Truthful Multi-Attribute Auction with Discriminatory Pricing in Cognitive Radio Networks

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ABSTRACT

In this paper, we design a market-based channel allocation scheme for cognitive radio networks by exploiting multi-attribute channel-aware auctions to consider channel diversity in frequency, time, and space domains. Different from existing research, our objective is to maximize the winning SUs' service satisfaction degree while enhancing the utilities of winning PUs and SUs, which can effectively encourage them to join the auction and improve the sustainability of the spectrum market. Based on an elaborately devised preference function, we allocate channels to SUs satisfying their demands while considering spatial and temporal channel reuse to enhance channel utilization. Moreover, we propose a discriminatory pricing method to enhance the utilities of winning PUs and SUs. A comprehensive analysis indicates that our multi-attribute auction is individually-rational, ex-post budget balanced, value-truthful, and attribute-truthful. Our simulation results indicate that the proposed multi-attribute auction can significantly increase the winners' utilities and ensure SUs' service satisfaction.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design

Keywords

Discriminatory pricing, multi-attribute auction, cognitive radio networks

1. INTRODUCTION

In cognitive radio networks (CRNs), primary users (PUs) intend to temporarily lease their idle licensed spectrum to secondary users (SUs) to improve spectrum utilization. To achieve this objective, auctions have been applied to construct market-based mechanisms for spectrum allocation [1, 6, 7, 9, 10, 12, 18, 19, 21, 24–26]. But most existing research overlooks the following important issues in a spectrum trading market.

- **Attribute-Aware Transaction Matching:** Although a few previous research [6, 12] has considered channel diversity, they do not fully investigate the demand diversity (channel bandwidth and available duration) among the SUs. Thus when purchasing channels, SUs need to consider not only the price, but also the bandwidth and the start and end of the available time. Since most existing auctions mainly aim at maximizing either social welfare or auction revenue, their matching results may degrade the service satisfaction degree of the SUs.
- **Attribute-Based Discriminatory Pricing:** To clear the market and avoid untruthful bidding, McAfee-style auctions [6,9,12,13,25] adopt uniform pricing such that all winning sellers are equally paid and all winning buyers are equally charged. On the other hand, VCG-style auctions [1, 7, 10, 19, 24, 26] do employ discriminatory pricing, which determines the critical price for each buyer (seller) to win the auction if bids higher (lower); but the revenue of the auctioneer may be negative [12], and the price does not taken into account the channel attributes. But adopting such pricing mechanisms does ignore some important properties in a real-world spectrum market: the price of a channel should be related to the bandwidth of the channel and the satisfaction degree of the users.

In this paper, we intend to improve buyers' service satisfaction degree and winners' utilities to attract the participation of PUs and SUs so that the spectrum market can sustainably survive in a long term. We will address the following problems: (i) how to quantitively summarize SUs' requirements when considering multiple channel attributes (e.g., bandwidth and time duration)? (ii) how to allocate channels to maximize the wireless service satisfaction degree of the SUs? (iii) how to design an attribute-aware discriminatory pricing to enhance the utilities of winning PUs and SUs while achieving truthfulness?

We propose a truthful multi-attribute double auction scheme with discriminatory pricing to overcome these challenges. First, we propose an attribute-based preference function to quantify the preference of an SU over a channel of a PU based on the channel bandwidth and available time. After determining the winning PUs and SUs based on a sophisticated boundary determination algorithm, we perform

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a maximum preference matching to identify the winning PU-SU trading pairs in both spatial and temporal domains. Furthermore, we devise a discriminatory pricing scheme for the winners by taking into account the *competitive power* of winning PUs and the *satisfying cost* of winning SUs. Through theoretical analysis, we prove that our auction scheme can achieve individual-rationality, ex-post budget balance, and truthfulness. Extensive simulation results indicate that the proposed multi-attribute auction can significantly improve SUs' received service satisfaction degree, and that our discriminatory pricing can effectively enhance all winners' utilities. The multifold contributions of the paper are summarized as follows:

- To our best knowledge, this paper is the first to apply multi-attribute double auction for channel allocation in CRNs. Our auction scheme can effectively assign channels to SUs satisfying their requirements by taking into account channel diversity in frequency, time, and space domains, and channel reuse in both time and space domains.
- Compared with the traditional pricing policies, our discriminatory pricing can simultaneously increase the utilities of the winning PUs and SUs.
- Besides individual-rationality, ex-post budget balance, and value-truthfulness, our auction possesses the property of attribute(bandwidth and time)-truthfulness.
- Extensive simulation results, which indicate the significant improvements of winners' utilities and SUs' received service satisfaction, confirm the effectiveness of our auction scheme.

The rest of the paper is organized as follows. The network model and problem formulation are described in Section 2. Our multi-attribute auction scheme and the corresponding economic property analysis are detailed in Section 3 and 4, respectively. After presenting our simulation study in Section 5, we briefly summarize the most related research in Section 6. The paper is concluded with a future research discussion in Section 7.

2. NETWORK MODEL AND PROBLEM FORMULATION

2.1 Network Model

Let $\mathcal{M} = \{\mathrm{PU}_1, \mathrm{PU}_2, \ldots, \mathrm{PU}_M\}$ and $\mathcal{N} = \{\mathrm{SU}_1, \mathrm{SU}_2, \ldots, \mathrm{SU}_N\}$ be respectively the sets of PUs and SUs participating in a spectrum trading market. An auctioneer is willing to observe the auction process in which the sellers (the PUs) and buyers (SUs) can trade spectrum (channels) such that the channels unused by the PUs can be leased to the SUs to simultaneously gain monetary profits for the PUs and wireless services for the SUs. The auctioneer receives a revenue that is defined to be the difference between the total charge from the buyers and the total payment to the sellers. Each channel is associated with three attributes: *bandwidth*, *starting available time*, and *ending available time*.

We assume that the channels belonging to different PUs may have different available bandwidths and different available time. We also assume that all the PU channels are orthogonal; but they can be reused by multiple SUs that do not interfere with each other in the spatial or temporal domain. The channel supply information (ask) of PU_i is denoted by $S_i(q_i^S, \langle b_i^S, w_i^S, [f_i^S, t_i^S]\rangle)$, indicating that PU_i has q_i^S channels to sell at a price of b_i^S per channel per unit time, and each channel has a bandwidth of w_i^S and can be continuously used from the starting time f_i^S to the ending time t_i^S. This indicates that the available channels belonging to the same PU are homogeneous. Heterogeneous channels can be considered by submitting multiple asks, with each describing one type of channel. Due to the fact that the information of the PUs' licensed channels, including available bandwidth and channel usage, is generally public, we assume that PUs do not report untruthful channel attributes to the auctioneer. Such an assumption is also adopted by [18].

An SU needs to consider not only the price, but also the attributes of a channel to satisfy its service requirement. Let $D_j(q_j^D, \langle b_j^D, w_j^D, [f_j^D, t_j^D]\rangle)$ denote the channel demand information (bid) of SU_j, which indicates that SU_j intends to buy q_j^D channels with the following properties: each channel should cost at most b_j^D per unit time, should be continuously available from f_j^D to t_j^D without any disruption, and should have a bandwidth of at least w_j^D.

