A Resource Allocation Algorithm for Users with Multiple Applications in 4G-LTE

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ABSTRACT

In this paper, we consider resource allocation optimization problem in fourth generation long term evolution (4G-LTE) with users running multiple applications. Each mobile user can run both delay-tolerant and real-time applications. In every user equipment (UE), each application has a application-status differentiation from other applications depending on its instantaneous usage percentage. In addition, the network operators provide subscriber differentiation by assigning each UE a subscription weight relative to its subscription. The objective is to optimally allocate the resources with a utility proportional fairness policy. We propose an algorithm to allocate the resources in two-stages. In the first-stage, the UEs collaborate with the evolved node B (eNodeB) that allocates the optimal rates to users according to that policy. In the second-stage, each user allocates its assigned rate internally to its applications according to their usage percentage. We prove that the two-stage resource allocation algorithm allocates the optimal rates without eNodeB knowledge of the UEs utilities. Finally, numerical results on the performance of the proposed algorithm are presented.

Categories and Subject Descriptors

K.6.2 [**Management of Computing and Information Systems**]: Installation Management—*pricing and resource allocation*; C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design—*wireless communication*

Keywords

Two-Stage Resource Allocation; Multiple Utilities; Subscriber Differentiation

1. INTRODUCTION

In recent years, there has been a rapid growth in mobile broadband service. This rapid growth is in both the number of subscribers and the traffic of each subscriber. The wireless network providers are moving from single service (e.g. Internet access) to multiple service offering (e.g. multimedia telephony and mobile-TV) [1]. Mobile subscribers are running multiple applications on their smart phones, simultaneously. These different applications and services have different performance requirements, for example, some are delay-tolerant and some are real-time applications. Therefore, they require different bit-rates and packet delays. Due to the different nature of different applications, service-offering differentiation needs to be taken into consideration when allocating the resources for different users. The usage percentage of each application on the UE requires an additional differentiation that we call application-status differentiation. In addition, network providers are recently providing subscriber differentiation [1], i.e. different users requesting the same service receive different treatment. Defined by the network providers, subscriber differentiation could be between corporate and private subscribers, post- and pre-paid subscribers, and/or privileged and roaming subscribers.

In [2, 3], the authors present an optimal rate allocation algorithm for users with delay-tolerant or real-time applications. The optimal rates are achieved by formulating the rate allocation optimization problem in a convex optimization framework. The authors use logarithmic and sigmoidal-like utility functions to represent delay-tolerant and real-time applications, respectively. In [2, 3], the rate allocation algorithm gives priority to real-time applications over delay-tolerant applications when allocating resources as the utility proportional fairness rate allocation policy is used. This resource allocation guarantees service-offering differentiation when allocating resources.

In this paper, we formulate the resource allocation optimization problem with service-offering differentiation, application status differentiation and subscriber differentiation that is casted in a convex optimization framework. In our system model, each subscriber has a subscription weight set by the network. In addition, each subscriber can run multiple applications, each with its own utility function, on his smart phone. The applications running on the phone have different application-status depending on their instantaneous usage percentage and importance to the subscriber. For example, the application running on the foreground, such a voice call, has higher application-status than the application running on the background, such as an automatic application update. Finally, the service-offering differentiation which gives priority to real-time applications over delay-

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tolerant applications is inherited in the utility proportional fairness rate allocation policy.

Our resource allocation algorithm is performed in two-stages. The first-stage is for allocating the users rates and this is performed collaboratively between the eNodeB and the UEs. The second-stage is for allocating the applications rates and this is performed internally in each UE.

1.1 Related Work

A non-convex optimization formulation for maximization of utility functions in wireless networks is presented in [4, 5]. Both elastic and sigmoidal-like utility functions are used. The authors present the algorithm to solve it optimally when the duality gap is zero and include a fair allocation heuristic to ensure network stability.

A utility max-min fairness resource allocation for users, with elastic and real-time traffic, sharing a single path in the network is proposed in [6]. In [7], the authors propose a utility proportional fair optimization formulation for high-SINR wireless networks using utility max-min architecture. A comparison between their algorithm and the traditional bandwidth proportional fair algorithms in [8] is presented and a closed form solution that prevents oscillations in the network is proposed.

A distributed power allocation algorithm for a mobile cellular system is proposed in [9]. The authors used non-concave sigmoidal-like utility functions. The algorithm provides an approximation to the global optimal solution and therefore could drop some users to maximize the overall system utilization. Therefore, it does not guarantee a minimum QoS for all users.

A weighted aggregation of elastic and inelastic utility functions for each UE is proposed in [10]. This aggregated utility functions are then approximated to the nearest strictly concave utility function from a set of functions using minimum mean-square error. These approximated utility functions are solved using a modified version of the distributed rate allocation algorithm by Frank Kelly [11]. Therefore, the allocated rates are approximations of the optimal rates.

