

## LEARNING GROUP ON CONDENSED MATHEMATICS BEIJING, FALL 2024

The goal of this learning group is to learn about basics of condensed mathematics. The scope will include but not be restricted to the theory of solid abelian groups.

### ORGANIZERS

Heng Du, Shizhang Li, Yihang Zhu, Foling Zou

### TIME AND PLACE

Mondays 4:20 pm, MCM 110.

### REFERENCES

The main references:

- [Cam24] Camargo, *Notes on Solid Geometry*. available here: <https://blogs.cuit.columbia.edu/jr4460/seminar-on-solid-geometry/>
- Clausen and Scholze, *24 lectures on Analytic Stacks, at IHES and Bonn, Fall 2023*. Videos available online:  
[https://www.youtube.com/watch?v=YxSZ1mTIpaA&list=PLx5f8Ie1FRgGmu6gmL-Kf\\_Rl\\_6Mm7juZ0&index=1](https://www.youtube.com/watch?v=YxSZ1mTIpaA&list=PLx5f8Ie1FRgGmu6gmL-Kf_Rl_6Mm7juZ0&index=1)  
or  
[https://www.bilibili.com/video/BV1MDvCeHEWd/?spm\\_id\\_from=333.337.search-card.all.click&vd\\_source=a5be8a75eb29fe979a159e395ace04ae](https://www.bilibili.com/video/BV1MDvCeHEWd/?spm_id_from=333.337.search-card.all.click&vd_source=a5be8a75eb29fe979a159e395ace04ae)

Complementary reference:

- [CS19] Clausen and Scholze, *Lectures on Condensed Mathematics, 2019*. available <https://people.mpim-bonn.mpg.de/scholze/Condensed.pdf>

**\* Some main definitions in [CS19] have been significantly updated in the newer version of the theory, as presented in the two main references.**

### TALK SCHEDULE

All references are to [Cam24].

**Talk 1, 10/14.** 2.1, 2.2. You can also include some background discussion following Lecture 1 of [CS19].

**Speaker:** Foling Zou (AMSS)

**Title:** Light profinite sets and condensed sets

**Talk 2, 10/21.** 2.3

**Speaker:** Yihang Zhu (Tsinghua)

**Title:** Light condensed abelian groups

**Talk 3, 10/28.** 3.1, 3.2.1, 3.2.2, Proof of 3.2.4 until the end of part (1). If you still have time, state the full 3.2.3.

**Speaker:** Heng Du (Tsinghua)

**Title:** Solid abelian groups

**Talk 4, 11/4.** The rest of the proof of 3.2.4. 3.2.5.

**Speaker:** Kehao Cheng (Peking University)

**Title:** Basic properties of solid abelian groups

**Abstract:** In the previous lecture, we have seen the definition of solid abelian groups. In this lecture, we will firstly focus on the formal properties of the category of solid abelian groups i.e. Thm 3.2.3 (1)-(8) except(4). Then we will give explicit computations of (derived) solidification of real number  $\mathbb{R}$ ,  $P$  and  $\mathbb{Z}[S]$  when  $S$  is a light profinite set and the (derived) solid tensor of countable product of  $\mathbb{Z}$  with itself, which will give the remaining part of Thm 3.2.3 except (14). Finally, we will present a computation of derived solidification of  $\mathbb{Z}[X]$  when  $X$  is a CW complex.

**11/11.** \* No talk.

**Talk 5, 11/18.** 3.3.

**Speaker:** Chenglong Ma (Peking University)

**Title:** Computing measures in solid abelian groups

**Abstract:** This talk is a continuation of last time where we finish the proof of Theorem 3.2.3. Ingredients include computation of the (derived) solidification of  $\mathbb{Z}[S]$  (where  $S$  is a light profinite set),  $P$ , and the (derived) solid tensor product of countable product of  $\mathbb{Z}$  with itself. As an application, we also prove that solidification computes singular cohomology of CW complexes.

**Talk 6, 11/25.** 3.4.

**Speaker:** Yu Xiao (AMSS)

**Title:** Structure of the category Solid and some examples for solid tensor products.

**Abstract:** In previous lectures we've seen the category Solid is a symmetric monoidal abelian category. In this talk, we will describe the objects in Solid as inductive limits of finite presented objects and find a flat object  $\prod_{\mathbb{N}} \mathbb{Z}$  for the solid tensor product. Finally, we will list some examples of solid abelian group such as  $\mathbb{Q}_p$ -Banach spaces and Fréchet spaces and their solid tensor products to see how the solid theory plays a role in non-archimedean functional analysis.

**Talk 7, 12/1.** .

**Speaker:** Fanyi Li (Tsinghua)

**Title:** Derived completeness in condensed settings.

**Abstract:** I will talk about derived completeness in condensed settings as an extension of an example (3.5.3) at the end of last time. In the first half I will talk about the derived completeness in module category. In the second half I will extend these to the condensed settings and prove that the solid tensor product of two connective derived complete solid complexes is also derived complete.

**Talk 8, 12/9.** .

**Speaker:** Kehao Cheng (Peking University)

**Title:** Animation

**Abstract:** 1. The notion of compactly projective generated category. 2. Basic infinity category theory. 3. Animation 4. Condensed anima and comparison of two ways to view a CW complex as a condensed anima.