Note that the ask price of a PU and the bid price of an SU is given with respect to "per channel per unit time", not "per unit bandwidth per unit time", because in practice the cost of a PU channel paid to FCC is nonlinear to its bandwidth, and the pricing policy needs to consider different parameters for PUs and SUs. Thus we assume that PUs and SUs provide ask and bid prices in terms of "per channel per unit time", and we term such a price a "unit price".

We also need to consider the interference among the SUs that reside in each other's interference range: two SUs that may interfere with each other should receive different channels. For this purpose we assume that an interference graph among the SUs, denoted by $G^I(\mathcal{N}, E^I)$, is available to the auctioneer. Such a graph is not hard to obtain. For example, G^I can be constructed based on the primary interference model if all SUs provide their location and transmission range information when registering with the auctioneer.

2.2 Problem Formulation

In order to express the preference of a buyer towards a channel with multiple attributes, an attribute-based preference function [3] is needed. Let δ_{ij} be the *preference degree* of SU_j to PU_i. Define

$$\delta_{ij} = \begin{cases} 0, \text{ if } w_j^D > w_i^S \text{ or } [f_j^D, t_j^D] \not\subseteq [f_i^S, t_i^S]; \\ \frac{1}{1+e^{-(w_i^S - w_j^D)}} \\ \times \frac{1}{1+e^{\{-(f_i^S - f_j^D)-(t_i^S - t_j^D)-[(t_i^S - f_i^S)-(t_j^D - f_j^D)]\}}}, \text{o.w.} \end{cases} \tag{1}$$

The definition of δ_{ij} is motivated by the Sigmoid function that can be utilized to assess the satisfaction degree of the obtained service [16]. In our case, the value of $\delta_{ij} \in \{0\} \bigcup [0.25, 1)$ quantifies the buyer SU_j's preference level to a channel of PU_i with respect to the wireless service that can be obtained from the channel. If $\delta_{ij} = 0$, PU_i's channel cannot satisfy the requirement of SU_j. When $w_j^D \le w_i^S$ and $[f_j^D, t_j^D] \subseteq [f_i^S, t_i^S]$, $\delta_{ij} \in [0.25, 1)$, and a larger value indicates a higher preference degree of SU_j to PU_i. We observe that δ_{ij} increases with the increase of $(w_i^S - w_j^D)$, $(f_i^S - f_j^D)$, $(t_i^S - t_j^D)$, and $(t_i^S - f_i^S) - (t_j^D - f_j^D)$, indicating that SU_j prefers the channel of PU_i with a higher bandwidth, a closer starting time, a longer residual time, and a larger time duration difference, which facilitates temporal channel reuse.

Let q_{ij} be the number of channels traded between PU_i and SU_j. Then the utility of PU_i can be calculated as

$$U_i^S = \sum_{\mathrm{SU}_j \in \mathcal{N}} [p_i^S - b_i^S] \times (t_j^D - f_j^D) \times q_{ij}, \tag{2}$$

where p_i^S is the unit price paid to PU_i by the auctioneer when it successfully sells channels to some SUs in the auction. Similarly, the utility of SU_j can be defined by

$$U_j^D = \sum_{\mathrm{PU}_i \in \mathcal{M}} [b_j^D - p_j^D] \times (t_j^D - f_j^D) \times q_{ij}, \tag{3}$$

with p_j^D being the unit price charged from SU_j by the auctioneer when SU_j successfully purchases channels from some PUs. The definitions of p_i^S and p_j^D, together with their corresponding properties, will be described in Section 3.3.

Accordingly, the revenue of the auctioneer, which is defined to be the difference between the total charges from the SUs and the total payments to the PUs, is given by

$$V = \sum_{\mathrm{PU}_i \in \mathcal{M}^w} \sum_{\mathrm{SU}_j \in \mathcal{N}^w} (p_j^D - p_i^S) \times (t_j^D - f_j^D) \times q_{ij}, \tag{4}$$

where $\mathcal{M}^w$ is the set of winning PUs and $\mathcal{N}^w$ is the set of winning SUs in the auction.

Additionally, considering the relationship between the supply and the demand in spectrum trading, the following two constraints must be held:

(i) $$\sum_{\mathrm{SU}_j \in \mathcal{N}} q_{ij} \leq q_i^S, \text{ for all } \mathrm{PU}_i \in \mathcal{M}$$

This constraint is resulted from the fact that the number of channels sold by PU_i cannot exceed its supply.

(ii) $$\sum_{\mathrm{PU}_i \in \mathcal{M}} q_{ij} \leq q_j^D, \text{ for all } \mathrm{SU}_j \in \mathcal{N}$$

which indicates that it is allowed to partially meet the demand of buyer SU_j in terms of the number of required channels.

2.3 Economic Properties

The goal of this paper is to design a multi-attribute double auction scheme to simultaneously achieve the following three critical economic properties: *individual-rationality*, *ex-post budget balance*, and *truthfulness*. These properties are required for the so-called *economically-robust double auction* [2], and are briefly described as follows.

- **Individual-Rationality:** An auction is *individually rational* if no winner's utility is negative; that is, $U_i^S \geq 0$ ($\forall \mathrm{PU}_i \in \mathcal{M}^w$) and $U_j^D \geq 0$ ($\forall \mathrm{SU}_j \in \mathcal{N}^w$). Achieving individual-rationality can attract PUs and SUs to participate in the auction for non-zero utilities.

- **Ex-post Budget Balance:** For an auction, *ex-post budget balance* requires that the auctioneer's revenue is not negative, i.e., $V \geq 0$. This property ensures that the auctioneer has incentive to organize the auction.

- **Truthfulness:** In our multi-attribute auction, two types of truthfulness need to be considered, i.e., *value-truthfulness* and *attribute-truthfulness*. Let $\bar{b}_i^S$ and $\bar{b}_j^D$ be the true unit prices of PU_i and SU_j, respectively. Denote by $\bar{w}_j^D$, $\bar{f}_j^D$, and $\bar{t}_j^D$ the true values of SU_j's attributes. An auction is *value-truthful* if no PU_i (or SU_j) can improve its utility by lying an ask price b_i^S with $b_i^S \neq \bar{b}_i^S$ (or a bid price b_j^D with $b_j^D \neq \bar{b}_j^D$); an auction is *attribute-truthful* if no SU_j can enhance its utility by reporting untruthful attributes $w_j^D \neq \bar{w}_j^D$, or $f_j^D \neq \bar{f}_j^D$, or $t_j^D \neq \bar{t}_j^D$. Note that an attribute-truthful auction must be time-truthful. Also note that attribute-truthfulness is defined for SUs only, as PUs have no incentive to lie about the available times and bandwidths of their channels since such information is public [18].

3. MULTI-ATTRIBUTE AUCTION

In this section, we propose a truthful multi-attribute double auction scheme termed "**M**ulti-attrib**U**te auction with di**S**criminatory pr**IC**ing" (**MUSIC**), which consists of three major stages: *potential winner determination*, *preference-based transaction matching*, and *attribute-based discriminatory pricing*.

3.1 Potential Winner Determination

Although the McAfee double auction scheme [13] can identify the winners, it cannot be directly applied to our model because it only considers single-unit homogeneous goods. To effectively obtain the sets of winning sellers and buyers, we propose the following three-step *potential winner determination* algorithm – a *potential winner* becomes a *true winner* only if it can successfully sell or purchase at least one channel in the stage of preference-based transaction matching.

