Hunting for Primordial Black Holes with Stochastic Gravitational-Wave Background

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Oral Defense for the Degree of PhD

OUTLINE

OVERVIEW: BASIC ASPECTS OF GRAVITATIONAL–WAVE ASTRONOMY

- Einstein Field Equation, Gravitational Wave, Sources, Data Analysis

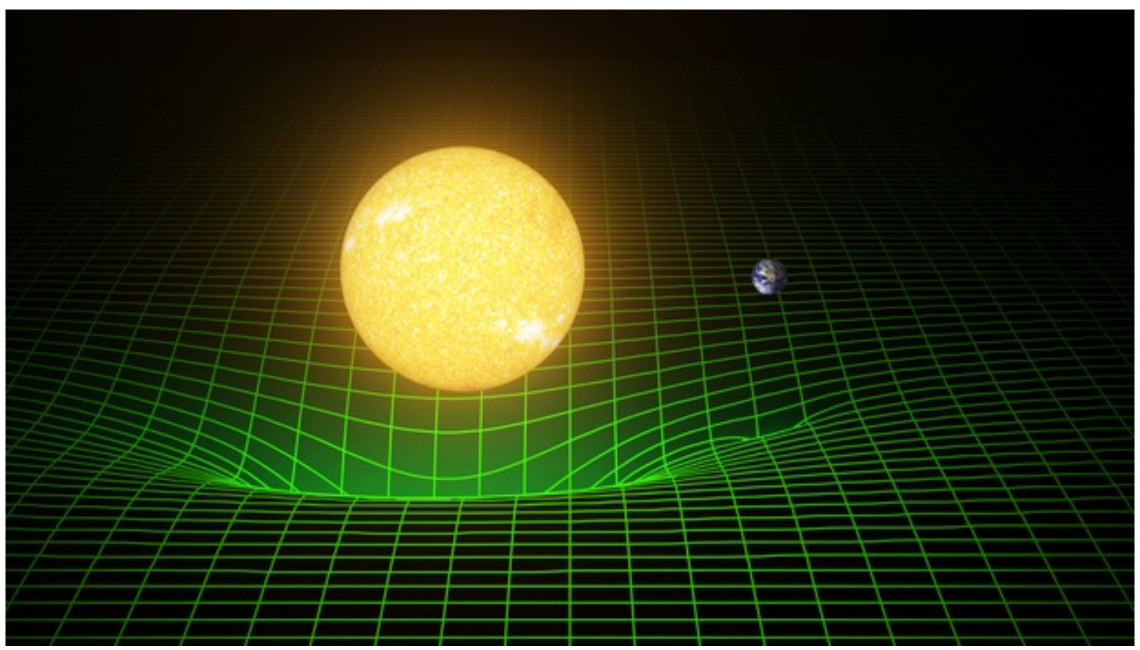
HUNTING FOR PRIMORDIAL BLACK HOLES WITH STOCHASTIC GRAVITATIONAL–WAVE BACKGROUND

- #1: Constraining Primordial Black Holes Using Stochastic Gravitational-Wave Background with Ground-Based Detectors
- #2: Constraining Primordial Black Holes Using Stochastic Gravitational-Wave Background with Space-Based Detectors

- Einstein Field Equation: $$\boxed{R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R} = \frac{8\pi G}{c^4}\boxed{T_{\mu\nu}}$$

Geometry of Spacetime

Energy, Matter



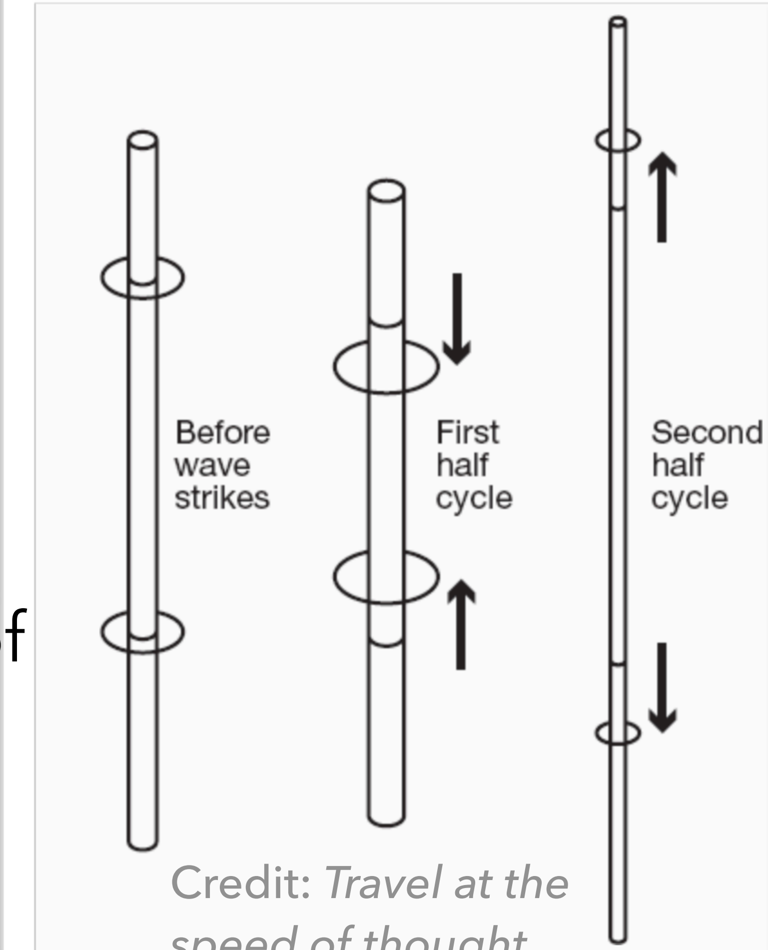
"Space tells matter how to move, matter tells space how to curve. "

- Linearization: $$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4}T_{\mu\nu}$$

Wave Equation!

General Relativity and Gravitational Wave

- 1915: General Relativity
- 1936: Einstein: GW doesn't exist
- 1957: Bondi thought experiment
- 1968: Weber's (failed) detection of gravitational waves
- 1978: Hulse-Taylor Binary Pulsar Orbital Decay (indirect evidence for gravitational waves)
- 2015: LIGO directly detected gravitational waves



Selected for a Viewpoint in *Physics*
PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

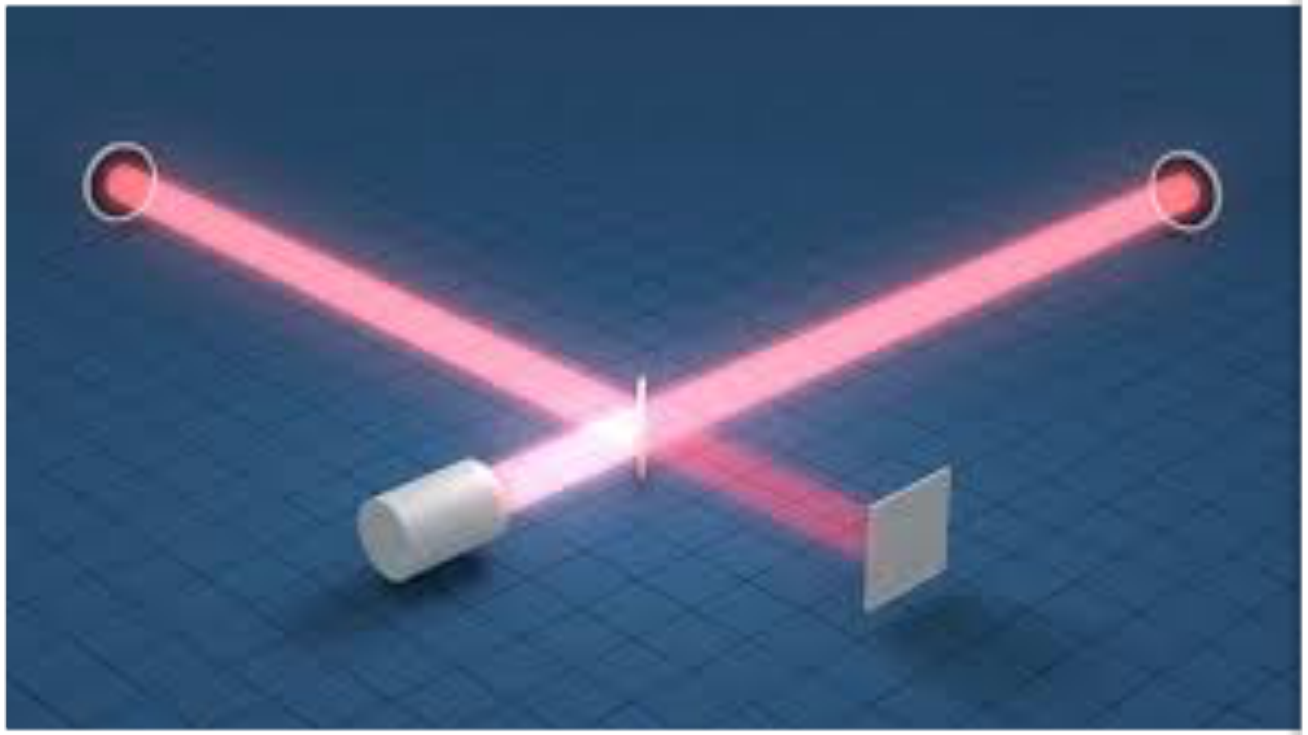
B. P. Abbott *et al.**
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_\odot$ and $29^{+4}_{-4} M_\odot$, and the final black hole mass is $62^{+4}_{-4} M_\odot$, with $3.0^{+0.5}_{-0.5} M_\odot c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