1.2 Our Contributions

Our contributions in this paper are summarized as:

- We present a novel two-stage method for allocating the optimal rates for users running multiple applications. In the first-stage, the eNodeB and the UEs collaborate to allocate the optimal rate to each UE. In the second-stage, each UE internally distributes its rate optimally to the different applications running on it.
- We prove that the new two-stage resource allocation optimization problem is equivalent to the one-stage resource allocation convex optimization problem that allocates rates directly to applications. We present the algorithm for solving the two-stage optimization problem and its simulation results.

The remainder of this paper is organized as follows. Section 2 presents the problem formulation. Section 3 proves that our novel two-stage allocated optimal rates are equivalent to one-stage allocated optimal rates. In Section 4, we present our two-stage rate allocation algorithm for the utility proportional fairness policy. Section 5 discusses simulation setup and provides quantitative results along with discussion. Section 6 concludes the paper.

2. PROBLEM FORMULATION

We consider single cell 4G-LTE mobile system consisting of a single eNodeB and M UEs. The rate allocated by the eNodeB to i^{th} UE is given by r_i. Each UE has its own utility function $V_i(r_i)$ that corresponds to the applications running of the UE. Our objective is to determine the optimal rates the eNodeB shall allocate to the UEs. We assume the user utility function $V_i(r_i)$ of i^{th} UE is given by:

$$V_i(r_i) = \prod_{j=1}^{N_i} U_{ij}^{\alpha_{ij}}(r_{ij}) \tag{1}$$

where $U_{ij}(r_{ij})$ is the j^{th} application utility function, r_{ij} is the rate allocated to the j^{th} application, and α_{ij} is the j^{th} application usage percentage on the i^{th} UE (i.e. $\sum_{j=1}^{N_i} \alpha_{ij} = 1$). We assume that $U_{ij}(r_{ij})$ is a strictly concave or a sigmoidal-like function. The utility functions $U(r)$ have the following properties:

- $U(0) = 0$ and $U(r)$ is an increasing function of r.
- $U(r)$ is twice continuously differentiable in r and bounded above.

In our model, we use the normalized sigmoidal-like utility function, as in [9], that can be expressed as

$$U(r) = c\Big(\frac{1}{1+e^{-a(r-b)}} - d\Big) \tag{2}$$

where $c = \frac{1+e^{ab}}{e^{ab}}$ and $d = \frac{1}{1+e^{ab}}$. So, it satisfies $U(0) = 0$ and $U(\infty) = 1$. The inflection point of normalized sigmoidal-like function is at $r^{\inf} = b$. In addition, we use the normalized logarithmic utility function, as in [7], that can be expressed as

$$U(r) = \frac{\log(1+kr)}{\log(1+kr_{max})} \tag{3}$$

where r_{max} is the maximum required rate for the user to achieve 100% utilization and k is the rate of increase of utilization with the allocated rate r. So, it satisfies $U(0) = 0$ and $U(r_{max}) = 1$. The inflection point of normalized logarithmic function is at $r^{\inf} = 0$.

The basic formulation of the resource allocation problem is given by the following optimization problem:

$$\begin{aligned} \max_{\mathbf{r}} \quad & \prod_{i=1}^{M}\Big(\prod_{j=1}^{N_i} U_{ij}^{\alpha_{ij}}(r_{ij})\Big)^{\beta_i} \\ \text{subject to} \quad & \sum_{i=1}^{M}\sum_{j=1}^{N_i} r_{ij} \leq R, \\ & r_{ij} \geq 0, \quad i = 1,2,...,M, \\ & \qquad\qquad j = 1,2,...,N_i. \end{aligned} \tag{4}$$

where R is the maximum achievable rate of the eNodeB, M is the number of UEs in the coverage ara of the eNodeB, and N_i is the number of applications running in the i^{th} UE.

Corollary 2.1. *The optimization problem (4) is a convex optimization problem and there exists a unique tractable global optimal solution.*

Proof. The objective function in optimization problem (4) given by $\prod_{i=1}^{M}\Big(\prod_{j=1}^{N_i} U_{ij}^{\alpha_{ij}}(r_{ij})\Big)^{\beta_i}$ is equivalent to the objective function $\sum_{i=1}^{M}\beta_i\sum_{j=1}^{N_i}\alpha_{ij}\log U_{ij}(r_{ij})$, so the optimization problem (4) can be written as follows:

$$\begin{aligned} \max_{\mathbf{r}} \quad & \sum_{i=1}^{M}\beta_i\sum_{j=1}^{N_i}\alpha_{ij}\log U_{ij}(r_{ij}) \\ \text{subject to} \quad & \sum_{i=1}^{M}\sum_{j=1}^{N_i} r_{ij} \le R, \\ & r_{ij}\ge 0, \quad i=1,2,...,M, \\ & \qquad\qquad j=1,2,...,N_i. \end{aligned} \tag{5}$$