1. **Ask & Bid Ordering**
We sort the ask prices of all PUs in a non-decreasing order and the bid prices of all SUs in a non-increasing order. Without loss of generality, we assume that the ordered prices are denoted by $b_1^S \leq b_2^S \leq \ldots \leq b_M^S$, and $b_1^D \geq b_2^D \geq \ldots \geq b_N^D$.

2. **Boundary Pair Selection**
In this step, we first add a virtual PU and a virtual SU (lines 3-6) to facilitate the boundary seller-buyer pair identification by setting $b_{M+1}^S = b_N^D$ and $b_{N+1}^D = b_M^S$ if $b_M^S < b_N^D$. Next we perform an exhaustive search to identify the PU and SU pairs with the largest j values satisfying $b_{i-1}^S \leq b_j^D \leq b_i^S \leq b_{j-1}^D$ for $2 \leq i \leq M+1$ and $2 \leq j \leq N+1$ (lines 7-12). The *boundary seller-buyer pair*, $(\mathrm{PU}_x, \mathrm{SU}_y)$, is the one corresponding to the largest number of winners (lines 13-14). Accordingly, b_x^S and b_y^D are respectively the *boundary ask price* and *boundary bid price*. Note that because the boundary seller-buyer pair selection is independent of the users' attributes, both b_x^S and b_y^D are independent of all users' attributes. The detailed algorithm is presented in Alg. 1, which can be performed with a time complexity of $O(MN)$.

3. **Potential Winner Determination**
After obtaining the boundary pair $(\mathrm{PU}_x, \mathrm{SU}_y)$, the set of potential winning sellers, $\mathcal{M}^w$, and the set of potential winning buyers, $\mathcal{N}^w$, can be determined by $\mathcal{M}^w = \{\mathrm{PU}_i : i = 1, 2, \ldots, x-1\}$ and $\mathcal{N}^w = \{\mathrm{SU}_j : j = 1, 2, \ldots, y-1\}$, respectively. According to Alg. 1, the bid prices and ask prices of the winning buyers and sellers have the following critical properties:

Algorithm 1 Boundary Pair Selection

Input: Sorted ask sequence $\{b_1^S \leq b_2^S \leq \ldots \leq b_M^S\}$ and sorted bid sequence $\{b_1^D \geq b_2^D \geq \ldots \geq b_N^D\}$.
Set Pair $= \{(0,0)\}$;
Set $b_{M+1}^S = 0$, $b_{N+1}^D = 0$; //Add a virtual PU and a virtual SU
if $b_M^S < b_N^D$ **then**
  Set $b_{M+1}^S = b_N^D$, $b_{N+1}^D = b_M^S$;
end if
for $2 \leq i \leq M+1$ **do**
  $j = \arg\max_{2 \leq j \leq N+1} \{b_{i-1}^S \leq b_j^D \leq b_i^S \leq b_{j-1}^D\}$;
  if $(i,j) \neq (M+1, N+1)$ **then**
    Pair $=$ Pair $\cup\, (i,j)$;
  end if
end for
Pick up all $(i,j) = \arg\max_{(k,h) \in \text{Pair}} \{\min\{k,h\}\}$;
Set $(x,y) = \arg\max_{(i,j)} \{i+j\}$;
Output (x,y).

(a) $b_i^S \leq b_j^D$, where $1 \leq i \leq x-1$ and $1 \leq j \leq y-1$; that is, any winning PU's ask price is not more than any winning SU's bid price.

(b) $b_i^S \leq b_y^D$ for $1 \leq i \leq x-1$, and $b_j^D \geq b_x^S$ for $1 \leq j \leq y-1$.

(c) $b_k^S \geq b_x^S \geq b_y^D$, where $x \leq k \leq M$, indicating that all losing PUs' ask prices are larger than the boundary ask price and boundary bid price.

(d) $b_h^D \leq b_y^D \leq b_x^S$, with $y \leq h \leq N$, which means that all losing SUs' bid prices are less then the boundary ask price and boundary bid price.

3.2 Preference-Based Transaction Matching

In the second stage, we associate the potential winning SUs to the potential winning PUs based on SUs' preference values to determine the supply-demand trading pairs. This matching process is implemented by an iterative algorithm, shown in Alg. 2, to consider spatial and temporal reuse of the PU channels. It contains three steps detailed as follows.

1. **Bipartite Graph Construction**
We construct a bipartite graph $G(\mathcal{M}^c, \mathcal{N}^c, E^c)$, where

$$\mathcal{M}^c = \{\text{PU}_{ik} : i = 1, 2, \ldots, x-1, k = 1, 2, \ldots, q_i^S\},$$

$$\mathcal{N}^c = \{\text{SU}_{jh} : j = 1, 2, \ldots, y-1, h = 1, 2, \ldots, q_j^D\},$$

and E^c is the set of edges. This implies that G contains q_i^S nodes for each PU_i in $\mathcal{M}^c$ and q_j^D nodes for each SU_j in $\mathcal{N}^c$. An edge $e(\text{PU}_{ik}, \text{SU}_{jh})$ exists between PU_{ik} and SU_{jh} if and only if $w_j^D \leq w_i^S$ and $[f_j^D, t_j^D] \subseteq [f_i^S, t_i^S]$, indicating that the kth channel of PU_i can be sold/allocated to SU_j as its hth channel. Each edge $e(\text{PU}_{ik}, \text{SU}_{jh})$ is associated with a weight $\delta_{(ik,jh)}^c$ and a variable $\tau_{(ik,jh)}^c$ defining the set of time intervals at which SU_j is allowed to use PU_i's channel. Initially, $\delta_{(ik,jh)}^c = \delta_{ij}$ and $\tau_{(ik,jh)}^c = \{[f_i^S, t_i^S]\}$.

2. **Maximum Preference Matching**
In the matching process, we intend to maximize the sum of the winning SUs' preference values, which implies that our auction scheme aims at providing all winning SUs with as good service as possible via the maximization of their total preference degrees. Therefore, we can obtain the following optimization problem:

$$\max \sum_{\text{PU}_{ik} \in \mathcal{M}^c} \sum_{\text{SU}_{jh} \in \mathcal{N}^c} \delta_{(ik,jh)}^c q_{(ik,jh)}^c \tag{5a}$$

$$\text{s.t.} \sum_{\text{SU}_{jh} \in \mathcal{N}^c} q_{(ik,jh)}^c \leq 1, \forall\, \text{PU}_{ik} \in \mathcal{M}^c, \tag{5b}$$

$$\sum_{\text{PU}_{ik} \in \mathcal{M}^c} q_{(ik,jh)}^c \leq 1, \forall\, \text{SU}_{jh} \in \mathcal{N}^c, \tag{5c}$$

$$q_{(ik,jh)}^c \in \{0,1\}, \forall e(\text{PU}_{ik}, \text{SU}_{jh}) \in E^c. \tag{5d}$$

where $q_{(ik,jh)}^c = 1$ if and only PU_{ik} is matched with SU_{jh}. It can be seen that (5) describes the traditional maximum weighted matching problem in a bipartite graph, in which there is at most one edge between any PU_{ik} and any SU_{jh}. Thus each PU_i has at most q_i^S edges and each SU_j has at most q_j^D edges, which is consistent with the two constraints described in Section 2.2. After obtaining the solution $\{q_{(ik,jh)}^c\}$ (line 5 of Alg. 2), we can compute $q_{ij} = \sum_{k=1}^{q_i^S} \sum_{h=1}^{q_j^D} q_{(ik,jh)}^c$.