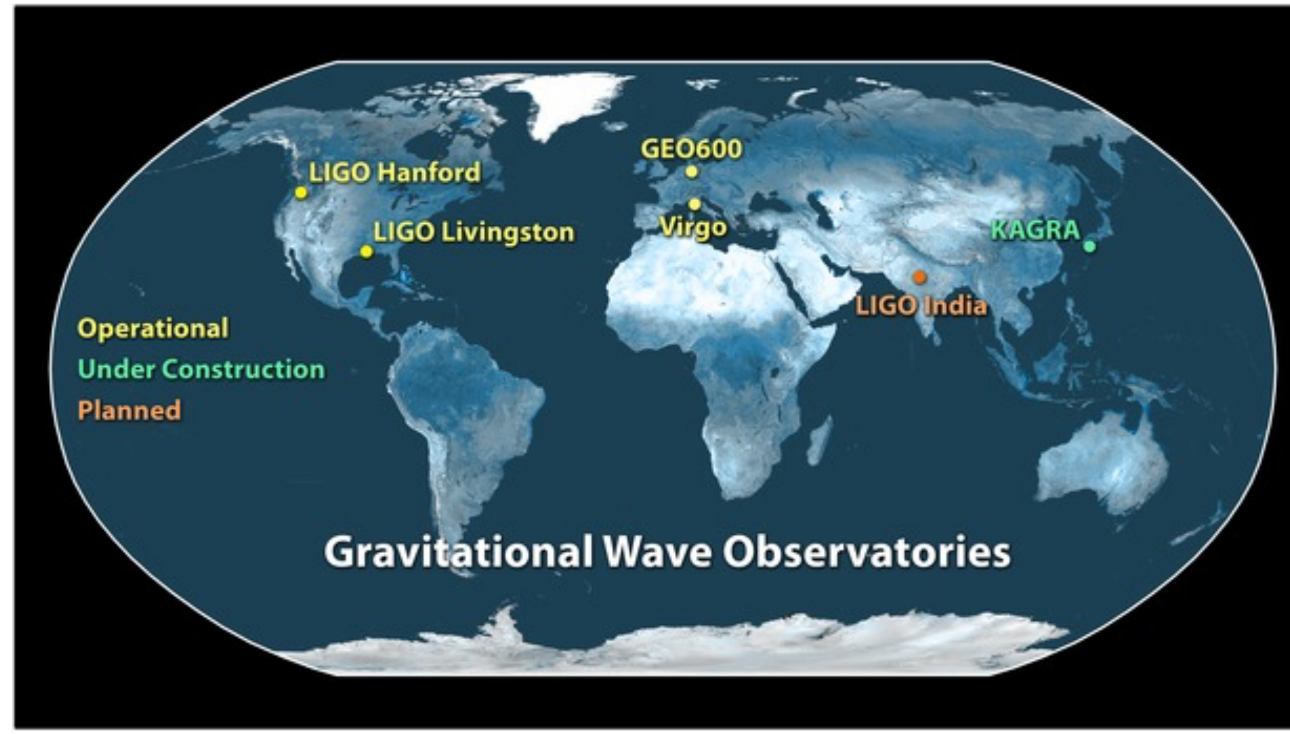
DOI: 10.1103/PhysRevLett.116.061102

Gravitational-Wave Detectors and Strain

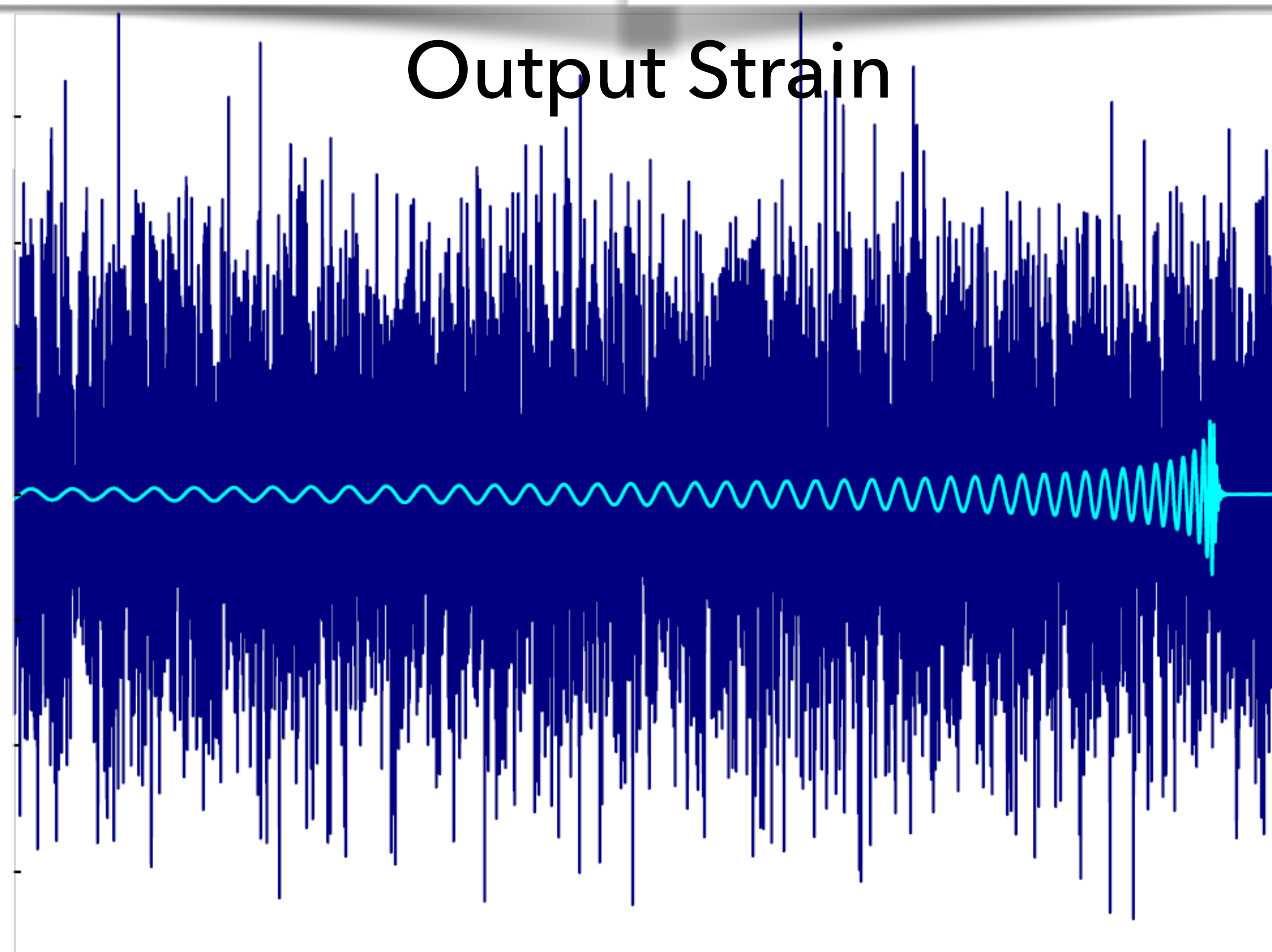
Laser Interferometer



Global Network

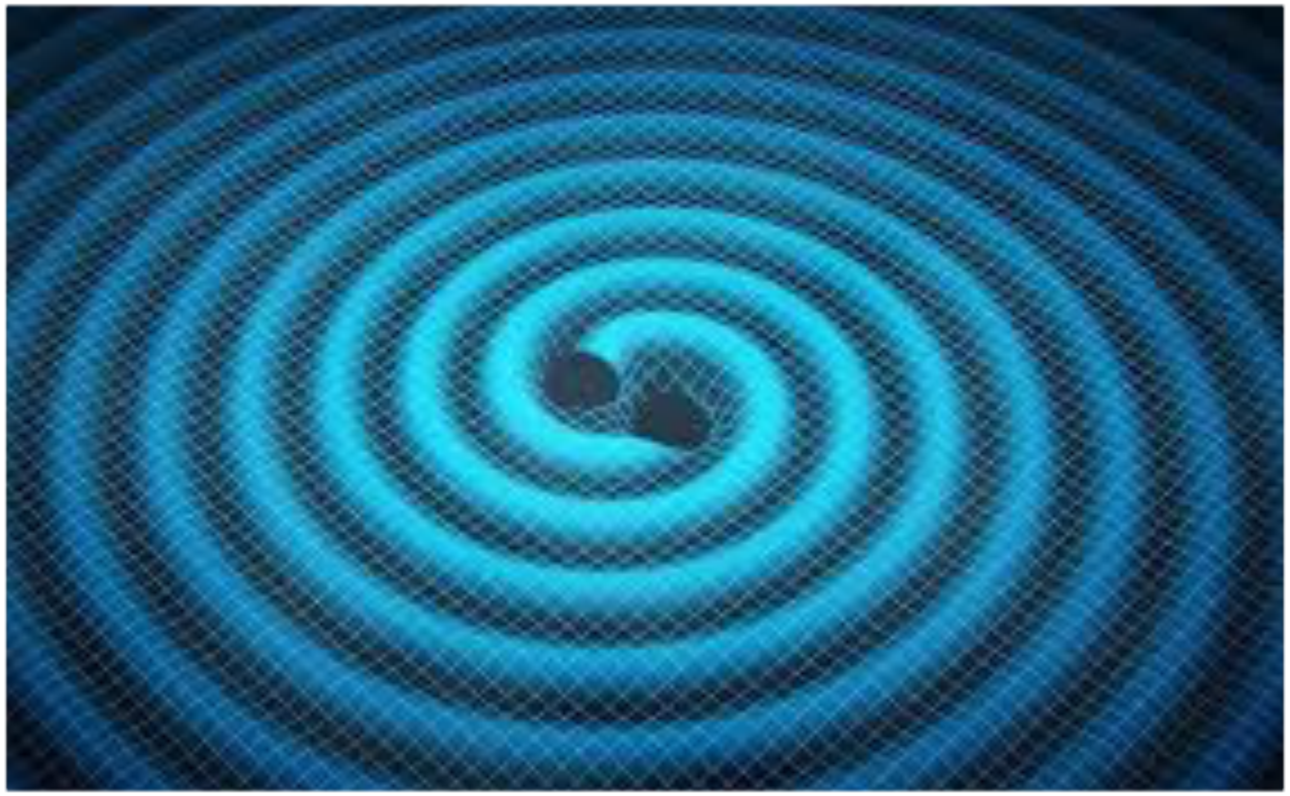


Output Strain

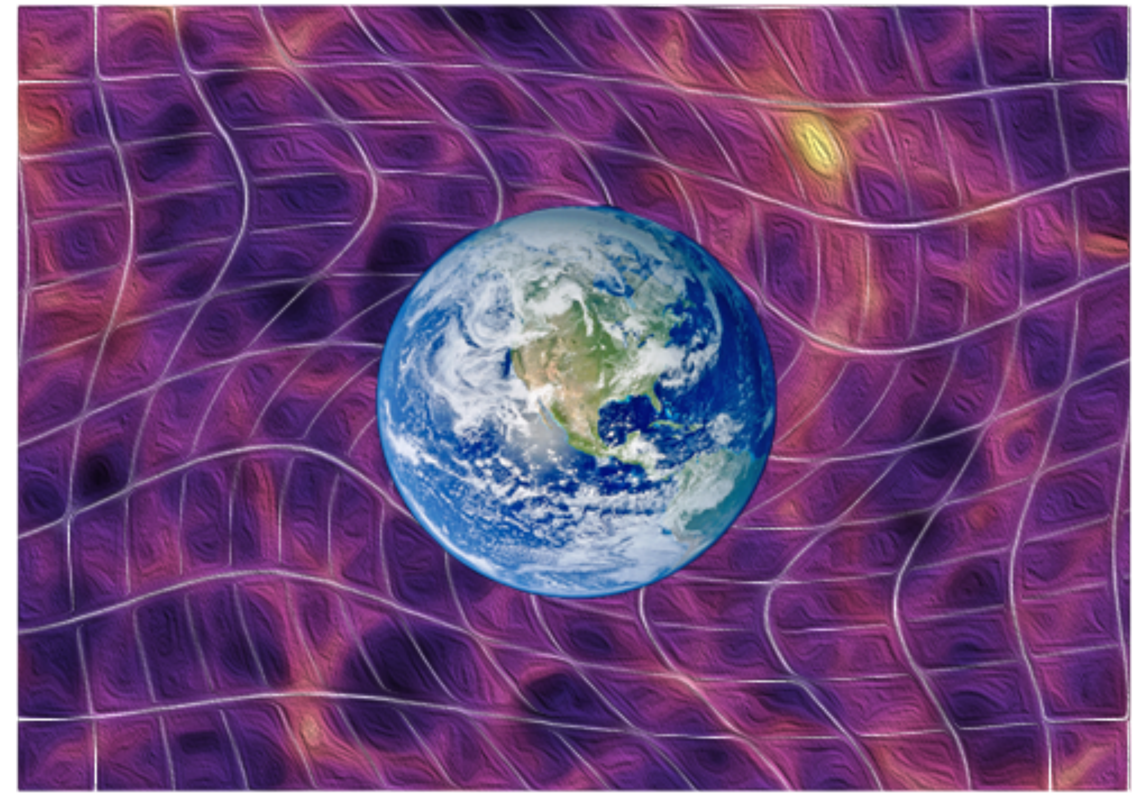


$$h(t) = \frac{\delta L}{L}$$

GRAVITATIONAL WAVE SOURCES



Compact Binary Coalescence
(Black Holes, Neutron Stars)

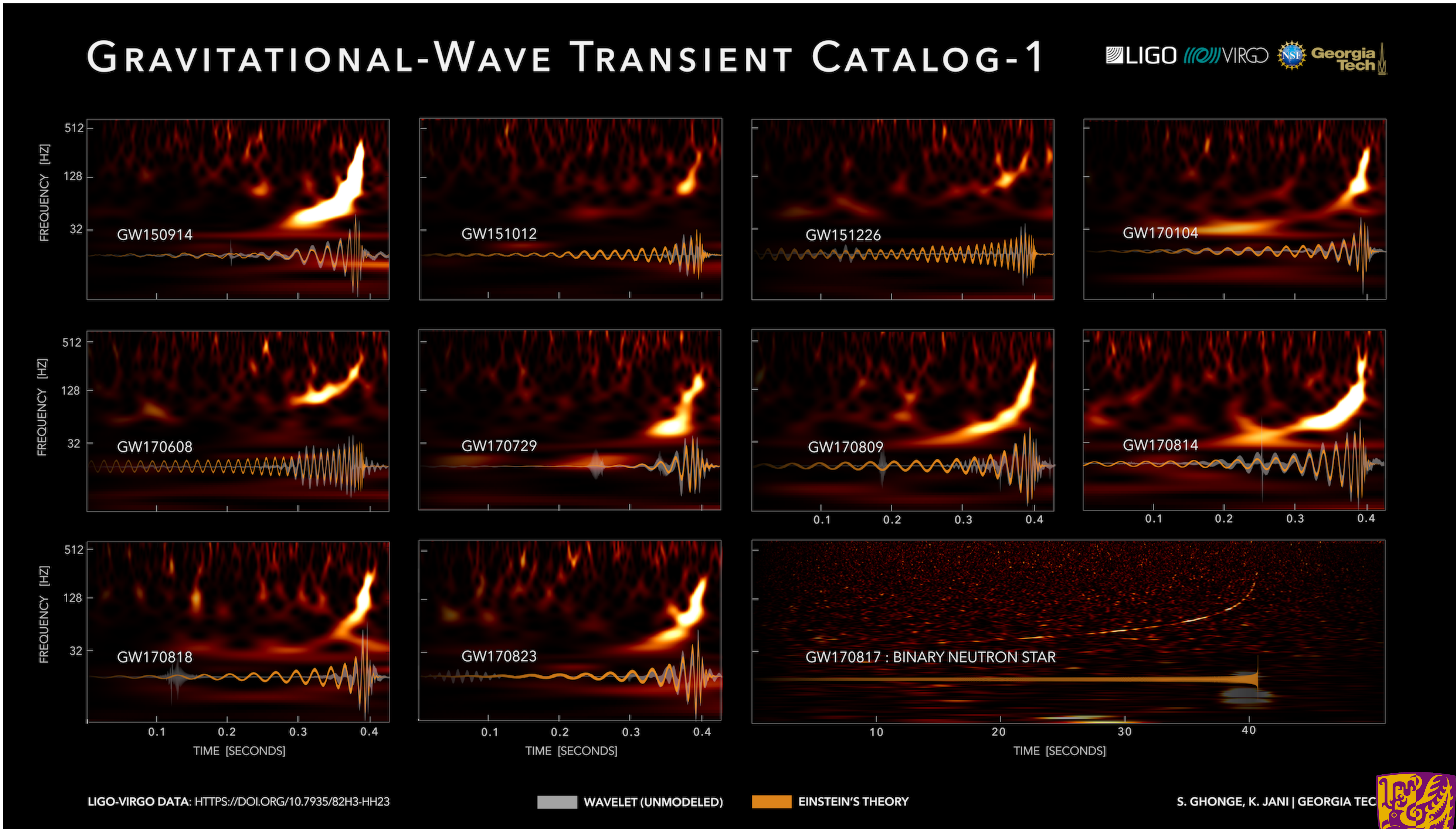


Stochastic Gravitational-Wave
Background

Other sources: Continuous Waves, Burst ...

