Looking for statistical dependence of beyond general relativity parameters among multiple gravitational-wave events

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Introduction

General relativity has passed all experimental tests up to now, especially the recent tests from gravitational wave events in genuinely strong gravitational field. In the current implementation of so-called test infrastructure of general relativity (TIGER), the gravitational wave (GW) data is compared with the generalized waveform templates containing one or more phenomenological beyond-general relativity (GR) parameters. However, if a violation of GR is indeed contained in the GW observation, it is reasonable to anticipate the degree of violation has dependency on certain physical parameters of the source, such as mass, spin, distance or any other additional charge in a particular way.

In this work, we use:

- mutual information
- Gaussian process regression

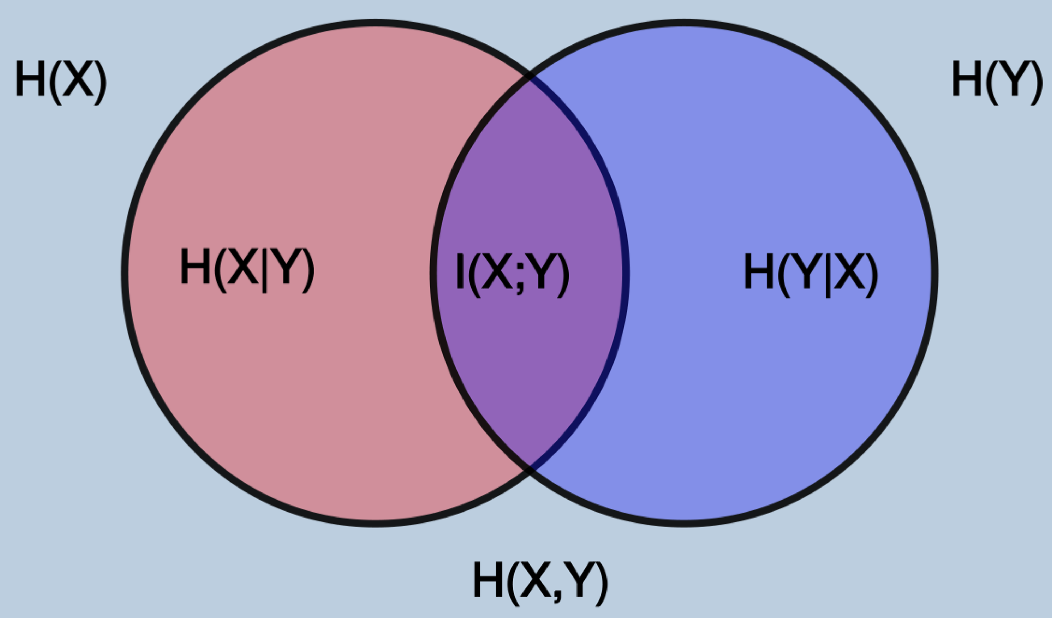
to look for the statistical dependence of beyond-GR parameters and reconstruct the functional form if violation of GR appears in GW data.

Statistical Tools

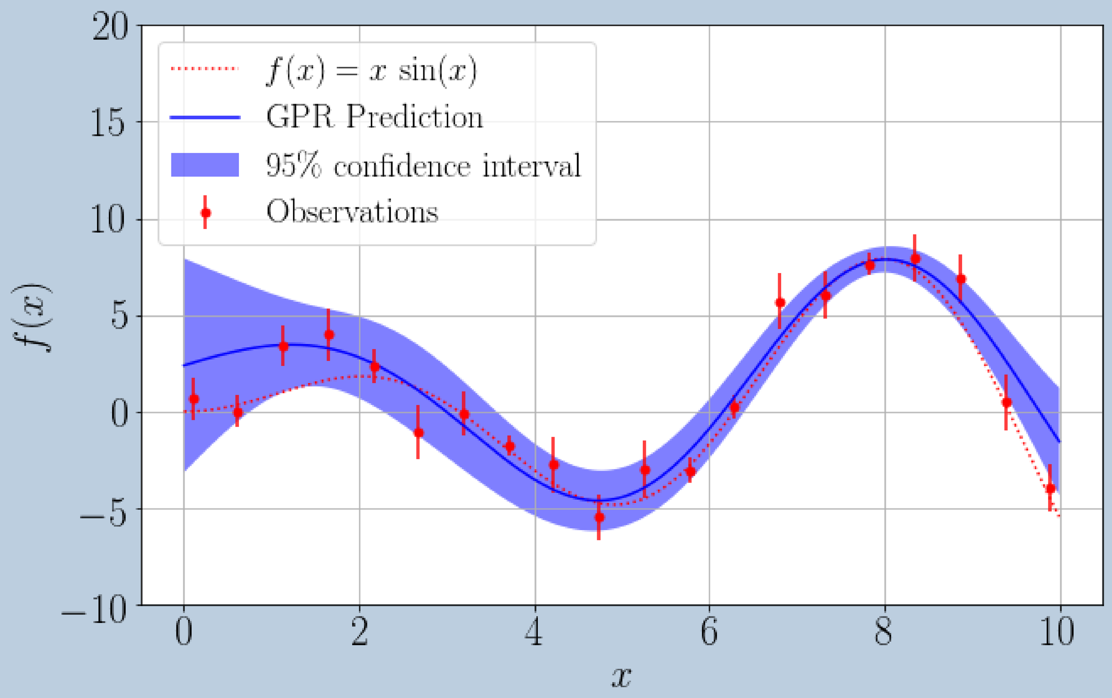
- The entropy $H(X)$ is a measure of uncertainty.

