One-layer trade cumulus boundary layer model

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This is a one-layer boundary layer model that solves the surface wind and associated convergence given the surface temperature distribution, based on the work by Lindzen and Nigam (1987) (hereafter LN87).

1. Linear wind-balanced equations

The steady-state surface wind is balanced by three forces: surface pressure gradient force, Coriolis force, and drag force. By specifying the temperature profile and assuming the existence of "back-pressure", which reflects the variation of boundary layer depth, the eddy sea-level pressure (perturbation from zonal mean) can be determined by the surface temperature distribution and "back-pressure" field (equation 9b in LN87):

$$p'_{SL} = g\rho_0 n H_0 \left(\frac{\gamma}{2} - 1\right) T'_S + g\rho_0 \left(2 - n\overline{T_S} + n\alpha H_0\right) h' \tag{1}$$

Where,

$g = 9.8\ m\ s^{-2}$, $\rho_0 = 1.225 kg\ m^{-3}$, $n = 1/T_0$, $T_0 = 288K$, $\gamma = 0.3$, $\alpha = 0.003 K\ m^{-1}$, $H_0 = 3km$,

$\overline{T_S}$ and T_S' are the zonal mean and perturbation of surface temperature, and h' is the eddy "back-pressure" field. In practice, the surface temperature is replaced by virtual temperature.

Substitute the above equation into the linearized steady-state wind balanced equation (equation 4 and 5 in LN87) and parameterize the drag force as a linear function of wind, we can obtain the linear balanced equations as below (having fixed the errors in equation 10 and 7c in LN87):

$$\varepsilon u' - f v' + \frac{A}{\cos\theta}\frac{\partial h'}{\partial \lambda} = \frac{B}{\cos\theta}\frac{\partial T_S'}{\partial \lambda} \tag{2}$$

$$f u' + \varepsilon v' + A\frac{\partial h'}{\partial \theta} - \frac{gn}{2a}\frac{\partial \overline{T_S}}{\partial \theta} h' = B\frac{\partial T_S'}{\partial \theta} \tag{3}$$

$$\frac{\partial u'}{\partial \lambda} + \frac{\partial\left(v'\cos\theta\right)}{\partial \theta} + \frac{a\cos\theta}{\tau_C H_0} h' = 0 \tag{4}$$

Where, $A = \frac{g}{a}\left(2 - n\overline{T_S} + n\alpha H_0\right)$, $B = \frac{gnH_0}{2a}\left(1 - \frac{2}{3}\gamma\right)$,

$\varepsilon = 2.5\ day^{-1}$, $a = 6371 km$, $\tau_C = 30 \min$, $f = 2\Omega\sin\theta$, $\Omega = 7.272\times10^{-5} s^{-1}$,

λ is the longitude, θ is the latitude, and u', v' are the eddy zonal and meridional wind.

Note: The drag coefficient ε and adjustment time scale τ_C are tunable parameters.

2. Semi-spectral approach

As suggested by LN87, the above equations (2), (3), and (4) can be solved by using Fourier transform along longitude and finite difference method along latitude. Recall that any horizontal field (globally) can be expanded by Fourier series:

$$f(\lambda,\theta) = \sum_{m=-\infty}^{\infty} F^m(\theta) e^{im\lambda} \tag{5}$$

And the differential relationship:

$$\frac{\partial f(\lambda,\theta)}{\partial \lambda} = \sum_{m=-\infty}^{\infty} imF^m(\theta) e^{im\lambda} \tag{6}$$

Where, m is the zonal wave number. In practice, it is truncated by [-M, M], where M is the maximum wavenumber.

