \cdot \frac{\epsilon^2 n}{P} \quad (5)$$

A crucial observation is that Equation 5 holds even if network coding is used [1]. Thus there must be a set Π of $\frac{k_r}{4} \cdot \frac{\epsilon^2 n}{P}$ arc-disjoint paths from the set of \mathcal{S}_A -vertices to the set of \mathcal{D}_A -vertices. Any such path π alternates isolated in-the-air arcs with sequences of one or more piggybacking arcs (we only consider *maximal* sequences, i.e. sequences that are not proper subsequences of other sequences of piggybacking arcs in the path). Let the *range* of an in-the-air arc simply be its length, and let the range of a piggybacking sequence be the distance between its first and last vertex (rather than the sum of the lengths of individual arcs, which could be much larger by Lemma 1). Denote by $\delta_a(\pi)$ the sum of the in-the-air arc ranges of π , and by $\delta_b(\pi)$ the sum of the piggybacking sequence ranges. Then, remembering that each vertex in \mathcal{S}_A has distance at least r from every vertex in \mathcal{D}_A :

$$\delta_a(\pi) + \delta_b(\pi) \geq r \quad (6)$$

The total number of in-the-air arcs over any set of arc-disjoint paths cannot exceed $\frac{n}{P} \cdot \frac{2}{\epsilon}$ (since the entire network cannot transmit more than $\frac{n}{P}$ packets per unit of time), so the total number of piggybacking sequences cannot exceed $2 \cdot \frac{2n}{\epsilon P}$. For any arbitrarily small $\eta > 0$, by Lemma 1 for all sufficiently large n the maximum distance between two positions of a node during the interval $[t, t + \frac{2}{\epsilon}]$ is in expectation less than η . Then, in expectation for a sufficiently small $\eta = O(\epsilon^{\frac{7}{2}})$:

$$\begin{aligned} \sum_{\pi \in \Pi} \delta_a(\pi) &\geq r|\Pi| - \sum_{\pi \in \Pi} \delta_b(\pi) \\ &\geq r \left(\frac{k_r}{4} \cdot \frac{\epsilon^2 n}{P} \right) - \eta \cdot \frac{4n}{\epsilon P} \geq \frac{r|\Pi|}{2} = \Omega \left(\frac{\epsilon^{\frac{5}{2}} n}{P} \right) \end{aligned} \quad (7)$$

We easily prove that Equation 7 implies the existence of some $\eta > 0$ such that, for all n , at least ηn nodes can simultaneously transmit (successfully) at distance at least η . Suppose this were not the case: remembering that the diameter of the deployment area is 1 and that during the interval $[t, t + \frac{2}{\epsilon}]$ each node in the network transmits at most $\frac{2}{\epsilon P}$ packets, the sum of the distances of “long-range” transmissions would be less than $1 \cdot \frac{2}{\epsilon P} \cdot \eta n$, while the sum of the distances of all other transmissions would be less than $\eta \cdot \frac{2}{\epsilon P} \cdot n$ – yielding $\sum_{\pi \in \Pi} \delta_a(\pi) < \eta \cdot \frac{4n}{\epsilon P}$, which would violate Equation 7 for all sufficiently small η .

But we can prove, both in the SNR model and in the RP model, that no given $\eta > 0$ allows ηn nodes to successfully transmit to distance at least η for all n – leading to a contradiction and proving the theorem. In the RP model, each node successfully receiving from distance η precludes any other successful transmission within distance $(1 + \Delta)\eta$, so there are at most $O(\eta^2) = o(\eta n)$ such nodes at any given time in a deployment area of diameter 1. In the SNR model, if nodes $u_1, \dots, u_{\eta n}$ are all transmitting simultaneously at time t to a range between η and 1, assuming without loss of generality that the transmission power of $u_{\eta n}$ is no higher than that of $u_1, \dots, u_{\eta n-1}$ and that its destination is some node u_d , we have:

$$\frac{N_0 + \sum_{h=1}^{\eta n-1} S_{h,d}(t)}{S_{\eta n,d}(t)} \geq \sum_{h=1}^{\eta n-1} \left(\frac{1}{\eta} \right)^{-\gamma_{\max}} \quad (8)$$

and the last term is greater than $\frac{1}{\beta}$ for all sufficiently large n , violating Equation 2. \square

4. $\Theta(1)$ DELAY WITH $\Theta(1)$ THROUGHPUT

This section presents a simple routing scheme that achieves $\Theta(1)$ delay and $\Theta(1)$ throughput/node under any *ballistic* mobility model (and requires each node to store at most $O(1)$ units of information at any given time). The key to our scheme’s performance is exploiting information about node positions for packet forwarding. The only previous work to do so is [2], which in fact achieves $\Theta(1)$ delay with only $\text{polylog } n$ throughput degradation (due in part to an overaggressive transmission policy). But [2] assumes (and relies heavily on) *static* destinations; without this assump-

tion our task is far more challenging since node mobility makes positional information become quickly obsolete.

Subsection 4.1 describes the scheme and Subsection 4.2 analyses its performance, assuming (as most previous work) that packet destinations are chosen uniformly and independently at random, and that packets are the largest possible size (i.e. only $\Theta(1)$ packets, rather than $\Omega(1)$, can be exchanged between a node and its nearest neighbour in the $\Theta(1/\sqrt{n})$ time units before they move apart). Subsection 4.3 shows how these two assumptions can be easily removed.