  Mutual information $I(X;Y) = H(X) + H(Y) - H(X,Y)$

  $I(X;Y) = 0$ if and only if X and Y are independent.



- Gaussian process regression is a non-parametric fitting method which assigns a multi-dimensional Gaussian distribution to observations, i.e., beyond-GR parameters in our case.

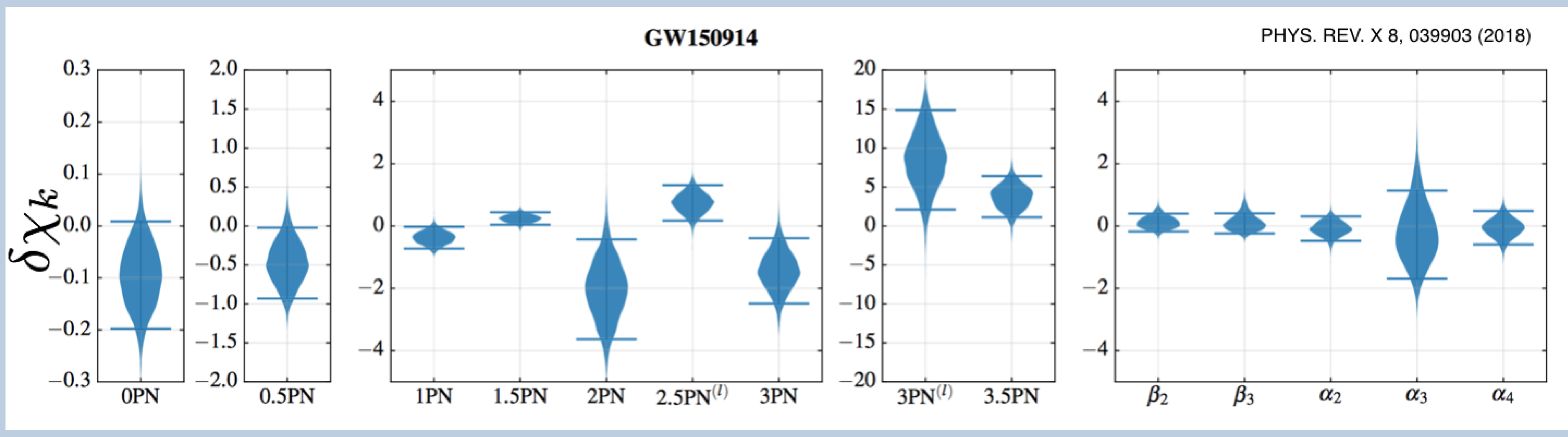


Parametrized Tests of General Relativity

- In the parametrization of TIGER, one or more phenomenological beyond-GR parameters $\delta\chi_k$ are introduced as a fractional deviation for coefficient at each PN order