Given the problem formulation in Section 2, we know that the utility functions $U_{ij}(r_{ij})$ are strictly concave or sigmoidal-like functions. From Lemma (III.1) in [2], $\log U_{ij}(r_{ij})$ is a strictly concave function for a strictly concave or sigmoidal-like utility function $U_{ij}(r_{ij})$. It follows that optimization problem (5) is convex and as a result optimization problem (4) is also convex. Therefore, there exists a tractable global optimal solution for optimization problem (4). $\square$

3. TWO-STAGE OPTIMIZATION PROBLEM

We divide optimization problem (4) in two optimization problems to be solved into two stages and allocate the same optimal rates as optimization problem (4). In the first-stage, the rates r_i are allocated to the users by the eNodeB and the solution is achieved collaboratively between the eNodeB and the UEs. We call this stage the eNodeB-UE rate allocation (EURA) stage. In the second-stage, the rates r_{ij} are allocated to applications and the allocation is done internally in the UE. We call this stage the internal UE rate allocation (IURA) stage.

3.1 EURA Optimization Problem

EURA optimization problem that is solved collaboratively between the eNodeB and the UEs and can be written as:

$$\begin{aligned} \max_{\mathbf{r}} \quad & \prod_{i=1}^{M} V_i^{\beta_i}(r_i) \\ \text{subject to} \quad & \sum_{i=1}^{M} r_i \le R, \\ & r_i \ge 0, \quad i=1,2,...,M. \end{aligned} \tag{6}$$

where $V_i=\prod_{j=1}^{N_i}U_{ij}^{\alpha_{ij}}(r_{ij})$ and $\mathbf{r}=\{r_1,r_2,...,r_M\}$ and M is the number of UEs in the coverage area of the eNodeB. In the optimization problem (6), since the objective function $\arg\max_{\mathbf{r}}\prod_{i=1}^{M}V_i^{\beta_i}(r_i)$ is equivalent to $\arg\max_{\mathbf{r}}\sum_{i=1}^{M}\beta_i\log(V_i(r_i))$, then optimization problem (6) can be written as:

$$\begin{aligned} \max_{\mathbf{r}} \quad & \sum_{i=1}^{M}\beta_i\log(V_i(r_i)) \\ \text{subject to} \quad & \sum_{i=1}^{M} r_i \le R, \\ & r_i \ge 0, \quad i=1,2,...,M. \end{aligned} \tag{7}$$

Corollary 3.1. *The optimization problem (6) is a convex optimization problem and there exists a unique tractable global optimal solution.*

Proof. From equation (1), we have that $\log(V_i(r_i)) = \sum_{j=1}^{N_i}\alpha_{ij}\log(U_{ij}(r_{ij}))$, and given the problem formulation in Section 2, then optimization problem (6) is convex (steps similar to Corollary 2.1) $\square$

3.2 IURA Optimization Problem

IURA optimization problem is solved internally in every UE and can be written for the i^{th} UE as follows:

$$\begin{aligned} \max_{\mathbf{r}_i} \quad & \prod_{j=1}^{N_i} U_{ij}^{\alpha_{ij}}(r_{ij}) \\ \text{subject to} \quad & \sum_{i=1}^{N_i} r_{ij} \le r_i^{\text{opt}}, \\ & r_{ij}\ge 0, \quad j=1,2,...,N_i. \end{aligned} \tag{8}$$

where $\mathbf{r}_i=\{r_{i1},r_{i2},...,r_{iN_i}\}$ and r_i^{opt} is the rate allocated by eNodeB to the i^{th} UE. In the optimization problem (8), since the objective function $\arg\max_{\mathbf{r}_i}\prod_{j=1}^{N_i}U_{ij}^{\alpha_{ij}}(r_{ij})$ is equivalent to $\arg\max_{\mathbf{r}_i}\sum_{j=1}^{N_i}\alpha_{ij}\log(U_{ij}(r_{ij}))$, then optimization problem (8) can be written as:

$$\begin{aligned} \max_{\mathbf{r}_i} \quad & \sum_{j=1}^{N_i}\alpha_{ij}\log U_{ij}(r_{ij}) \\ \text{subject to} \quad & \sum_{i=1}^{N_i} r_{ij} \le r_i^{\text{opt}}, \\ & r_{ij}\ge 0, \quad j=1,2,...,N_i. \end{aligned} \tag{9}$$

Corollary 3.2. *The optimization problem (8) is a convex optimization problem and there exists a unique tractable global optimal solution.*

Proof. Given the problem formulation in Section 2, then optimization problem (8) is convex (steps similar to Corollary 2.1). $\square$

3.3 Equivalence

In this section, we show the equivalence of EURA optimization problem (6) and IURA optimization problem (8) to optimization problem (4).

