Generic machine learning inference on heterogeneous treatment effects in randomized experiments, with an application to immunization in India

Victor Chernozhukov et al. (2024), Econometrica.

Naoki Eguchi

2025.7.9 ミクロ計量経済学

Faculty of Medicine, Kyoto University

Introduction

Motivation

- When conducting RCT (Randomized Controlled Trails), reseachers and policy makers are often curious about not only ATE but also HTE (Heterogeneous Treatment Effects).
- In such cases, ML methods are good for estimating

Proposed estimator

-

Main identification results and estimation strategies

BLP (Best Linear Predictor)

- Firstly, we obtain the estimator $S(Z)$ for CATE $s_0(Z)$ by some ML method using the auxilirary samples $\mathcal{A}$.
- BLP is defined as the projection of $s_0(Z)$ on the linear span of 1 and $S(Z)$ in $L^2(P)$.

$$\mathrm{BLP}[s_0(Z)|S(Z)] = \arg \min_{f(Z) \in \mathrm{Span}(1, S(Z))} \mathbb{E}[(s_0(Z) - f(Z))^2]$$

  - This equals to the solution of $\arg \min_{b_1, b_2} \mathbb{E}[(s_0(Z) - b_1 - b_2 S(Z))^2]$.
    $\rightarrow \beta_1 = \mathbb{E}[s_0(Z)], \beta_2 = \dfrac{\mathrm{Cov}(s_0(Z), S(Z))}{\mathrm{Var}(S(Z))}$

First strategy : Weighted Residual BLP

- Consider the regresson model with the moment condition as follows.

$$Y = \alpha' X_1 + \beta_1(D - p(Z)) + \beta_2(D - p(Z))(S(Z) - \mathbb{E}[S(Z)]) + \epsilon, \mathbb{E}[w(Z)\epsilon X] = 0$$

$$\text{where } w(Z) = \frac{1}{p(Z)(1 - p(Z))}, X = (X_1, X_2),$$

$$X_1 = (1, B(Z)), X_2 = (D - p(Z), (D - p(Z)(S(Z) - \mathbb{E}[S(Z)]))).$$

Theorem 1

- Consider $z \mapsto S(z)$ and $z \mapsto B(z)$ as fixed maps. (known function)
- Assume that Y and X have finite second moments, $\mathbb{E}[XX']$ is full rank, and $\text{Var}(S(Z)) > 0$.
- Then, $(\beta_1, \beta_2)' = \arg\min_{b_1, b_2} \mathbb{E}[(s_0(Z) - b_1 - b_2 S(Z))^2]$ (Identified)

Second strategy : Horvitz-Thompson BLP

Theorem 2

- Consider $z \mapsto S(z)$ and $z \mapsto B(z)$ as fixed maps. (known function)
- Assume that Y has finite second moments, $\mathbb{E}[\tilde{X}\tilde{X}']$ is finite and full rank, and $\text{Var}(S(Z)) > 0$.
- Then, $(\beta_1, \beta_2)' = \arg\min_{b_1, b_2} \mathbb{E}[(s_0(Z) - b_1 - b_2 S(Z))^2]$ (Identified)

GATES (sorted Group Average Treatment Effects)

- Firstly, we build the groups by the estimated value $S(Z)$ of $s_0(Z)$.

$$G_k = \{S(Z) \in I_k\}, k = 1, ..., K, I_k = [l_{k-1}, l_k) : \text{disjoint}$$

- The estimand "GATES" is defined as $\mathbb{E}[s_0(Z)|G_k]$ for $k = 1, ..., K$.

Two strategies : Weihgted Residual and Horvitz-Thompson GATES

- Consider the regression model with the moment condition as follows.

$$Y = \alpha' X_1 + \sum_{k=1}^{K} \gamma_k (D - p(Z)) \mathbf{1}_{G_k} + \nu, \mathbb{E}[w(Z)\nu W] = 0$$

$$\text{where} W = (X_1, W_2')', W_2 = \{(D - p(Z))\mathbf{1}_{G_k}\}_{k=1}^{K}.$$

Identification of GATES

Theorem 3

- Consider $z \mapsto S(z)$ and $z \mapsto B(z)$ as fixed maps. (known function)
- Assume that Y has finite second moments and both $\mathbb{E}[WW']$ and $\mathbb{E}[\tilde{W}\tilde{W}']$ are finite and full rank.
- Then, $\gamma = \{\gamma_k\}_{k=1}^K$ defined in two different ways are equivalent and identified.

$$\gamma_k = \mathbb{E}[s_0(Z)|G_k]$$

CLAN (Classification Analysis)

- When BLP and GATES reveal substantial heterogeneity, it is interesting to know the properties of the subpopulations that are most and least affected.
    - We focus on the "least affected group" G_1 and "most affected group" G_K.
- Let $g(Y, Z)$ be a vector of characteristics of an observational unit. The estimands are

$$\delta_1 = \mathbb{E}[g(Y, Z)|G_1] \quad \text{and} \quad \delta_K = \mathbb{E}[g(Y, Z)|G_K]$$

- These parameters are identified with no assumption because they are just averages of observed variables.
- We compare δ_1 and δ_K to detect (single out) the covariates which causes the heterogeneity.
    - We can extend the comparison of not only averages but also variances or distributions.

”Variational” estimation and inference methods

Uncertainty

- Let θ denote a generic target parameter such as BLP β_2 or GATE γ_k.
- There are two principal sources of sampling uncertainty.
  - Estimation uncertainty regarding the parameter θ, conditional on the data subpopulations
  - Uncertainty or ”variation” induced by the data splitting
- Actually, estimation uncertainty is a standard topic, so, as usual, we solve this problem by the Gaussian approximation to construct a confidence interval.
- On the other hands, data-splitting uncertainty is a novel topic, which is solved by taking a median of any estimators in permuated splitting.

Estimation uncertainty in single split

-

Applicatoion

Final marks

References

References

- Victor Chernozhukov, Mert Demirer, Esther Duflo, and Iván Fernández-Val (2024), *Generic machine learning inference on heterogeneous treatment effects in randomized experiments, with an application to immunization in India*, Econometrica.
- Kosuke Imai and Michael Lingzhi Li (2025), *Statistical Inference for Heterogeneous Treatment Effects Discovered by Generic Machine Learning in Randomized Experiments*, Journal of Business and Economic Statistics.
- 末石直也 (2024), データ駆動型回帰分析, 日本評論社.