Algorithm 2 Preference-Based Transaction Matching

1: **Input**: $\mathcal{M}^w = \{\text{PU}_i\}$, $S = \{S_i\}$, $\mathcal{N}^w = \{\text{SU}_j\}$, $D = \{D_j\}$, $G^I(\mathcal{N}, E^I)$.
2: Construct a bipartite graph $G(\mathcal{M}^c, \mathcal{N}^c, E^c)$;
3: Set $\tau_{(ik,jh)}^c = \{[f_i^S, t_i^S]\}$, for $\forall$ $\text{PU}_{ik} \in \mathcal{M}^c$;
4: **while** $E^c \neq \emptyset$ **do**
5:   Obtain $\{q_{(ik,jh)}^c\}$ by computing a maximum weighted matching on G^c;
6:   **for** Each $q_{(ik,jh)}^c = 1$ **do**
7:     Remove SU_{jh} from $\mathcal{N}^c$ and all edges $e(\text{PU}_{ik}, \text{SU}_{jh''})$ from E^c that connect PU_{ik} to existing SU_j nodes (denoted by $\text{SU}_{jh''}$);
8:     **for** Each $e(\text{PU}_{ik}, \text{SU}_{j'h'})$ with $e(\text{SU}_j, \text{SU}_{j'}) \in E^I$ **do**
9:       Update $\tau_{(ik,j'h')}^c$;
10:       Remove $e(\text{PU}_{ik}, \text{SU}_{j'h'})$ from E^c if $[f_{j'}^D, t_{j'}^D]$ is not covered by $\tau_{(ik,j'h')}^c$;
11:       Update $\delta_{(ik,j'h')}^c$ if $e(\text{PU}_{ik}, \text{SU}_{j'h'})$ still exists;
12:     **end for**
13:   **end for**
14: **end while**
15: Set $q_{ij} = \sum_{k=1}^{q_i^S} \sum_{h=1}^{q_j^D} q_{(ik,jh)}^c$;
16: Output $\{q_{ij}\}$.

3. **Spatial & Temporal Channel Reuse**
In order to promote channel reuse in both the spatial and the temporal domain, we compute matched PU and SU pairs iteratively and update the bipartite graph G at each iteration according to the interference graph G^I. This process is illustrated by the outmost **For** loop (lines 6-13) in Alg. 2.

In this **For** loop, we check each PU_{ik} with a matching SU node (i.e., $q_{(ik,jh)}^c = 1$). First we remove all the matched SU nodes (SU_{jh}) from $\mathcal{N}^c$ and the relevant edges (the edges that connect PU_{ik} with $\text{SU}_{jh''}$ for any valid h'' in G) from E^c to avoid self-interference (line 7). The removal of $e(\text{PU}_{ik}, \text{SU}_{jh})$ implies that the hth required channel of SU_j is satisfied by the kth channel of PU_i. Then we use another **For** loop (lines 8-12) to process all the SU nodes (denoted by $\mathrm{SU}_{j'}$, $\mathrm{SU}_{j'h'}$) that may interfere with SU_j in both the temporal and spatial domains according to the following procedure: For each edge $e(\mathrm{PU}_{ik}, \mathrm{SU}_{j'h'})$, we first update its $\tau^c_{(ik,j'h')}$ by removing from $\tau^c_{(ik,j'h')}$ the time interval $[f^D_j, t^D_j]$ during which $\mathrm{SU}_{j'}$ interferes with SU_j if they use the same channel and keeping only those that can cover $[f^D_{j'}, t^D_{j'}]$ (line 9); then we remove $e(\mathrm{PU}_{ik}, \mathrm{SU}_{j'h'})$ if $\tau^c_{(ik,j'h')}$ is empty (line 10), which indicates that the kth channel of PU_i can not be used as the h'th required channel of $\mathrm{SU}_{j'}$ because SU_j and $\mathrm{SU}_{j'}$ interfere each other in both the spatial domain and temporal domain over the kth channel of PU_i; and finally we compute a preference value for each time interval in $\tau^c_{(ik,j'h')}$ based on (1) and assign the largest value to $\delta^c_{(ik,j'h')}$ (line 11).

In the following we use $\mathrm{Match}(\mathrm{SU}_j)$ to denote the set of PUs that are matched with SU_j according to Alg. 2, i.e., $\mathrm{PU}_i \in \mathrm{Match}(\mathrm{SU}_j)$ if and only if $q^c_{ij} > 0$. Note that there may exist some winning PUs and SUs that may not have any transaction matching; thus a checking and updating process to determine the final true winner sets needs to be performed. One such a procedure is outlined as follows: For each $\mathrm{PU}_i \in \mathcal{M}^w$, set $\mathcal{M}^w = \mathcal{M}^w \setminus \mathrm{PU}_i$ if $\mathrm{PU}_i \notin \bigcup_{\mathrm{SU}_j \in \mathcal{N}^w} \mathrm{Match}(\mathrm{SU}_j)$, i.e., remove the PU that cannot sell any channel to the winning SUs. Similarly, for each $\mathrm{SU}_j \in \mathcal{N}^w$, set $\mathcal{N}^w = \mathcal{N}^w \setminus \mathrm{SU}_j$ if $\mathrm{Match}(\mathrm{SU}_j) \cap \mathcal{M}^w = \emptyset$; that is, an SU is removed if it cannot obtain any channel from the winning PUs. This checking and updating process can guarantee that each winner has at least one transaction matching.

In Alg. 2, we can utilize the Hungarian algorithm to find out the maximum weighted matching on G^c (line 5) with a time complexity of $\max\{O(|\mathcal{M}^c|^3), O(|\mathcal{N}^c|^3)\}$, where $|\mathcal{M}^c| = \sum_{i=1}^{x-1} q^S_i$ and $|\mathcal{N}^c| = \sum_{j=1}^{y-1} q^D_j$. The process of channel reuse checking (lines 6-13) is performed at most $|\mathcal{M}^c||\mathcal{N}^c|$ times. Note that we update E^c at each iteration, and there are at most $|\mathcal{N}^c|$ iterations in the while-loop (lines 4-14). Thus, the time complexity of Alg. 2 is $\max\{O(|\mathcal{M}^c|^3|\mathcal{N}^c|), O(|\mathcal{N}^c|^4)\}$.

PROPERTY 1. *The changes of winning PUs' ask prices $\{b^S_i\}$ ($\mathrm{PU}_i \in \mathcal{M}^w$) and winning SUs' bid prices $\{b^D_j\}$ ($\mathrm{SU}_j \in \mathcal{N}^w$) do not impact on the transaction matching result.*

3.3 Attribute-based Discriminatory Pricing

Considering the diversity of the spectrum access services, we intend to design a discriminatory pricing policy in our multi-attribute auction, which pays the PUs and charges the SUs discriminatively. Compared with the traditional bid-independent uniform pricing model employed by previous work [6, 9, 12, 21, 25], our discriminatory pricing scheme can enhance the user utility by increasing the channel prices of the PUs that can provide a good service and by reducing the channel charges of the SUs that receive a bad service. Before introducing our pricing protocol, we first present two concepts: **Normalized Competitive Power** and **Normalized Satisfying Cost**.