GWTC (GW TRANSIENT CATALOG)-1 RESULTS:

- 10 BBH + 1 BNS: approximately one events/15 days



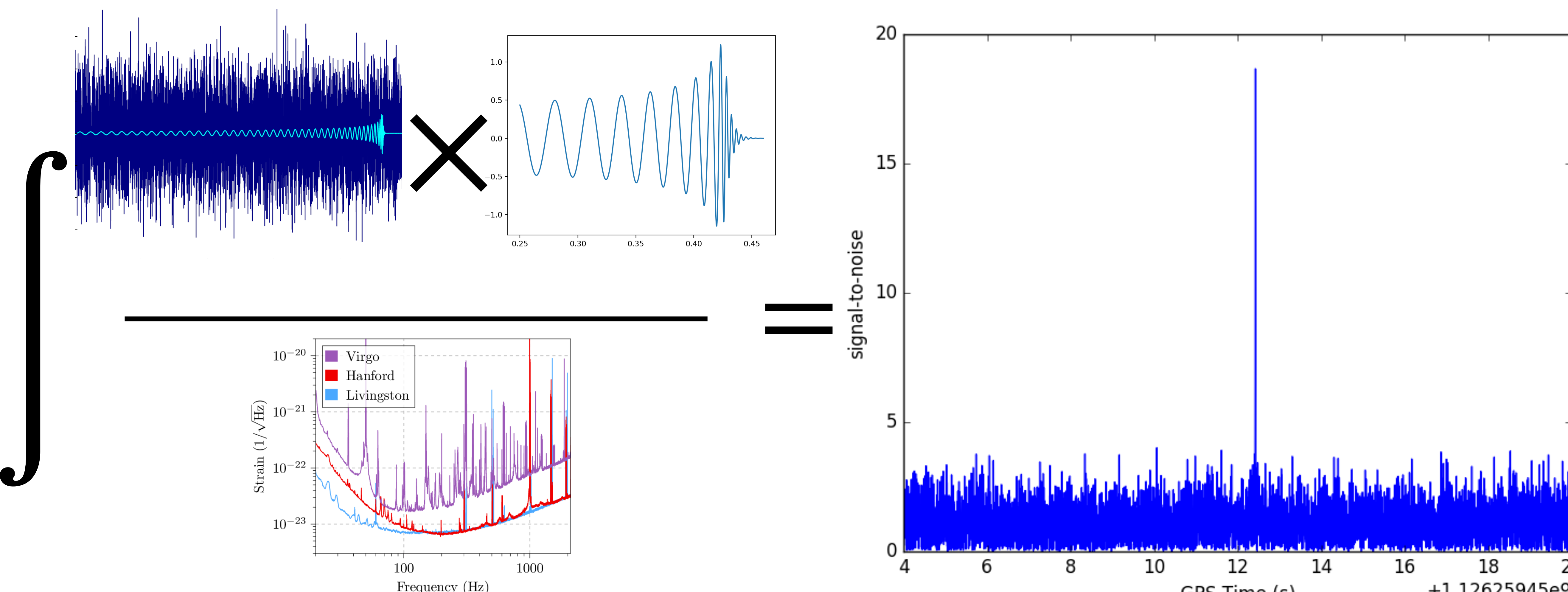
- How to perform parameter estimation? By match filtering

Signal

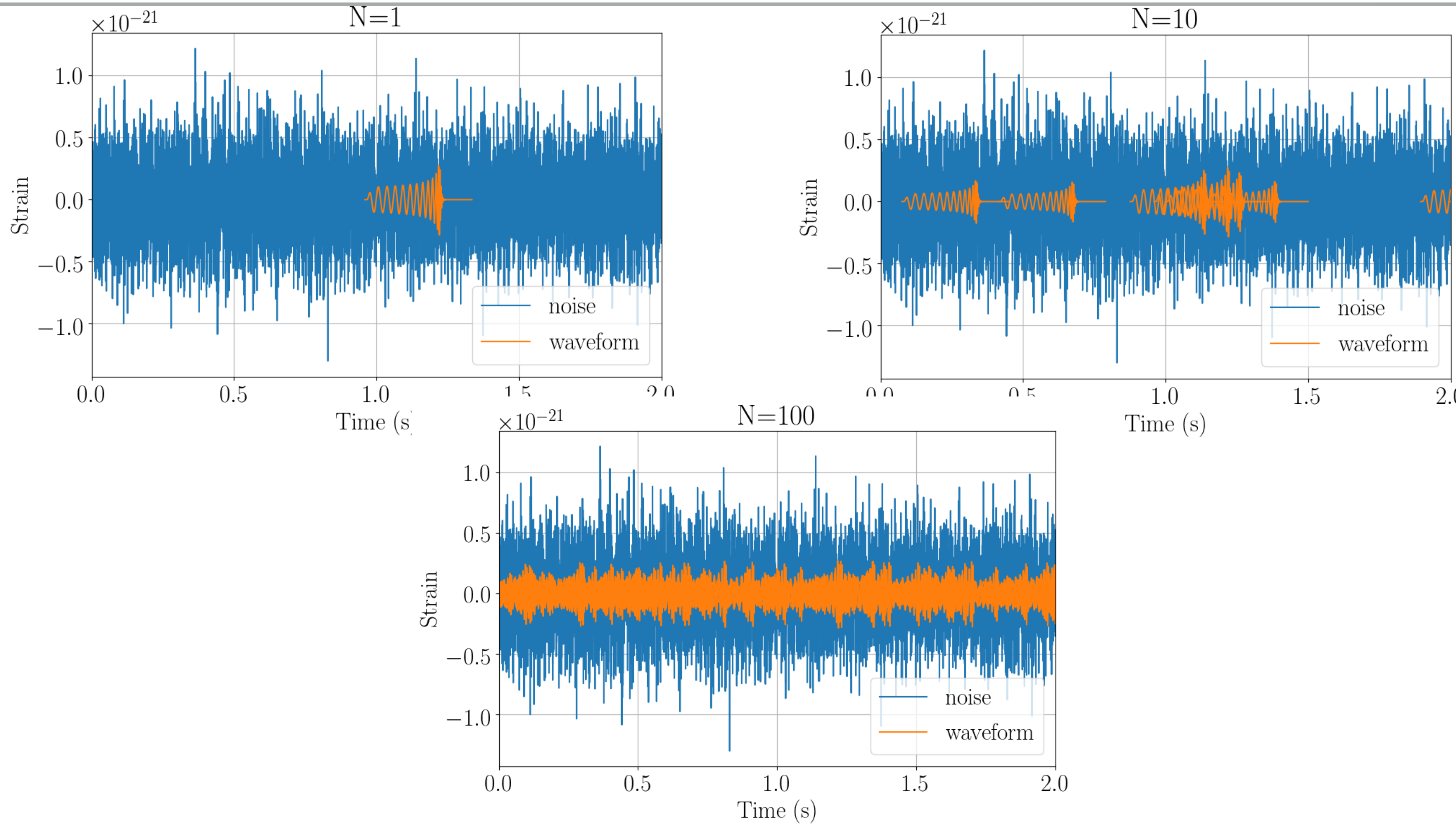
Template

$$\frac{S}{N}(t) = 4\Re \int_0^\infty \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_n(f)} \exp^{2\pi i f t}\, df$$

Noise power spectrum



STOCHASTIC GRAVITATIONAL-WAVE BACKGROUND(SGWB)



- Incoherent superposition of a large number of independent, unresolved gravitational waves

e.g. Primordial GWs,

GWs from many binary black hole coalescences

STOCHASTIC GRAVITATIONAL-WAVE BACKGROUND(SGWB)

- Mathematical characteristics: Energy density spectrum

$$\Omega_{\rm gw}(\nu) = \frac{1}{\rho_c}\frac{d\rho_{\rm gw}}{d\ln\nu}\,.$$

- Detection method: Cross correlation with multiple detectors

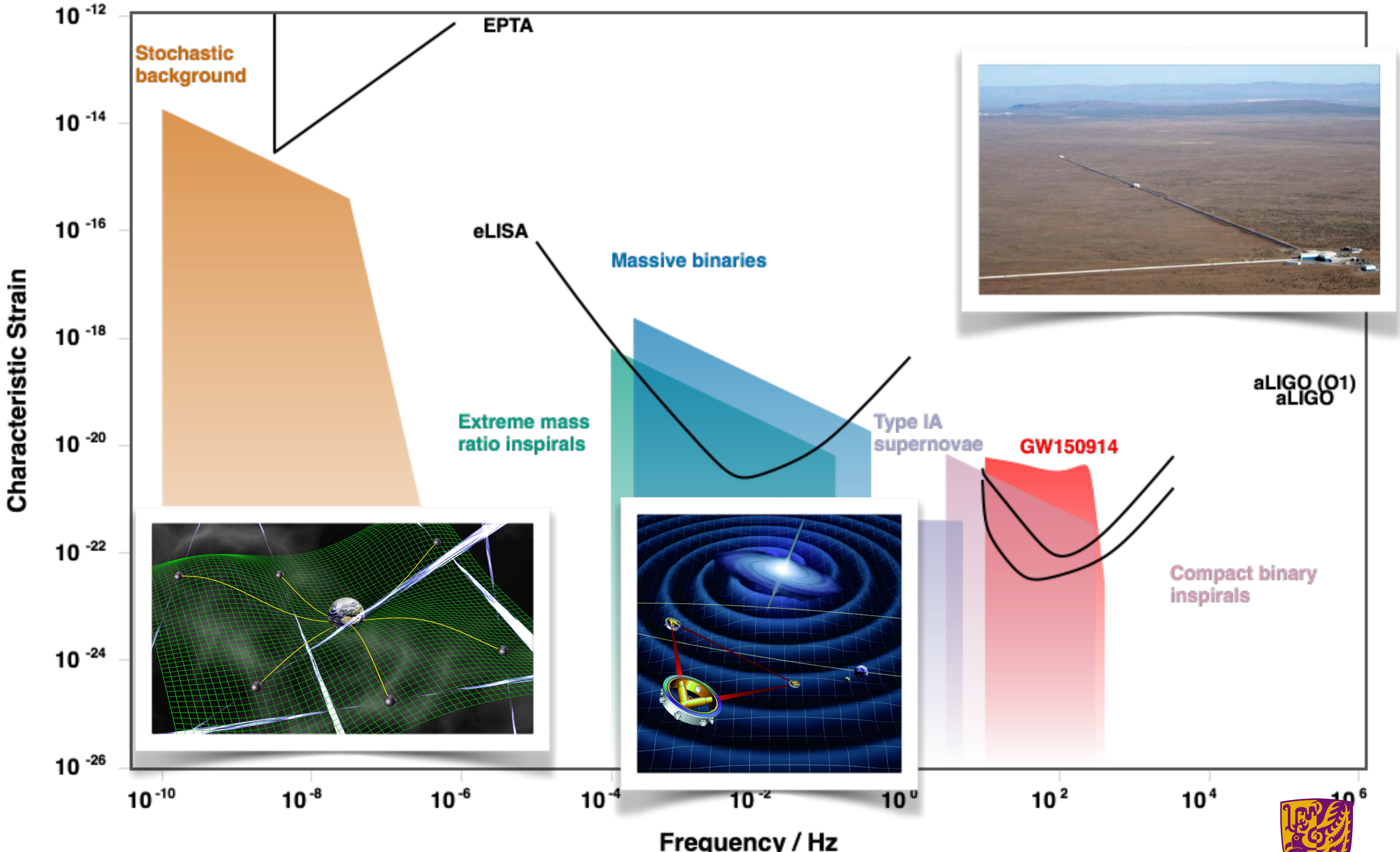
Signal

$$\frac{S}{N} = \frac{\int_{-\infty}^{\infty}\left\langle \tilde{s}_1{}^*(f)\tilde{s}_2(f)\right\rangle \tilde{Q}(f)df}{\sqrt{\frac{T}{4}\int_{-\infty}^{\infty} df P_1(|f|)P_2(|f|)|\tilde{Q}(f)|^2}}.$$

Filter function

Noise power spectrum

Stochastic Gravitational-Wave Background(SGWB)



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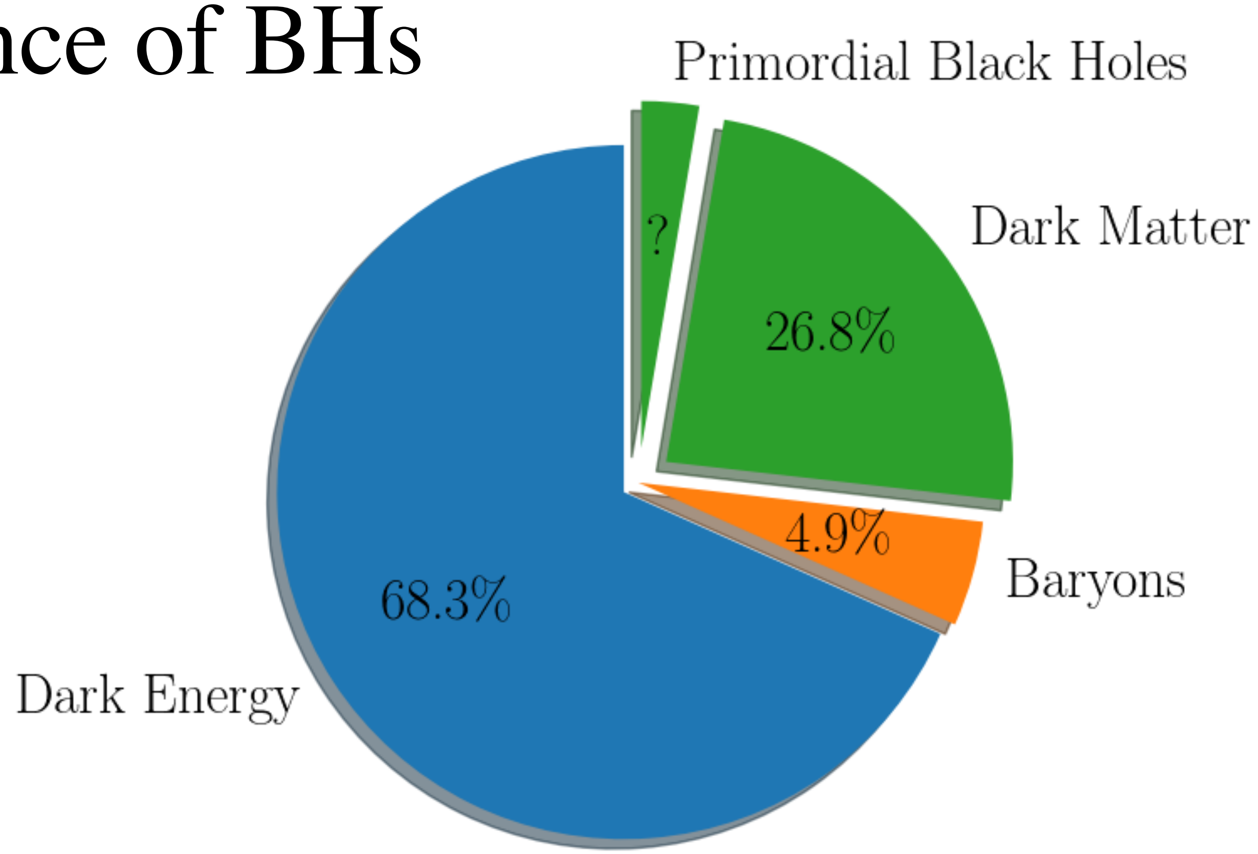
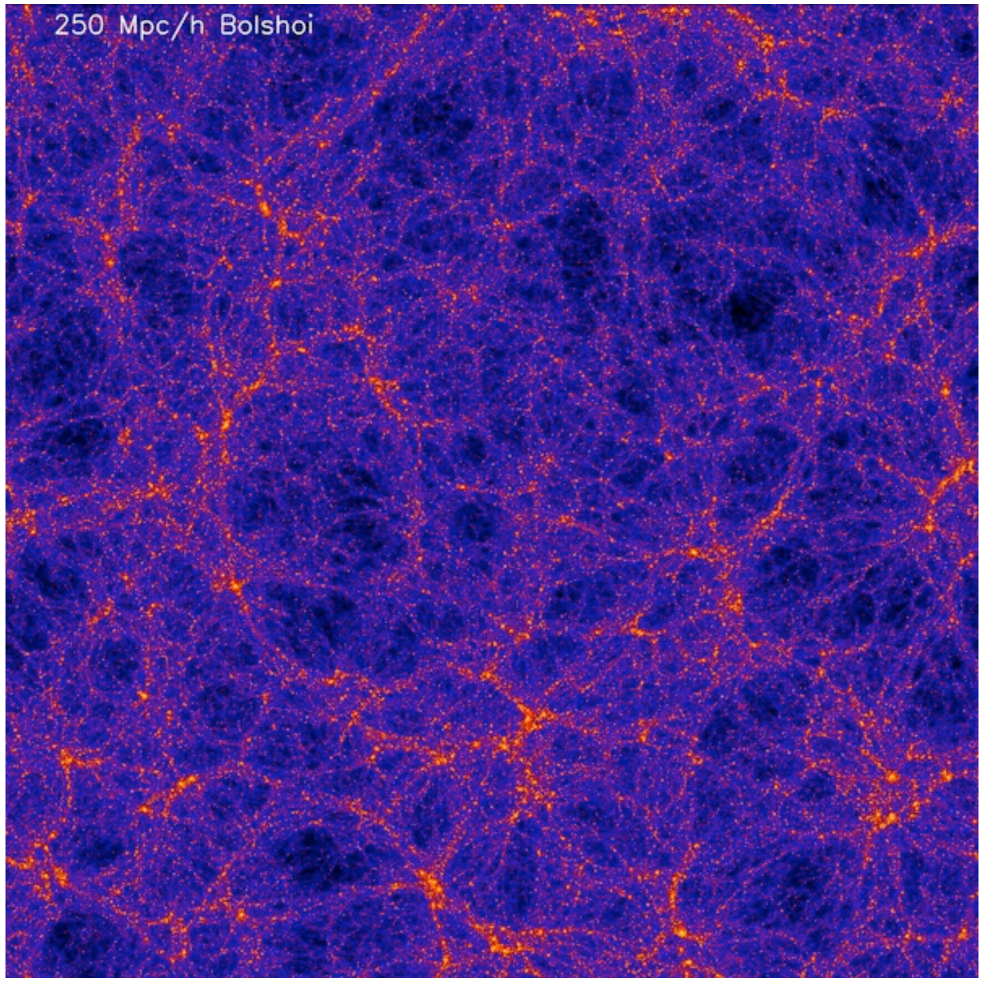
- Physical Review Letters 120, 191102, Sai Wang, Yifan Wang, Qing-Guo Huang, Tjonnie G.F. Li

