Generic machine learning inference on heterogeneous treatment effects in randomized experiments, with an application to immunization in India

Victor Chernozhukov et al. (2024), Econometrica.

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2025.7.9 ミクロ計量経済学

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Introduction

Motivation

- When conducting RCT (Randomized Controlled Trails), reseachers and policy makers are often curious about not only ATE but also HTE (Heterogeneous Treatment Effects).
- In such cases, ML methods are good for estimating

Proposed estimator

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Main identification results and estimation strategies

BLP (Best Linear Predictor)

- Firstly, we obtain the estimator $S(Z)$ for CATE $s_0(Z)$ by some ML method using the auxilirary samples $\mathcal{A}$.
- BLP is defined as the projection of $s_0(Z)$ on the linear span of 1 and $S(Z)$ in $L^2(P)$.

$$\text{BLP}[s_0(Z)|S(Z)] = \arg \min_{f(Z) \in \text{Span}(1, S(Z))} \mathbb{E}[(s_0(Z) - f(Z))^2]$$

  - This equals to the solution of $\arg \min_{b_1, b_2} \mathbb{E}[(s_0(Z) - b_1 - b_2 S(Z))^2]$.

    $\rightarrow \beta_1 = \mathbb{E}[s_0(Z)], \beta_2 = \dfrac{\text{Cov}(s_0(Z), S(Z))}{\text{Var}(S(Z))}$

First strategy : Weighted Residual BLP

- Consider the regresson model with the moment condition as follows.

$$Y = \alpha' X_1 + \beta_1(D - p(Z)) + \beta_2(D - p(Z))(S(Z) - \mathbb{E}[S(Z)]) + \epsilon, \mathbb{E}[w(Z)\epsilon X] = 0$$

$$\text{where } w(Z) = \frac{1}{p(Z)(1 - p(Z))}, X = (X_1, X_2),$$

$$X_1 = (1, B(Z)), X_2 = (D - p(Z), (D - p(Z)(S(Z) - \mathbb{E}[S(Z)]))).$$

Theorem 1

- Consider $z \mapsto S(z)$ and $z \mapsto B(z)$ as fixed maps. (known function)
- Assume that Y and X have finite second moments, $\mathbb{E}[XX']$ is full rank, and $\text{Var}(S(Z)) > 0$.
- Then, $(\beta_1, \beta_2)' = \arg\min_{b_1, b_2} \mathbb{E}[(s_0(Z) - b_1 - b_2 S(Z))^2]$ (Identified)

Second strategy : Horvitz-Thompson BLP

Theorem 2

- Consider $z \mapsto S(z)$ and $z \mapsto B(z)$ as fixed maps. (known function)
- Assume that Y has finite second moments, $\mathbb{E}[\tilde{X}\tilde{X}']$ is finite and full rank, and $\text{Var}(S(Z)) > 0$.
- Then, $(\beta_1, \beta_2)' = \arg\min_{b_1, b_2} \mathbb{E}[(s_0(Z) - b_1 - b_2 S(Z))^2]$ (Identified)

GATES (sorted Group Average Treatment Effects)

- Firstly, we build the groups by the estimated value $S(Z)$ of $s_0(Z)$.

$$G_k = \{S(Z) \in I_k\}, k = 1, ..., K, I_k = [l_{k-1}, l_k) : \text{disjoint}$$

- The estimand "GATES" is defined as $\mathbb{E}[s_0(Z)|G_k]$ for $k = 1, ..., K$.

Two strategies : Weihgted Residual and Horvitz-Thompson GATES

- Consider the regression model with the moment condition as follows.

$$Y = \alpha' X_1 + \sum_{k=1}^{K} \gamma_k (D - p(Z)) \mathbf{1}_{G_k} + \nu, \mathbb{E}[w(Z)\nu W] = 0$$

$$\text{where} W = (X_1, W_2')', W_2 = \{(D - p(Z))\mathbf{1}_{G_k}\}_{k=1}^{K}.$$

Identification of GATES

Theorem 3

- Consider $z \mapsto S(z)$ and $z \mapsto B(z)$ as fixed maps. (known function)
- Assume that Y has finite second moments and both $\mathbb{E}[WW']$ and $\mathbb{E}[\tilde{W}\tilde{W}']$ are finite and full rank.
- Then, $\gamma = \{\gamma_k\}_{k=1}^K$ defined in two different ways are equivalent and identified.

$$\gamma_k = \mathbb{E}[s_0(Z)|G_k]$$

CLAN (Classification Analysis)

- When BLP and GATES reveal substantial heterogeneity, it is interesting to know the properties of the subpopulations that are most and least affected.
  - We focus on the ”least affected group” G_1 and ”most affected group” G_K.
- Let $g(Y, Z)$ be a vector of characteristics of an observational unit. The estimands are

$$\delta_1 = \mathbb{E}[g(Y,Z)|G_1] \quad \text{and} \quad \delta_K = \mathbb{E}[g(Y,Z)|G_K]$$

- These parameters are identified with no assumption because they are just averages of observed variables.
- We compare δ_1 and δ_K to detect (single out) the covariates which causes the heterogeneity.
  - We can extend the comparison of not only averages but also variances or distributions.

”Variational” estimation and inference methods

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Applicatoion

Final marks

References

References

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