Lemma 3.3. *For strictly concave or sigmoidal-like utility functions $U_{ij}(r_{ij})$, the slope of natural logarithm of utility functions $p=S_{ij}(r_{ij})=\frac{\partial\log U_{ij}(r_{ij})}{r_{ij}}$ are invertible and the inverse functions $r_{ij}=S_{ij}^{-1}(p)$ are strictly decreasing functions.*

Proof. For the strictly concave utility function $U_{ij}(r_{ij})$ case and from the utility function properties in Section 2, the utility function is positive $U_{ij}(r_{ij})>0$, increasing and twice differentiable with respect to r_{ij}. Then, it follows that $U'_{ij}(r_{ij})=\frac{\partial U_{ij}(r_{ij})}{\partial r_{ij}}>0$ and $U''_{ij}(r_{ij})=\frac{\partial^2 U_{ij}(r_{ij})}{\partial r_{ij}^2}<0$. It follows that, we have $S_{ij}(r_{ij})=\frac{\partial\log(U_{ij}(r_{ij}))}{\partial r_{ij}}=\frac{U'_{ij}(r_{ij})}{U_{ij}(r_{ij})}>0$ and $\frac{\partial S_{ij}(r_{ij})}{\partial r_{ij}}=\frac{U''_{ij}(r_{ij})U_{ij}(r_{ij})-U'^2_{ij}(r_{ij})}{U^2_{ij}(r_{ij})}<0$. Therefore, $S_{ij}(r_{ij})$ of any strictly concave utility function is strictly decreasing function.

For the sigmoidal-like utility function $U_{ij}(r_{ij})$ case, the utility function of the normalized sigmoidal-like function is given by equation (2). For $0 < r_{ij} < R$, we have the first and second derivative as

$$\begin{aligned}\frac{\partial}{\partial r_{ij}} S_{ij}(r_{ij}) =& \frac{a_{ij} d_{ij} e^{-a_{ij}(r_{ij}-b_{ij})}}{1-d_{ij}(1+e^{-a_{ij}(r_{ij}-b_{ij})})} \\ &+ \frac{a_{ij} e^{-a_{ij}(r_{ij}-b_{ij})}}{(1+e^{-a_{ij}(r_{ij}-b_{ij})})} > 0 \\ \frac{\partial^2}{\partial r_{ij}^2} S_{ij}(r_{ij}) =& \frac{-a_{ij}^2 d_{ij} e^{-a_{ij}(r_{ij}-b_{ij})}}{c_{ij}\Big(1-d_{ij}(1+e^{-a(r_{ij}-b_{ij})})\Big)^2} \\ &+ \frac{-a_{ij}^2 e^{-a_{ij}(r_{ij}-b_{ij})}}{(1+e^{-a_{ij}(r_{ij}-b_{ij})})^2} < 0.\end{aligned}$$

It follows that $S_{ij}(r_{ij})$ of any sigmoidal-like utility function is strictly decreasing function.

As a result, $S_{ij}(r_{ij})$ of all the utility functions in Section 2 are strictly decreasing functions. Therefore, $S_{ij}(r_{ij})$ functions are invertible and $r_{ij} = S_{ij}^{-1}$ are strictly decreasing functions. □

Corollary 3.4. *The i^{th} user optimal rate r_i allocated by optimization problem (6) is equal to the i^{th} user aggregated applications rates $\sum_{j=1}^{N_i} r_{ij}$ allocated by optimization problem (4).*

Proof. From optimization problem (5), we have the Lagrangian:

$$L_T(r_{ij}) = (\sum_{i=1}^{M} \beta_i \sum_{j=1}^{N_i} \alpha_{ij} \log U_{ij}(r_{ij})) - p_T(\sum_{i=1}^{M}\sum_{j=1}^{N_i} r_{ij} - R + z) \tag{10}$$

where $z \geq 0$ is the slack variable and p_T is the Lagrange multiplier which corresponds to the total price per bandwidth for the M channels (i.e. shadow price [2]). So, we have

$$\frac{\partial L_T(r_{ij})}{\partial r_{ij}} = \beta_i \alpha_{ij} \frac{U'_{ij}(r_{ij})}{U_{ij}(r_{ij})} - p_T = 0 \tag{11}$$

$$p_T = \beta_i \alpha_{ij} \frac{U'_{ij}(r_{ij})}{U_{ij}(r_{ij})} = f_{ij}(r_{ij}) \tag{12}$$

$$r_{ij} = f_{ij}^{-1}(p_T) \tag{13}$$

$$r_i = \sum_{j=1}^{N_i} r_{ij} = \sum_{j=1}^{N_i} f_{ij}^{-1}(p_T). \tag{14}$$

From optimization problem (7), we have the Lagrangian:

$$L_S(r_i) = (\sum_{i=1}^{M} \beta_i \log V_i(r_i)) - p_S(\sum_{i=1}^{M} r_i - R + z) \tag{15}$$