Primordial Black Hole as Dark Matter

- A hypothetical BH produced by direct gravitational collapse of a primordial overdensity in the early Universe
- Candidate for cold dark matter
- GWs provide a new observational window to probe the existence of BHs



Primordial Black Hole as Dark Matter



(Credit: Bolshoi Simulation)

Search for Weakly Interacting Massive Particles (WIMPs), $\boxed{\text{Null Results!}}$

- Particle and Astrophysical Xenon Detector (PandaX-II)
- Large Underground Xenon dark matter experiment (LUX)
- Large Hadron Collider (LHC)
- Alpha Magnetic Spectrometer (AMS-02)
- Fermi Large Area Telescope (Fermi LAT).

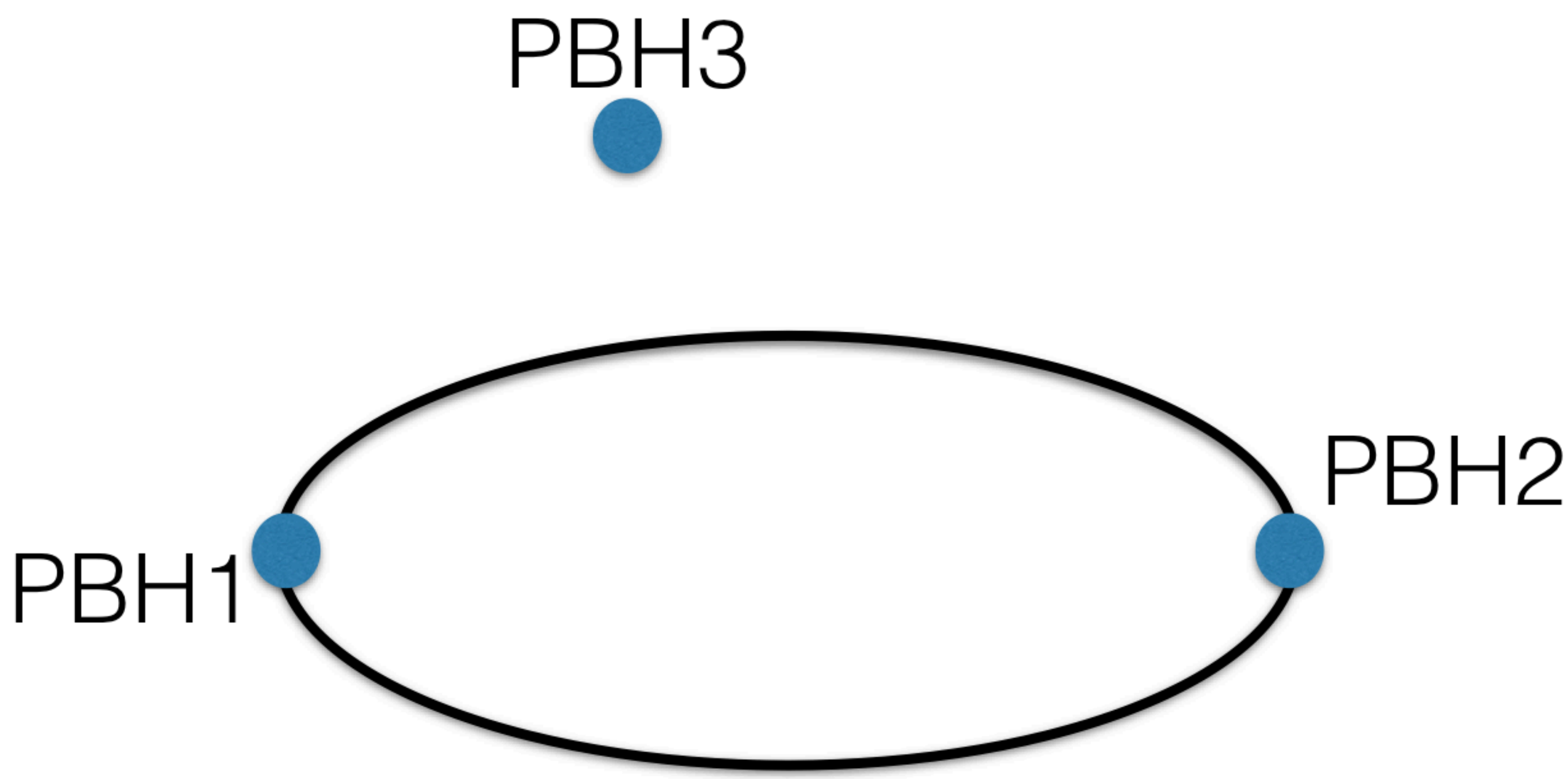
$\boxed{\text{Primordial Black Hole is a Candidate within the Standard Model.}}$

Primordial Black Hole as Dark Matter

1 Solar Mass Primordial Black Hole
radius ~ 3 km

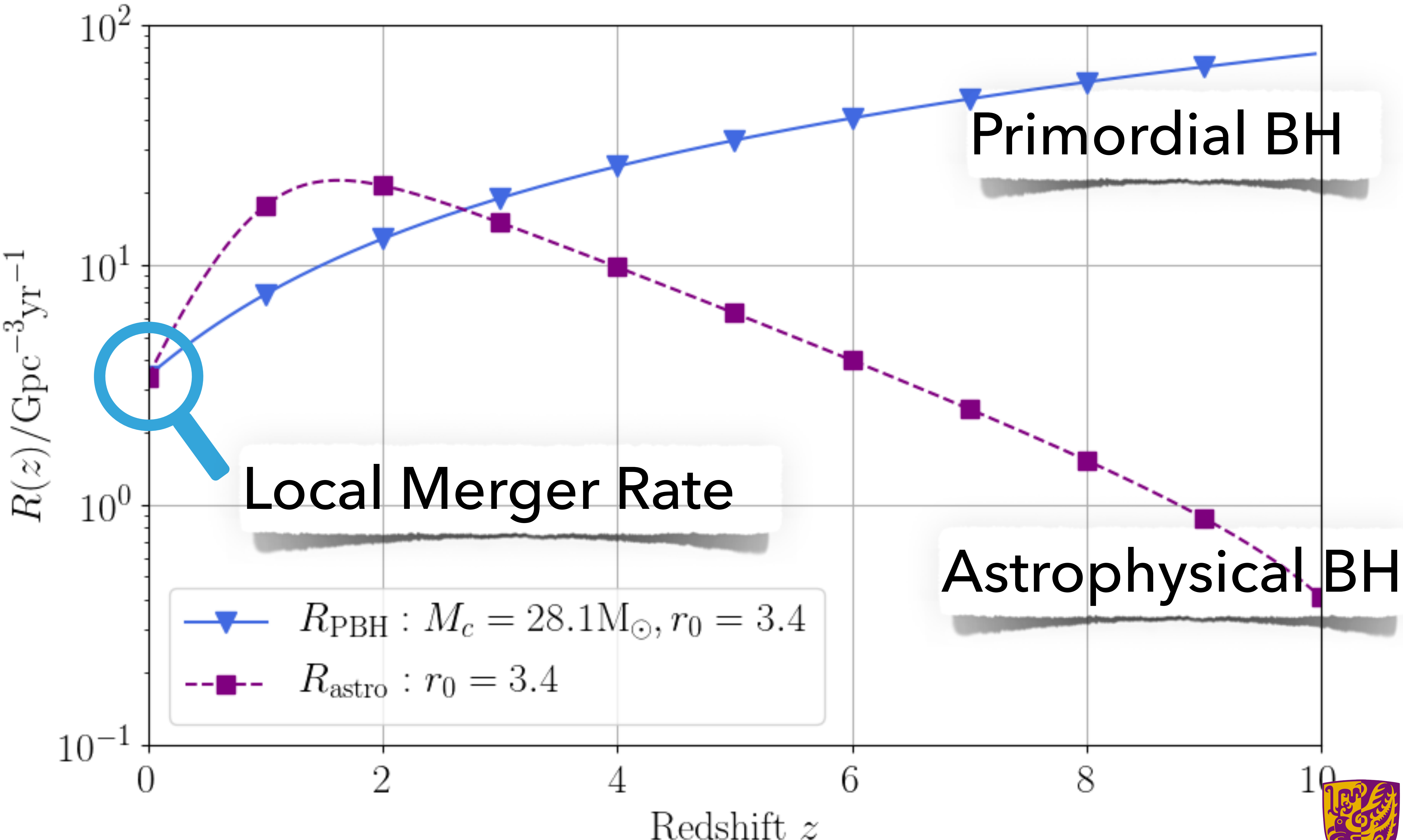
Event Rate

M. Sasaki, et al, Phys. Rev. Lett. 117, 061101



- Primordial Black Hole can form Binaries
- The orbital parameters are determined by the initial separation and the tidal force from a third body

Event Rate



Primordial BH
Local Merger Rate
Astrophysical BH
$R_{\rm PBH}$: $M_c = 28.1 M_\odot, r_0 = 3.4$
$R_{\rm astro}$: $r_0 = 3.4$
10^2
10^1
10^0
10^{-1}
$R(z)/{\rm Gpc}^{-3}{\rm yr}^{-1}$
0
2
4
6
8
10
Redshift z

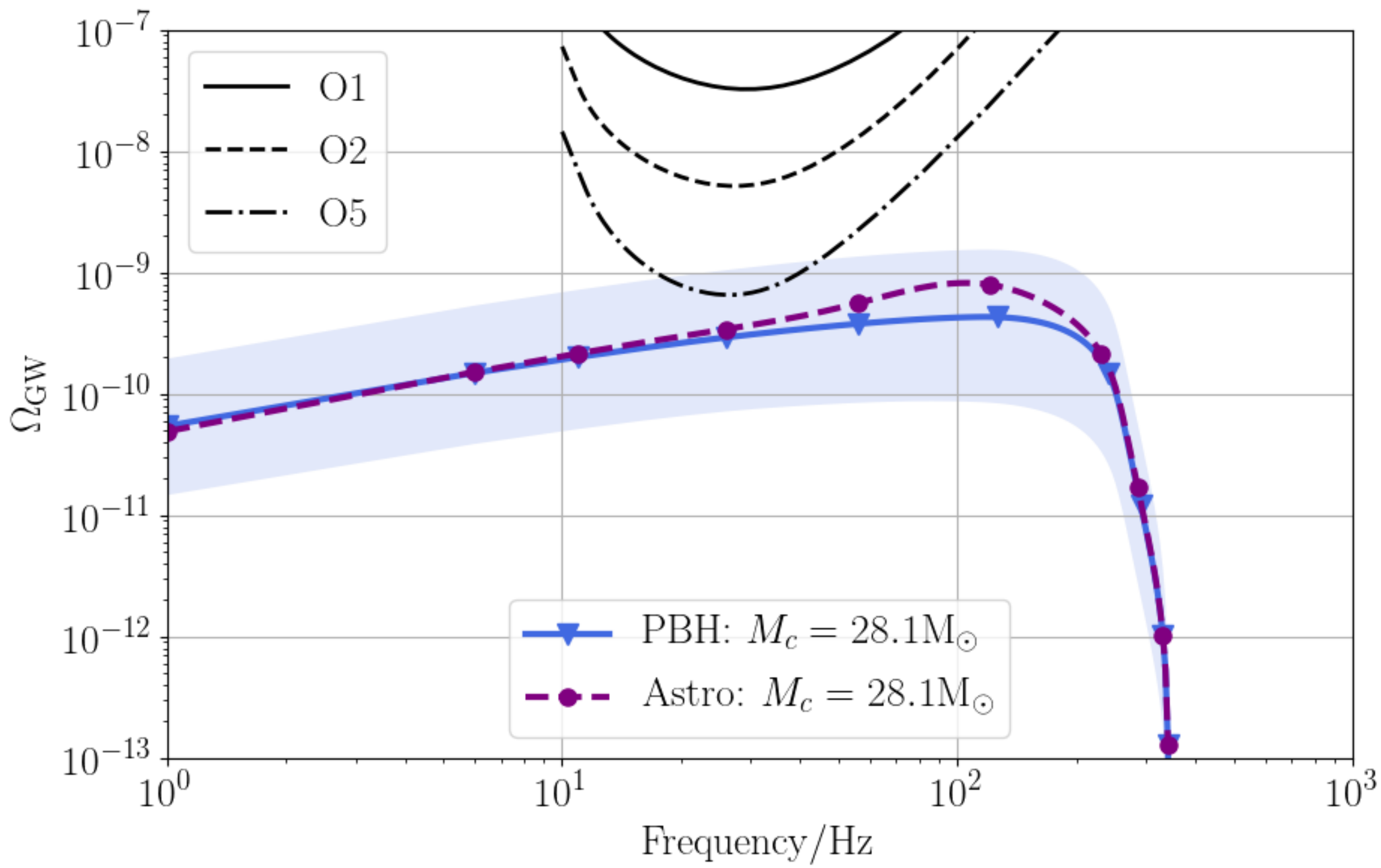
Derive SGWB Spectrum

$$\Omega_{\rm GW} = \frac{\nu}{\rho_{\rm c}} \frac{d\rho_{\rm GW}}{d\nu}$$

$$= \frac{\nu}{\rho_{\rm c} H_0} \int_0^{z_{\rm sup}} \boxed{\frac{R_{\rm PBH}(z; M_{\rm PBH}, f)}{(1+z)E(z)}} \times \boxed{\frac{dE_{\rm GW}}{d\nu_{\rm s}}(\nu_{\rm s})} dz$$