4.1 Colouring, Sighting, Routing

We assume each node has access to an accurate clock and can assess the position and velocity vector *relative to itself* of any node it successfully communicates with. Note that nodes need not know their “global” position, the boundaries of the deployment area, the specific mobility pattern (as long as it is ballistic), the position of their next waypoint, or any other information about other nodes.

We assume direct communication between nodes takes place through a particular version of the Reservation Aloha protocol [4], where time is partitioned into slots of duration k_{slot}/\sqrt{n} with $k_{slot} = \Theta(1)$, and each slot is in turn partitioned into 10 subslots. On the first subslot each node transmits a packet of the subslot’s duration with independent probability $p_{tr} = \Theta(1)$; if the transmission is successful, the recipient transmits a reply on the second subslot, and then initiator and recipient alternate for the remaining subslots. If all 10 packets are successfully transmitted (note that due to mobility the “reservation” protocol may fail on any subslot), we say that the two nodes *sight* each other during that slot. Successful communication is obviously strongly dependent on distance: if two nodes are within distance k_{enc}/\sqrt{n} (for some $k_{enc} = \Theta(1)$) at some instant t , we say they *encounter* each other at t .

For simplicity of exposition we associate to each integer from 1 to n (and thus, implicitly, to each node) a unique *colour* from a set C . Packets exchanged in our scheme are of two types, *messages* and *sightings*. Messages are simply the data packets that our scheme must route from source to destination; each message is coloured with the same colour as its destination. Sightings carry information about node positions in the network: whenever a node u_s sights a node u_i , u_s generates a sighting of u_i – i.e. a packet carrying the time of the encounter as well as the position and velocity vector of u_i at that time. The latter are initially relative to u_s , but each subsequent node that receives u_i ’s sighting recalculates them relative to itself, and adjusts the sighting accordingly, from the position and velocity relative to itself of the node transmitting the sighting. Each sighting is coloured with the same colour as the “sighted” node.

In addition to its permanent colour, each node “flies a banner” of $k_b\sqrt{n}$ colours changing over time, for some $k_b = \Theta(1)$ – a new colour is added to the banner at each time slot, replacing the oldest colour, according to a hash function $\mathcal{H} : C \times \mathbb{N} \rightarrow C$ shared by all nodes in the network that associates to each $(node, slot)$ pair the colour entering the node’s banner in that slot. While a node u_i carries a colour in its banner we say u_i is a *banneret* for that colour and for the corresponding node. The use of a hash function allows a banneret to efficiently communicate its banner colours to the nodes it meets, since those nodes can reconstruct all the $\Theta(\sqrt{n})$ colours in the banner from the colour of the banneret.

Algorithm 1 Routing with ballistic mobility

each node generates new messages for random destinations at an information rate $\Theta(1)$
at any time slot \mathcal{S} , any node u_i keeps a set $M_i(\mathcal{S})$ of coloured messages and a set $\Sigma_i(\mathcal{S})$ of coloured sightings stored, and is associated to a banner $B_i(\mathcal{S})$
for each time slot \mathcal{S} **do**
 if u_i and u_j successfully communicate during \mathcal{S} **then**
 spotter/destination interaction:
 u_i stores in $\Sigma_i(\mathcal{S})$ a sighting of u_j
 u_j stores in $\Sigma_j(\mathcal{S})$ a sighting of u_i
 spotter/banneret interaction:
 u_i transmits to u_j a (random) sighting in $\Sigma_i(\mathcal{S})$ whose colour is in $B_j(\mathcal{S})$, if any
 u_j transmits to u_i a (random) sighting in $\Sigma_j(\mathcal{S})$ whose colour is in $B_i(\mathcal{S})$, if any
 source/banneret interaction:
 u_i transmits to u_j a (random) message in $M_i(\mathcal{S})$ whose colour is in $B_j(\mathcal{S})$, if any
 u_j transmits to u_i a (random) message in $M_j(\mathcal{S})$ whose colour is in $B_i(\mathcal{S})$, if any
 match message/sighting pairs:
 u_i matches messages in $M_i(\mathcal{S})$ with sightings in $\Sigma_i(\mathcal{S})$ of the same colour
 u_j matches messages in $M_j(\mathcal{S})$ with sightings in $\Sigma_j(\mathcal{S})$ of the same colour
 banneret/relay interaction:
 u_i transmits to u_j a (random) message in $M_i(\mathcal{S})$ whose destination u_j is projected to encounter, if any
 u_j transmits to u_i a (random) message in $M_j(\mathcal{S})$ whose destination u_i is projected to encounter, if any
 relay/destination interaction:
 u_i transmits to u_j a (random) message in $M_i(\mathcal{S})$ with destination u_j , if any
 u_j transmits to u_i a (random) message in $M_j(\mathcal{S})$ with destination u_i , if any
 end if
end for

This is the linchpin of our construction. Informally, a node’s banner states the colours of sightings and messages that node is “interested” in and will collect from those nodes it meets. Bannerets for a colour then act as rendezvous points for the sightings and messages of that colour, allowing each message to be routed in the “right” direction.