where $z \geq 0$ is the slack variable. So, we have

$$\frac{\partial L_S(r_i)}{\partial r_i} = \beta_i \frac{V'_i(r_i)}{V_i(r_i)} - p_S = 0 \tag{16}$$

$$p_S = \beta_i \frac{V'_i(r_i)}{V_i(r_i)} \tag{17}$$

using $\log V_i(r_i) = \sum_{j=1}^{N_i} \alpha_{ij} \log U_{ij}(r_{ij})$ from equation (1) and $r_i = \sum_{j=1}^{N_i} r_{ij}$ we have

$$\begin{aligned}\frac{\partial \log V_i(r_i)}{\partial r_{ij}} =& \frac{\partial}{\partial r_{ij}}\Big(\sum_{j=1}^{N_i} \alpha_{ij} \log U_{ij}(r_{ij})\Big) \\ \frac{\partial \log V_i(r_i)}{\partial r_i}\frac{\partial r_i}{\partial r_{ij}} =& \alpha_{ij} \frac{\partial \log U_{ij}(r_{ij})}{\partial r_{ij}} \\ \frac{V'_i(r_i)}{V_i(r_i)} =& \alpha_{ij} \frac{U'_{ij}(r_{ij})}{U_{ij}(r_{ij})}\end{aligned} \tag{18}$$

substituting in (17) we have

$$p_S = \beta_i \alpha_{ij} \frac{U'_{ij}(r_{ij})}{U_{ij}(r_{ij})} = p_T = f_{ij}(r_{ij}) \tag{19}$$

$$r_i = \sum_{j=1}^{N_i} r_{ij} = \sum_{j=1}^{N_i} f_{ij}^{-1}(p_T) \tag{20}$$

so the shadow prices p_T and p_S of optimization problem (4) and (6) are equal and so are the rates of equation (14) and (20). □

Corollary 3.5. *The j^{th} application in i^{th} user optimal rate r_{ij} allocated by optimization problem (8) is equal to the j^{th} application in i^{th} user optimal rate r_{ij} allocated by optimization problem (4).*

Proof. From optimization problem (9), we have the Lagrangian:

$$L_I(r_{ij}) = (\sum_{j=1}^{N_i} \alpha_{ij} \log U_{ij}(r_{ij})) - p_I(\sum_{j=1}^{N_i} r_{ij} - r_i^{\text{opt}} + z) \tag{21}$$

where $z \geq 0$ is the slack variable and p_I is the Lagrange multiplier which corresponds to the internal price per bandwidth for the total i^{th} user applications (i.e. internal shadow price).

$$\frac{\partial L_I(r_{ij})}{\partial r_{ij}} = \alpha_{ij} \frac{U'_{ij}(r_{ij})}{U_{ij}(r_{ij})} - p_I = 0 \tag{22}$$

$$\beta_i p_I = \beta_i \alpha_{ij} \frac{U'_{ij}(r_{ij})}{U_{ij}(r_{ij})} = f_{ij}(r_{ij}) \tag{23}$$

using constraints of equation (9) then we have

$$r_i^{\text{opt}} = \sum_{j=1}^{N_i} r_{ij} = \sum_{j=1}^{N_i} f_{ij}^{-1}(\beta_i p_I) \tag{24}$$

from equation (20) then $p_T = \beta_i p_I$. From equation (12) and (23), then optimal rates r_{ij} allocated by optimization problem (8) are equal to optimal rates r_{ij} allocated by optimization problem (4). □

Theorem 3.6. *Optimization problems (6) and (8) are equivalent to optimization problem (4).*

Proof. It follows from Corollary 3.4 and 3.5 that optimization problems (6) and (8) are equivalent to optimization problem (4). □

4. ALGORITHMS

The optimal rates are allocated in two-stages. In the first-stage, EURA algorithm allocates the users rates r_i. In the second-stage, IURA algorithm allocates the applications rates r_{ij}.

4.1 EURA Algorithm

In this section, we present the first-stage of resource allocation where the rates r_i are allocated to the UEs. The algorithm is divided into a UE algorithm shown in Algorithm (1) and a eNodeB algorithm shown in Algorithm (2). The algorithm is a modification of the distributed algorithms in [2].

Algorithm 1 UE Algorithm

Send initial bid $w_i(1)$ to eNodeB
loop
  Receive shadow price $p_S(n)$ from eNodeB
  if STOP from eNodeB **then**
    Calculate allocated rate $r_i^{\text{opt}} = \frac{w_i(n)}{p_S(n)}$
  else
    Solve $r_i(n) = \arg\max_{r_i} \Big(\beta_i \log V_i(r_i) - p_S(n) r_i\Big)$
    Send new bid $w_i(n) = p_S(n) r_i(n)$ to eNodeB
  end if
end loop

In Algorithm (1) and (2), each UE starts with an initial bid $w_i(1)$ which is transmitted to the eNodeB. The eNodeB calculates the difference between the received bid $w_i(n)$ and the previously received bid $w_i(n-1)$ and exits if it is less than a pre-specified threshold δ. We set $w_i(0) = 0$. If the value is greater than the threshold, eNodeB calculates the shadow price $p_S(n) = \frac{\sum_{i=1}^{M} w_i(n)}{R}$ and sends that value to all the UEs. Each UE receives the shadow price to solve the rate r_i that maximizes $\log \beta_i V_i(r_i) - p_S(n) r_i$. That rate is used to calculate the new bid $w_i(n) = p_S(n) r_i(n)$. Each UE sends the value of its new bid $w_i(n)$ to eNodeB. This process is repeated until $|w_i(n) - w_i(n-1)|$ is less than the threshold δ.