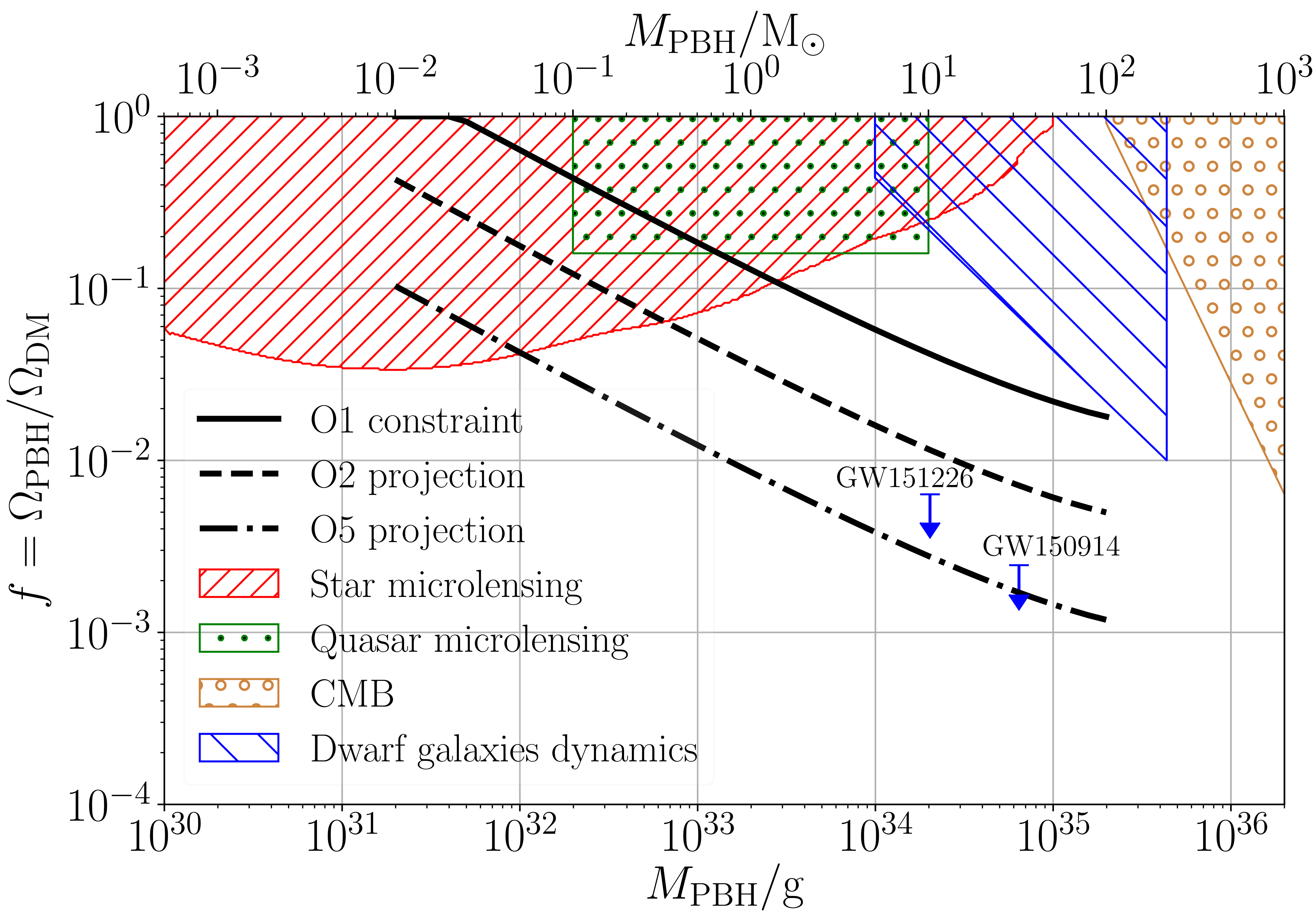
Event Rate GW energy per source

- Results:



CONSTRAINING ABUNDANCE

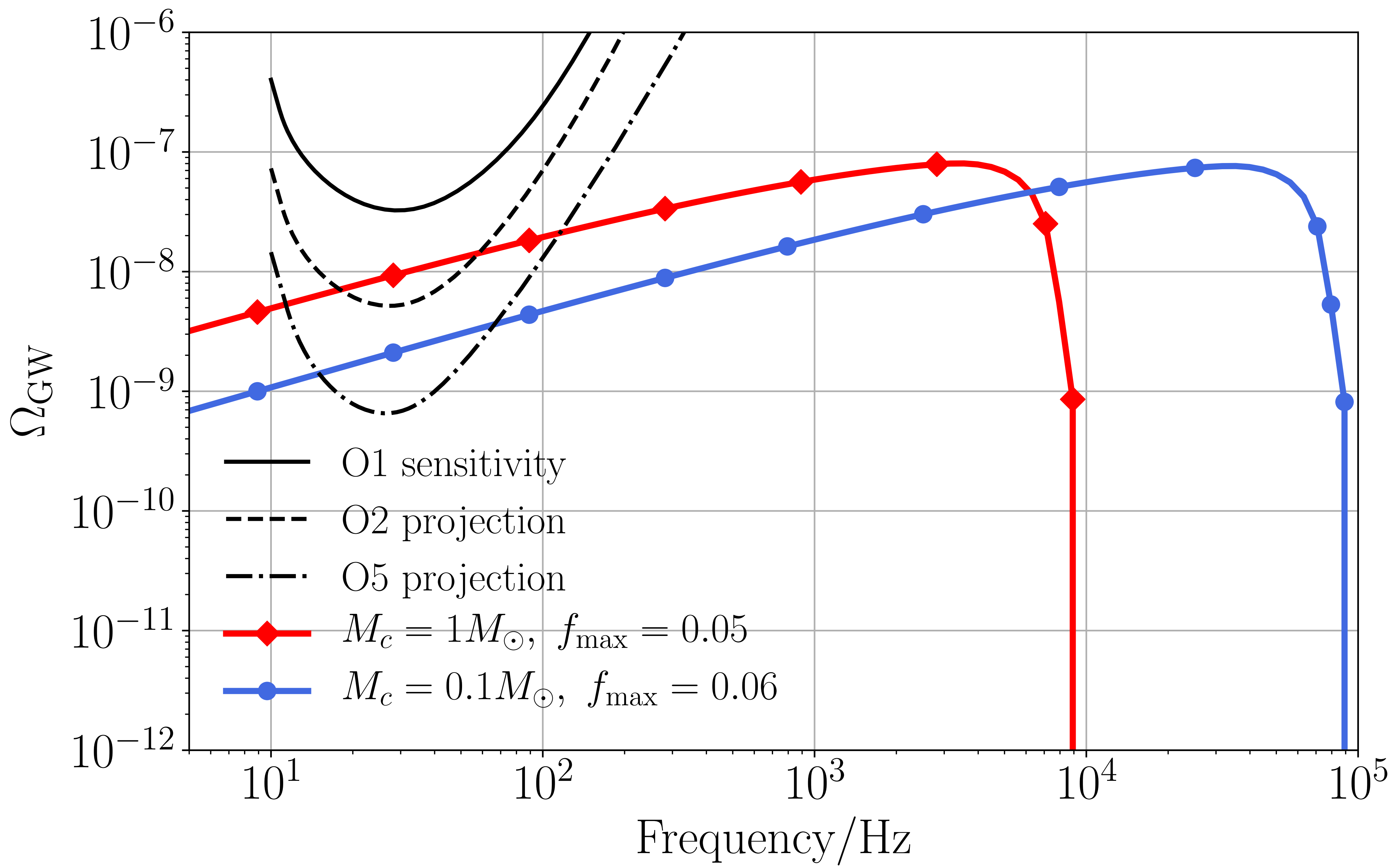
$$\Omega_{GW}\,(\nu; f, M_{\mathrm{PBH}}) \leq \text{Advanced LIGO's O1 constraint}$$



- Phys. Rev. Lett. 120, 191102

Primordial Black Hole SGWB

- Stochastic Background from sub-solar mass binary primordial black hole coalescence



Conclusions:

1. The first constraint on primordial BH abundance using gravitational waves
2. Better than other constraining methods (e.g. lensing) in mass range [1,100] solar mass.
3. We may find hints for sub-solar mass BHs from stochastic background.

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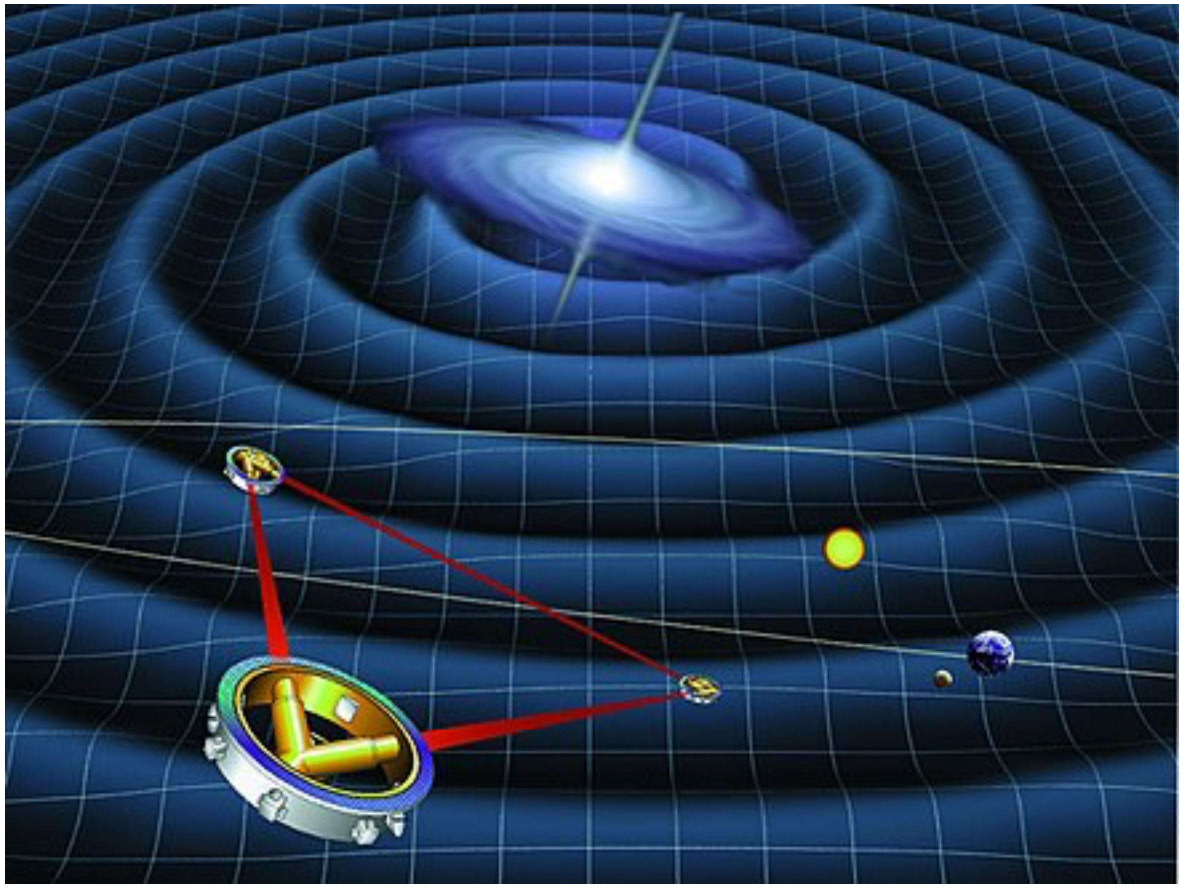
Massive Black Holes and EMRIs

- In this project we calculate the SGWB from Massive BH-sub solar mass primordial BH interacting system in the space-based detector frequency band (10^-5 Hz - 10^-1Hz)