More precisely, after a node u_s generates a sighting of u_i , it retains the sighting for some time k_{ret} (with $k_{ret} = \Theta(1)$) and shares it with any banneret for u_i sighted during that time; we say u_s is acting as a *spotter* for u_i . If u_s could share multiple sightings it has collected, it chooses a random one. Similarly, any source node that generates a message for a destination u_d retains the message for time k_{ret} , and shares it with any banneret for u_d sighted during that time, choosing a random message if multiple ones could be shared.

Once a banneret u_b obtains both a message and a sighting of the same colour as a destination u_d , it can assess for each node u_r it sights whether that node will encounter u_d on its current trajectory (assuming u_d maintains the same velocity vector it had when sighted); if so, u_b transfers the message to u_r . In this case we say that u_r is acting as a *relay* for the message to u_d . Again, if a banneret could transfer multiple messages (for which it has obtained the corresponding

sightings) to a potential relay, it chooses a random one; and similarly, when a relay u_r sights a destination for one or more messages it is relaying, it delivers to that destination one of those messages at random. Like spotters, bannerets retain sightings for time k_{ret} before discarding them; and similarly, bannerets and relays retain messages for time k_{ret} before discarding them.

Then, in addition to a source of a message and its destination, three more nodes are involved in the delivery of a message: the banneret, the spotter (which must encounter the destination before it encounters the banneret), and the relay (which must encounter the banneret after the source and spotter do, and before it encounters the destination). Note that each node, at any given time, is simultaneously acting in all five roles for many (in fact, $\Theta(\sqrt{n})$) messages.

Algorithm 1 shows the pseudocode of our routing scheme, while Figure 2 provides a visual intuition; finally, Table 3 summarizes the scheme's constants introduced in this subsection and used throughout the rest of the analysis.

Table 3: Scheme constants.

Constant	Description
k_{slot}	Slot duration is k_{slot}/\sqrt{n}
k_b	Each banner holds $k_b\sqrt{n}$ colours
k_{enc}	Encounter distance is k_{enc}/\sqrt{n}
p_{tr}	Each node has probability p_{tr} of transmitting in any given time slot
k_{ret}	Sightings and messages are each retained for time k_{ret} by any given node before being discarded by that node

4.2 Analysis

This subsection is devoted to proving that, if each node generates messages for random destinations at an information rate $\Theta(1)$, then each message has probability $\Theta(1)$ of being delivered to its destination within time $\Theta(1)$.

Our proof proceeds in three steps. First, we show that with probability $\Theta(1)$ within time $\Theta(1)$ the source and a spotter from the destination both encounter a banneret, which in turn then encounters a relay to the destination. Then, we show that the probability that any of these encounters does not result in a sighting (i.e. that nodes involved fail to exchange information because of interference from other nodes) is bounded away from 1. Finally, we prove that the probability that during any such sighting the nodes involved fail to transmit the necessary data (because of conflicts with other data that should be transmitted in the same sighting) is also bounded away from 1.

To prove the first step we rely on two lemmas, informally stating that, given two nodes on “long” trajectories, any other node has probability $\Omega(1/\sqrt{n})$ of encountering either (Lemma 2, a result analogous to that in [11], but under a more general mobility model), and probability $\Omega(1/n)$ of encountering *both* (Lemma 3).

LEMMA 2. *Consider an arbitrary node u_i . Let $\mathcal{E}_i(t)$ be the event that at time t u_i is at distance $\Delta s = \Theta(1)$ from the deployment area's border (if any) on a normal trajectory it will maintain at least until time $t + \Delta t$ for some sufficiently small $\Delta t = \Theta(1)$. Choose another arbitrary node u_j . Conditioned on $\mathcal{E}_i(t)$, the probability that u_j encounters u_i during the interval $[t, t + \Delta t]$ is $\Omega(1/\sqrt{n})$.*

PROOF. Consider the trajectories of u_i and u_j during the interval $[t, t + \Delta t]$. With probability $\Omega(1)$ that of u_j is normal and without waypoints. Conditioned on this event, simple geometric considerations show that with probability $\Omega(1)$ the trajectories of u_i and u_j meet at some point in space \mathbf{x}_{enc} , which u_j has probability $\Omega(1)$ of reaching before the time t_{enc} when u_i does. By the mode condition in Subsection 2.1, with probability $\Omega(1/\sqrt{n})$ u_j travels at some speed $s \in \left[\frac{|\mathbf{x}_i(t_{enc}) - \mathbf{x}_j(t)|}{t_{enc} - t} - \frac{k_{enc}}{\sqrt{n}}, \frac{|\mathbf{x}_i(t_{enc}) - \mathbf{x}_j(t)|}{t_{enc} - t} + \frac{k_{enc}}{\sqrt{n}} \right]$ – in which case it is within distance $\frac{k_{enc}}{\sqrt{n}}$ of u_i at t_{enc} . \square

LEMMA 3. *Consider a distance $\Delta s = \Theta(1)$ and a duration $\Delta t = \Theta(1)$ sufficiently small that, choosing a node uniformly at random, with probability $\epsilon = \Theta(1)$ it is travelling with speed greater than $\frac{\Delta s}{\Delta t}$ to a waypoint at distance greater than Δs (note that Δs and Δt must exist if mobility is ballistic).*

Choose two arbitrary nodes u_h and u_i . Let $\mathcal{E}_{h,i}(t)$ be the event that they remain for (at least) an interval $[t, t + \Delta t]$ on two normal trajectories such that $\mathbf{x}_h(t + \frac{\Delta t}{3})$ and $\mathbf{x}_i(t + 2\frac{\Delta t}{3})$ are on a line at an angle $\theta = \Theta(1)$ with both trajectories, and are at distance $\frac{\Delta s}{3}$ from each other and at least $\frac{\Delta s}{3}$ from the border of the deployment area (if any). Choose another arbitrary node u_j . Conditioned on $\mathcal{E}_{h,i}(t)$, the probability that u_j encounters u_h during the interval $[t, t + 2\frac{\Delta t}{3}]$ and u_i during the interval $[t + \frac{\Delta t}{3}, t + \Delta t]$ is $\Omega(1/n)$.