Algorithm 2 eNodeB Algorithm

loop
  Receive bids $w_i(n)$ from UEs {Let $w_i(0) = 0 \ \forall i$}
  if $|w_i(n) - w_i(n-1)| < \delta \ \forall i$ **then**
    STOP and allocate rates (i.e r_i^{opt} to user i)
  else
    Calculate $p_S(n) = \frac{\sum_{i=1}^{M} w_i(n)}{R}$
    Send new shadow price $p_S(n)$ to all UEs
  end if
end loop

4.2 IURA Algorithm

In this section, we present the second-stage of resource allocation where the rates r_{ij} are allocated internally in the UE to its applications. The algorithm is shown in Algorithm (3). The UE uses the allocated rate in the first-stage r_i^{opt} and solves the maximization problem that is given by $\mathbf{r}_i = \arg\max_{\mathbf{r}_i} \sum_{j=1}^{N_i} (\alpha_{ij} \log U_{ij}(r_{ij}) - p_I r_{ij}) + p_I r_i^{\text{opt}}$. Finally, the UE allocates the rates r_{ij} to the corresponding applications.

5. SIMULATION RESULTS

Algorithm (1), (2) and (3) were applied to various logarithmic and sigmoidal-like utility functions with different parameters in MATLAB. The simulation results showed convergence to the optimal global point. In this section, we present the simulation results of six utility functions, similar to [2, 3], shown in Figure 1, corresponding to three UEs with aggregated utilities shown in Figure 2. We use three normalized sigmoidal-like functions that are expressed by equation (2) with different parameters, $a = 5$, $b = 10$ which is an approximation to a step function at rate $r = 10$ (e.g. VoIP), $a = 3$, $b = 20$ which is an approximation of an adaptive real-time application with inflection point at rate $r = 20$ (e.g. standard definition video streaming), and $a = 1$, $b = 30$ also is an approximation of an adaptive real-time application with inflection point at rate $r = 30$ (e.g. high definition video streaming). These sigmoidal-like utility functions are running in User (i.e UE) 1, 2, and 3, respectively. We use three logarithmic functions that are expressed by equation (3) with $r_{max} = 100$ and different k_i parameters which are

Algorithm 3 Internal UE Algorithm

loop
  Receive r_i^{opt} from eNodeB {by Algorithm (1) and (2)}
  Solve
  $\mathbf{r}_i = \arg\max_{\mathbf{r}_i} \sum_{j=1}^{N_i} (\alpha_{ij} \log U_{ij}(r_{ij}) - p_I r_{ij}) + p_I r_i^{\text{opt}}$
  $\{\mathbf{r}_i = \{r_{i1}, r_{i2}, ..., r_{iN_i}\}\}$
  Allocate r_{ij} to the j^{th} application
end loop

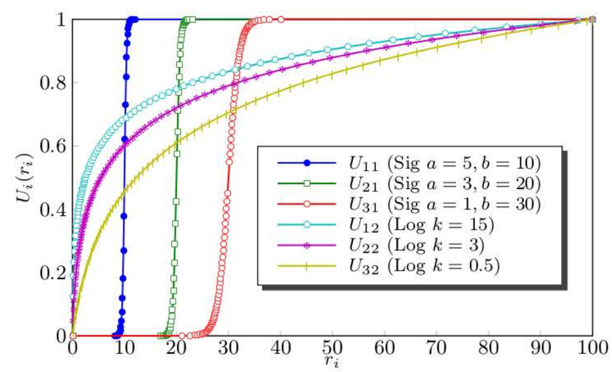


Figure 1: The applications utility functions $U_{ij}(r_{ij})$.

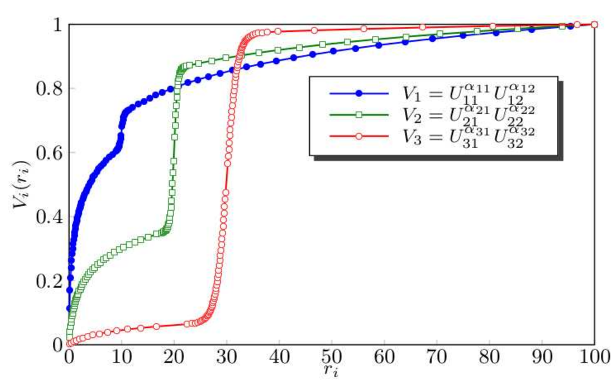


Figure 2: The aggregated utility functions $V_i(r_i)$ corresponding to the i^{th} user.