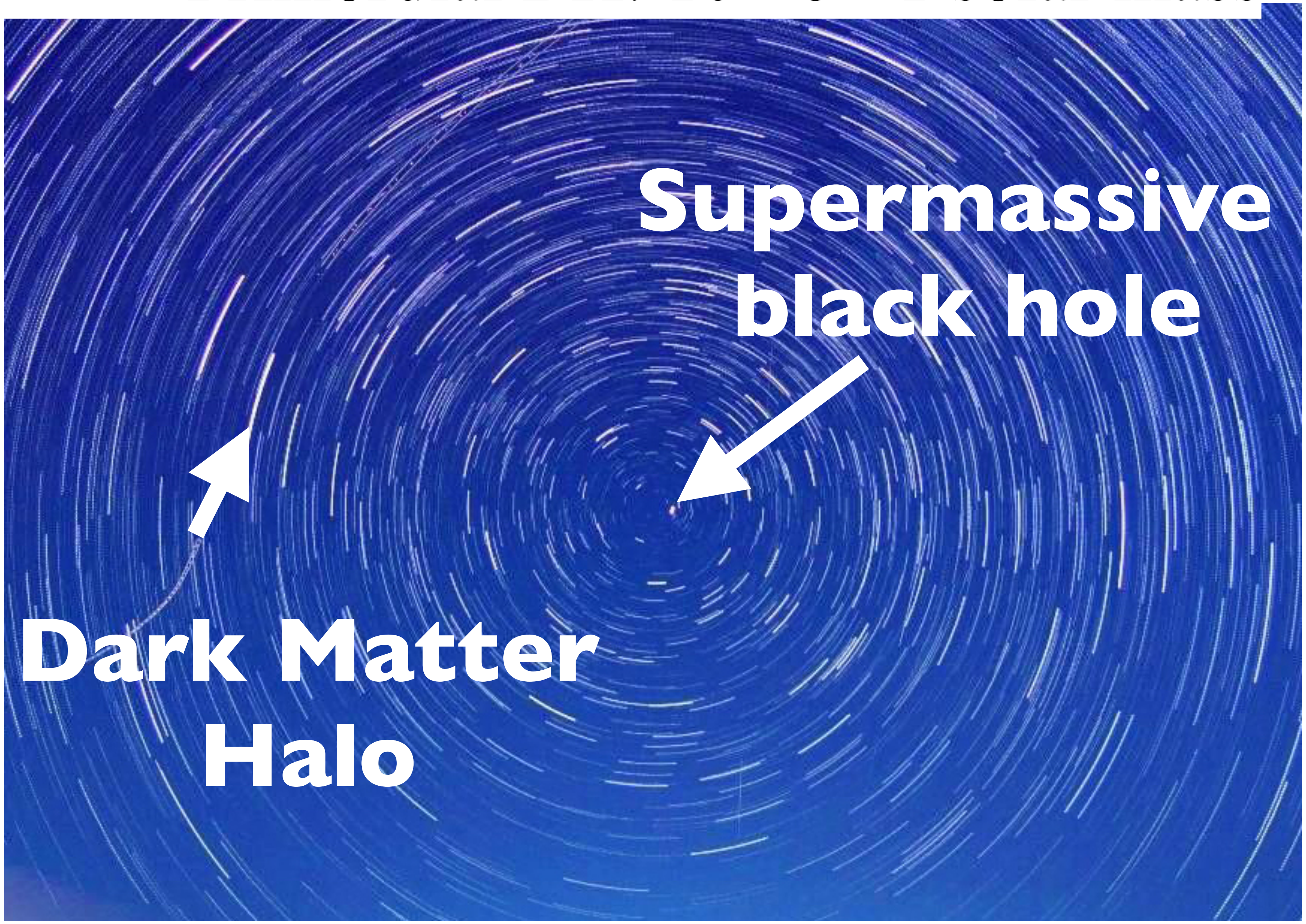


Credit: https://en.wikipedia.org/wiki/Laser_Interferometer_Space_Antenna

EXTREME MASS-RATIO INSPIRALS(EMRIS)

- Massive BHs are ubiquitous at the galactic center
- We consider: Massive BH: 10^4-10^7 solar mass

  Primordial BH: 10^-8 - 1 solar mass



SGWB FROM A SINGLE HALO

- Assumption: Primordial black holes are in circular orbit

$$\Omega_{\rm gw}(\nu) = \frac{1}{\rho_c}\frac{d\rho_{\rm gw}}{d\ln\nu}\,. \qquad \rho_{\rm GW}(\nu) = F_{\rm GW}(\nu)/c.$$

- Energy flux:

fraction of PBH in DM

$$F_{\rm GW}(\nu) = \int \frac{f_{\rm PBH}\rho_{\rm DM}(r; M_{\rm MBH})}{m_{\rm PBH}} \frac{dE/dt(r; M_{tot}, \eta)}{d_L^2} r^2 dr,$$

- Quadrupole formula for GWs, assuming circular motion

$$\frac{dE}{dt}(r; M_{tot}, \eta) = \frac{32}{5}\frac{G^4}{c^5}\eta^2 \left(\frac{M_{tot}}{r}\right)^5 .$$

$$r(\nu, M_{tot}) = \left[\frac{(GM_{tot})^{1/2}}{\pi(1+z)\nu}\right]^{\frac{2}{3}} ,$$

SGWB FROM A SINGLE HALO

- The gravitational wave power:

$$\frac{dF_{\rm GW}}{d\nu}(\nu) = \frac{64\pi}{15}\frac{(1+z)G^{7/2}}{c^5}\frac{f_{\rm PBH}\rho_{\rm DM}m_{\rm PBH}M_{\rm MBH}^{5/2}}{r^{1/2}d_L^2},$$

- Stochastic gravitational-wave energy density spectrum

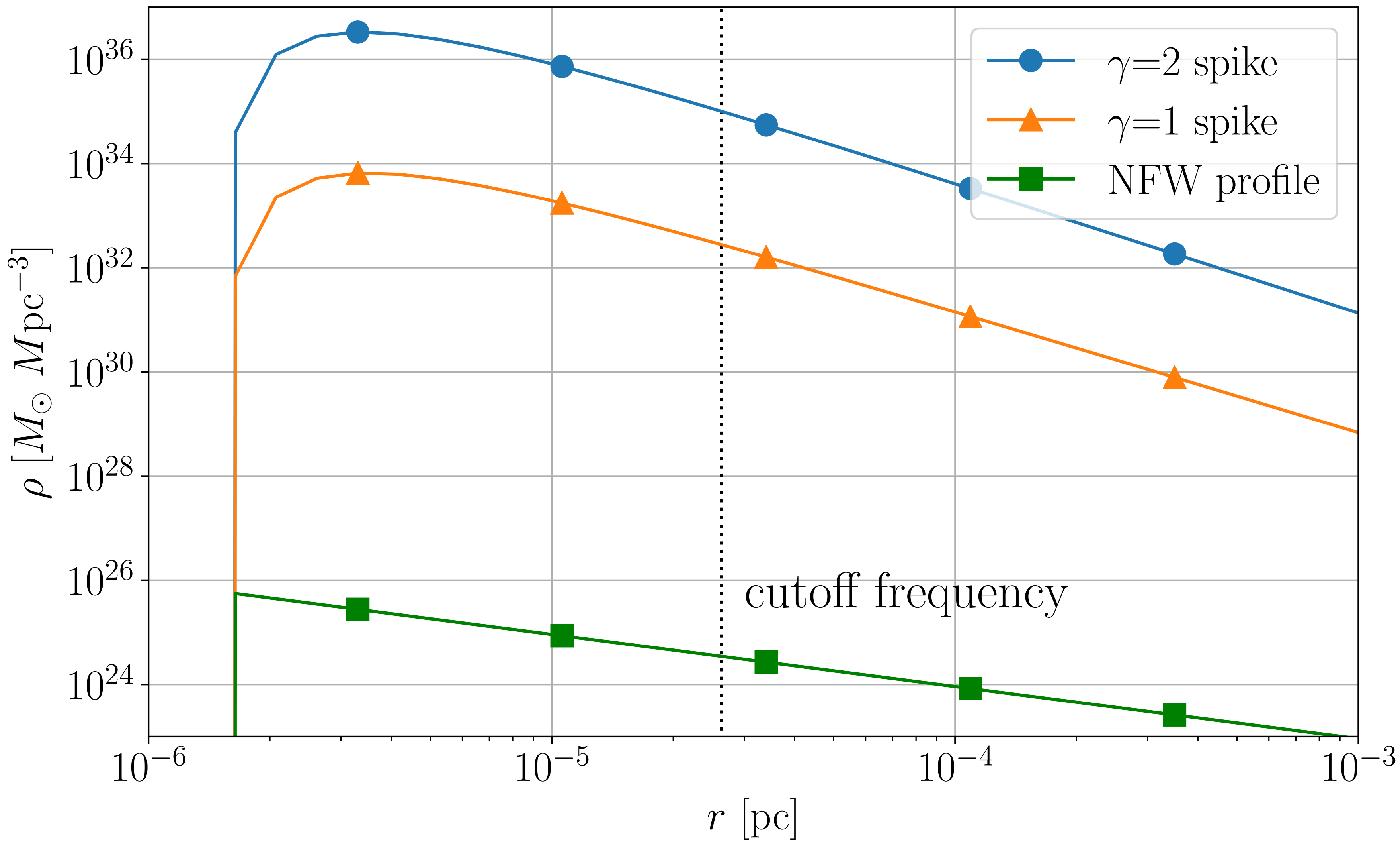
$$\Omega_{\rm GW}^{SgrA^*}(\nu) = \frac{\nu}{\rho_c}\frac{64\pi G^{7/2}}{15c^6}\frac{f_{\rm PBH}m_{\rm PBH}M_{\rm MBH}^{5/2}\rho_{\rm DM}}{r^{1/2}d_L^2}.$$

$\rho_{\rm DM}$?

SGWB FROM A SINGLE HALO

- Dark matter spike distribution:

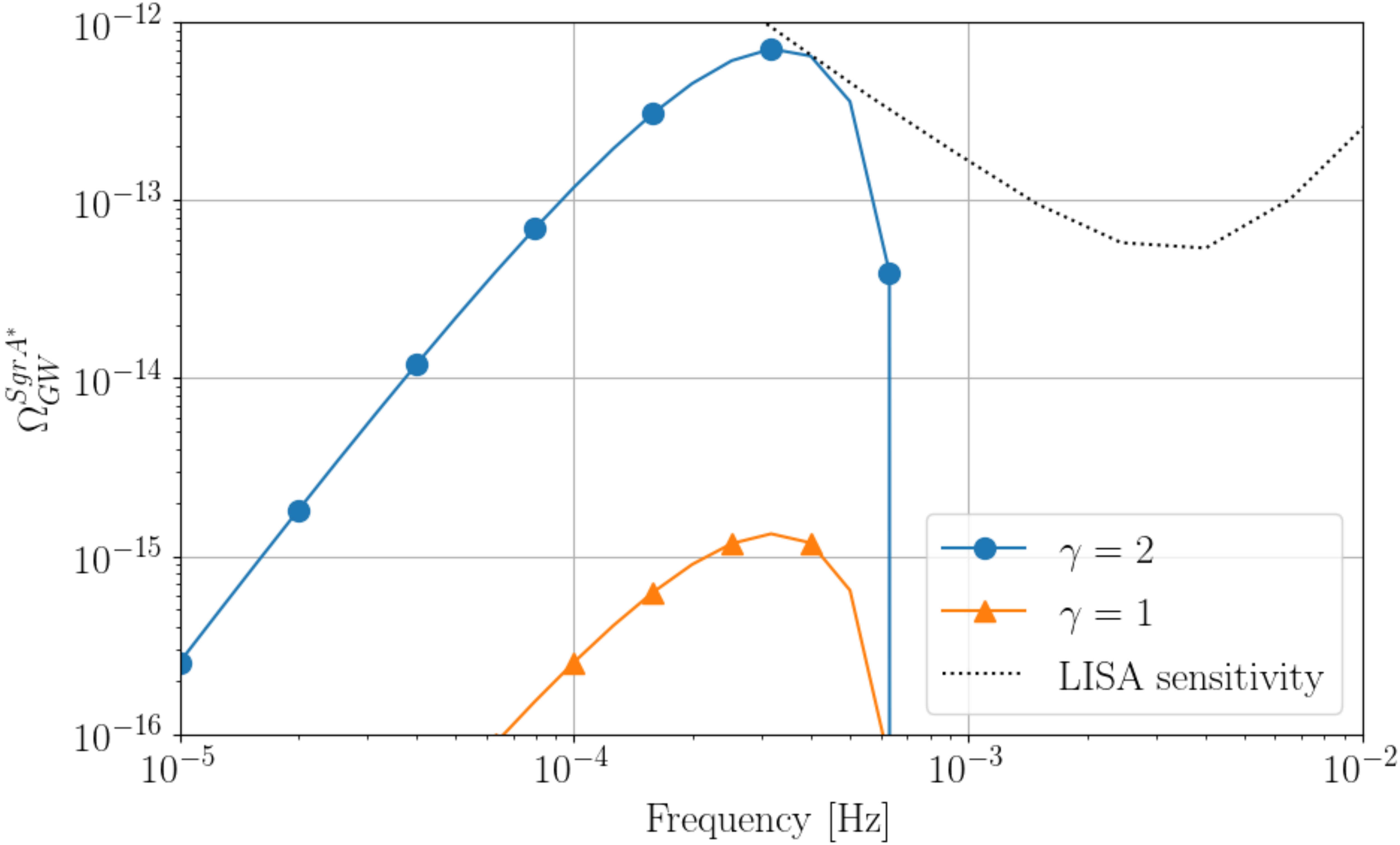
$$\rho_{\rm sp}(r) = \rho_R \left(1 - \frac{4R_s}{r}\right)^3 \left(\frac{R_{\rm sp}}{r}\right)^{\gamma_{\rm sp}}$$



- Cutoff frequency: the maximum orbit that can be detected by LISA emitted by an EMRI whose $M_{\rm SMBH} = 4 \times 10^6 M_\odot$

SGWB from a Single Halo

$$\Omega_{\rm GW}^{SgrA^*}(\nu) = \frac{\nu}{\rho_c}\frac{64\pi G^{7/2}}{15c^6}\frac{f_{\rm PBH}m_{\rm PBH}M_{\rm MBH}^{5/2}\rho_{\rm DM}}{r^{1/2}d_L^2}.$$