PROOF. With probability $\Omega(1)$ the trajectory of u_j is normal and without waypoints during $[t, t + \Delta t]$. Simple geometric considerations show that, for any arbitrarily small $\delta = \Theta(1)$, there is a probability $\Omega(1)$ that the line of the trajectory of u_j will intersect the lines of the trajectories of u_h and u_i within distance δ respectively of $\mathbf{x}_h(t + \frac{\Delta t}{3})$ and of $\mathbf{x}_i(t + 2\frac{\Delta t}{3})$. Furthermore, for any arbitrarily small $\delta = \Theta(1)$ there is some sufficiently small $\eta = \Theta(1)$ such that, choosing any point \mathbf{x}^1 on the trajectory of u_h within distance η of $\mathbf{x}_h(t + \frac{\Delta t}{3})$ and any point \mathbf{x}^2 on the trajectory of u_i within distance η of $\mathbf{x}_i(t + 2\frac{\Delta t}{3})$, we have that $(1 - \delta)\frac{\Delta s}{3} < |\mathbf{x}^2 - \mathbf{x}^1| < (1 + \delta)\frac{\Delta s}{3}$. Note that u_h and u_i reach \mathbf{x}^1 and \mathbf{x}^2 within time η of $t + \frac{\Delta t}{3}$ and of $t + 2\frac{\Delta t}{3}$ (respectively). Then, by the mode condition in Subsection 2.1, for any such pair of points \mathbf{x}^1 and \mathbf{x}^2 the probability density of the speed of u_j within the interval $\left[\frac{|\mathbf{x}^2 - \mathbf{x}^1| - 2\eta}{\Delta t/3 + 2\eta}, \frac{|\mathbf{x}^2 - \mathbf{x}^1| + 2\eta}{\Delta t/3 - 2\eta} \right]$ is $\Theta(1)$.

Assume then that the line of u_j 's trajectory does indeed intersect the trajectories of u_h and u_i within distance η of $\mathbf{x}_h(t + \frac{\Delta t}{3})$ and of $\mathbf{x}_i(t + 2\frac{\Delta t}{3})$ – which, as we said, happens with probability $\Theta(1)$ – at two points \mathbf{x}_{enc}^1 and \mathbf{x}_{enc}^2 that u_h and u_i reach, respectively, at time t_{enc}^1 and t_{enc}^2 . The interval of speeds that guarantee u_j covers the distance $|\mathbf{x}_{enc}^2 - \mathbf{x}_{enc}^1|$ in time $(t_{enc}^2 - t_{enc}^1) \pm \frac{k_{enc}}{2\sqrt{n}}$ has size $\Omega(1/\sqrt{n})$, and thus the probability that u_j chooses such a speed v is $\Omega(1/\sqrt{n})$. Conditioned on such a choice, the probability that the position of u_j (on the line of its trajectory) at time t is within distance $v \cdot (t_{enc}^1 - t) \pm \frac{k_{enc}}{2\sqrt{n}}$ of \mathbf{x}_{enc}^1 is also $\Omega(1/\sqrt{n})$ – meaning that the probability that u_j will pass within distance $\frac{k_{enc}}{2\sqrt{n}}$ of u_h and within distance $\frac{k_{enc}}{2\sqrt{n}} + \frac{k_{enc}}{2\sqrt{n}} = \frac{k_{enc}}{\sqrt{n}}$ of u_i is $\Omega(1/n)$. \square

Given a message generated at time t_{gen} by a source u_{src} with destination u_d , arbitrarily partition the set of all other