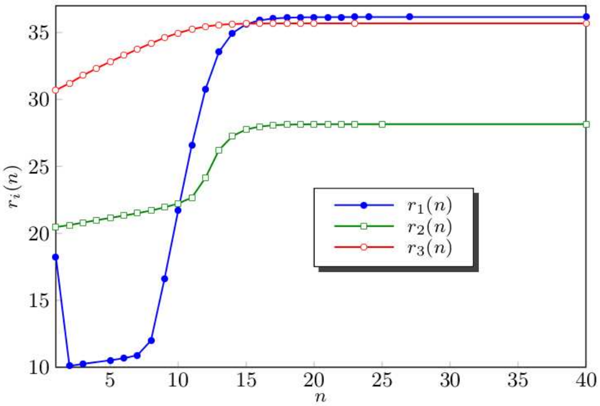


Figure 3: The users rates convergence $r_i(n)$ with number of iterations n for $R = 100$ (EURA algorithm).

approximation for delay tolerant applications (e.g. FTP). We use $k = \{15, 3, 0.5\}$. These logarithmic utility functions are running in User 1, 2, and 3, respectively. We set $\beta_i = 1 \ \forall \ i$. Let $\boldsymbol{\alpha} = \{\alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22}, \alpha_{31}, \alpha_{32}\}$ be the set of application-status weights.

5.1 Convergence Dynamics for R =100

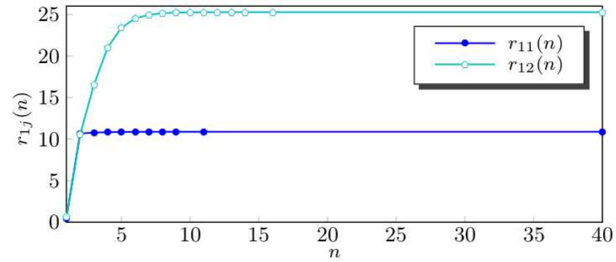
In the following simulations, we set $R = 100$, application-status weights $\boldsymbol{\alpha} = \{0.1, 0.9, 0.5, 0.5, 0.9, 0.1\}$, and number of iterations $n = 40$. In Figure 3, we show the allocated rates r_i of different users with the number of iterations n. This is the solution of optimization problem (6) using EURA algorithm. The user rates are used to solve optimization problem (8) using IURA algorithm to achieve the optimal applications rates. Figure 4, we show the allocated application rates r_{ij} for each user with the number of iterations n. This solution is equivalent to solving optimization problem (4).

5.2 For $10 \leq R \leq 105$

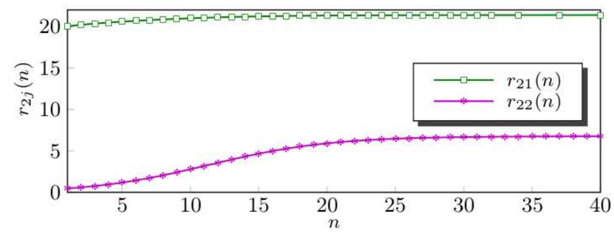
In the following simulations, we set $\delta = 10^{-3}$ and the eNodeB total rate R takes values between 10 and 105 with step of 5. In Figure 5, we show the final users rates r_i with different eNodeB total rate R. This is the solution of optimization problem (6) using EURA algorithm. Figure 6, we show the final applications rates r_{ij} of different users with different eNodeB total rate R. This is the solution of optimization problem (8) using IURA algorithm.

5.3 Sensitivity to change in α

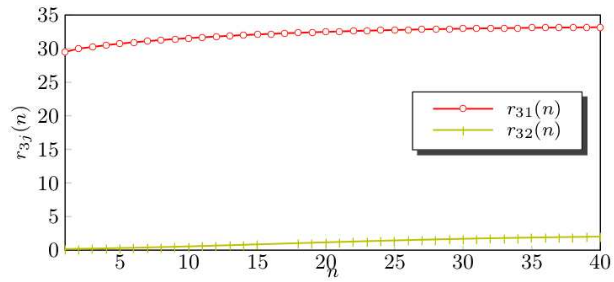
In the following simulations, we set $\delta = 10^{-3}$ and the total achievable rate of the eNodeB $R = 100$. We measure the sensitivity of the change in the usage percentages (corresponding to application-status differentiation) of the application running in the UEs. The users switch between



(a) The application rates $r_{1j}(n)$ of the 1^{st} user.



(b) The application rates $r_{2j}(n)$ of the 2^{nd} user.



(c) The application rates $r_{3j}(n)$ of the 3^{rd} user.