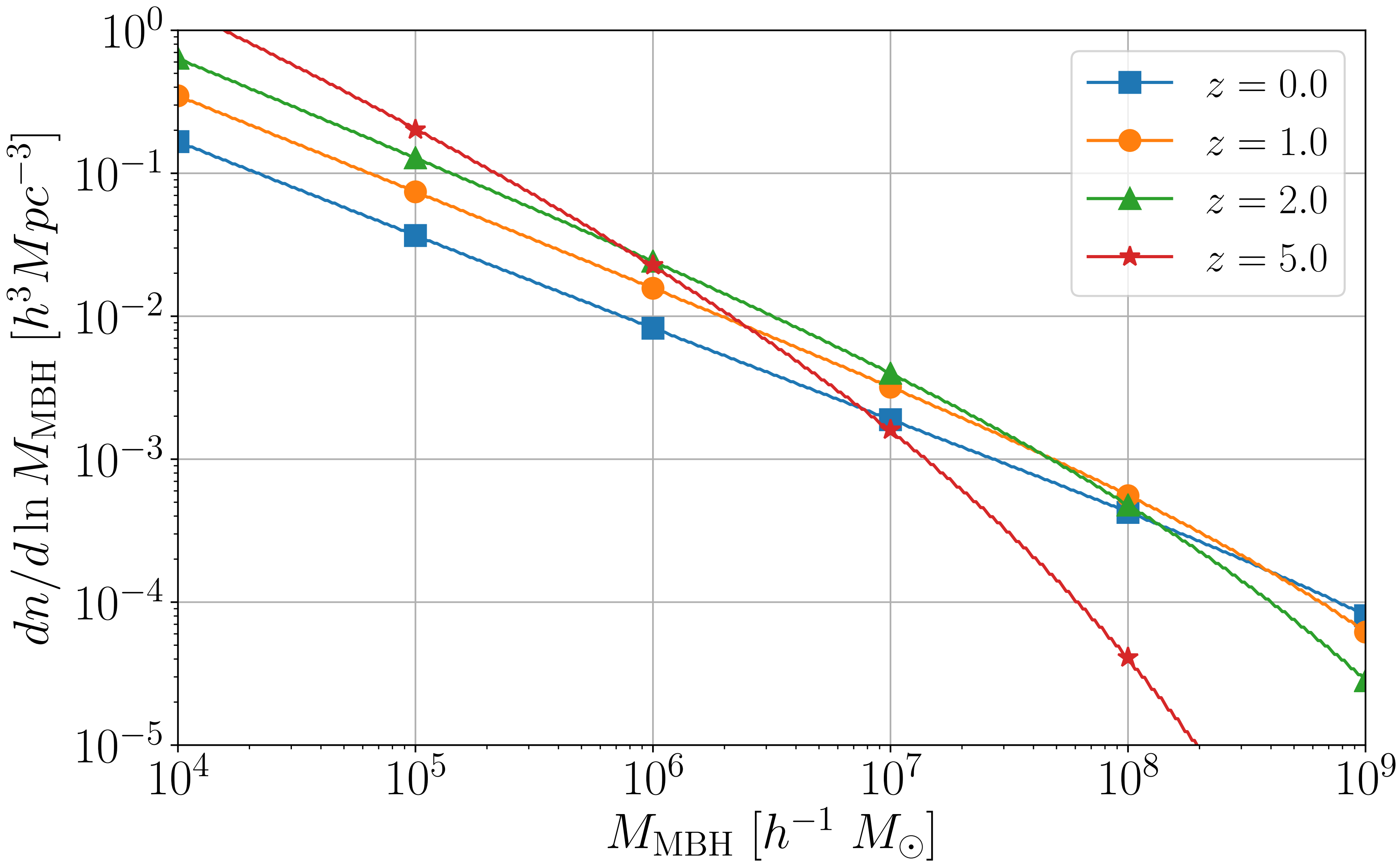


EXTRAGALACTIC CONTRIBUTION

$$\Omega_{\mathrm{GW}}(\nu) = \frac{\nu}{\rho_c}\frac{4\pi}{c}\iint \frac{dF_{\mathrm{GW}}}{d\nu}\boxed{\frac{dn}{dM_{\mathrm{MBH}}}dM_{\mathrm{MBH}}}\chi^2 d\chi,$$

Massive black hole mass density modeling:

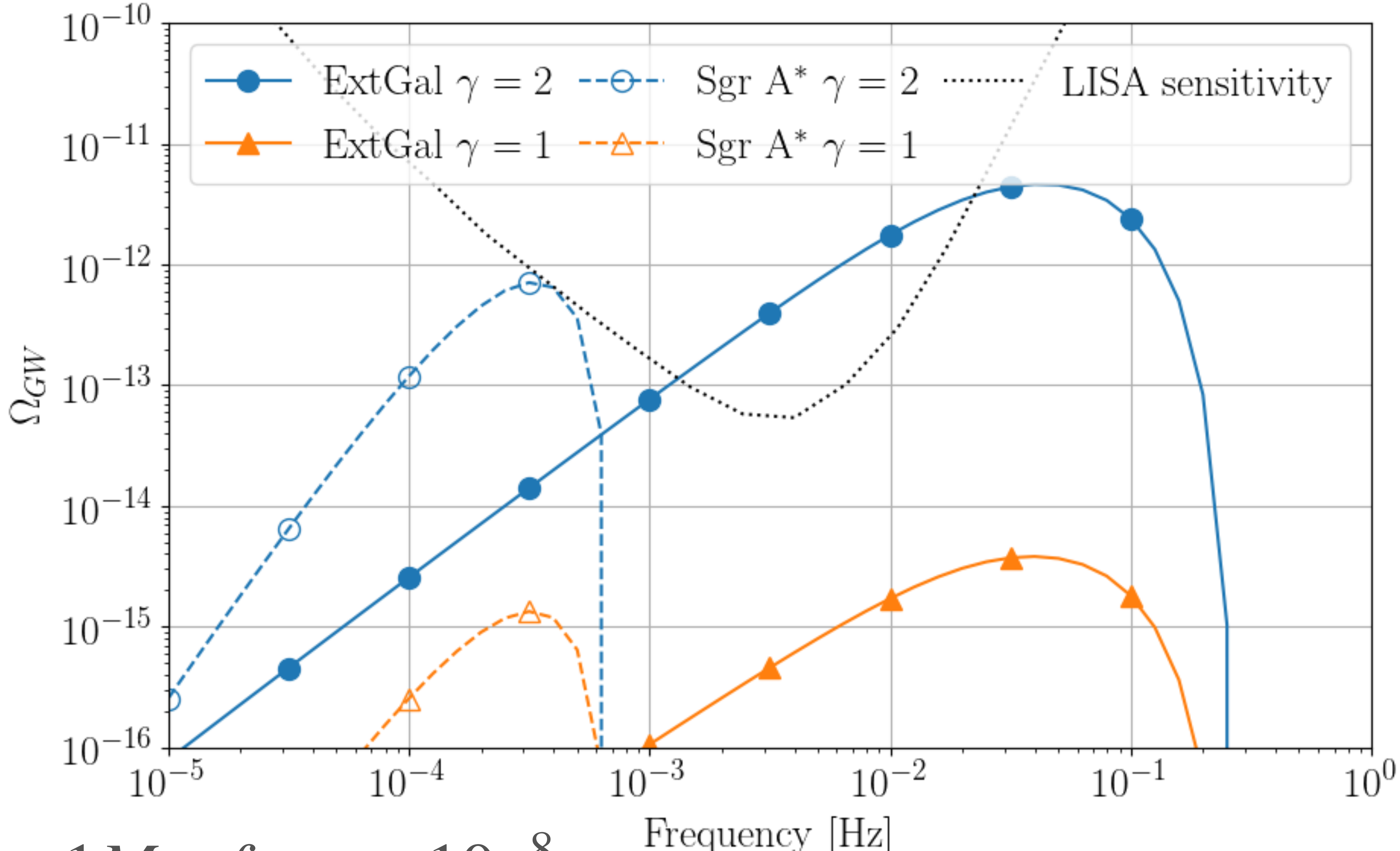
$$\log\left(\frac{M_{\mathrm{MBH}}}{10^8 h^{-1} M_\odot}\right) = (-2.66 \pm 0.33) + (1.39 \pm 0.22) \times \log\left[\beta^3 H(z)\left(\frac{M_{vir}}{10^{13} h^{-1} M_\odot}\right)\right].$$



EXTRAGALACTIC CONTRIBUTION

$$\Omega_{\rm GW}(\nu) = f_{\rm PBH} m_{\rm PBH} \frac{\nu}{\rho_c} \frac{256\pi^2 G^{7/2}}{15c^7} \times \int \frac{dz}{(1+z)H(z)} \int \frac{\rho_{\rm DM} M_{\rm MBH}^{5/2}}{r^{1/2}} \frac{dn}{dM_{\rm MBH}} dM_{\rm MBH}.$$

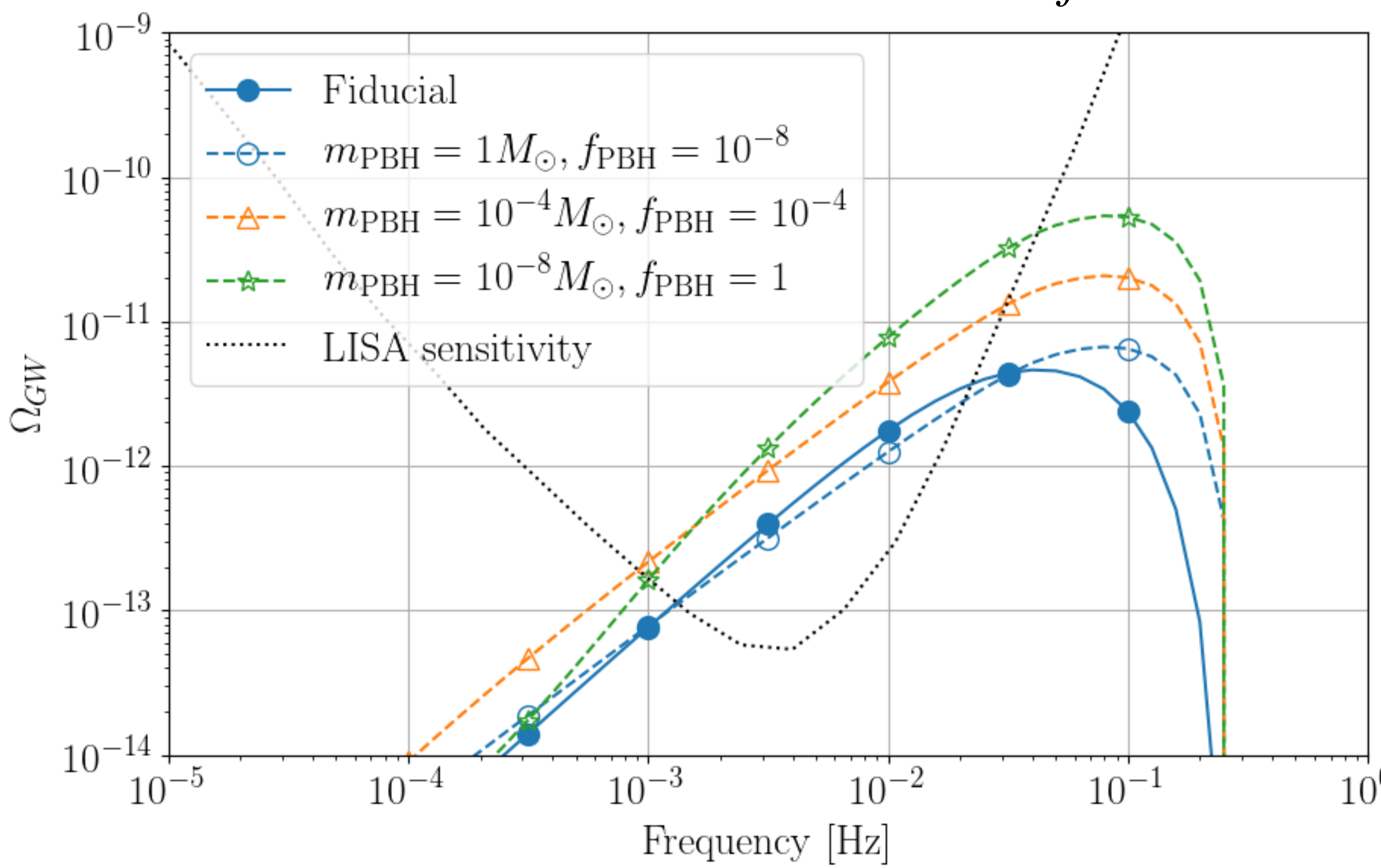
Results:



$m_{\rm PBH} = 1 M_\odot, f_{\rm PBH} = 10^{-8}$

Density Enhancement due to GW Dissipation

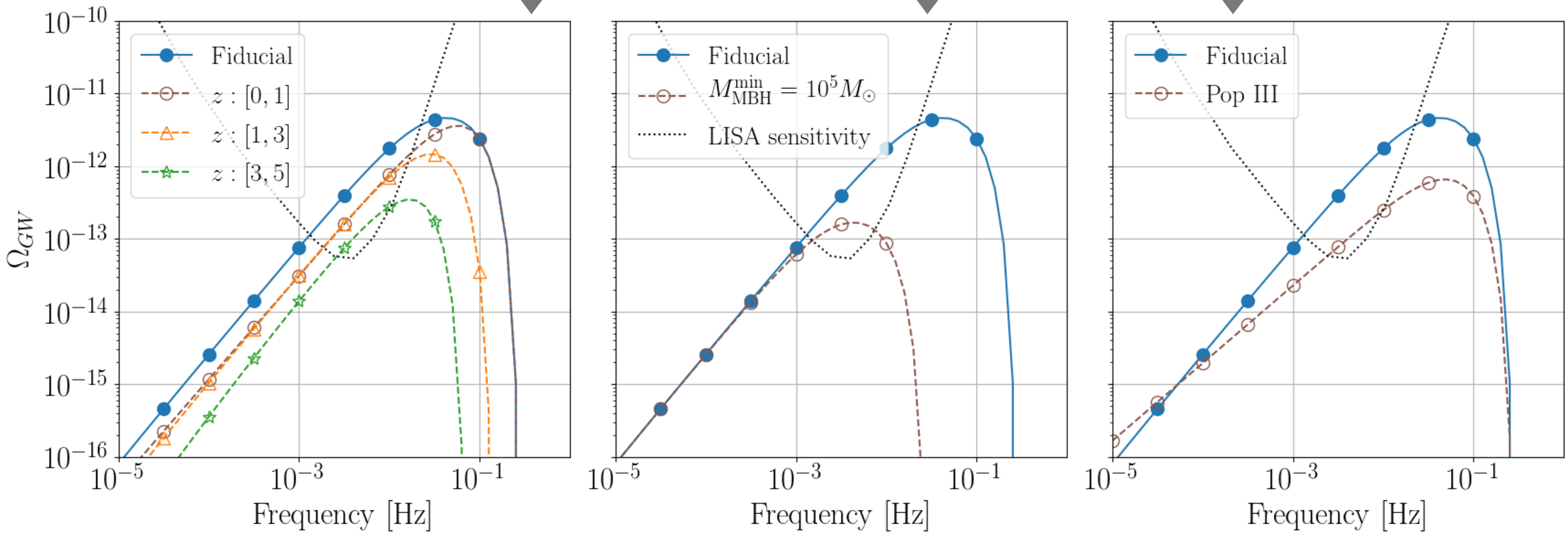
- Orbital decay duration: $$\Delta t = \frac{5}{256}\frac{c^5}{G^3}\frac{1}{\eta M_{tot}^3}(r_i^4 - r_f^4),$$
- Mass conservation relation: $$\rho_f(r_f) = \frac{r_i^2}{r_f^2}\rho_i(r_i)$$