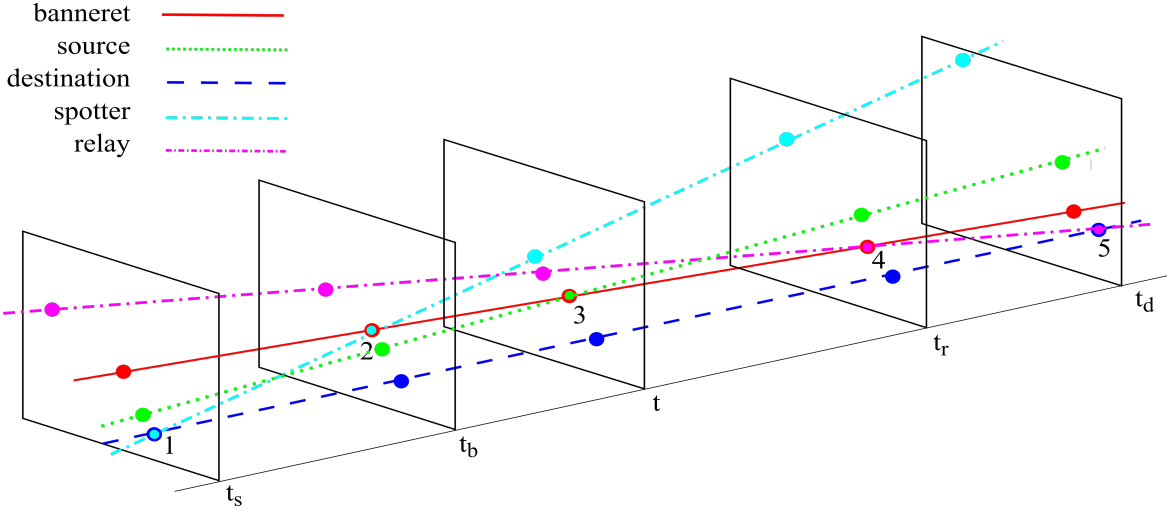


Figure 2: Delivery of a message: 1. the spotter generates a sighting of the destination 2. the spotter transmits the sighting to the banneret 3. the source transmits the message to the banneret 4. the banneret transmits the message to the relay 5. the relay transmits the message to the destination.

nodes into three subsets U_1, U_2, U_3 each of size at least $\lfloor \frac{n-2}{3} \rfloor$. Then, for some $k_1 \in \Theta(1)$, in U_1 there are with probability $1 - o(1)$ at least $(k_1 - o(1))\sqrt{n}$ bannerets for u_d that will remain bannerets for u_d for the next interval of time of duration k_1 . With ballistic mobility u_{src} has probability $\Theta(1)$ of having the properties that make Lemma 2 applicable. Then Lemma 2 ensures that, with probability $\Theta(1)$, one banneret u_b encounters u_{src} at some time $t < t_{gen} + k_{s-b}$ for $k_{s-b} = \Theta(1)$.

Similarly, u_d has probability $\Theta(1)$ of having a trajectory, relative to u_b , that makes Lemma 3 applicable. Then with probability $\Theta(1)$, by Lemma 3, in U_2 one node u_s (a spotter) will encounter u_d at time t_s and then u_b at time $t_b > t_s$, and in U_3 one node u_r (a relay) will encounter u_b at some time $t_r > t_b$ and then u_d at some later time $t_d > t_r$.

Given the trajectories of the source, destination, banneret, spotter and relay, we now prove that with strictly positive probability the destination sights the spotter and the relay, and the banneret sights the spotter, the source, and the relay – all in the “right” order. Note that these five nodes have some positional constraints, which *do* alter the probability distribution of the remaining nodes. The following lemma allows us to prove that such a perturbation is not too large:

LEMMA 4. *Assume that n balls are ρ -uniformly distributed in a set of m bins. Let \mathcal{E}_a^i be the event that bin i holds at least a balls. Then, for any $u \geq n/m$ and any $d \geq 0$, $\Pr(\mathcal{E}_{u+d}^i | \mathcal{E}_u^i) \leq \Pr(\mathcal{E}_u^i)$.*

PROOF. Since the distribution is ρ -uniform, we can denote by k_i/m the probability that a ball chooses bin i , for some $k_i = \Theta(1)$. The probability $p_i(x)$ that bin i has exactly x balls is

$$p_i(x) = \binom{n}{x} \left(\frac{k_i}{m} \right)^x \left(1 - \frac{k_i}{m} \right)^{n-x}, 0 \leq x \leq n \quad (9)$$

We must prove that

$$\frac{\sum_{x=u+d}^n p_i(x)}{\sum_{x=d}^n p_i(x)} \leq \sum_{x=u}^n p_i(x) \quad (10)$$

Equation 10 is immediately satisfied if $d = 0$. We then prove that for $d > 0$

$$\frac{\sum_{x=u+d}^n p_i(x)}{\sum_{x=d}^n p_i(x)} \leq \frac{\sum_{x=u+d-1}^n p_i(x)}{\sum_{x=d-1}^n p_i(x)} \quad (11)$$

Note that the right side of Equation 11 is a linear convex combination of the term on the left side and $\frac{p_i(u+d-1)}{p_i(d-1)}$. Also, the left side is a linear convex combination of $n-d+1$ terms, where the j^{th} term is $\frac{p_i(u+d+j)}{p_i(d+j)}$ if $j \in [0, n-u-d]$, and 0 if $j \in [n-u-d+1, n-d]$. Therefore, it is enough to show that $\frac{p_i(u+d)}{p_i(d)} \leq \frac{p_i(u+d-1)}{p_i(d-1)}$ – the proof that $\frac{p_i(u+d+j)}{p_i(d+j)} \leq \frac{p_i(u+d+j-1)}{p_i(d+j-1)}$ for $j \geq 0$ is virtually identical. By Equation 9, this is true if:

$$\frac{\binom{n}{u+d-1}}{\binom{n}{d-1}} \geq \frac{\binom{n}{u+d}}{\binom{n}{d}} \quad (12)$$

and since Equation 12 can be simplified into $u(n+1) \geq 0$, which always holds given that $u, n \geq 0$, our claim follows. \square

By Lemma 4 then, at any given time and in any region of space where the expected number of nodes would be m and the probability that such a number exceeded $m' \geq m$ would be $p(m')$ in the absence of any constraints placed by the selection of source, destination, banneret, spotter and relay, the probability of witnessing more than $m' + 5$ nodes does not exceed $p(m')$. By allowing nodes to communicate, at any time slot, with a sufficiently low transmission probability p_{tr} , we can lower to an arbitrarily small ϵ the probability that, in any given time slot where a node *does* transmit, the transmission is unsuccessful because of interference from other nodes – while still guaranteeing a probability bounded away from 0 of a transmission. More formally:

LEMMA 5. *For any arbitrarily small $\epsilon > 0$ and arbitrarily large ρ , there exists a sufficiently small $p_{tr} = \Theta(1)$ such that, for all sufficiently large n , the following is true. Assume n nodes are placed independently at random according to some*

distribution with an expected density no greater than ρn at any point of the deployment area, subject to arbitrary position and velocity constraints on 5 nodes u_1, \dots, u_5 . Then if u_1 encounters some node u_i at some time t_{enc} during a slot $[t, t + \frac{k_{slot}}{\sqrt{n}}]$ and chooses to transmit to u_i during the slot, the probability of an unsuccessful transmission is less than ϵ , both in the SNR and in the RP model.

PROOF. First of all, note that if two nodes are at distance d of each other at some time during a slot, they are never at distance greater than $d + \frac{2k_{slot}}{\sqrt{n}}$ or smaller than $d - \frac{2k_{slot}}{\sqrt{n}}$ during the same slot.

Let us consider the RP protocol. If u_1 encounters u_i , their distance is never greater than $\frac{k_{enc} + 2k_{slot}}{\sqrt{n}}$ at any time during the slot. Thus, only nodes that, at time t , are within distance $r_0(n) = (1 + \Delta) \frac{k_{enc} + 2k_{slot}}{\sqrt{n}} + 2 \frac{k_{slot}}{\sqrt{n}}$ of u_i can interfere. The expected number of such nodes, including u_i and the arbitrarily positioned u_2, u_3, u_4, u_5 , transmitting during the slot is by Lemma 4 at most $p_{tr} (5 + \rho n \cdot 3.1416 (r_0(n))^2)$ – which can be made *arbitrarily* smaller than ϵ by a suitable choice of p_{tr} and is obviously an upper bound on the probability of at least one such node being present.

Let us leverage this result to prove the lemma in the SNR model, remembering that the choice of $\Delta > 0$ in the RP model is arbitrary, and that for all sufficiently large n and any τ in the slot $\frac{S_{1,i}(\tau)}{N_0} > \frac{\beta}{2}$ – so we need only prove that $\frac{S_{1,i}(\tau)}{\sum_{j \neq 1} S_{j,i}(\tau)} \geq \frac{\beta}{2}$ with probability greater than $(1 - \epsilon)$. Consider a set of circles C_0, C_1, \dots , all centred on the position of u_i at time t , with the radius of C_j being $r_j(n) = 2^j \cdot (1 + \Delta) \frac{k_{enc} + 2k_{slot}}{\sqrt{n}} + 2 \frac{k_{slot}}{\sqrt{n}}$ (note that this matches the previous definition of $r_0(n)$). By the same arguments used for the RP model, any node outside C_j at time t will have a distance at least $2^j(1 + \Delta)$ times that of u_1 from u_i throughout the slot.

Denote by M_j the set of nodes that at time t are inside C_j but not inside C_{j-1} and will transmit during the slot. Noting that $r_{i+1} < 2r_i$, in the worst case scenario where u_2, \dots, u_5 are inside C_0 , we have that $E[|M_j|] \leq 4E[|M_{j-1}|]$. For any τ in the slot we then have:

$$\begin{aligned} E \left[\max_{\tau} \frac{\sum_{j=1}^{\infty} \sum_{u_h \in M_j} S_{h,i}(\tau)}{S_{1,i}(\tau)} \right] &\leq \sum_{j=1}^{\infty} E \left[\max_{\tau} \frac{\sum_{u_h \in M_j} S_{h,i}(\tau)}{S_{1,i}(\tau)} \right] \\ &\leq \sum_{j=1}^{\infty} E \left[\frac{|M_j|}{(2^j(1 + \Delta))^{\gamma_{min}}} \right] \leq \frac{E[|M_0|]}{(1 + \Delta)^{\gamma_{min}}} \sum_{j=1}^{\infty} \left(\frac{4}{2^{\gamma_{min}}} \right)^j \\ &\leq \frac{E[|M_0|]}{(1 + \Delta)^{\gamma_{min}}} \cdot \frac{2^{2 - \gamma_{min}}}{1 - 2^{2 - \gamma_{min}}} \end{aligned} \quad (13)$$

and for a sufficiently small $E[|M_0|]$ the last term is less than $\frac{\epsilon}{2} \cdot \frac{2}{\beta}$, and thus $\Pr \left[\max_{\tau} \frac{\sum_{j=1}^{\infty} \sum_{u_h \in M_j} S_{h,i}(\tau)}{S_{1,i}(\tau)} \geq \frac{2}{\beta} \right]$ by Markov's inequality is less than $\frac{\epsilon}{2}$. Remembering (from the RP analysis) that all sufficiently small $p_{tr} = \Theta(1)$ can make $E[|M_0|]$ arbitrarily small for all n , and that the probability of *any* transmission (and thus noise) from nodes in M_0 is less than $E[|M_0|]$, all sufficiently small $p_{tr} = \Theta(1)$ yield:

$$\Pr \left[\max_{\tau} \frac{\sum_{j=0}^{\infty} \sum_{u_h \in M_j} S_{h,i}(\tau)}{S_{1,i}(\tau)} \geq \frac{2}{\beta} \right] < \frac{\epsilon}{2} + \frac{\epsilon}{2} \quad (14)$$

making less than ϵ even in the SNR model the probability of excessive interference. \square

By Lemma 5 then, the probability of having excessive interference during at least *one* of the 5 encounter slots is at most 5ϵ ; and since in each encounter the probability that the appropriate node transmits is $p_{tr} = \Theta(1)$, the probability that *all* five encounters entail successful communication is at least $(1 - 5\epsilon)p_{tr}^5$ which is bounded away from 0 for all n .