Definition 1. The *normalized competitive power* of a winning $\mathrm{PU}_i \in \mathcal{M}^w$ among all winning PUs is defined as

$$\alpha_i = \frac{w^S_i - \min_{\mathrm{PU}_k \in \mathcal{M}^w}\{w^S_k\}}{\max_{\mathrm{PU}_k \in \mathcal{M}^w}\{w^S_k\} - \min_{\mathrm{PU}_k \in \mathcal{M}^w}\{w^S_k\}}.$$

It can be observed that $\alpha_i \in [0, 1]$ is monotonically increasing with the increase of w^S_i, indicating that the better service in terms of the available channel bandwidth a PU provides, the higher competitive power it has.

Definition 2. The *normalized satisfying cost* of a winning $\mathrm{SU}_j \in \mathcal{N}^w$ among all winning SUs is defined to be

$$\beta_j = \frac{w^D_j - \min_{\mathrm{SU}_h \in \mathcal{N}^w}\{w^D_h\}}{\max_{\mathrm{SU}_h \in \mathcal{N}^w}\{w^D_h\} - \min_{\mathrm{SU}_h \in \mathcal{N}^w}\{w^D_h\}}.$$

We observe that $\beta_j \in [0, 1]$ monotonically increases when w^D_j increases, which means that the more bandwidth an SU demands, the larger satisfying cost it has to pay. This reflects the fact that the difficulty degree of satisfying an SU demanding more bandwidth becomes higher due to the limited amount of bandwidth resource.

Based on the common sense that *supplying more receives more and demanding more pays more*, we can calculate the payments of the PUs and the charges from the SUs in the following way.

- **Payments of the PUs**
  If PU_i sells channels to some SUs, the unit price (per channel per unit time) paid from the auctioneer to PU_i is
  $$p^S_i = \alpha_i \frac{b^S_x + b^D_y}{2} + (1 - \alpha_i) b^D_y. \quad (6)$$
  Since $\alpha_i \in [0, 1]$, we have $p^S_i \in [b^D_y, \frac{b^S_x + b^D_y}{2}]$. Note that the better service a PU offers the higher utility it can gain. Also note that with a uniform pricing, all PUs obtain the same unit price b^D_y for each channel, reducing the utilities of the PUs providing better service.
- **Charges from the SUs**
  If SU_j purchases channels from some PUs, the unit price charged by the auctioneer is
  $$p^D_j = \beta_j b^S_x + (1 - \beta_j) \frac{b^S_x + b^D_y}{2}. \quad (7)$$
  Accordingly, we have $p^D_j \in [\frac{b^S_x + b^D_y}{2}, b^S_x]$. Note that in discriminatory pricing, an SU can achieve a higher utility by paying less for smaller channel bandwidth, while all SUs have to pay b^S_x for each channel if a uniform pricing is adopted. Thus, our discriminatory pricing can save payments of the SUs that have smaller channel bandwidth demands.

PROPERTY 2. *The unit price p^S_i (p^D_j) monotonically increases as α_i (β_j) increases, and is independent of the transaction matching result, b^S_i, and b^D_j.*

3.4 An Example for Illustration

We present a simple example to illustrate the main stages of our auction algorithm MUSIC. Suppose that there are three PUs and three SUs in the spectrum trading market, with the supply $S_i(q^S_i, \langle b^S_i, w^S_i, [f^S_i, t^S_i]\rangle)$ and the demand

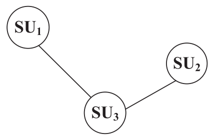


Figure 1: Graph G^I.

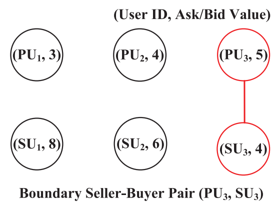


Figure 2: Boundary Pair

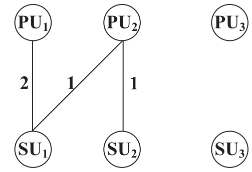


Figure 3: Matching Result

Table 1: Users' Supply & Demand Information

PU ID	Supply	SU ID	Demand
PU_1	$(2, \langle 3, 2, [0, 3]\rangle)$	SU_1	$(3, \langle 8, 2, [0, 1]\rangle)$
PU_2	$(1, \langle 4, 3, [0, 1]\rangle)$	SU_2	$(1, \langle 6, 1.5, [0, 1]\rangle)$
PU_3	$(1, \langle 5, 3, [1, 3]\rangle)$	SU_3	$(2, \langle 4, 1, [2, 3]\rangle)$

$D_j(q_j^D, \langle b_j^D, w_j^D, [f_j^D, t_j^D]\rangle)$ listed in Table 1. The interference graph G^I is shown in Fig. 1.

First we find out the boundary seller-buyer pair as shown in Fig. 2. Accordingly, the set of potential winning sellers is $\mathcal{M}^w = \{\mathrm{PU}_1, \mathrm{PU}_2\}$ and the set of potential winning buyers is $\mathcal{N}^w = \{\mathrm{SU}_1, \mathrm{SU}_2\}$. Then we perform Alg. 2 to establish the PU-SU trading pairs. The final transaction matching result is presented in Fig. 3, where the numbers labeling the lines are the values of q_{ij}. Since SU_1 and SU_2 do not interfere in spatial domain, they can use the channel provided by PU_2 at the same time. According to (6) and (7), the clearing prices are set to be $p_1^S = 4$, $p_2^S = 4.5$, $p_1^D = 5$, and $p_2^D = 4.5$. Correspondingly, we can obtain the following utility values: $U_1^S = (p_1^S - b_1^S) \times 1 \times 2 = 2$, $U_2^S = (p_2^S - b_2^S) \times 1 \times 2 = 1$, $U_1^D = (b_1^D - p_1^D) \times 1 \times 3 = 9$, and $U_2^D = b_2^D - p_2^D = 1.5$; and the revenue of the auctioneer is $V = (p_1^D - p_1^S) \times 2 + (p_1^D - p_2^S) + (p_2^D - p_2^S) = 2.5$.

4. ECONOMIC PROPERTY ANALYSIS

In this section, we present an in-depth theoretical analysis on our proposed auction scheme MUSIC.