Effects of Integral Limits

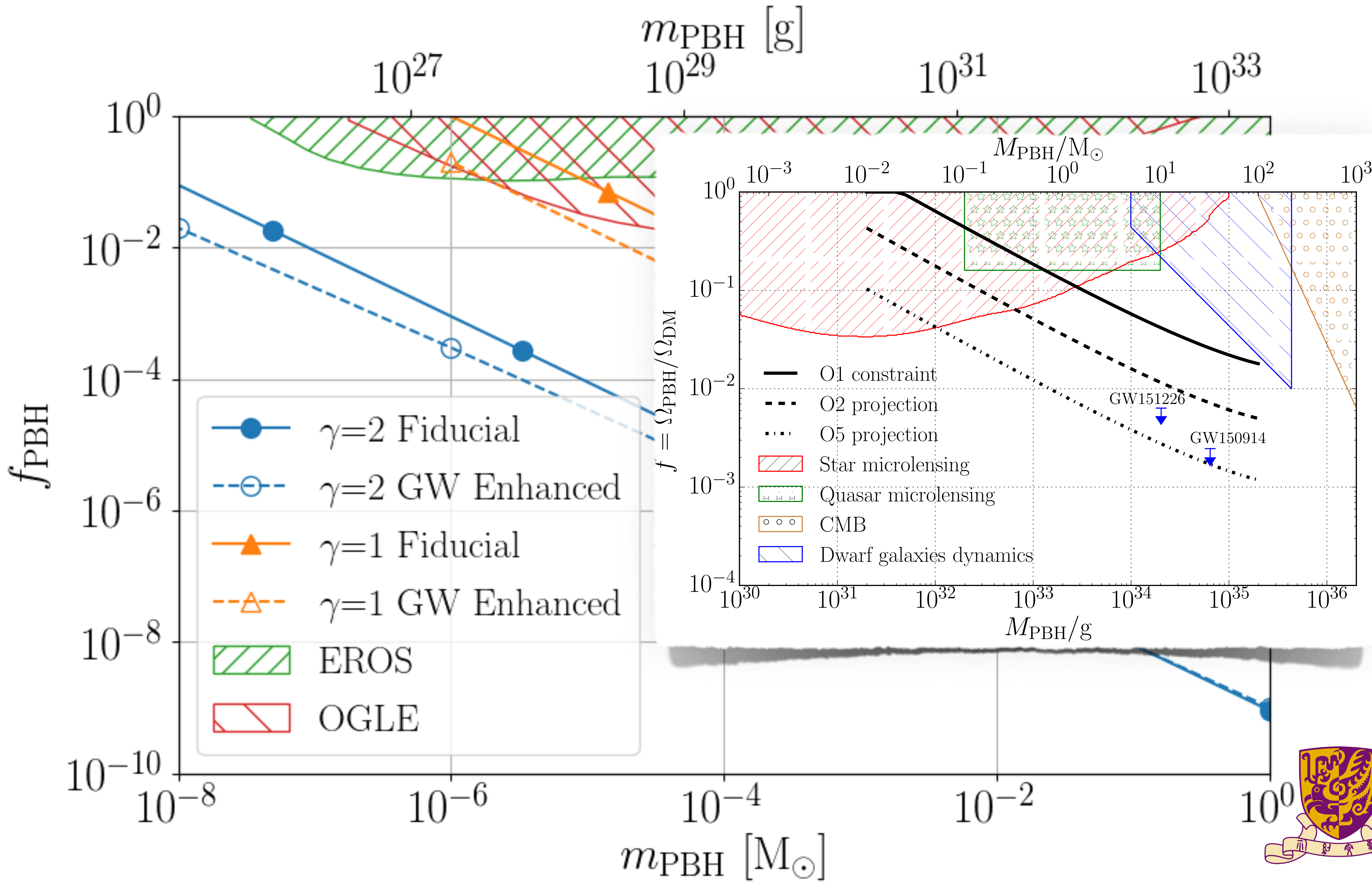
$$\Omega_{\rm GW}(\nu) = f_{\rm PBH} m_{\rm PBH} \frac{\nu}{\rho_c} \frac{256\pi^2 G^{7/2}}{15c^7}$$

$$\boxed{\times \int \frac{dz}{(1+z)H(z)}} \boxed{\int \frac{\rho_{\rm DM} M_{\rm MBH}^{5/2}}{r^{1/2}}} \boxed{\frac{dn}{dM_{\rm MBH}} dM_{\rm MBH}}.$$



CONSTRAINTS FOR PBH ABUNDANCE

$$\Omega_{GW}(\nu; m_{PBH}, f_{PBH}) \leq \Omega_{GW}^{\mathrm{LISAsensitivity}}$$



CONCLUSIONS:

- The contributions from Sagittarius A* and extragalactic massive black holes peak at different characteristic frequencies. Future LISA detection may use this feature to probe primordial black holes

- The stochastic gravitational-wave background can be detected if the abundance of primordial black holes with 1 solar mass exceeds a minimum value ranging from 10^-6 to 10^-9, given the modeling uncertainty

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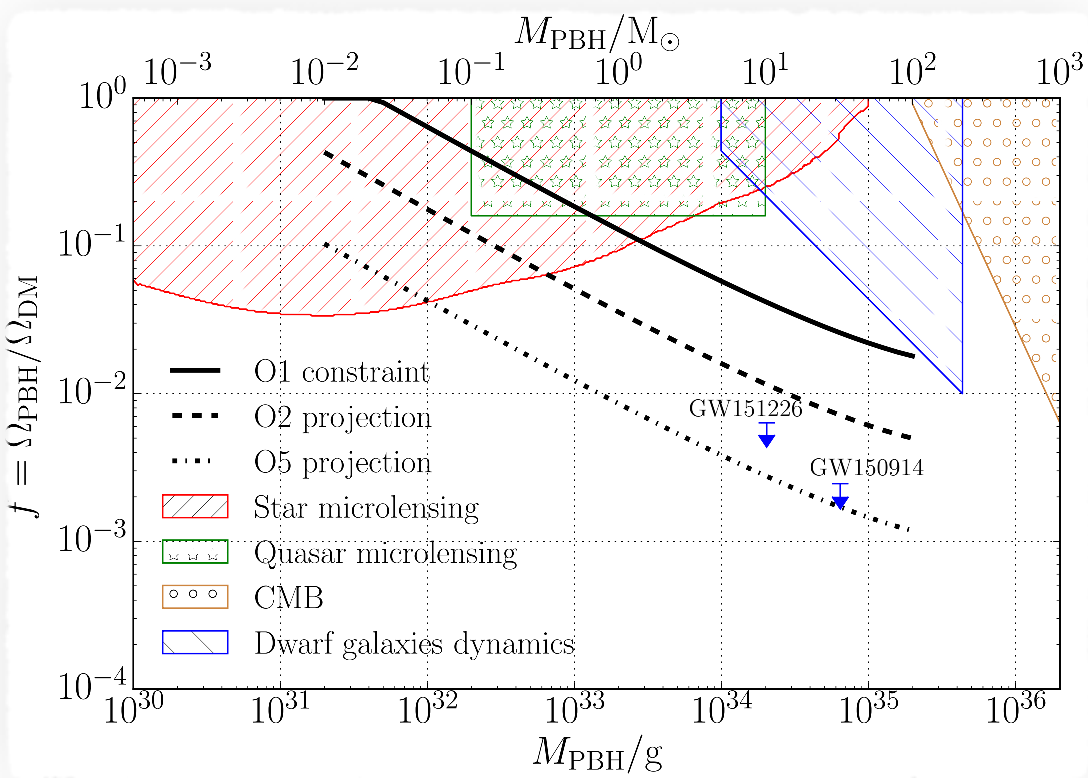

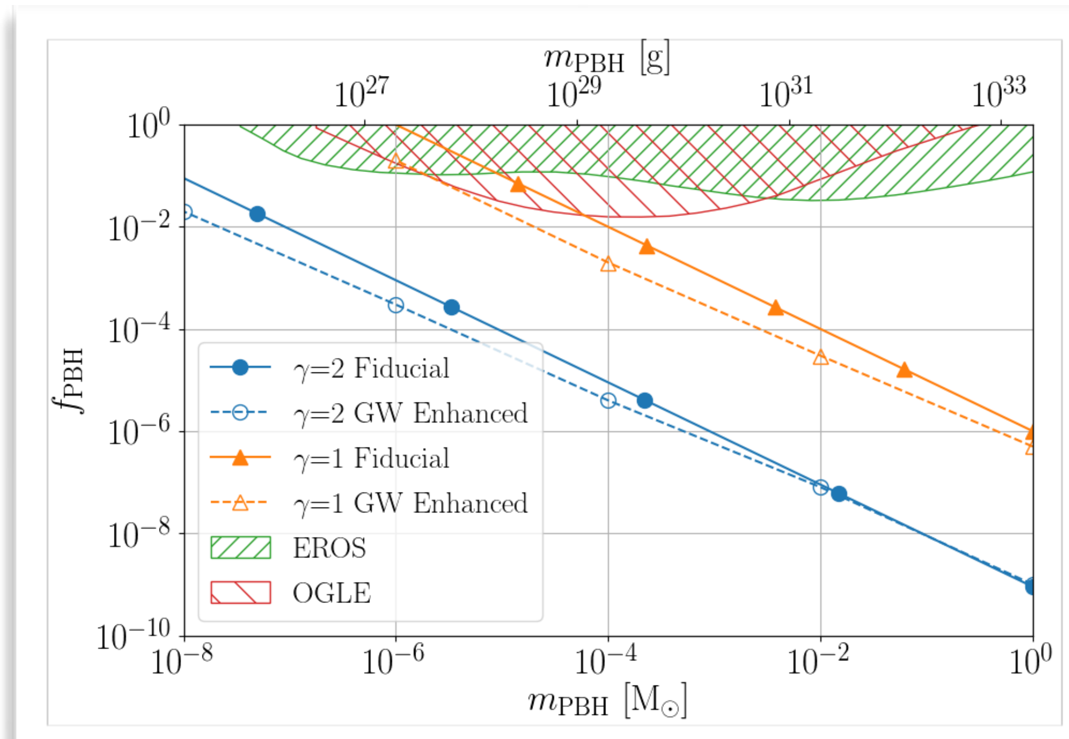

Future Plan

- Anisotropic Stochastic Gravitational-Wave Background
- Waveforms for Binary Black-Hole Coalescence with Eccentricity
- Directional Search for Stochastic Gravitational-Wave Background with Space-Based Detectors

Thank you!

BACKUP

- Gravitational wave emission power with eccentricity

$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{32}{5} \frac{G^4 m_1^2 m_2^2 (m_1+m_2)}{c^5 a^5 (1-e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right).$$

- Eccentricity decay rate

$$\left\langle \frac{de}{dt} \right\rangle = -\frac{19}{12} \frac{\beta}{c_0^4} \frac{e^{-29/19} (1-e^2)^{3/2}}{[1+(121/304)e^2]^{1181/2299}}. \quad (5.13)$$

Backup

- Binary orbital life time with eccentricity

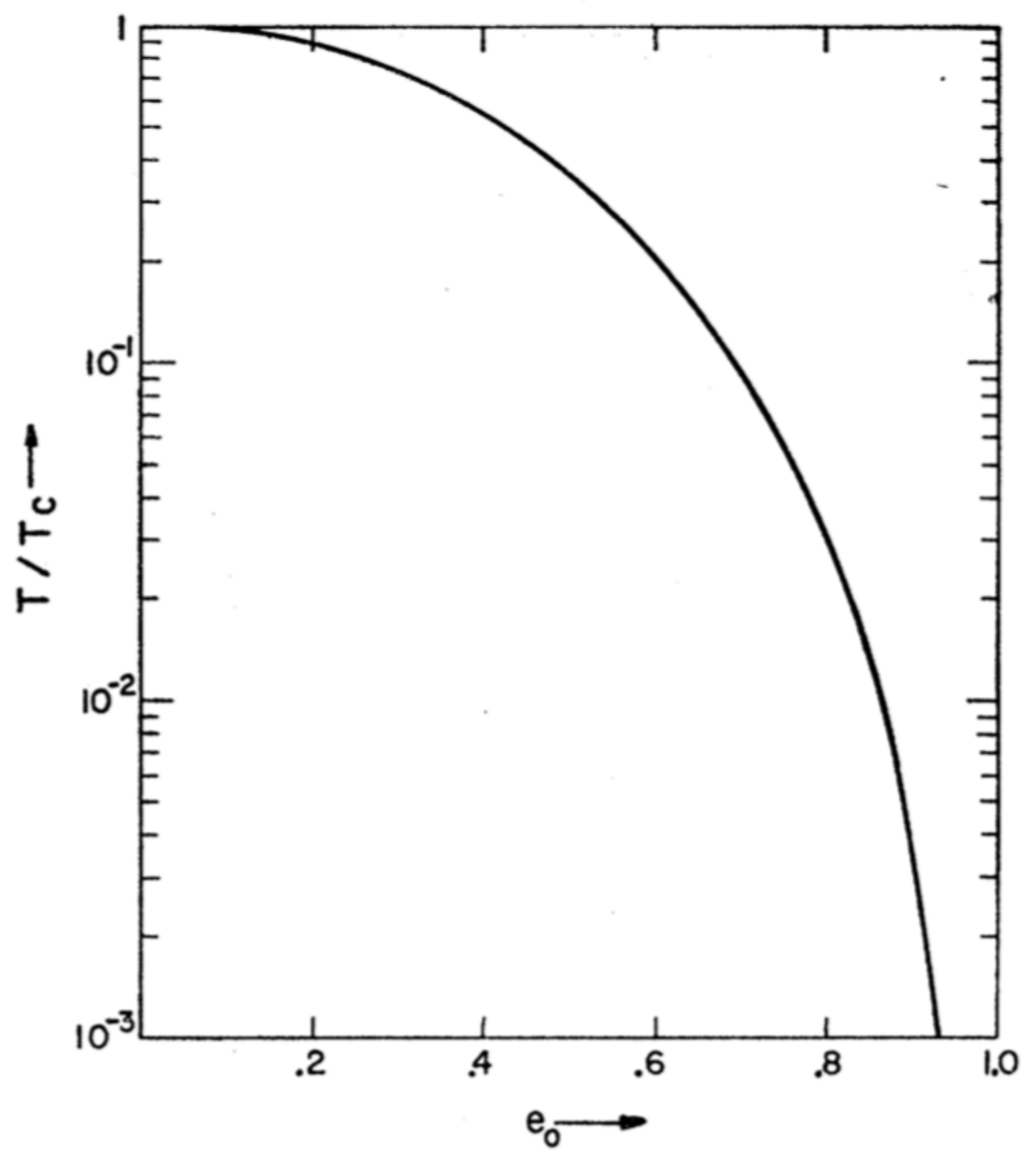

FIG. 2. The ratio of the lifetime of an eccentric system to that of a circular one plotted against the initial eccentricity. This ratio is independent of the initial value of the semimajor axis.