Expand u, v, and h as Fourier series according to (5) and (6), then we can get the balance equations on each wavenumber m (because the Fourier series are orthogonal):

$$\varepsilon u^m - fv^m + \frac{imA}{\cos\theta} h^m = \left(\frac{B}{\cos\theta}\frac{\partial T_S'}{\partial \lambda}\right)^m \tag{7}$$

$$fu^m + \varepsilon v^m + A\frac{\partial h^m}{\partial \theta} - \frac{gn}{2a}\frac{\partial \overline{T_S}}{\partial \theta} h^m = \left(B\frac{\partial T_S'}{\partial \theta}\right)^m \tag{8}$$

$$imu^m + \frac{\partial\left(v^m \cos\theta\right)}{\partial \theta} + \frac{a\cos\theta}{\tau_C H_0} h^m = 0 \tag{9}$$

Where,

$$u'(\lambda,\theta) = \sum_{m=-\infty}^{\infty} u^m(\theta) e^{im\lambda},\ v'(\lambda,\theta) = \sum_{m=-\infty}^{\infty} v^m(\theta) e^{im\lambda},\ h'(\lambda,\theta) = \sum_{m=-\infty}^{\infty} h^m(\theta) e^{im\lambda}$$

Center-difference scheme is used in calculating the meridional gradient:

$$\varepsilon u_j^m - fv_j^m + \frac{imA_j}{\cos\theta} h_j^m = \left(\frac{B}{\cos\theta}\frac{\partial T_S'}{\partial \lambda}\right)_j^m \tag{10}$$

$$fu_j^m + \varepsilon v_j^m - \frac{A_j h_{j-1}^m}{\theta_{j+1}-\theta_{j-1}} - \frac{gn}{2a}\left(\frac{\partial \overline{T_S}}{\partial \theta}\right)_j h_j^m + \frac{A_j h_{j+1}^m}{\theta_{j+1}-\theta_{j-1}} = \left(B\frac{\partial T_S'}{\partial \theta}\right)_j^m \tag{11}$$

$$imu_j^m - \frac{\cos\left(\theta_{j-1}\right)}{\theta_{j+1}-\theta_{j-1}} v_{j-1}^m + \frac{\cos\left(\theta_{j+1}\right)}{\theta_{j+1}-\theta_{j-1}} v_{j+1}^m + \frac{a\cos\theta_j}{\tau_C H_0} h_j^m = 0 \tag{12}$$

Where, j is the latitude index, excluding the most south and north points.

The boundary conditions are given as:

$$u^m = v^m = h^m = 0 \ \ at\ \ \theta = \pm 90^o \tag{13}$$

Rearrange u^m, v^m, and h^m as one column vector, the above algebra equations (10~12) can be solved numerically, as illustrated by the Figure 1.

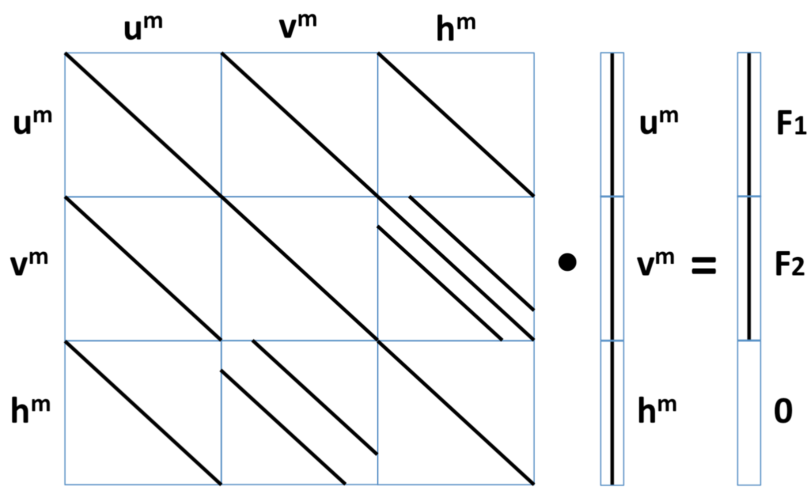

Figure 1. Schematics of the algebra equations (10~12). The block tridiagonal matrix on the left hand side represents the coefficients of u^m, v^m, and h^m; the column vector $[F_1, F_2, 0]^T$ represents the forcing terms on the right hand side of the equations (10~12); no-zero values are shown as black solid line, while zeros are shown as margin.

References:

Lindzen, R. S., and S. Nigam, 1987: On the role of sea surface temperature gradients in forcing low-level winds and convergence in the Tropics. *J. Atmos. Sci.*, **44**, 2418–2436.