The third and final step in our proof is showing that the three sightings of the banneret and the sighting of the relay and destination each have an independent probability bounded away from 0 of transmitting the necessary data (note that if the encounter between the spotter and the destination results in a sighting, the spotter automatically obtains the needed data, i.e. the sighting of the destination).

The union B of all the colours in the banners flown by the banneret u_b during the interval from the encounter with the spotter to that with the relay has cardinality $O(\sqrt{n})$. Similarly, the cardinality of the union A of all the colours of the spotter's sightings, of the source's messages, and of the nodes encountered by the relay is $O(\sqrt{n})$ with probability $1 - o(1)$. Remembering that, other than the colour of u_d , every remaining colour in B is chosen uniformly and independently at random, the probability that $|B \cap A| = 1$ is $\Theta(1)$. Since the probability that the source carries a second message of the same colour as u_d is $O(1/\sqrt{n})$, the probability of the event \mathcal{E}_b that at most one message needs to be exchanged at each of the u_b 's sightings with the spotter, source and relay in their respective roles is $\Theta(1)$.

Finally, let t_d be the time the relay u_r encounters the destination u_d , and consider the set U of nodes other than u_b encountered by u_r during the time interval $[t_d - k_{ret}, t_d]$. U has cardinality $O(\sqrt{n})$ with probability $1 - o(1)$, and obviously includes all bannerets that may transmit to u_r a message for u_d still retained by u_r at t_d since messages are discarded after time k_{ret} . For the same reason, denoting by B' the union of all colours in the banners of U during the interval $[t_d - 2k_{ret}, t_d]$, only messages whose colour is in B' can be transmitted to u_r in $[t_d - k_{ret}, t_d]$. And since banner colours are chosen uniformly and independently at random, the event \mathcal{E}_r that the colour of u_d is not in B' is independent of \mathcal{E}_b and has probability $\Theta(1)$.

Then, each message has probability $\Theta(1)$ of being successfully delivered to its destination; and if this happens, it must happen within time $3k_{ret} = O(1)$ since the source, banneret and relay can each hold messages for at most time k_{ret} before discarding them.

4.3 Small Packets, Non-uniform Destinations

We have so far assumed that the size (in information units) of each packet is $\Theta(1/\sqrt{n})$ – the maximum that still guarantees $\Omega(1)$ packet exchanges between a node and its nearest neighbour before they move apart. It is extremely easy to see that the routing scheme of the previous section can be extended to deal with packets that are smaller by an arbitrary factor m (as long as packets contain at least $k_{id} \log n$ bits, see Subsection 2.2) by simply having every

node simulate m virtual nodes, each of which receives an identical fraction $1/m$ of the host's bandwidth and storage. A virtual node uses the same unit of space (the diameter of the deployment area) and the same unit of time (the time required to span 1 unit of space at maximum speed) as the host, but its unit of information (that transmissible at full bandwidth in 1 unit of time) is $(1/m)^{th}$ of that of the host; so its storage requirements are also $(1/m)^{th}$ of the host's and it can transmit up to $\Theta(1)$ packets per encounter.

We have also assumed so far (as in virtually every other work on the subject) that the destination of each message is chosen uniformly and independently at random. This may not always be realistic. For example, a node sending a large file to another node may well be addressing 100% of its outgoing traffic to that destination until the file has been sent. Note that if a node generates m messages for a given destination during an interval of duration 1, each has only a (non-independent) $\Theta(1/m)$ probability of being transmitted to a banneret and then delivered to the destination.

This issue can be easily addressed with a minor modification to our scheme: every message from a source to a destination is routed through an intermediate random node, in a fashion similar to that proposed in [23] for hyper-cubic networks. More precisely, we can assume that each node splits in two its communication bandwidth, reserving half for source-intermediary routing, and the other half for intermediary-destination routing. A message is routed from its source to a randomly chosen intermediary with the basic scheme. After being delivered to the intermediary at time t , it is stored and is scheduled to be "regenerated" at a time chosen uniformly and independently at random in $[t + \Delta^{min}, t + \Delta^{max}]$, with $\Delta^{min}, \Delta^{max} = \Theta(1)$. If multiple messages are scheduled to be regenerated simultaneously, all but (a randomly chosen) one are discarded. A message that is not discarded is finally routed to the destination with the basic scheme (see Algorithm 2). One can then easily prove the following:

THEOREM 2. *Let $\Delta t = \Delta^{max} + 6k_{ret} = \Theta(1)$, and consider a generic interval of time $[t, t']$ with $t' - t = \Theta(1)$. If there exists some $m \geq 1$ such that during the interval $[t - \Delta t, t' + \Delta t]$ (of duration $\Theta(1)$) messages for a total of at most m units of information are generated in the network for a destination u_i , then choosing a banner hash function uniformly at random guarantees that each such message is delivered by Algorithm 2 within time Δt of its generation with probability $\Omega(1/m)$.*

PROOF. Each message that successfully reaches its destination may spend at most time $3k_{ret}$ before reaching its intermediary, then at most time Δ^{max} before being regenerated, and then at most time $3k_{ret}$ before reaching its destination – for a total of at most Δt time units before generation and delivery. Therefore, the only messages that may coexist at any point in time with those generated during $[t, t']$ are those generated during $[t - \Delta t, t' + \Delta t]$.