THEOREM 1. *The MUSIC auction scheme is individually-rational for both PUs (sellers) and SUs (buyers).*

PROOF. According to (6) and (7), since $b_i^S \le b_y^D$ ($\mathrm{PU}_i \in \mathcal{M}^w$) and $b_j^D \ge b_x^S$ ($\mathrm{SU}_j \in \mathcal{N}^w$), we have $p_i^S \ge b_y^D \ge b_i^S$ and $p_j^D \le b_x^S \le b_j^D$. Therefore, based on the definitions of PU's utility and SU's utility (see (2) and (3)), we have $U_i^S \ge 0$ and $U_j^D \ge 0$, which confirms the individual-rationality of our auction scheme. □

THEOREM 2. *Our auction scheme MUSIC can achieve ex-post budget balance for the auctioneer.*

PROOF. Since $b_y^D \le p_i^S \le p_j^D \le b_x^S$, we have $V \ge 0$. This indicates that our auction scheme can satisfy ex-post budget balance. □

LEMMA 1. *If PU_i wins the auction with an ask price b_i^S, it can also win by asking $b_i^S - \varepsilon$ ($\varepsilon > 0$).*

PROOF. Since the old position of PU_i with b_i^S in the sorted ask sequence is $i \le x-1$ and its new position after reporting $b_i^S - \varepsilon$ is $i^- \le i \le x-1$, the boundary seller-buyer pair $(\mathrm{PU}_x, \mathrm{SU}_y)$ remains unchanged when b_i^S is reduced to $b_i^S - \varepsilon$. Thus, PU_i can still be selected as a potential winner. On the other hand, according to Property 1, PU_i can obtain the same transaction matchings in the auction since transaction matching is independent of bid prices. □

THEOREM 3. *The MUSIC auction scheme is value-truthful for all PUs (sellers).*

PROOF. Proving the value-truthfulness for the PUs is equivalent to showing that $U_i^S \le \bar{U}_i^S$ if $b_i^S \ne \bar{b}_i^S$ for all $\mathrm{PU}_i \in \mathcal{M}$. All possible cases are discussed below when $b_i^S \ne \bar{b}_i^S$.

Case 1: $b_i^S < \bar{b}_i^S$.

(1) According to Property 1 and Lemma 1, PU_i can also win the auction with b_i^S if it wins with $\bar{b}_i^S$ while obtaining the same set of transaction matchings. Then, we have $U_i^S = \bar{U}_i^S$ as p_i^S is independent of b_i^S and $\bar{b}_i^S$ (see Property 2).

(2) If PU_i wins with b_i^S and loses with $\bar{b}_i^S$ in the auction, we have $\bar{b}_i^S \ge b_x^S \ge p_i^S$, which leads to $U_i^S \le 0 = \bar{U}_i^S$.

(3) In addition, $U_i^S = \bar{U}_i^S = 0$ if PU_i loses when asking both b_i^S and $\bar{b}_i^S$.

Case 2: $b_i^S > \bar{b}_i^S$.

(1) Similar to the analysis of Case 1, $U_i^S = \bar{U}_i^S$ if PU_i wins (or loses) with both b_i^S and $\bar{b}_i^S$.

(2) If PU_i loses by asking b_i^S and wins by asking $\bar{b}_i^S$, from Theorem 1, we have $\bar{U}_i^S \ge 0 = U_i^S$.

Therefore, we can conclude that our auction scheme MUSIC achieves value-truthfulness for all PUs. □

LEMMA 2. *If SU_j wins the auction with a bid b_j^D, it can also win by bidding $b_j^D + \varepsilon$ ($\varepsilon > 0$).*

PROOF. The boundary seller-buyer pair $(\mathrm{PU}_x, \mathrm{SU}_y)$ is unchanged when SU_j bids at b_j^D and $b_j^D + \varepsilon$, because the old position of SU_j with b_j^D in the sorted bid sequence is $j \le y-1$ and its new position after reporting $b_j^D + \varepsilon$ is $j^- \le j \le y-1$. Thus, SU_j can still win the auction. □

THEOREM 4. *The auction MUSIC is value-truthful for all SUs (buyers).*

PROOF. In order to prove the value-truthfulness of SUs, we need to show that $U_j^D \le \bar{U}_j^D$ if $b_j^D \ne \bar{b}_j^D$ for all $\mathrm{SU}_j \in \mathcal{N}$, which is presented as follows.

Case 1: $b_j^D > \bar{b}_j^D$.

(1) Property 1 and Lemma 2 indicate that SU_j can also win the auction with b_j^D if it wins with $\bar{b}_j^D$ while obtaining the same set of transaction matchings. Then, we have $U_j^D = \bar{U}_j^D$, because p_j^D is independent of b_j^D and $\bar{b}_j^D$ (see Property 2).

(2) If SU_j wins with b_j^D and loses with $\bar{b}_j^D$ in the auction, we have $\bar{b}_j^D \le b_y^D \le p_j^D$, resulting in $U_j^D \le 0 = \bar{U}_j^D$.

(3) Moreover, $U_j^D = \bar{U}_j^D = 0$ if SU_j loses when bidding both b_j^D and $\bar{b}_j^D$.

Case 2: $b_j^D < \bar{b}_j^D$.

(1) By using the similar analysis in Case 1, $U_j^D = \bar{U}_j^D$ if SU_j wins (or loses) with both b_j^D and $\bar{b}_j^D$.

(2) If SU_j loses by submitting b_j^D and wins by bidding $\bar{b}_j^D$, from Theorem 1, we have $\bar{U}_j^D \geq 0 = U_j^D$.

Thus we can conclude that our auction MUSIC ensures value-truthfulness for all SUs. □

THEOREM 5. *MUSIC is an attribute-truthful auction for all SUs (buyers).*

PROOF. Note that SU_j's communications can be disrupted by the PUs if its reported attributes are less than the true values; thus SU_j could only submit attributes with larger values in the auction, i.e, $w_j^D > \bar{w}_j^D$ or $[f_j^D, t_j^D] \supset [\bar{f}_j^D, \bar{t}_j^D]$.

Let $\bar{q}_{ij}$ and $\bar{p}_j^D$ be the number of allocated channels and the unit price when SU_j bids truthfully, respectively. The number of channels assigned to SU_j might be reduced, if the untruthful values of the attributes become larger, i.e., $q_{ij} \leq \bar{q}_{ij}$. The main reason is that higher values of required attributes lead to fewer satisfied channels and smaller preference values δ_{ij} (see (1)), obtaining fewer transaction matchings in the matching stage. Moreover, according to Property 2, $p_j^D \geq \bar{p}_j^D$ because of the increase of the required channel bandwidth. Thus, if SU_j bids $w_j^D > \bar{w}_j^D$ or $[f_j^D, t_j^D] \supset [\bar{f}_j^D, \bar{t}_j^D]$, we have

$$\begin{aligned} U_j^D &= \sum_{\mathrm{PU}_i \in \mathcal{M}^w} [b_j^D(\bar{t}_j^D - \bar{f}_j^D) - p_j^D(t_j^D - f_j^D)]q_{ij} \\ &< \sum_{\mathrm{PU}_i \in \mathcal{M}^w} [b_j^D - \bar{p}_j^D](\bar{t}_j^D - \bar{f}_j^D) = \bar{U}_j^D. \end{aligned}$$

This means that no SU_j can improve its utility via lying values of its attributes. □

5. SIMULATION RESULTS

5.1 Simulation Settings

In this study, we simulate a wireless cognitive radio network within a square area of 200m×200m, in which all SUs are uniformly distributed at random. To investigate the impact of PU density, the number of PUs varies from 10 to 100 at a step size of 10 while the number of SUs is fixed to 50. All users' channel supplies and demands, including the ask and bid prices, channel quantity, bandwidth, starting time, and ending time, are randomly and uniformly chosen within certain ranges listed in Table 2. Particularly, the value of the maximum bid is changed from 3 to 8 at a step size of 1 so that the impact of bid distribution can be evaluated.

To evaluate the performance of our auction scheme MUSIC, we choose the following auction algorithms for the purpose of comparison.