Figure 4: The applications rates convergence $r_{ij}(n)$ with number of iterations n for $R = 100$ (IURA algorithm).

their applications with the following usage percentages

$$\boldsymbol{\alpha}(t) = \begin{cases} \{0.1, 0.9, 0.5, 0.5, 0.9, 0.1\} & ; 0 \leq t \leq 40 \\ \{0.5, 0.5, 0.3, 0.7, 0.2, 0.8\} & ; 40 < t \leq 80 \\ \{0.1, 0.9, 0.9, 0.1, 0.9, 0.1\} & ; 80 < t \leq 120 \\ \{1.0, 0.0, 0.9, 0.1, 0.8, 0.2\} & ; 120 < t \leq 160 \\ \{0.5, 0.5, 0.9, 0.1, 0.8, 0.2\} & ; 160 < t \leq 200. \end{cases} \tag{25}$$

In Figure 7, we show the users rates r_i convergence with time for the changing usage percentages given by $\boldsymbol{\alpha}(t)$.

6. CONCLUSION

In this paper, we proposed a novel two-stage approach for resource allocation in 4G-LTE. In the first-stage, eNodeB collaborates with the UEs to allocate the rates to all the UEs in its coverage area. In the second-stage, each UE internally allocates rates to its applications. We proved that this resource allocation is optimal and that it is equivalent to the direct allocation of rates to applications by eNodeB. Our proposed algorithm takes into consideration service-offering differentiation (real-time and delay-tolerant applications), application-status differentiation (usage percentage of every application within a UE) and subscriber differentiation (subscribers priority within a network). We showed through

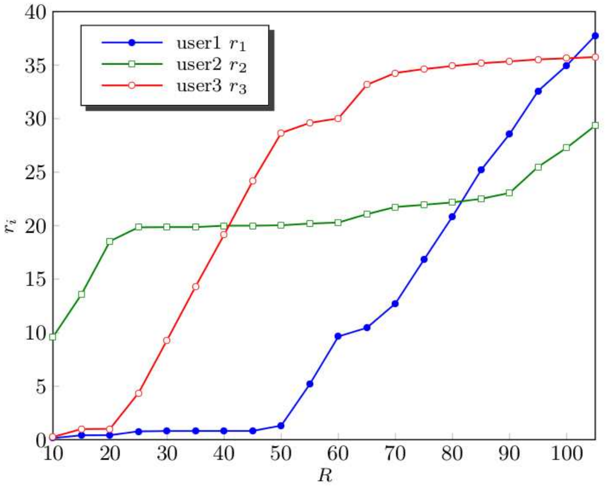


Figure 5: The users rates r_i are the solution to optimization problem (6) for different values of R.

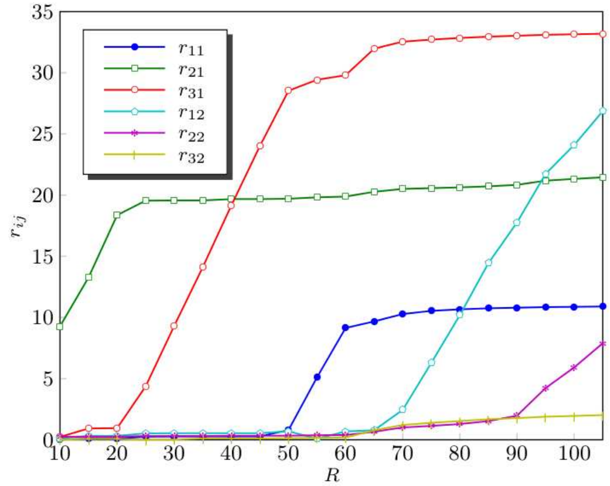


Figure 6: The applications rates r_{ij} are the solution to optimization problem (8) using optimal user rates r_i of optimization problem (6) or the solution of optimization problem (4) for different values of R.

simulations that our two-stage algorithm converges to the optimal rates.

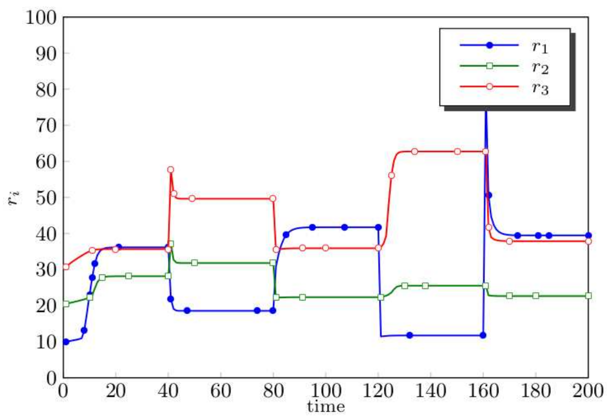


Figure 7: The user rates r_i convergence with the change in applications usage percentages $\boldsymbol{\alpha}(t)$.

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