P159: Threshold

$$g(k) = \binom{n_1 - c_2}{k} \left\{ \frac{q}{(1-q)^2(1-p)^2} \right\}^k \binom{n_1 - k}{r_2 + c_3}$$

$$f(k) = \mathrm{Binom}(\bar{r}, q_2) = \binom{\bar{r}}{k} {q_2}^k (1 - q_2)^{\bar{r} - k}$$

$$q_2 = \frac{q}{q + (1-q)^2(1-p)^2}$$

$$1 - q_2 = \frac{(1-q)^2(1-p)^2}{q + (1-q)^2(1-p)^2}$$

Then,

$$f(k) = \binom{\bar{r}}{k} \left\{ \frac{q}{q + (1-q)^2(1-p)^2} \right\}^k \left\{ \frac{(1-q)^2(1-p)^2}{q + (1-q)^2(1-p)^2} \right\}^{\bar{r} - k}$$

$$\frac{g(k)}{f(k)} = \frac{\binom{n_1 - c_2}{k}\binom{n_1 - k}{r_2 + c_3}}{\binom{\bar{r}}{k}} \left\{ \frac{q}{(1-q)^2(1-p)^2} \right\}^k \left\{ \frac{q + (1-q)^2(1-p)^2}{q} \right\}^k \left\{ \frac{q + (1-q)^2(1-p)^2}{(1-q)^2(1-p)^2} \right\}^{\bar{r} - k}$$

$$\frac{g(k)}{f(k)} = \frac{\binom{n_1 - c_2}{k}\binom{n_1 - k}{r_2 + c_3}}{\binom{\bar{r}}{k}} \left\{ \frac{q + (1-q)^2(1-p)^2}{(1-q)^2(1-p)^2} \right\}^{\bar{r}}$$

$$q_1 = \frac{q}{(1-q)^2(1-p)^2}$$

Then,

$$\frac{g(k)}{f(k)} = \frac{\binom{n_1 - c_2}{k}\binom{n_1 - k}{r_2 + c_3}}{\binom{\bar{r}}{k}} \{q_1 + 1\}^{\bar{r}}$$

$$\bar{r} = \min(n_1 - r_2 - c_3, n_1 - c_2)$$