Note that since intermediate nodes are chosen uniformly and independently at random, the analysis of the source-to-intermediary phase proceeds exactly as in the basic case regardless of m . Then each message generated during $[t, t']$ has probability $\Theta(1)$ of reaching an intermediary that receives at most $O(m/n)$ units of information, i.e. $O(m/(nP))$ messages, for u_i generated during $[t - \Delta t, t' + \Delta t]$. Each such message has then at least probability $\Omega(1/m)$ of being

Algorithm 2 Routing to non-uniform destinations

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for each slot  $\mathcal{S}$  do
  act as source
   $M_{gen} \leftarrow \{\text{messages generated as source in } \mathcal{S}\}$ 
  if  $M_{gen} \neq \emptyset$  then
    select uniformly at random  $msg \in M_{gen}$ 
    select uniformly at random an intermediary node  $u_j$ 
    route  $msg$  to  $u_j$  with Algorithm 1
  end if
  act as intermediary
  for each message  $msg$  received at  $t \in \mathcal{S}$  do
    select uniformly at random  $t_{reg} \in [t + \Delta^{min}, t + \Delta^{max}]$ 
    schedule  $msg$  for regeneration at  $t_{reg}$ 
  end for
   $M_{reg} \leftarrow \{\text{messages scheduled for regeneration in } \mathcal{S}\}$ 
  if  $M_{reg} \neq \emptyset$  then
    select uniformly at random  $msg \in M_{reg}$ 
    route  $msg$  to its original destination with Algorithm 1
  end if
end for

```

successfully transmitted to an appropriate banneret before being discarded. Conditioned on such an event, by exactly the same arguments used for the basic case, the message has probability $\Omega(1)$ of reaching its destination – proving the theorem. \square

By Theorem 2 then, if the information rate at which the network generates messages for a given destination is $O(1)$, each will be delivered with probability $\Theta(1)$; if it is m times higher, only a fraction $\Theta(1/m)$ of the messages will be delivered. This is clearly the best that can be achieved, since a single node cannot receive $\omega(1)$ information per unit of time. Note that the bounds on the rate need only hold over a window of duration $\Theta(1)$; shorter peaks are absorbed by the routing delay. Furthermore, note our scheme is “fair” in the sense that *each* message has a probability within a constant factor of every other for that destination of being delivered, regardless of where they are generated (as long as they are generated within the same timeframe).

5. CONCLUSIONS

This work does not add one more specimen to the already overpopulated “zoo” of mobility models for delay-tolerant networks. Instead, it considers the entire universe of all models satisfying some extremely mild and “natural” conditions (including all models that are present in the literature, and some that are not, like Levy runs) and draws a simple line: that of ballistic vs. non-ballistic mobility. Networks without ballistic mobility, as the number of nodes grows, witness either per-node throughput become vanishingly small, or communication delays grow infinitely large – regardless of centralized control or complex network coding schemes. Networks with ballistic mobility can instead adopt the routing scheme we propose and achieve simultaneously non-vanishing throughput and bounded delay.

Our scheme uses only simple routing, without any central coordination, network coding, or packet replication, and does not require nodes to adjust their transmission power. Each node needs only store $O(1)$ units of information at any given time, have access to an accurate clock, and gauge the position and velocity vectors *relative to itself* of those

nodes it successfully communicates with. Also, our scheme works equally well for all ballistic mobility patterns, without requiring any knowledge of the distance and speed distribution – but still asymptotically outperforms all existing schemes, none of which achieves simultaneously non-vanishing throughput and bounded delay.

We remark that this work, for space limitations, only focuses on throughput and delay *asymptotics*. In particular, our goal is to highlight the role of ballistic mobility and to make our routing scheme and analysis as simple as possible. Thus, we omit considering a number of optimizations entailing only constant factors, even though such factors might be large and could easily spell the difference between a practical scheme and one of purely theoretical interest. Exploring a more practical version of our scheme (including not only performance optimizations, but also robustness to failing and/or malicious nodes) on real mobility traces, from people to sensors drifting in fluid media, is certainly one of the most obvious directions for future research.

From a theoretical point of view, one important assumption of our analysis (and of virtually all pre-existing literature) is that message destinations are chosen uniformly at random. Our scheme can be easily modified to deal with non-uniform destinations without asymptotic loss of performance by routing messages through random intermediate nodes. However, the lower bound of Section 3 does not hold if message sources choose very close destinations. In other words, if the majority of traffic is local, ballistic mobility is still sufficient to guarantee non-vanishing throughput and bounded delay, but may not be *necessary*. Is it possible to generalize our results, by relating the distance distribution of the “next waypoint” to the distance distribution of message destinations, and proving that if the two coincide one can simultaneously achieve $\Theta(1)$ per-node throughput and $\Theta(1)$ delay even in the absence of ballistic mobility?

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