- **SPRITE:** [9] a truthful single-attribute multi-unit auction scheme with uniform pricing, which consists of three phases: buyer group construction, transaction set formation, and market clearing price determination. For each group, group members cooperatively bid the same channels.
- **Revised McAfee double auction (ReMcAfee):** the traditional McAfee auction [13] is a classic truthful single-attribute auction scheme with uniform pricing. For fair comparison, we combine our matching algorithm (Alg. 2) with McAfee to achieve multi-unit trading and channel reuse (ReMcAfee).
- **Our revised multi-attribute auction with uniform pricing (MAUP):** In MAUP, all winning SUs are charged a unit price of a_x^S and all winning PUs are paid at a unit price of b_y^D. The only difference between MAUP and MUSIC is the pricing policy.

Table 2: Parameters

Parameters	Values
Number of PUs (M)	$[10:10:100]$
PUs' asks (a_i^S)	$(0,3]$
Number of SUs (N)	50
SUs' bids (b_j^D)	$(0, B_{max}]$
B_{max}	$[3:1:8]$
Channel quantity (q_i^S, q_j^D)	$[1,5]$
Channel bandwidth (w_i^S, w_j^D)	$[1,20]$MHz
Starting time (f_i^S, f_j^D)	$[0,10]$s
PU Ending time (t_i^S)	$[0,30]$s
SU Ending time (t_j^D)	$[0,50]$s
Interference Range	100m

By comparing with SPRITE, ReMcAfee, and MAUP, the differences between single-attribute and multi-attribute auctions, and those between uniform and discriminatory pricing, can be effectively demonstrated. Note that the term "single-attribute" indicates that price is the only factor considered by the buyers in the auction. Thus, the SUs only concern about whether a channel can satisfy their price constraints without considering the channel diversity in terms of bandwidths and available times. Accordingly, the weights of all edges in G in the process of transaction matching are equal for ReMcAfee.

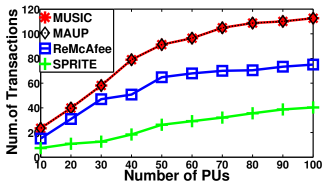
These auction schemes will be examined according to the following performance metrics: (i) **Num. of transactions**, i.e., the number of transactions between winning PUs and SUs. With a fixed channel supply, more transactions indicate higher channel utilization (and channel reuse degree). (ii) **Per-Win-PU (SU) utility**, a short term of "per winning PU's (SU's) utility". (iii) **Per-Win-SU satisfaction**, an abbreviation of "per winning SU's satisfaction degree" (λ_j for SU_j), is calculated based on the Sigmoid function [16]:

$$\lambda_j = \frac{\sum_{\mathrm{PU}_i \in \mathcal{M}^c} q_{ij}\lambda_{ij}}{q_j^D}, \tag{8}$$

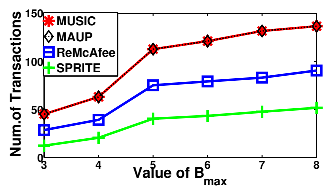
where $\lambda_{ij} = \begin{cases} 0, \text{ if } [f_j^D, t_j^D] \not\subseteq [f_i^S, t_i^S]; \\ \frac{1}{1+e^{-(w_i^S - w_j^D)}}, \text{o.w.} \end{cases}$ This means that the satisfaction degree $\lambda_j \in [0,1)$ of a winning SU_j increases with the number of channels it successfully purchases and the bandwidth of the channels. Moreover, a higher λ_j indicates a better received service.

5.2 Simulation Results

In this subsection, we analyze the simulation results by examining the impact of the number of PUs and the bid distribution when the number of SUs is fixed to 50. Note that the number of PUs is fixed at 100 when B_{max} is varied, while $B_{max} = 5$ when the number of PUs is changed.

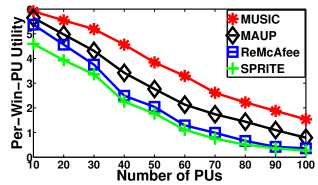


(a) Impact of number of PUs

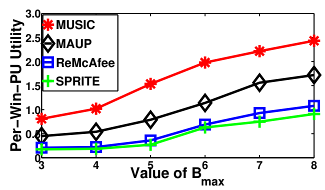


(b) Impact of maximum bid

Figure 4: Num. of transactions.

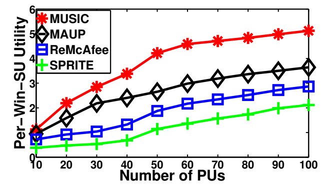


(a) Impact of Number of PUs

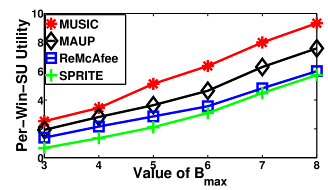


(b) Impact of maximum bid

Figure 5: PU utility.

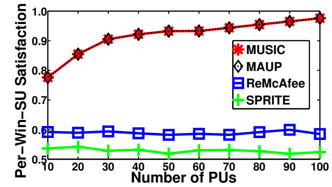


(a) Impact of number of PUs

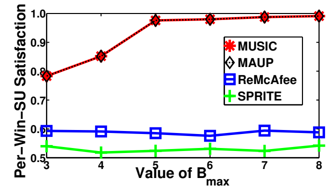


(b) Impact of maximum bid

Figure 6: SU utility.



(a) Impact of number of PUs



(b) Impact of maximum bid

Figure 7: SU Satisfaction.

First, we examine the number of successful transactions. From Fig. 4(a), we observe that the number of transactions increases when the number of PUs grows. Since the available number of channels supplied by each PU is limited, more PUs can offer more channels, leading to an increase in the trading volume. On the other hand, with a bigger value of B_{max}, the purchasing power of the SUs is enhanced, resulting in a larger number of transaction matchings traded in the auction (see Fig. 4(b)).

Different from SPRITE that considers channel reuse only in space domain, MUSIC, MAUP and ReMcAfee, which perform channel reuse in both time and space domains, can obtain more transaction matchings in the auction. Although the channel reuse algorithm (Alg. 2) is implemented in ReMcAfee, its channel utilization is still worse than those of MUSIC and MAUP because the diversity of the channel available times is ignored. In addition, both MUSIC and MAUP select the boundary seller-buyer pair by taking into account the number of winners; therefore they are able to get more winning transactions compared with ReMcAfee.

Next, the impacts of the number of PUs and the bid distribution on the users' utilities are respectively presented in Figs. 5 and 6. As more and more PUs participate in the auction, the competition among the PUs for selling their channels becomes fiercer, resulting in the reduction of the boundary ask price of each channel. Thus, as shown in Fig. 5(a), the utility of each winning PU is decreased. While for the winning SUs, a larger number of PUs indicates that more channels (and matchings) can be obtained at lower prices, leading to an improvement of winning SUs' utilities (see Fig. 6(a)). From Figs. 5(b) and 6(b), it can be seen that the utilities of all winning PUs and all winning SUs are enhanced with higher bids. This is because the winning PUs can receive higher payments from the auctioneer and the winning SUs can buy more channels in the auction.