$$\psi_k \to \psi_k(1+\delta\chi_k). \quad (1)$$

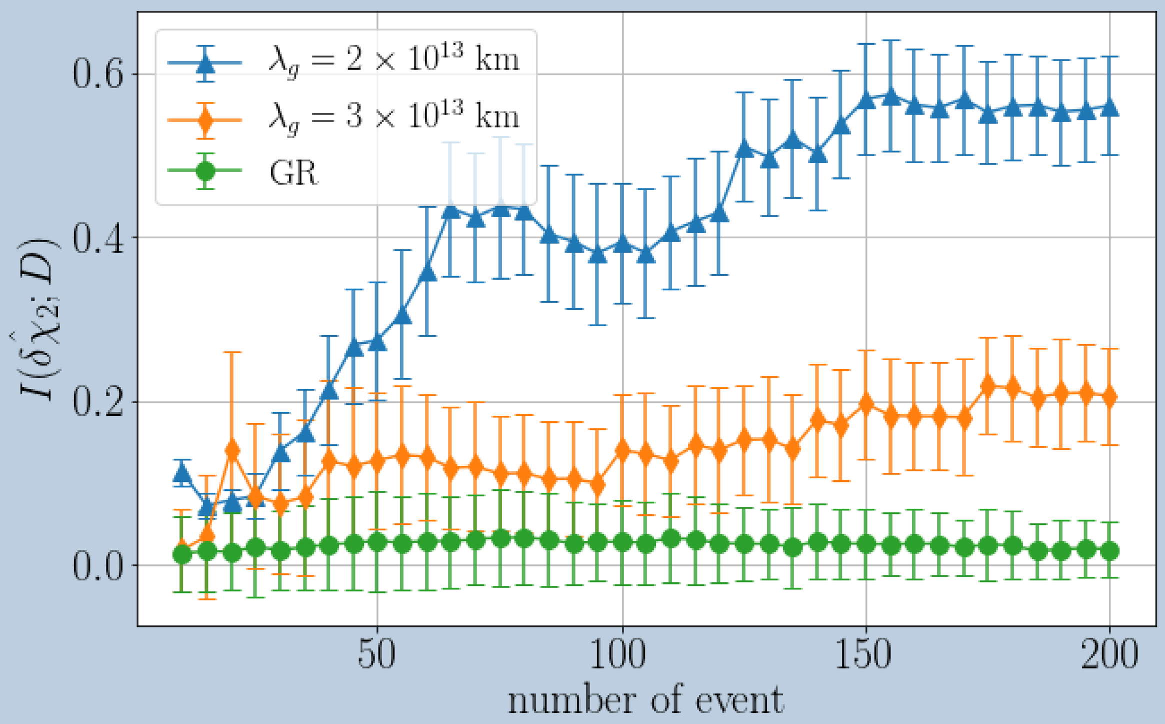
where ψ_k is the waveform template coefficient predicted by GR.



Looking for possible dependence of $\delta\chi_k$ and GW source parameters.