When comparing these four auction schemes, we obtain two observations. First, MUSIC outperforms MAUP. By exploiting attribute-based discriminatory pricing, MUSIC can enhance the payments of the winning PUs that provide the channels with better attribute values and simultaneously reduce the charges of the winning SUs that are allocated the channels with worse attribute values, improving all winners' utilities. Although this improvement of the winners' utilities is achieved at the cost of a decrease in the auctioneer's revenue, all PUs and SUs have great incentives to participate in the auction for high utilities, leading to an enhanced channel utilization. The second observation is that both MUSIC and MAUP are superior over ReMcAfee and SPRITE. As analyzed before, both MUSIC and MAUP can obtain more transactions than ReMcAfee and SPRITE do. Thus the utilities of the winning PUs and SUs can be increased, which is consistent with the conclusion drawn from Fig. 4.

Finally, we report the values of the winning SUs' satisfaction degree in Fig. 7, to investigate the difference between multi-attribute and single-attribute auctions. In our consideration, the channel diversity in terms of available bandwidths and available time durations becomes larger when the number of PUs increases. Under such a situation, ReMcAfee and SPRITE, which ignores the channel diversity, cannot upgrade the received wireless service for the winning SUs. As a result, the SUs' satisfaction degrees remain at a low level. In contrast, both MUSIC and MAUP match SUs to PUs with the goal of improving all SUs' received service. Thus, with more channels supplied by the PUs, MUSIC and MAUP can enhance SUs' received service by selecting as many good channels as possible. For example, as shown in Fig. 7(a), the satisfaction degree can be increased to more than 0.9 when the number of PUs is larger than 30. Similarly, with higher bid prices, the winning SUs can buy more good channels (and matchings). From Fig. 7(b), it can be seen that the satisfaction degree is getting closer to 1 as B_{max} changes from 1 to 5. Therefore, we can conclude that our multi-attribute auction MUSIC and its variant MAUP perform better than ReMcAfee and SPRITE in terms of winning SUs' satisfaction.

6. RELATED WORK

Almost all existing auction schemes proposed for the secondary spectrum market [1,6,7,9,10,12,19,21,24–26] perform the following two steps: *winner determination* and *pricing*. The first step computes the set of winning buyers and sellers while simultaneously making spectrum assignment for trading purpose (matching the winning sellers and buyers based on their bid prices and spectrum attributes) and the second step sets the trading price to clear the market. The two most famous truthful auction schemes are Vickrey-Clarke-Groves (VCG) [4, 8, 15] and McAfee [13]. VCG employs *discriminatory pricing*, which determines the critical price (*social opportunity cost*) for each buyer (seller) to win the auction if bids higher (lower), while McAfee takes *uniform pricing*, which sacrifices the least-profitable buyer-seller pair in the sorted bid lists, and pays the same price to the winning sellers and charges the same price from the winning buyers. According to the objectives and pricing policies, existing auction mechanisms follow either VCG-style [1,7,10,19,24,26] or McAfee-style [6,9,12,21,25]; but the direct adoption of VCG or McAfee is a very challenging problem due to spatial channel reuse and complicated channel attributes in secondary spectrum market. Most auction schemes proposed for secondary spectrum market take into account spatial reuse but not the spectrum attributes.

VCG-style double auctions intend to maximize system efficiency (social welfare or spectrum efficiency, etc, typically involving NP-hard problems) but budget-balance can not always be guaranteed. Thus most existing VCG-style auctions [1, 7, 10, 24, 26] address unilateral inputs (only buyers submit their bids and no seller gets involved) as VCG double auctions could not guarantee budget balance. These schemes consider spatial channel reuse but mainly assumes homogenous channels. VERITAS [24] is the first VCG-style unilateral truthful auction that pays each winner the price of its critical neighbor in the interference graph. A few works [1,7,10,26] consider Bayesian setting and design truthful auctions based on the Myerson's Optimal Mechanism (MOM) [14]. Bayesian setting implies that the true valuations of the bid prices are drawn from publicly known probability distributions, and thus a virtual bid can be computed from the original bid for each buyer. Then an optimization problem is formulated based on the virtual bids, and heuristics are exploited for computational efficiency. Pricing is based on MOM, which states that an auction is truthful if and only if there exists a critical bid for each buyer i such that i wins if bidding higher and loses otherwise, and the clearing price for i is independent of the bid of i. VCG-style auctions in [1, 7, 10, 26] differ in their approaches to determine the winners and their channels based on algorithms/heuristics with different computational complexity. Double auction based on MOM is studied in DISTRICT-D [19]. VCG-style online unilateral truthful auctions considering spectrum reuse at the spatial domain have been addressed in [5, 22, 23].

McAfee-style double auctions can achieve the economic properties of individual-rationality, ex-post budget balance, and truthfulness at the price of low efficiency. To support spatial reuse, [6,11,17,20,25] first perform a bid-independent buyer grouping based on the interference graph and then apply the McAfee-style winner determination and pricing mechanism. Since TAHES [6] considers non-uniform channels, a matching procedure is needed to match each buyer group to a channel before McAfee-style auction can be applied. A similar idea is explored in District-U [19], though grouping is not explicitly stated. These auction schemes achieve economic-robustness but lose their truthfulness when extended to heterogeneous multi-unit bids (a buyer purchases more than one channel at different bid prices) as the boundary buyer who wins in one channel and loses in another can enhance its revenue by lying about the bid of the losing channel demand [20]. A remedy approach is proposed in [20] to remove the channel demand that may cause such kind of internal collusion for truthfulness guarantee. A two-level McAfee-style auction scheme is studied in [12], which realizes an auction between PUs and access points (APs), and another auction between APs and regular wireless users. By observing that the buyer with a higher bid price can afford a higher price for the same channel, Xiang *et al.* [21] present a homogeneous single-item auction with a discriminatory pricing scheme for the SUs and a uniform pricing scheme for the PUs.

In this paper, we propose a McAfee-style double auction mechanism with discriminatory pricing to support channel reuse in both spatial and temporal domains with the considerations of channel diversity (bandwidth and available duration) and price diversity (discriminatory pricing). Compared to the existing McAfee-style auctions, we make the pricing and channel assignment attribute-aware, i.e., the buyers and sellers are matched based on their channel attributes to enhance the quality of wireless services for SUs, improve the channel utilization of the spectrum market, and enhance the revenue of the PUs with a better channel supply.

7. CONCLUSION

In this paper, we introduce a novel multi-attribute auction scheme to solve the problem of channel allocation in CRNs. With the consideration of channel diversity in frequency, time, and space domains, the diversity of SUs' demands can be accurately reflected via an attribute-based preference function in our auction design. Our proposed auction scheme can effectively assign channels while guaranteeing SUs' received service and achieving channel reuse in both time and space domains, largely increasing the channel utilization. Furthermore, the utilities of winning PUs and SUs can be significantly improved by employing discriminatory pricing in MUSIC. Through mathematical analysis, we prove that MUSIC can simultaneously satisfy individual-rationality, ex-post budget balance, value-truthfulness, and attribute-truthfulness. Our simulation results also indicate that the SUs' service satisfaction can be enhanced by considering multiple attributes of the channels, and that the winners' utilities can be improved by utilizing discriminatory pricing.

In our future research, we will investigate online multi-attribute auction to handle SU's dynamic access more effectively, and exploit various discriminative pricing policies to consider the impact of SU locations on pricing.

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