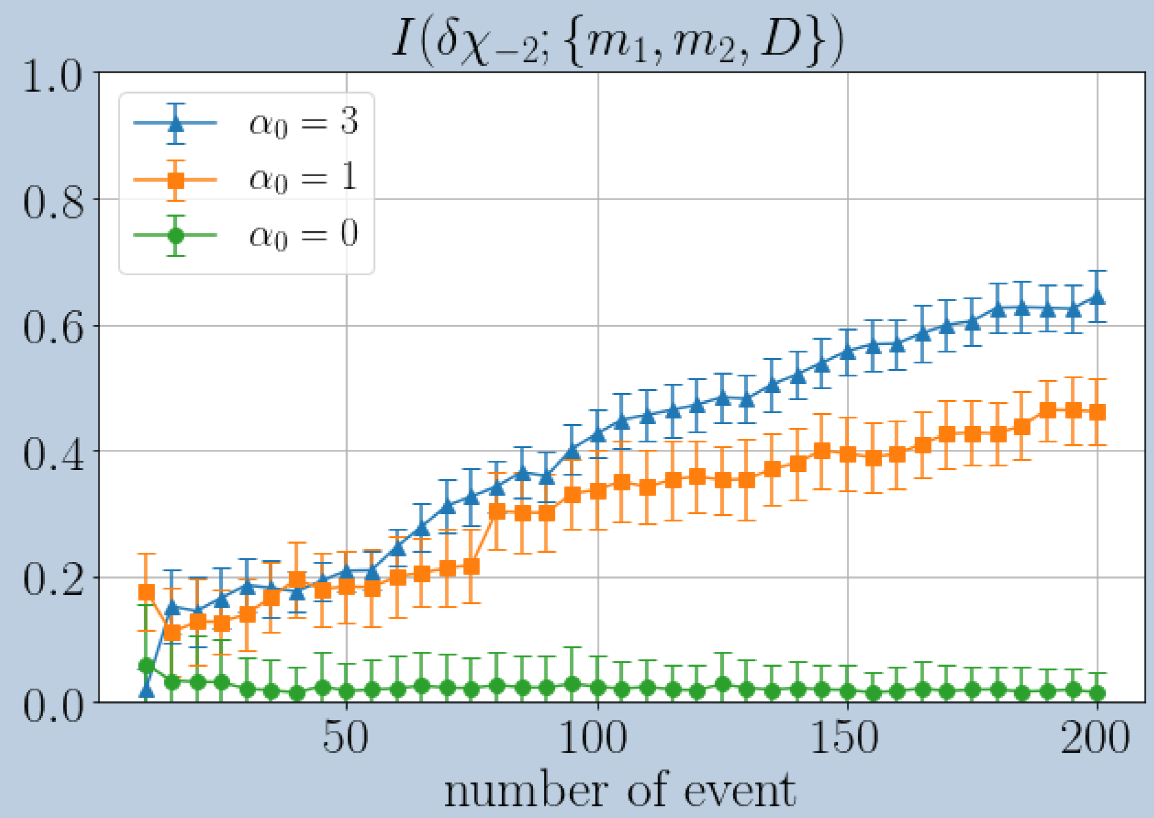
1st Example: Theory of Massive Gravity

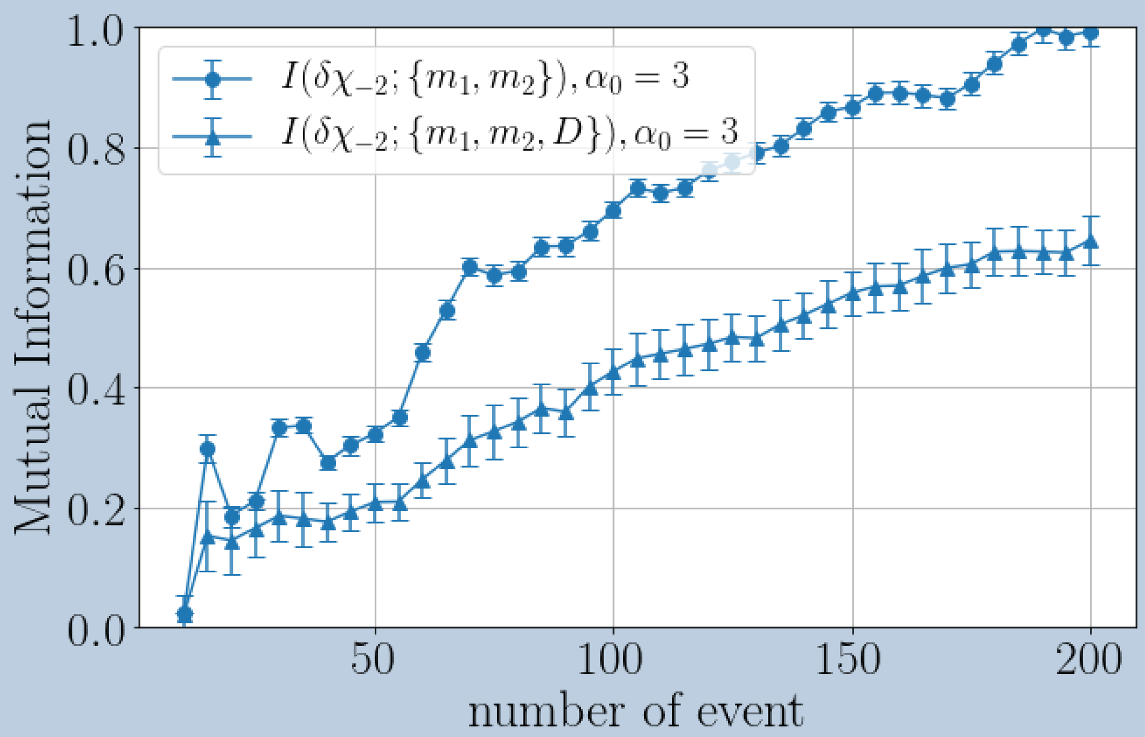
We simulate three catalogs of binary black hole signals with finite Compton length λ_g of massive graviton. The non-GR injections show clearly **non-zero mutual information** between $\delta\chi_2$ and the source distance D.



2nd Example: Brans-Dicke Gravity

The $\delta\chi_{-2}$ introduced by Brans-Dicke gravity depends non-linearly on the difference of component masses, and has no relation with the distance of sources.





$$\begin{cases} I(\delta\chi_{-2}; \{m_1, m_2, D\}) \neq 0, \\ I(\delta\chi_{-2}; D|\{m_1, m_2\}) = 0. \end{cases} \Rightarrow$$

	m_1	m_2	Distance
$\delta\chi_{-2}$	✓	✓	×